NOTICE

PORTIONS OF THIS REPORT ARE ILLEGIBLE. It
has been reproduced from the best available
copy to permit the broadest possible avail-
ability.

OPERATIONAL SAFETY SYSTEM RELIABILITY

Final Report

for the period of

Nov. 15, 1975 - Nov. 14, 1976

NOTICE
This report was prepared as an account of work
sponsored by the United States Government. Neither the
United States nor the United States Department of
Energy, nor any of their employees, nor any of their
contractors, subcontractors, or their employees, makes
any warranty, express or implied, or assumes any legal
liability or responsibility for the accuracy, completeness
or usefulness of any information, apparatus, product or
process disclosed, or represents that its use would not
infringe privately owned rights.

R. W. Hockenbury

Rensselaer Polytechnic Institute
Troy, NY 12181

December 1977

Prepared For

Department of Energy under Contract No. EY-76-S-02-2825.*000

NTIS MN ONLY**MASTER**

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

EB

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

ABSTRACT

This report describes a computerized system intended to be used in conjunction with operational data (observed failure rates) of nuclear power plant components. The codes convert the system description to its Boolean equivalent and obtain the system reliability vs. time. Additional codes provide a measure of the uncertainty in this calculated system reliability.

OPERATION SAFETY SYSTEM RELIABILITY

Introduction

A set of computer programs and a methodology representing an approach to the problem of operational reliability procedures have been completed. The scope and structure of the numerical calculations are such that they can be run on an intermediate size computer usually available at a reactor site for reactor data processing, with a capability of dealing with the main features and performance of a protection system. Implicit in this approach are plans for various aspects of data acquisition and processing, including a proposed method for continuously monitoring performance and signs of impending failure of sensors and (in principle) other components. Operating data of several types have been collected from industry sources, leading to a good perspective on data requirements and availability despite limitations on data presently available for model testing. Response of utility personnel to practical applications of the proposed procedures was solicited. On the basis of the current emphasis on improving nuclear plant availability, it can be said that the study was indeed worthwhile and timely.

The principal parts of the program are the following:

1. Determination of safety system reliability in operational nuclear power plants by on-site staff using the plant computers.
2. Use of component or sub-system failure data, from operational experience, in the codes of Item 1.
3. Development of methods to evaluate the uncertainty in component failure data and to combine the component uncertainties in order to determine the uncertainty or confidence limits of the system reliability.
4. Develop methods to statistically examine selected sensor variables in order to anticipate off-normal behavior.

The flow chart in Figure 1 is intended to show the overall analysis system. This system is intended for use by personnel in the plant with minimum training in reliability and on typical computers already present in nuclear plants. This computer requirement is in contrast to the very large computer memory capability required by more elaborate codes.

System Analysis

The structure of the system or fault tree is input to the BOOLEX code in a series of statements which are interpreted and converted to a Boolean expression. (See Appendix A for more details and an example.) This expression is then converted to exclusive Boolean form (equivalent to $\wedge$ a probability expression). BOOLEX then converts the exclusive Boolean expression to a FORTRAN subroutine called LOGIC.

LOGIC

This subroutine is compiled and stored in a binary format.

RELIA

This code used LOGIC and component failure rate parameters to calculate the system reliability at pre-selected time intervals. See Appendix A for an example.

APPROX

Details are presented in Appendix B with a sample calculation. A system of components in series has been treated by the method of moments using Mellin transforms¹ to combine the component uncertainties into a system distribution in reliability. The mean and variance of system reliability are found and also presented in Appendix B. The method has been programmed with further details in Appendix D.

EXACT

The EXACT subroutine in the flow chart refers to determination of the system reliability distribution by convolution and transformation of variables methods.

Our procedure here is to:

1. select appropriate probability density functions for component failure rates
2. compare results to those of approximate methods such as
 - (a) the method of moments of
 - (b) assuming each component reliability is normally distributed.
3. Extend the exact method eventually to larger systems.

As noted in our last progress report, the convolution of series components for the high reliability case, where the standard deviation of the component failure rate is small compared to the mean value, is simple if one assumes each component failure rate is normally distributed. This causes the component reliability to be lognormally distributed and leads to a system reliability which is also lognormally distributed. A sample calculation is given for four series components in Appendix C. Comparisons to the method of moments result (APPROX) are given in Table 1 and Figure 2.

For a large number of series components, a central limit theorem applies², namely, that the probability density function of the product of a large number of random variates is lognormally distributed, independent of the individual density functions.

Methods for On-Line Monitoring of Sensor Response

Experience gained from Monte Carlo simulation of sensor system response, as described in previous progress reports, indicates that a moments analysis of fluctuations in noise spectra is likely to be a useful way to provide routine on-line monitoring of this response. It is anticipated that with a suitable algorithm it will be feasible to automate this process so that a computer output will selectively identify those results which imply performance degradation indicative of near term failure. In this respect the proposed methodology is similar to the application of "security indices" to a power distribution system.

The method is based on the assumption that at some stage prior to failure of a sensor or signal processing component the uncertainty in its response will increase to the point of dominating the fluctuations in the response function. Obviously this has to be the case unless the failure is a completely abrupt transition from normal operation to inoperability, a condition not likely to be encountered except under a catastrophic condition such as fire. Therefore, the general approach is as follows: measure successive noise spectra, each expressed either as a time series or a power spectral density; it appears most convenient to monitor selected frequencies in the p.s.d., calculating, say, the first through fourth moments of the distribution of fluctuations in this deviation from the mean of the p.s.d. at a selected frequency. The first moment will be a response to the instrument drift as usually measured. However, the second moment (variance) of this distribution is more likely to contain information as to whether the drift is normal, and the third and fourth moments (related to skewness and kurtosis) can be expected to provide further indices relating to departure from normal response.

Work in progress is the development of a statistical algorithm to determine significance levels for the moments, and the testing of this by applying it to the analysis of the Monte Carlo simulations previously described (which can be run with a variety of uncertainty distributions to simulate possible approaches to failure). The final result will be an algorithm which continuously sorts noise spectra, calculates the appropriate moments and provides as output a "security index" as a measure of the failure probability.

The relationship of this analysis to the system reliability calculation is that it provides an update on the component or subsystem failure rate input data for the system analysis. It is a means of narrowing the range of uncertainty of the system reliability calculation. This appears to be a particularly fruitful methodology to explore in connection with other components. In general, it is expected that the same general considerations regarding approach to failure apply to components other than those in the sensor system.

SUMMARY

Work to date has made it clear that powerful reliability analyses can be carried out on intermediate size computers. The approximate methods described in this report have checked out well in the comparisons made so far. We consider that the present results clearly warrant the further development and refinement of the overall computer program as a working tool. Further liaison with operating plants would ensure that practical adaptations are made. It is expected that numerical results for a specific plant may lead to further simplifications in the sense of identifying an optimum schedule for running various parts of the overall reliability code and using empirical models representing further simplification in some cases.

More specifically, the present results include:

1. A plan for a master program which starts with system logic component failure rates and their uncertainties (defined by an appropriate distribution function) and produces, as output, system or subsystem reliability and measures of uncertainty or confidence levels for this calculation.
2. Exact convolution calculations (Appendix C) using lognormally distributed component reliabilities for comparison to the approximate results (Appendix B) from the moments methods. For the case of 4 series components, point by point comparison of $f(R)$ from the moments and exact methods can be made from Figure 2 or the listing in Table 1. It is more appropriate to integrate $f(R)$ in order to determine the probability of a reliability $\leq R_0$ where R_0 is a selected reference value. The crucial place to compare the exact and approximate results may very well be in the tails of the $f(R)$ distribution. This is precisely the region where the largest differences may occur between

the exact and approximate methods. Referring to Table 1, the Beta distribution (moments method) is higher than the lognormal result for $R < .9978$ and lower for $R < .9985$.

Future developments could include:

1. Make computer runs for larger systems, parallel systems, mixed types of distributions, small and large standard deviations and sensitivity to assumed distributions of failure rate uncertainties; use Monte Carlo techniques to verify some of the calculations.
2. Apply the methods to the protection systems in the FFTF PSAR and the CRBRP PSAR.
3. Run various parameters to determine the uncertainty in the confidence limits resulting from (a) and (b).
4. Develop response function codes for testing component performance on-line.
5. Review data from several plants and analyze.

References

1. W. C. Giffin, Transform Techniques for Probability Modeling, Academic Press, (1975).
2. J. Aitchinson and J.A.C. Brown, The Lognormal Distribution, Cambridge (1969).
3. R. G. Bennetts, I.E.E.E. Trans. on Rel., R-24, No. 3, 175(1975).
4. G. J. Hahn and S.S. Shapiro, Statistical Models in Engineering.
5. M. G. Kendall and A. Stuart, The Advanced Theory of Statistics, Vol. I, Hafner (1963).

Publication

R. W. Hockenbury and M. L. Yeater, "Reliability Models For Plant Protection Systems", Informational Meeting on Fast Reactor Safety and Related Physics, Oct. 5, 1976.

TABLE 1

Probability Distribution of Reliability For Four Components* in Series

<u>Reliability</u>	<u>Beta</u>	<u>Lognormal</u>
0.9977	0.40	0.2
.9978	8.69	6.9
.9979	106.26	104.5
.9980	700.52	729.8
.9981	2336.3	2353.1
.9982	3659.7	3504.9
.9983	2475.	2412.34
.9984	651.5	767.4
.9985	59.	112.9
.9986	1.6	7.67

Mean Reliability

Beta Distribution

$$\mu_1^1 = 0.99819869$$

Lognormal Distribution

$$\mu_1^1 = 0.99820162$$

Variance

Beta Distribution

$$\mu_2 = 1.1887 \times 10^{-8}$$

Lognormal Distribution

$$\mu_2 = 1.2953 \times 10^{-8}$$

* Four components were used in this test case; this calculation could have been performed for a larger number.

Figure 1. Flow Chart for Boolex Code

Figure 2 RELIABILITY

APPENDIX A

BOOLEX*

Alton P. Donnell, Jr.

The computer code BOOLEX is designed to develop a probability expression from system diagrams or fault trees. This code is now operational on a D.E.C. PDP-15 computer. This computer has a 32k memory, interactive graphics and a 250,000 word disk.

1. The methodology is based on an algorithm developed by Bennett's⁽³⁾. The following have been added to expand Bennett's algorithm:
 - a. Expansion of the AND and OR gates to accept up to 8 inputs/gate.
 - b. Inclusion of 2/3 logic gates (other m out of n gates may be added if needed).
2. Some of the specific aspects of the Fortran coding are as below:
 - a. The logical variables are input in 'R' format and stored as double integer values. Since each different logical variable has a double precision integer value that is unique, the variable may be identified and manipulated.
 - b. Complement values (eg. $\bar{A}$) are indicated by Z A. From the right, each variable name is allocated 3 spaces. The fourth is reserved for the complement indicator; the fifth left blank for spacing. The Z indicator is added or deleted by adding or subtracting the double precision integer value associated with Z .

*In partial fulfillment of Master of Engineering Project, R.P.I.

3. Code organization (BOOLEX) The exclusive Boolean code is organized as follows:

- POL:**
- 1) Converts the reverse Polish notation input to the RPN stack
 - 2) Manipulates NOR, NAND, XOR, OR2/3 into AND/OR form
 - 3) Calls BOOL
 - 4) Calls EXCL
 - 5) Calls WRLOG
- BOOL:**
- 1) Pops-up each term/element in order
 - 2) Performs AND/OR operations by calling MULT and/or ORCHEK
 - 3) Reduces the result to a non-redundant s.o.p. (sum of products) form
 - 4) Prints the non-exclusive Boolean equation
- MULT:**
- 1) Performs AND operations by multiplying term by term
 - 2) Eliminates terms using $A\bar{A}=\emptyset$, $AA=A$, $A1=A$, $A\emptyset=\emptyset$
- ORCHEK:**
- 1) Eliminates terms using $A+A=A$, $A+AB=A$, $XY+X\bar{Y}=X$
 - 2) Places terms in order of increasing cardinality (for greater efficiency)
- EXCL:**
- 1) Converts from Boolean to Exclusive Boolean Form
 - 2) Prints results
- WRLOG:**
- 1) Constructs a Fortran language sub routine called LOGIC for use by the code RELIA to calculate time-dependent
 - 2) Stores LOGIC on the disk memory of the PDP-15

An example of the test problems run in the development of Boolex and RELIA is given on the following pages. The input is drawn on page A4 as a success tree, the input could just as easily be viewed as a system diagram, with the top OR output being the system reliability. The input is coded as shown on page A5 in the form recognized by the reverse Polish notation algorithm. On page A6, the Boolean equivalent and the mutually exclusive forms are presented. From the latter, subroutine LOGIC is created and listed on page A7. The component names and input parameters are listed on page A8. For example, element A is denoted by AAA and $k=1$ indicating a constant probability (reliability) was chosen. Element E, denoted by EEE has $k=2$ indicating that its reliability is modeled by $e^{-\lambda t}$, with λ in column 4 equal to 0.32×10^{-4} per hour. Element G has $k = 3$ which is used to indicate that its reliability is modeled by a Weibull distribution in time with parameters alpha and beta in columns 5 and 6, respectively. The last line on page A8 indicated that the reliability will be calculated at 500 hour intervals up to 2000 hours. Pages A9-A13 give the calculated reliability starting at time zero and at 500 hour intervals afterwards, for each component and for the system. The portion of the output devoted "Section Reliability" corresponds to the seven lines or terms on page A6.

TEST 12

$$Z = [((A+B)(C+D))E][AB+FG][E+BC+AF] + (\bar{A} + \bar{B}\bar{C})$$

$$= A + AC + ABCE + BEFD + ABDF + ABED + ACDFE + AEDFC + BCDFG$$

$$Z = \bar{A} + A\bar{B}\bar{C} + AB\bar{E} + AB\bar{D}\bar{E}F + A\bar{B}C\bar{D}\bar{E}\bar{F} + A\bar{B}C\bar{E}FG + A\bar{B}C\bar{D}\bar{E}$$

Z	X1	X2	OR	(2)	FND	
X1	X4	X5	X6	AND	(3)	END
X2	ZAAA	X3	OR	(2)	FND	
X3	ZBBB	ZCCC	AND	(2)	FND	
X4	X7	X8	AND	(2)	FND	
X5	X11	X12	OR	(2)	FND	
X6	EEE	X13	X14	OR	(3)	END
X7	X9	X17	AND	(2)	FND	
X8	DDD	EFE	OR	(2)	FND	
X9	AAA	BBB	OR	(2)	FND	
X10	BBB	CCC	OR	(2)	FND	
X11	AAA	BBB	AND	(2)	FND	
X12	FFF	GGG	AND	(2)	FND	
X13	DDD	CCC	AND	(2)	FND	
X14	AAA	FFF	AND	(2)	FND	
END						

PRINT/TV ?

?

BOOLEAN RESULT IS AS FOLLOWS

Z =
 7AAA +
 ZCCC ZBBB +
 EEE RRR AAA +
 EEE GGG FFF RRR +
 FFF AAA RRR DDD +
 CCC DDD BRR AAA +
 EEE GGG FFF CCC AAA +
 FFF AAA GGG DDD CCC +
 CCC DDD GGG FFF BRR

PRINT/TV ?

↑

EXCLUSIVE BOOLEAN RESULT IS:

Z=
 ZAAA +
 ZCCC 7RRR AAA +
 EFE RRR AAA +
 FFF AAA RRR DDD 7FFF +
 CCC DDD RRR AAA 7FFF 7FFF +
 FEE GGG FFF CCC AAA 7RRR +
 FFF AAA GGG DDD CCC 7FEE 7RRR

PRINT/TV SUBROUTINE LOGIC ?

```

SUBROUTINE LOGIC (T,REL)
IMPLICIT DOUBLE PRECISION(A-Z)
COMMON/X1/X(50)
COMMON/X2/A(50,5)
AAA=FUN1( 1)
ZAAA=.1D01- AAA
BBB=FUN1( 2)
ZBBB=.1D01- BBB
CCC=FUN1( 3)
ZCCC=.1D01- CCC
DDD=FUN1( 4)
ZDDD=.1D01- DDD
EEE=FUN2( 5,T)
ZEEE=.1D01- EEE
FFF=FUN2( 6,T)
ZFFF=.1D01- FFF
GGG=FUN3( 7,T)
ZGGG=.1D01- GGG
X( 1)=.1D01
X( 1)=X( 1)* ZAAA
X( 2)=.1D01
X( 2)=X( 2)* ZCCC* ZBBB* AAA
X( 3)=.1D01
X( 3)=X( 3)* FFF* BBB* AAA
X( 4)=.1D01
X( 4)=X( 4)* FFF* AAA* BBB* DDD* ZEEE
X( 5)=.1D01
X( 5)=X( 5)* CCC* DDD* BBB* AAA* ZEEE* ZFFF
X( 6)=.1D01
X( 6)=X( 6)* FFF* GGG* FFF* CCC* AAA* ZBBB
X( 7)=.1D01
X( 7)=X( 7)* FFF* AAA* GGG* DDD* CCC* ZEEE* ZBBB
REL=0.0D00
REL=REL+X( 1)
REL=REL+X( 2)
REL=REL+X( 3)
REL=REL+X( 4)
REL=REL+X( 5)
REL=REL+X( 6)
REL=REL+X( 7)
RETURN
END

```

STOP 000000

DOS-15 124

5

INPUT DATA

NAME	K	CONST	P	LAMDA	ALPHA	BETA
AAA	1	0.9600	+20	0.0000	+00	0.0000
BBB	1	0.9800	+00	0.0000	+00	0.0000
CCC	1	0.9500	+00	0.0000	+00	0.0000
DDD	1	0.9400	+00	0.0000	+00	0.0000
EEF	2	0.0000	+00	0.3200	-04	0.0000
FFF	2	0.0000	+00	0.2000	-06	0.0000
GGG	3	0.0000	+00	0.0000	+00	0.0000

RUN TIME = 2000.00 DELTA T = 500.00

RESULTS OF RELIABILITY CALCULATION

TIME = 0.0 HOURS

COMPONENT	RELIABILITY
AAA	0.96000000+00
BBB	0.94000000+00
CCC	0.95000000+00
DDD	0.94000000+00
EEE	0.10000000+01
FFF	0.10000000+01
GGG	0.10000000+01

SECTION RELIABILITY

1	0.40000000-01
2	0.96000000-03
3	0.94000000+00
4	0.00000000+00
5	0.00000000+00
6	0.10000000-01
7	0.00000000+00

SYSTEM RELIABILITY= 0.10000000+01

RESULTS OF RELIABILITY CALCULATION

TIME : 538.0 HOURS

COMPONENT	RELIABILITY
AAA	0.95000000+00
BBB	0.98000000+00
CCC	0.95000000+00
DDD	0.94000000+00
EEE	0.98412730+00
FFF	0.99990000+00
GGG	0.97218730+00

SECTION	RELIABILITY
1	0.40000000-01
2	0.96000000+00
3	0.92586700+00
4	0.14035630-01
5	0.13334490-05
6	0.17449490-01
7	0.26455100-03

SYSTEM RELIABILITY= 0.00857800+00

RESULTS OF RELIABILITY CALCULATION

TIME = 1000.0 HOURS

COMPONENT	RELIABILITY
AAA	0.96000000D+00
BBB	0.99000000D+00
CCC	0.95000000D+00
DDD	0.94000000D+00
EEE	0.96850000D+00
FFF	0.99000000D+00
GGG	0.94132910D+00

SECTION	RELIABILITY
1	0.40000000D-01
2	0.96000000D-03
3	0.91117100D+00
4	0.27845700D-01
5	0.52912110D-05
6	0.16625700D-01
7	0.50819110D-03

SYSTEM RELIABILITY= 0.99711600D+00

APPROXIMATE METHODS TO DETERMINE THE DISTRIBUTION FUNCTION SYSTEM RELIABILITY

Because of the number of series elements in most protection systems, we have concentrated on this configuration. Each series component reliability is viewed as a random variable to be modeled by a function, for example, $R = \exp(-\lambda t)$ where the basic uncertainty is in the failure rate λ . Thus assuming a probability density function for λ , the probability density function of R , $f(R)$, is determined by transformation of variables. Then the distributions, $f(R)$, of all the series components must be combined to find the overall probability distribution in reliability of the series string of components. Since, for the series case, the overall reliability is the product of individual reliabilities, the problem is one of combining products of random variables. This is conveniently treated by the use of Mellin transforms. For example, for two components A and B in series, with reliability distributions $f_A(R)$ and $f_B(R)$, respectively, the system reliability distribution $f(R)$ is :

$$f(R) = \int_0^1 \frac{1}{v} f_A(v) f_B\left(\frac{R}{v}\right) dv$$

where:

$$R = R_A R_B$$

$$v = R_A$$

This equation is the Mellin convolution of $f_A(R)$ and $f_B(R)$ whose transform is:

$$F_m(s) = F_m^A(s) F_m^B(s)$$

where the Mellin transform of $f(x)$ is:

$$F_m(s) = \int_0^\infty x^{s-1} f(x) dx$$

Note that this integral is also recognized as the $(s-1)^{\text{th}}$ moment of $f(x)$ about the origin.

Because of its flexibility and range of variable ($0 \rightarrow 1$), the beta distribution is convenient to use for the component reliability. Thus we can write:

$$f(R) = \frac{\Gamma(a+b)}{A \Gamma(a) \Gamma(b)} R^{a-1} (1-R)^{b-1}$$

and similarly for $f(R)$; then $F_m(s)$ is :

$$F_m(s) = \frac{\Gamma(a+b) \Gamma(a+s-1)}{\Gamma(a+b+s-1) \Gamma(a)} \frac{\Gamma(c+d) \Gamma(c+s-1)}{\Gamma(c+d+s-1) \Gamma(c)}$$

where c, d refer to $f(r)$

From $F_m(s)$, we obtain the first moment(mean) of the system by letting $s=2$; the second moment, about the origin, of the system is found by setting $s=3$, and so on, for the third and higher moments. From the moments about the origin, the moments about the mean of the system are found directly as follows:

1. The mean or first moment about the origin = $F_m(2)$
2. The variance or second moment about the mean = μ_2 where:

$$\mu_2 = F_m(3) - [F_m(2)]^2 =$$

3. The third moment about the mean = μ_3 where:

$$\mu_3 = F_m(4) - 3 F_m(2) F_m(3) + 2 [F_m(2)]^3$$

4. The fourth moment about the mean = μ_4 where:

$$\mu_4 = F_m(5) - 4 F_m(2) F_m(4) + 6 [F_m(2)]^2 F_m(3) - 3 [F_m(2)]^4$$

The moments can be used to select an approximate distribution for the system reliability from the Pearson or Johnson family as described by Hahn and Shapiro or Kendall. This calculation has been done by hand for four components and has been programmed to handle a larger number. Failure rates were chosen for four components; each component reliability was modeled by a beta distribution with parameter a and b, determined by the mean and variance of each failure rate distribution. From $F_m(s)$ for the 4 component system, the first

* See Appendix D

four moments of the system were obtained as outlined above. These moments were used in a method of moments calculation to select an approximate distribution by Kendall's method. This calculation resulted in a choice of the type I or beta distribution for the system reliability distribution. The system beta distribution parameters a and b are determined directly from the system moments already calculated and the distribution is listed in Table 1 and plotted in Figure 2. Similar results are given for the case of lognormal component distributions. The lognormal case is discussed in another section (EXACT) of this memorandum. The means and variances of each calculation are also presented in Table 1.

Outline of Approximate Calculation - 4 Series Elements

<u>Element</u>	<u>Failure Rate Parameter</u>		<u>Beta Dist. Parameters</u>	
	<u>Mean</u>	<u>Variance</u>	<u>a</u>	<u>b</u>
1	10^{-6}	10^{-14}	994897	101
2	10^{-5}	10^{-12}	110778	111
3	5×10^{-6}	2.5×10^{-13}	209794	105
4	2×10^{-6}	4×10^{-14}	509797	102

System Moments From Mellin Transforms

Mean and Central Moments

$$\begin{aligned}
 s=2 \quad \mu_1^1 &= .99819869 \\
 s=3 \quad \mu_2 &= 1.1887 \times 10^{-8} \\
 s=4 \quad \mu_3 &= -9 \times 10^{11} \\
 s=5 \quad \mu_4 &= 6.7 \times 10^{-10}
 \end{aligned}$$

Higher Moments About Origin

$$\begin{aligned}
 s=3 \quad \mu_2^1 &= .996400639 \\
 s=4 \quad \mu_3^1 &= .9946058377 \\
 s=5 \quad \mu_4^1 &= .992814281
 \end{aligned}$$

Kendall's Method of Moments Application

$$\beta_1 = \mu_3^2 / \mu_2^3 = 4.82 \times 10^3$$

$$\beta_2 = \mu_4 / \mu_2^2 = 4.74 \times 10^6$$

$$K(\text{criterion}) = \frac{\beta_1 (\beta_2 + 3)^2}{4 (2\beta_2 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)}$$

$K = 150.7$ - Greater than unity; indicating a Type I (Beta) distribution is determined as the system distribution in reliability. The parameters of this system distribution are found from the Mellin transform:

$$F_m(s) = \frac{\Gamma(a+b) \Gamma(a+s-1)}{\Gamma(a+b+s-1) \Gamma(a)}$$

where if $s=2$,

$$a = \frac{\mu_1}{1 - \mu_1} \quad b$$

$$\text{and } b = \left[\frac{(u_2^1 - u_1^1)(1 - u_1^1)}{(u_1^1 - u_2^1) u_1^1} \right] \left[1 - \frac{u_2^1 (1 - u_1^1)}{(u_1^1 - u_2^1) u_1^1} \right]^{-1}$$

and

$$a = 151,083.98$$

$$b = 272.64$$

in:

$$f_g(R) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} R^{a-1} (1-R)^{b-1}$$

This function is shown in Figure 2 and listed in Table 1.

APPENDIX C

EXACT

If λ is normally distributed, then the component reliability, $R = \exp(-\lambda t)$ is lognormally distributed.* That is,

$$f(R) = \frac{\exp \left[- \frac{(-\ln R - \mu t)^2}{2 t^2 \sigma^2} \right]}{R t \sigma \sqrt{2\pi}}$$

For a system of four series components, the system distribution is then:

$$f_g(R) = \frac{\exp \left[- \frac{(\ln R + \sum_i \mu_i t)^2}{2 t^2 \sum_i \sigma_i^2} \right]}{R \sqrt{2\pi} \sqrt{\sum_i \sigma_i^2}}$$

where μ_i, σ_i^2 = mean and variance of i^{th} component.

This distribution has been calculated for the same four components in Appendix B with the results given in Table 1 and Figure 2.

*See previous Progress Report, May - August, 1976

APPENDIX D

User's Guide

Purpose: To compute moments about zero, moments about mean, β_1 , β_2 , Pearson criteria, system reliability distribution function and values of $f(R)$ for various R .

Method of Solution: Approximate method described in main text.

Restrictions:

- maximum number of elements is 999.

Programming Language: FORTRAN IV - WATFIV

Computer: IBM360-67

Inputs

Nomenclature:

- T - time interval (Hrs.)
N - number of elements
UP - mean failure rate (1 Hr.)
S - standard deviation (1 Hr.)
K - number of calculation points to right (or left) of mean.

Input Cards

First Card	T	Columns 1-13	Format D13.6
Second Card	N	Columns 1-3	Format I3
Next N Cards	UP	Columns 1-13	Format D13.6
	S	Columns 14-26	Format D13.6
Last Card	K	Columns 1-2	Format I2

Output

All input quantities, except K, are printed out for verification. In addition, A and B, the exponents of each element's beta distribution, are printed below the mean failure rate and standard deviation.

Calculated Output Consists of:

U(I) Ith moment about zero

UM(I) Ith moment about mean

BETA 1 Square of skewness

BETA 2 Kurtosis

KAPPA Pearson Criteria

P,Q Exponents for overall beta distribution:

$$f(R) = \frac{\Gamma(P+Q)}{\Gamma(P)\Gamma(Q)} R^{P-1} (1-R)^{Q-1}$$

LN(F(R)) = Natural Logarithm of Reliability distribution function.

Comments:

The program is written using double precision variables. In order to preserve accuracy while computing moments about zero, the numerator and denominator for each moment was computed first and then the division was performed, rather than performing the intermediate divisions. This was done in order to minimize the number of divisions, which introduce error.

In order to evaluate the expression:

$$f(R) = \frac{\Gamma(P+Q)}{\Gamma(P)\Gamma(Q)} R^{P-1} (1-R)^{Q-1} \quad (1)$$

one can use a logarithmic expansion formula for the gamma function. The procedure is as follows:

1. Taking the natural logarithm of equation (1) yields:

$$\ln(f(R)) = \ln[\Gamma(P+Q)] - \ln[\Gamma(P)] - \ln[\Gamma(Q)] \\ + [P-1] \ln R + [Q-1] \ln 1-R \quad (2)$$

2. The following expansion for $\Gamma(x)$ can then be used to evaluate $\Gamma(P+Q)$, $\Gamma(P)$ and $\Gamma(Q)$:

$$\ln(\Gamma(x)) = (x-\frac{1}{2}) \ln(x-1) + \frac{1}{2} \ln 2\pi$$

3. This can be substituted into equation (2) to yield:

$$\ln(f(R)) = (R+Q-\frac{1}{2}) \ln(P+Q-1) - (P+Q-1) - (P-\frac{1}{2}) \ln(P-1) + (P-1) - (Q-\frac{1}{2}) \\ \ln(Q-1) + (Q-1) - \frac{1}{2} \ln 2\pi.$$

4. This expression can readily be evaluated to yield $\ln(f(R))$ and $f(R)$ is then simply:

$$f(R) = \exp(\ln f(R)).$$

More details on input/output are shown on page D-4.

7 JUL \$\$\$\$\$\$ STRUMMATER
CFLCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

100-443887-100

THE INPUTS CONSIST OF:

C
C T TIME INTERVAL (HRS) C

C	N	NUMBER OF ELEMENTS (UP TO 9997)	C
C	UP	MEAN FAILURE RATE (/HR)	C

C	UP	MEAN FAILURE RATE (1/YR)	C
C	S	STANDARD DEVIATION	C

C	S	STANDARD DEVIATION	C
C	K	TAKES TEN STANDARD DEVIATIONS, ON BOTH	C

C SIDES OF THE MEAN, AND DIVIDES IT INTO K C

INTERVALS FOR CALCULATING THE RELIABILITY

C INPUT CARDS

C	FIRST CARD	7 (013.6)	C
C	SECOND CARD	8 (113)	C

C SECQNO CARD N (13)
NEXT N CARDS 47 5 12013-51

LAST CARD K (12)

100

3

C	U(1)	1-TH MOMENT ABOUT ZERO	C
C	UM(1)	1-TH MOMENT ABOUT MEAN	C

C	BETA1	SQUARE OF SKEWNESS	C
---	-------	--------------------	---

C	BETA2	KURTOSIS
---	-------	----------

C	DATA	EXPONENTS FOR OVERALL BETA DISTRIBUTION	C
C	P,Q		

C	LN(F(R))	LOGARITHMIC FUNCTION DESCRIBING THE	C
---	----------	-------------------------------------	---

C	KAPPA	PEARSON CRITERIA	C
---	-------	------------------	---

100-443887-100

THE PROGRAM FIRST COMPUTES THE BETA DISTRIBUTION
PARAMETERS FOR EACH ELEMENT. THE MELLIN TRANSFORM

IS THEN USED TO COMPUTE THE FIRST FOUR MOMENTS, FOR

C THE OVERALL DISTRIBUTION, ABOUT ZERO; THESE ARE THEN C

C CONVERTED TO MOMENTS ABOUT THE MEAN. USING ALL THIS INFORMATION THE PROGRAM THEN CALCULATES THE BETA

C INFORMATION, THE PROGRAM THEN CALCULATES THE BETA C
C DISTRIBUTION FUNCTION WHICH DESCRIBES THE OVERALL C

DISTRIBUTION FUNCTION WHICH DESCRIBES THE OVERALL
SYSTEM RELIABILITY.