LA 1679

LA-8451-MS

Informal Report

190
8-28-80
MMB

**On the Global Stiffness of
Hex-Can Assembly in a Uniform Force Field**

MASTER

University of California

LOS ALAMOS SCIENTIFIC LABORATORY

Post Office Box 1663 Los Alamos, New Mexico 87545

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

LA-8451-MS
Informal Report

UC-79p (Base Technology)
Issued: July 1980

On the Global Stiffness of Hex-Can Assembly in a Uniform Force Field

Frederick D. Ju*
Joel Bennett

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

*Short-Term Visiting Staff Member. Department of Mechanical Engineering, University of
New Mexico, Albuquerque, NM 87106.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

54

ON THE GLOBAL STIFFNESS OF HEX-CAN ASSEMBLY IN A UNIFORM FORCE FIELD

by

Frederick D. Ju and Joel Bennett

ABSTRACT

Two approximate constitutive equations are derived that can be used to represent the global stiffness of a hexagonal cross-section duct in a uniform force field. The first equation uses a single coefficient that can be determined from Poisson's ratio for the material and the duct geometry. This equation is useful for isothermal applications. The second equation can be used to account for temperature-varying material properties and requires that two coefficients be determined from Poisson's ratio and the duct geometry. These equations are useful both in reactor safety analysis and design.

I. INTRODUCTION

The cores of liquid metal fast breeder reactors (LMFBRs) are constituted of hexagonal cylindrical cans. The hex-cans, supported on inlet nozzles, are held in the vertical position in some designs by former rings at two levels of elevation - one at the above core load plane (ACLP) and another further up at the top load plane (TLP see Fig. 1). The rings are integral parts of the core barrel at those two levels (see Fig. 2). At the operating temperature, the hex-cans that are at considerably higher temperature than the ring-barrel assembly will expand more than the design clearance. The axisymmetric design

Fig. 1. CRBR fuel can assembly.

of the reactor thus results in a uniform planar central pressure field, which depends very much on both the temperature field and the global stiffness of the core. The structural analysis may be done elaborately using some available computer codes. However, because of the number of cans involved, even a one-twelfth segment of the core is generally impractical for such analysis. Yet because of the large number of hex-cans, we may treat the core assembly as a uniform and homogeneous solid. When the global stiffness of the hypothetical solid is known, the pressure may be computed with a much simplified mathematical model.

Fig. 2. Cross-section of fuel can assembly at ACLP and TLP.

The determination of the global stiffness of the core assembly will come from the analysis of a single hex-can. It is stipulated that the temperature variation is gradual with respect to the size of individual hex-cans to permit the assumption that an individual hex-can is in a "locally uniform field." The normal displacement and pressure relationship of the can will be expressed by an approximate equation; whereas, such a relationship is known exactly for a solid hexagonal cell. If the load and the displacement of both cells are made equal, the modulus of the solid cell becomes the equivalent modulus of the hollow hex can. This equivalent modulus is the global stiffness of the core assembly.

II. NUMERICAL ANALYSIS OF A HEX CAN

We assumed that at the temperature of interest the material behaves elastically. The input data of the modulus of elasticity and the boundary condition then may be chosen arbitrarily based on the principle of linear superposition. The finite element method is used for generating data toward an approximate constitutive equation. The resulting data should provide (1) the parameters pertinent to the deformation-load relationship, such as the size of the hex-can, the wall thickness ratio, and the Poisson's ratio, and (2) the distribution of load as a function of the thickness ratio and Poisson's ratio.

From the approximate mechanical constitutive equation, we may determine the equivalent stiffness of a corresponding solid cell. The global stiffness of the core assembly thus results as a function of the wall thickness and the Poisson's ratio.

The analysis is basically one of elastic equilibrium. A stability analysis is necessary in the pressure field to determine the range for which the equivalent stiffness is valid.

A. Mathematical Model

The cross section of a hex-can at the load planes is represented by a region bounded by two concentric regular hexagons. The wall thickness of the hex-can is the normal distance between the hexagons as in Fig. 3. The elastic region is assumed to be in a state of plane strain.

The interior surface of the hexagonal cylinder is traction free. For the global uniform pressure field on the outer boundary, we require only that the total pressure per side is uniform for the single hex-can. Because the matching sides of adjacent cells must maintain contact throughout deformation, the sides of hex-cans will remain flat; thus, the outer boundary is subjected to a uniform normal displacement. There is no shear on the surface. These assumptions hold provided that the wall is not so thin as to cause stability problems.

The exaggerated representation of Fig. 4 shows a hex-can deforming under a uniform pressure and a uniform normal displacement.

Because of symmetry, the analysis may be carried out for a one-twelfth segment (Fig. 5). The boundary values of this quadrilateral region are prescribed according to the boundary members shown in the circles of Fig. 5.

Fig. 3. Hex-can cross section.

Fig. 4. Hex-can deformation under
(a) uniform pressure and
(b) uniform normal displacement.

Fig. 5. Typical finite element mesh
of a twelfth section of
the hex can.

(1) On Boundary 1, the mixed boundary condition is prescribed by a uniform normal displacement and a free tangential traction.

(2) On Boundary 2, the condition is also of the mixed type, described by the zero normal displacement and zero tangential traction.

(3) Boundary 3 is traction free.

(4) The condition on Boundary 4 is the same as that of Boundary 2.

Figure 5 also shows a typical 80 element mesh generated for the Finite Element method. PSAAS,¹ a plane elasticity code, was used in the Finite Element analysis. The input data used in the parametric analysis are as follows:

E = the Young's modulus: 6.89×10^4 MPa (10^7 psi)

ν = the Poisson's ratio: 0.25, 0.3, 0.35, 0.4, 0.45

a = one half the cross flats dimension: 0.254 M (10 in.), 0.229 M (9 in.), 0.203 M (8 in.), 0.178 M (7 in.).

$\eta = b/a$ = the thickness index ($a-b$ is the hex-can thickness): 0.1, 0.2, 0.5, 0.75, 0.9, 0.91, 0.92, 0.93, 0.94, 0.95, 0.96.

Boundary conditions

on B-1: $U_x = -.254$ mm (- .01 in.), $R_y = 0$

on B-2: $U_n = 0$, $R_t = 0$

on B-3: $R_x = R_y = 0$

on B-4: $R_x = 0$, $U_y = 0$.

B. Pressure on the Hex-can

The pressure on the boundary of a homogeneous solid cell will be uniform corresponding to a uniform normal displacement as based on the classical solution of a plane region of any shape subjected to a planar hydrostatic loading. However, for a hollow hexagonal cylinder, the distribution of the pressure on the outer wall is not expected to be uniform as may be qualitatively

deduced from the shape of deformation shown in Fig. 4a. The pressure on the outer edge of the hex-can from the PSAAS solution will be given as nodal point restraint forces, (P_i) . The totality of the nodal forces thus gives the total pressure P on the half length of the side, as shown in Fig. 6a. The corresponding uniform pressure p for a solid cell also may be related to the total pressure P as shown in Fig. 6b. The distributed pressure on a hex-can may be approximated by assuming a linear distribution between node points. The magnitude of the distributed pressures at node points $\{p_i\}$ are solved from the linear transform

$$\tilde{T} \{p_i\} = \{P_i\} \quad (1)$$

Fig. 6. Pressure distribution for uniform normal displacement
(a) hollow hex cylinder (b) solid hex cylinder.

where

$$T_2 = \frac{C}{6} \begin{bmatrix} 2 & 1 & & & \\ 1 & 4 & 1 & & 0 \\ \hline 0 & & 1 & 4 & 1 \\ & & & & 2 \end{bmatrix}$$

Figures 7 and 8 show the boundary pressures obtained for two different wall thicknesses and five different values of Poisson's ratio. The nodal point restraining pressure P_i are shown in solid lines. The corresponding distributed pressures are shown in dash lines. It is seen that, for a specific wall thickness ratio, the distributed pressure is fundamentally similar. The increasing magnitude corresponding to increasing Poisson's ratio is attributed to the resistance of the material to the volumic compression. The distribution of pressure definitely depends on the wall thickness ratio.

In the limit when the region is a solid hexagonal cell, the pressure distribution is uniform as shown in Fig. 6b. For the hollow hexagonal cylinders, the pressure distribution favors the region near the vertices of the hexagonal cylinder. As the wall thickness ratio reduces, the pressure distribution shifts toward the corner of the hex can as shown in Fig. 9. The size of the hex-can is not a factor.

C. Stability

Figures 7 through 9 show that the nodal pressure rapidly attenuates to zero toward the center section of a side of a hex-can. The stress state is that of a pure compression. For thin-walled hex-cans, the problem of stability soon arises, reducing further the effective stiffness of the core assembly.

The buckling configuration definitely affects the critical load for individual hex-cans. Figure 10 illustrates two types of buckling configurations. In Fig. 10a, the buckling configuration is localized. To maintain the corner angle, the two adjacent sides of neighboring cans separate in the middle. Figure 10b shows that a group of cans may undergo congruent deformation while maintaining the rigidity of the vertex angles.

The first configuration in Fig. 10a can be represented by a plate with opposite clamped edges, while the other two edges are free. In-plane forces are applied to the clamped edges. The corresponding critical membrane force is

Fig. 7. Pressure distribution on hex cans $(\frac{b}{a}) = 0.9$.

Fig. 8. Pressure distribution on hex cans $(\frac{b}{a}) = 0.95$.

Fig. 9. Pressure distribution for various wall thickness.

Fig. 10. Hex can buckling configuration (a) wall separation and (b) congruent buckling.

$$N_{cr} = \frac{12\pi^2 D}{(1+\eta)^2 a^2} , \quad (2)$$

and the critical compressive stress is

$$\sigma_{cr} = \frac{N_{cr}}{h} = \pi^2 \left(\frac{1-\eta}{1+\eta} \right)^2 \frac{E}{1-\nu^2} , \quad (3)$$

where $\eta = b/a$ is the wall-thickness index, such that $(1 - \eta)$ is the wall thickness ratio.

In Fig. 10b, the buckling mode may be extended from a regular hexagonal frame for which the critical load is²

$$N_{cr} = 3 \left(\frac{2\pi}{3} \right)^2 D / \left[(1+\eta)^2 a^2 \right] . \quad (4)$$

The corresponding compressive stress is

$$\sigma_{cr} = \frac{\pi^2}{9} \left(\frac{1-\eta}{1+\eta} \right)^2 \frac{E}{1-\nu^2} . \quad (5)$$

The critical compressive stresses in Eqs.(3) and (5) represent the high and the low limits of the stress level in the side of a hex-can for which stability becomes the significant concern. As a conservative estimate, we shall use the lower limit for the admissible compressive stress.

Figure 11 gives the critical stress, Eq. (5), as a function of the wall thickness ratio for five values of Poisson's ratio. Figure 12 shows that the stability criterion may be expressed for the global pressure, which is readily related to design problems.

Fig. 11. Critical compressive stress.

Fig. 12. Critical nominal pressure.

III. THE GLOBAL STIFFNESS

In Article 2.0, a numerical analysis model was set up to compute the pressure on the sides of a hexagonal container that will cause a given uniform normal displacement. The set of data from the numerical solution in (η, ν, P) will be used in developing an approximate equation relating those parameters for a given material stiffness. The form of the approximate equation, but with unknown coefficients will be assumed the same as a combination of the known exact solution of a solid hexagonal region and of a hollow circular cylinder under similar loads.

The approximate equation then is compared to the mechanical constitutive equation of an equivalent homogeneous solid hexagonal cell. The stiffness of this equivalent solid cell results in corresponding normal displacement and total pressure P and is the equivalent stiffness of the hexagonal containers. Thus the global stiffness of the core assembly is

$$\mu_e = \mu_g .$$

A. Pressure Displacement Equation for Hex-Cans

The analytical equation for a hexagonal cylinder cannot be obtained in the general form. Hence, an approximate form will be proposed as guided by the following considerations.

(1) The equation should reduce to the exact form when the interior cavity approaches zero.

(2) The general form might be similar to that of a hollow circular cylinders, for which the exact equation is known.

The exact equations for a solid hexagonal cell and a hollow cylinder are respectively

$$2\mu u = -(1 - 2\nu) pa , \tag{6}$$

and

$$2\mu u = \frac{-pa}{1-\eta^2} \left[(1 - 2\nu) + \eta^2 \right] , \tag{7}$$

where μ is the modulus of rigidity, ν is the Poisson's ratio, a is one half the across-flats dimension, p is the uniform pressure, η is the thickness index, and u is the normal displacement.

The proposed equation for $pa = \sqrt{3P}$ then becomes,

$$2\mu u = - \frac{pa}{1-\eta^2} [(1-2\nu)(1+A\eta+B\eta^2)+C\eta+D\eta^2]. \quad (8)$$

Equation (8) reduces to (6) when $\eta \rightarrow 0$. The coefficients are determined by the least square method, using data obtained for a set of specific values of the wall-thickness index (η) and the Poisson's ratio (ν). Altogether five sets of values for the coefficients are obtained corresponding to the thickness ranges of the cylinder.

Case 1: $\eta = [0.9, 0.95]$

Case 2: $\eta = [0.75, 0.95]$

Case 3: $\eta = [0.5, 0.95]$

Case 4: $\eta = [0.2, 0.95]$

Case 5: $\eta = [0.1, 0.95]$

Equation (8) is written as

$$F = \frac{1+\nu}{1-\eta^2} [(1-2\nu) + (1-2\nu)\eta A + (1-2\nu)\eta^2 B + \eta C + \eta^2 D] - \frac{Eu}{\sqrt{3P}} = 0 \quad (8a)$$

where E is the Young's modulus and $2P$ is the total pressure per side. The values of E and u are input as invariant. P is obtained as the total pressure corresponding to different values of ν and η , as shown in Figs. 7 and 8. The coefficients (A, B, C, D) then are determined through minimization of $(I = \Sigma F^2)$ where the summation, Σ , is over the entire set of (ν, η, P) for each of the five cases considered. The results are given in Table I. The different sets of values of (A, B, C, D) are substituted into Eq. (8) to compute the total pressure P so that the accuracy can be measured in comparison to the computer solution by PSAAS. From 8a, we obtain

$$P = \frac{-Eu(1-\eta^2)}{\sqrt{3}(1+\nu)} / [(1-2\nu)(1+A\eta+B\eta^2)+C\eta+D\eta^2].$$

TABLE I
COEFFICIENTS FOR EQ. (8) FOR VARIOUS THICKNESS RANGES OF A HEXAGONAL CYLINDER

Cases	1	2	3	4	5
A	0.170564	-0.072849	-0.421802	-0.577368	-0.619726
B	-0.168732	0.090048	0.466526	0.635616	0.681800
C	0.242867	0.320882	0.433272	0.4828430	0.495012
D	0.753850	0.671002	0.549882	0.496424	0.482682

In case 1, when only data from $\eta=[0.9, 0.95]$ are used, the values of P for any ν within the thickness range are accurate at the order of one ten-thousandth, which is well below the error by the finite element method. For thickness outside the range, the error is less than one per cent at $\eta = 0.75$, 4.3% at $\eta = 0.5$, 7.3% at $\eta = 0.2$, and 5% at $\eta = 0.1$.

In case 2, when a single set of data in (η, ν, P) is added for $\eta = 0.75$, the accuracy at $\eta = 0.75$ is improved to 0.1% while the errors at other lower value of η are dropped by a full percent without significant reduction of accuracy in the range $\eta = [0.9, 0.95]$.

When the data at the low end of η are included, the accuracy of P improves steadily there with only a slight rise in error for $\eta = [0.9, 0.95]$. In Case 5, the error at $\eta = 0.1$ and 0.2 is only around 3% while the accuracy at $\eta = [0.9, 0.95]$ is still about one-thousandth.

B. Equivalent Stiffness - Unicoefficient Model

With the determination of Eq. (8), we may begin to consider an equivalent solid-cell model for which the constitutive equation is known as

$$2\mu_e u = -(1 - 2\nu)pa \quad . \quad (9)$$

Thus, Eq. (9), using the equivalent modulus of rigidity, must result in the same displacement (u) under the same load (p) as Eq. (8) using the true modulus of rigidity. A direct ratio yields the stiffness ratio

$$\frac{\mu_e}{\mu} = (1 - 2\nu)(1 - \eta^2)/[(1 - 2\nu)(1 + A\eta + B\eta^2) + C\eta + D\eta^2] \quad . \quad (10)$$

The equivalent stiffness μ_e is a function of the Poisson's ratio and the wall thickness. This stiffness ratio is alternately expressed as

$$\frac{E_e}{E} = \frac{\mu_e}{\mu} , \quad (11)$$

where E_e is the equivalent Young's Modulus. Figures 13 and 14 show the stiffness ratio for cases 1 and 5. Those two sets of curves differ somewhat for low values of η , but that for $\eta = [0.9, 0.95]$ there is little difference. Thus, only one set of curves for the range $\eta = [0.9, 0.95]$ is therefore shown in Fig. 15.

Figure 15 may be used for a quick design aid. For instance, an application in the liquid metal fast breeder reactor design for a thickness ratio of $\frac{h}{a} = 0.086$ (or $\eta = 0.914$) and $\nu = 0.321$, the corresponding equivalent stiffness is $\mu_e = 0.0482\mu$ as demonstrated in Fig. 15.

Equation (10) or its graphic representation in Figs. 13 through 15, is essentially the only equivalent stiffness that can be developed from a uniform field model. The model is simple to use, as the designer may always use the original values of the Poisson's ratio. Yet, the simplicity may very well be its shortcoming when dealing with a thermal stress problem in which the Poisson's ratio varies because of the temperature variation. For that reason, an artificial dual-coefficient model is proposed.

C. Equivalent Stiffness - Dual-Coefficient Model

The dual-coefficient model introduces an equivalent Poisson's ratio so that an equivalent modulus of rigidity may be determined as a function of thickness only. For the dual-coefficient model, we shall express the constitutive equation for the uniform field as

$$2\bar{\mu}_e u = - (1 - 2\mu_e) p a . \quad (12)$$

Equation (8) may be rearranged to the form

$$2\mu u = \frac{pa}{1 - \eta^2} [1 + (A + C)\eta + (B + D)\eta^2] \left[1 - 2 \frac{1 + A\eta + B\eta^2}{1 + (A + C)\eta + (B + D)\eta^2} \right] \nu$$

Fig. 13. Equivalent stiffness for case 1.

Fig. 14. Equivalent stiffness for case 5.

$$2\alpha\mu u = -pa(1 - 2\gamma\nu), \quad (13)$$

where

$$\alpha = \frac{1 - \eta^2}{1 + (A + C)\eta + (B + D)\eta^2}, \quad \gamma = \frac{1 + A\eta + B\eta^2}{1 + (A + C)\eta + (B + D)\eta^2}. \quad (14)$$

Fig. 15. Equivalent stiffness for thin wall hex can.

In comparing Eqs. (12) and (13), we may write

$$\frac{\bar{\mu}_e}{\mu} = \alpha \quad \text{and} \quad \frac{\nu_e}{\nu} = \gamma. \quad (15)$$

Both coefficients are then functions of the thickness index η . Figure 16 through 20 express the Eqs. (14) and (15) for the five cases of coefficients (A,B,C,D). The single case for the range $\eta = [0.9, 0.95]$ is shown is Fig. 21.

IV. APPLICATION

As an illustration, we shall apply the method to determine the core pressure required to expell the upper subassembly region (upper axial blanket plus fission gas plenum) from its original position during a core disruptive accident. The data for this application is based on the Clinch River Breeder Reactor.³

A. Given Data

The geometry may be related to Figs. 1 and 2 with numerical dimensions as follows:

(a) Hex-can (referred to the reinforced section, load pad, at ACLP and TLP:

across flats (2a)	0.12052 m (4.745 in.)
thickness (h)	0.00521 m (0.205 in.)
length of load pad (d)	0.1016 m (4 in.)
clearance between cans (G)	0.00038 m (0.015 in.)

Fig. 16. Equivalent dual coefficient case 1.

Fig. 17. Equivalent dual coefficient case 2.

Fig. 18. Equivalent dual coefficient case 3.

Fig. 19. Equivalent dual coefficient case 4.

Fig. 20. Equivalent dual coefficient case 5.

Fig. 21. Equivalent dual coefficient for thin-wall hex can.

(b) Core barrel and core former assembly:

outside radius (C)	1.91 m (75.25 in.)
effective inside radius (B)	1.81 m (71.25 in.)

(c) Core assembly

effective radius of shield-barrel interface (B)	1.81 m (71.25 in.)
effective radius of shield-core interface (A)	1.216 m (47.88 in.)

Let the hex cans and the ring-barrel assembly be made of 316 ss with the following material properties:⁴

(a) Thermal strain (ϵ_T)

@ 1173 K	$17.2 \times 10^{-3} \text{ mm/mm}$
@ 873 K	10.74×10^{-3}
@ 573 K	4.78×10^{-3}

(b) Yield strength at 0.2% (σ_y)

@ 1173 K	77 MPa ($11.17 \times 10^3 \text{ psi}$)
@ 873 K	114.4 MPa ($16.6 \times 10^3 \text{ psi}$)
@ 723 K	119.7 MPa ($17.36 \times 10^3 \text{ psi}$)

(c) Young's modulus (E)

@ 1173 K	$1.17 \times 10^5 \text{ MPa}$ ($17 \times 10^6 \text{ psi}$)
@ 1023 K	$1.332 \times 10^5 \text{ MPa}$ ($19.3 \times 10^6 \text{ psi}$)
@ 723 K	$1.637 \times 10^5 \text{ MPa}$ ($23.7 \times 10^6 \text{ psi}$)

(d) Poisson's Ratio (ν)

@ 1173 K	0.342
@ 1023 K	0.321
@ 723 K	0.298

(e) Coefficient of friction

Breakaway	0.68-1.74
Moving (kinematic)	0.28-0.62

B. Analysis

We assumed that during the core disruptive accident the above-core sub-assembly region is heated to the sodium boiling temperature. The boundary and interface temperatures (in parentheses) and the mean temperatures of a region (in brackets) are shown in Fig. 22. In that figure, the reactor core and the core assembly are simplified and modeled by a solid cylinder interior to two concentric hollow cylinders as distinguished by the geometry and the temperature differences.

1. Compatibility Equation. The displacement field is described completely by its radial component, which may be delineated into a part as a result of free thermal expansion and another pressure. Thus,

Fig. 22. Simplified model of hex-can assembly at ACLP and TLP.

$$u(r) = u_T(r) + u_p(r) \quad .$$

The compatibility equation at each interface must take into consideration the cumulative clearance (C_t) in each region. Hence,

$$u_p^I(A) + u_T^I(A) - C_t^I = u_p^{II}(A) + u_T^{II}(A) \quad , \quad (16)$$

$$u_p^{II}(B) + u_T^{II}(B) - C_t^{II} = u_p^{III}(B) + u_T^{III}(B) \quad , \quad \text{and} \quad (17)$$

$$C_t = C_t^I + C_t^{II} = 3.15 \times 10^{-3} B \quad , \quad (18)$$

where the superscripts of Roman numerals indicate the corresponding regions, and the clearance per unit radial length is

$$\frac{0.00038}{0.12090} = 3.15 \times 10^{-3} \text{ m/m} .$$

2. Thermal Expansion. The equation of axisymmetric thermal expansion is readily available in any elasticity text.⁵ For the boundary and the interface temperatures given in Fig. 22 and the corresponding thermal strain in 4.1, we can derive that

$$u_T^I(A) = \epsilon_T^A A = 17.2 \times 10^{-3} A , \quad (19)$$

$$u_t^{II}(A) = \frac{B^2 \epsilon_T^B - A^2 \epsilon_T^A}{B^2 - A^2} + \frac{\epsilon_T^A - \epsilon_T^B}{2 \log \left(\frac{B}{A} \right)} A = 13.546 \times 10^{-3} A , \quad (20)$$

$$u_t^{II}(B) = \frac{B^2 \epsilon_T^B - A^2 \epsilon_T^A}{B^2 - A^2} + \frac{\epsilon_T^A - \epsilon_T^B}{2 \log \left(\frac{B}{A} \right)} B = 13.546 \times 10^{-3} B , \quad \text{and} \quad (21)$$

$$u_t^{III}(B) = \frac{C^2 \epsilon_T^C - B^2 \epsilon_T^B}{C^2 - B^2} + \frac{\epsilon_T^B - \epsilon_T^C}{2 \log \left(\frac{C}{B} \right)} B = 7.705 \times 10^{-3} B . \quad (22)$$

3. Radial Displacement from Pressure. The packed hex-can assembly is simplified and modeled by an elastic continuum in a uniform pressure field. Hence,

$$p^I = p_i^{II} = p(A) = P; \quad p_o^{II} = p_i^{III} = p(B) = P; \quad p_o^{III} = p(C) = 0 ,$$

where the subscripts 'i' and 'o' designate the internal and the external pressures of the annular region.

In computing the global stiffness of each region, we shall refer to the individual hex-can of the region. There, we have

$$\eta = \frac{(a - h)}{a} = \frac{(0.06026 - 0.00521)}{0.06026} = 0.914 \quad .$$

The values of Poisson's ratio corresponding to the mean temperatures of the regions are

$$\nu^I = 0.342, \quad \nu^{II} = 0.321, \quad \nu^{III} = 0.298 \quad .$$

The global stiffness of the core regions I and II then may be obtained by using Eq. (10) or Fig. 15 for

$$\mu_g^I = 0.0444 \mu_{1173 \text{ K}} = 0.0444 E_{1173 \text{ K}} / 2(1 + \nu^I) = 1.935 \times 10^3 \text{ MPa} ,$$

$$\mu_g^{II} = 0.0485 \mu_{423} = 2.445 \times 10^3 \text{ MPa} , \text{ and}$$

$$\mu^{III} = \mu_{723} = 6.306 \times 10^4 \text{ MPa} .$$

The elastic equations of axisymmetrically pressurized solid and hollow cylinders are used,⁵ resulting in

$$u_p^I(A) = -7.966 \times 10^{-5} \text{ pA} , \quad (23)$$

$$u_p^{II}(A) = -7.366 \times 10^{-5} \text{ pA} , \quad (24)$$

$$u_p^{II}(B) = -7.366 \times 10^{-5} \text{ pB} , \text{ and} \quad (25)$$

$$u_p^{III}(B) = 10.45 \times 10^{-5} \text{ pB} . \quad (26)$$

4. Global Pressure and Friction Force. The expressions of radial displacements from thermal expansion and pressure, Eqs. (19) through (26), are substituted into the compatibility Eqs. (16) - (18), to result in

$$p = 28.25 \text{ MPa } (4.1 \times 10^3 \text{ psi}) .$$

We observe in Fig. 12 that for $\nu = 0.914$, the value of the global pressure is much below the critical pressure.

In computing the maximum friction force per unit area we take into consideration the surface condition, the environment, the temperature, and use the lower values for the breakaway and the kinematic coefficients of friction, 0.70 and 0.30 respectively. The maximum friction force per unit area are then

Breakaway	19.85 MPa (2880 psi)
Kinematic	9.45 MPa (1225 psi)

where breakaway denotes the initiation of, and kinematic the presence of, the motion.

5. Threshold Core Pressures for Upper Subassembly Expulsion. Three configurations of above-core subassembly expulsion to be considered include the entire core assembly (Regions I and II together), the inner core (Region I), and an individual hex can. The total friction surface in each case is measured by the actual contact peripheral length and the depth of the load pad at both load planes; ACLP and TLP. The respective areas for the three configurations are 264 m^2 , 1.957 m^2 , and 0.0851 m^2 . The expulsion force for each of the three cases is the maximum friction force of the respective configuration. Hence

(a) P_B (Breakaway)	52.5 MN ($11.7 \times 10^6 \text{ lb}$)
P_K (Kinematic)	22.4 MN ($4.92 \times 10^6 \text{ lb}$)
(b) P_B	38.8 MN ($8.7 \times 10^6 \text{ lb}$)
P_K	16.35 MN ($3.67 \times 10^6 \text{ lb}$)
(c) P_B	1.7 MN ($0.38 \times 10^6 \text{ lb}$)
P_K	0.72 MN ($0.16 \times 10^6 \text{ lb}$).

The core pressure is assumed to be generated totally in the fuel region of the core (Region I). The pressurized areas of Region I and an individual hex-can are respectively 4.646 m^2 and 0.0127 m^2 (assuming effectively blocked flow channels). The threshold core pressures for expulsion of the three configurations are then

(a) p_B	11.3 MPa (1530 psi)
p_K	4.82 MPa (680 psi)
(b) p_B	8.3 MPa (1210 psi)
p_K	3.5 MPa (510 psi)
(c) p_B	133.5 MPa (19300 psi)
p_K	57 MPa (8130 psi).

Thus, we can conclude that the critical core pressure is 8.3 MPa when the upper core subassembly structure is expelled. Thereafter, it takes only a core pressure of 3.5 MPa to sustain the lifting motion. However, only some extremely high localized pressure could expell an individual hex-can.

The containment failure was analyzed with a very simple mathematical model. Consideration may be given to the core weight, omitted here, as secondary effect.

6. Discussion. In this analysis, we developed an approximate constitutive equation for the hex-can subjected to a central pressure field, which corresponds to a uniform normal displacement on sides. The least-square method is the most convenient one to use for determining the coefficients in the constitutive equation. The result reduces to the exact form when the inside wall dimension, the half cross-flat b , approaches zero. Because of relatively more data in the range of η : [0.9, 0.95], the accuracy is very high there. Also, the thickness range includes those used in the LMFBR. However, even with the small amount of data for thicker walls, the accuracy is admissible in comparison to the numerical method used in obtaining the data. Better accuracy at lower values of η may be accomplished by incorporating more data from $\eta=0.1$ and 0.2. For the present application, the values of the coefficients in the approximate constitutive equation are considered adequate.

As a design aid and also for simplification of computations for problems involving an assembly of a large number of hex cans in a central pressure field, the hollow can may be replaced by an equivalent solid hexagonal cell.

The equivalent stiffness of the solid cell was obtained for two models. One is a uni-coefficient model, which yields a single equivalent stiffness (either the modulus of rigidity μ_e or the Young's modulus E_e). The stiffness is a function of the wall thickness and the Poisson's ratio. In the dual coefficient model, an equivalent modulus of rigidity $\bar{\mu}_e$ and an equivalent ν_e are obtained. These are artificially created to delineate the effect of Poisson's ratio, such that both are functions of thickness only but are independent of each other.

However, where Poisson's ratio is a constant, the uni-coefficient model should be used for conceptual simplicity. However, when Poisson's ratio varies, as in a varying temperature field, the dual coefficient model is preferred.

We shall now analyze the structural strength of the hex-can in resisting a 28.25 MPa global pressure. On the surface the pressure is less than a third of the yield strength in bearing. However, consider the fact that at the wall thickness prescribed the pressure is concentrated near the vertex over a region less than a third of the half wall length in a near triangular distribution. Even though a conservative one third of the half wall length is used, the maximum pressure would be approximately six times the global pressure. The corner would yield under such concentrated pressure.

Then the compressive force (N in Figs. 7 and 8) is by equilibrium equal to $\sqrt{3}P$ (= pa). The corresponding compressive stress therefore

$$\sigma_c = p/(1-\eta) = 328 \text{ MPa } (47.6 \times 10^3 \text{ psi}).$$

The value is much too high above the yield strength.

Furthermore, in the inner core region the Young's Modulus at the high temperature is only 1.17×10^5 MPa. Correspondingly

$$\frac{\rho}{E} = 0.24 \times 10^{-3}.$$

We observe in Fig. 12, the value is just above the stability curve for $\nu = 0.342$. While the confinement from lifting may be achieved, the hex-can will fail structurally.

REFERENCES

1. R. V. Browning, D. G. Miller and C. A. Anderson, TSAAS: Finite Element Thermal and Stress Analysis of Axisymmetric Solids with Orthotropic Temperature-Dependent Material Properties, Los Alamos Scientific Laboratory Report LA-5599-MS, May 1974.
2. S. P. Timoshenko and J. M. Gere, Theory of Elastic Stability, McGraw-Hill, 1961, pp 150-151.
3. CRBRP, Project Management Corporation, Preliminary Safety Analysis Report (1975).
4. Nuclear System Materials Handbook, V. 1, Bk. 1, Part 1, Group I, Sec. 4.
5. S. P. Timoshenko and J. N. Goodier, Theory of Elasticity, McGraw-Hill, 1970, pp 69-71, 448-450.