Parametric Systems Analysis of the Modular Stellarator Reactor (MSR)

Los Alamos
Los Alamos National Laboratory
Los Alamos, New Mexico 87545

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R. L. Miller
R. A. Krakowski
C. G. Bathke

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## ABSTRACT

I. INTRODUCTION 1

II. PARAMETRIC SYSTEMS MODEL 5
   A. Plasma Model 5
   B. Overall Plant Performance 8
   C. First-Wall/Blanket/Shield (FW/B/S) Model 12
   D. Coil-Set Model 13
   E. Summary of Systems Model 16

III. RESULTS 19
   A. General Parametric Results 19
   B. Focused Parametric Results 25

IV. CONCLUSIONS 38

REFERENCES 39
PARAMETRIC SYSTEMS ANALYSIS
OF THE MODULAR STELLARATOR REACTOR (MSR)

by

R. L. Miller, R. A. Krakowski, and C. G. Bathke

ABSTRACT

The close coupling in the stellarator/torsatron/heliotron (S/T/H) between coil design (peak field, current density, forces), magnetics topology (transform, shear, well depth), and plasma performance (equilibrium, stability, transport, beta) complicates the reactor assessment more so than for most magnetic confinement systems. In order to provide an additional degree of resolution of this problem for the Modular Stellarator Reactor (MSR), a parametric systems model has been developed and applied. This model reduces key issues associated with plasma performance, first-wall/blanket/shield (FW/B/S), and coil design to a simple relationship between beta, system geometry, and a number of indicators of overall plant performance. The results of this analysis can then be used to guide more detailed, multidimensional plasma, magnetics, and coil design efforts towards technically and economically viable operating regimes. In general, it is shown that beta values > 0.08 may be needed if the MSR approach is to be substantially competitive with other approaches to magnetic fusion in terms of system power density, mass utilization, and cost for total power output around 4.0 GWe; lower powers will require even higher betas.

I. INTRODUCTION

Recently completed systems studies of the stellarator/torsatron/heliotron (S/T/H) approaches to fusion power\(^1\) have identified a rich and interrelated ensemble of basic physics parameters that potentially can lead to a variety of reactor embodiments. The use of deformed toroidal-field coils to generate a magnetics topology similar to that found in a classical stellarator has added to recently renewed interest in these approaches as power reactors.\(^2\)\(^-\)\(^4\) Table IA summarizes key reactor parameters projected from a range of S/T/H reactor studies; an intercomparison with other leading toroidal fusion concepts is also made in Table IB.
### TABLE IA
**SUMMARY OF STELLARATOR/TORSATRON/HELIOTRON (S/T/H) FUSION REACTOR CONCEPTS**

<table>
<thead>
<tr>
<th></th>
<th>T-15</th>
<th>HELIOTRON-H1</th>
<th>UWTOR-M2</th>
<th>MSR3,4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>z=3</td>
<td>z=2</td>
<td>z=3</td>
<td>z=2</td>
</tr>
<tr>
<td></td>
<td>m=16</td>
<td>m=15</td>
<td>m=6</td>
<td>m=6</td>
</tr>
<tr>
<td>N=20</td>
<td></td>
<td></td>
<td>N=18</td>
<td>N=18</td>
</tr>
</tbody>
</table>

| Plasma radius (m)   | 2.3   | 1.8         | 1.72     | 2.11   |
| Major radius (m)    | 29.2  | 21.0        | 24.1     | 23.24  |
| Plasma volume (m³)  | 3049  | 1343        | 1830     | 2050   |
| Average density (10²⁰/m³) | 1.33 | 1.2     | 1.56     | 1.50   |
| Average temperature (keV) | 7.3  | 13       | ~10      | 8.0    |
| Lawton parameter (10²⁰ s/m³) | 3.0  | --       | 1.7      | 3.7    |
| Average beta        | 0.035 | 0.08      | 0.05     | 0.04   |
| Plasma power density (MW/m³) | 1.4  | --       | 3.91     | 2.34   |
| Magnetic field (T)  | 5.0   | 4.0        | 5.5      | 6.0    |
| Neutron current (MW/m²) | 1.1  | 1.3       | 1.8      | 1.3    |
| Thermal power (MWt) | 4340  | 3600       | 5500     | 4800   |
| Net power (MWe)     | 1400  | 1260       | 1760     | 1530   |
| System power density (MWt/m³) | 0.35 | 0.43      | 0.35     | 0.26   |
| Recirculating power fraction | 0.08 | 0.05     | 0.08     | 0.08   |
| Net plant efficiency (n_TH = 0.35) | 0.32 | 0.33      | 0.32     | 0.32   |

### TABLE IB
**COMPARISON OF THE MODULAR STELLARATOR REACTOR (MSR) WITH OTHER TOROIDAL FUSION REACTOR CONCEPTS**

<table>
<thead>
<tr>
<th>MSR3,4</th>
<th>STARFIRE6</th>
<th>EBTR7</th>
<th>RFPR8</th>
</tr>
</thead>
<tbody>
<tr>
<td>z=2</td>
<td>m=6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N=18</td>
<td>N=12</td>
<td>N=36</td>
<td>N=20</td>
</tr>
</tbody>
</table>

| Plasma radius (m) | 2.11 | 2.4 | 1.0 | 1.2 |
| Major radius (m)  | 23.24| 7.0 | 35.0| 12.7|
| Plasma volume (m³) | 2050 | 781 | 691 | 564 |
| Average density (10²⁰/m³) | 1.5 | 0.8 | 0.95 | 2.00 |
| Average temperature (keV) | 8.0 | 22 | 29 | 15-20 |
| Lawton parameter (10²⁰ s/m³) | 3.7 | 3.0 | 1.7 | 2.0 |
| Average beta | 0.04 | 0.067 | 0.17 | 0.30 |
| Plasma power density (MW/m³) | 2.34 | 4.5 | 4.1 | 4.50 |
| Magnetic field (T) | 6.0 | 5.8 | 5.0/2.25 | 3.0 |
| Neutron current (MW/m²) | 1.3 | 3.6 | 1.4 | 2.7 |
| Thermal power (MWt) | 4800 | 4033 | 4028 | 3000 |
| Net power (MWe) | 1530 | 1200 | 1214 | 750 |
| System power density (MWt/m³) | 0.26 | 0.30 | 0.24 | 0.50 |
| Recirculating power fraction | 0.08 | 0.7 | 0.15 | 0.17 |
| Net plant efficiency (n_TH = 0.35) | 0.32 | 0.30 | 0.30 | 0.25 |
| Unit direct cost ($/kWe) (1980) | 1547 | 1438 | 1737 | 1335 |
| Construction time (years) | 10 | 6 | 5 | 10 |
| COE (mills/kWeh) | 94 | 67 | 72 | 80 |

---

*To which is added indirect charges (typically 23%), interest during construction (typically 25% for a 5-year construction period), or escalation during construction (typically 16% for a 5-year construction period).*
One of the more perplexing uncertainties that inhibits an unambiguous resolution of the "optimal" S/T/H reactor, in general, or the optimum configuration for a Modular Stellarator Reactor (MSR), in particular, is the close coupling of key issues related to physics (beta, stability/equilibrium, transport, magnetics topology) and engineering (first-wall/blanket/shield design, coil design, system power density, design modularity, cost). Unresolved relationships between magnetics (i.e., rotational transform, shear, depth of magnetic well), stability/equilibrium beta limits, and overall plasma performance, particularly, complicates the S/T/H reactor assessment. The intent of the present study is to elucidate by means of an analytic parametric model the effects of key constraints on the viability of the MSR approach.

In its simplest form, a systems approach to fusion reactor design characterizes the total system in terms of the plasma, the first-wall/blanket/shield (FW/B/S), the coil set, and the overall plant performance. Figure 1 illustrates these four essential elements as applied to S/T/H reactors. In addition to introducing key notation, Fig. 1 also identifies a number of interrelated subelements that should be considered by a parametric systems study. A similar approach was used in a previous study, with relationships between beta, plasma magnetics, and coil design being dictated by a quantitative stability/equilibrium model based on diffusion-driven plasma currents. Although self-consistent in the sense that beta and, hence, plasma performance were coupled directly to the coil design, this approach can legitimately be criticized as overly constraining the reactor design on the basis of a simplified and experimentally unverified model for plasma stability/equilibrium. Because the state-of-knowledge in this crucial area is not sufficient to provide a better or more accepted relationship between beta, magnetics, and coil design, an alternative approach would simply treat beta parametrically. This "beta-decoupled" mode of parametric systems analysis is adopted here, with the general formalism and assumptions described in Ref. 4 being retained where possible. The present inability to relate the assumed beta to the actual coil configuration through desired or required rotational transform, shear, and magnetic well depth must ultimately be addressed by the results of ongoing work. For the interim, the present approach is used to bridge the scoping study reported in Ref. 4 to the conceptual engineering design study presently in progress.
CHARACTERIZE PLASMA
- Particle and energy transport
- Stability and equilibrium
- Pressure balance, power density and β
- Ignition condition
- Plasma edge condition (limiter or divertor)

CHARACTERIZE FIRST-WALL/BLANKET/SHIELD
- Neutron wall loading, $I_w$
- Blanket/shield thickness, $\delta_b$
- Blanket power density, $P_{TH}/B = (N + 1/4)(1 + \Delta_b)/(1 + \Delta_b/2r_w)$
- Impurity control scheme
  $(r_s ~ r_w ~ r_c)$

CHARACTERIZE COIL SET
- Geometry
  $(r_c, d/r_c, \delta_c, \delta_b)$ (continuous helix or modular)
- Field ripple
- Peak field at coil, $B_{CM}$
- Conductor current density, $J_c$
- Mechanical design

CHARACTERIZE OVERALL PLANT PERFORMANCE
- Total thermal power, $P_{TH} = (N + 1/4)(2n)^2 I_w r_w R_T$
- Recirculating power fraction ($\epsilon$) and net electric power, $P_E = (1-\epsilon)P_{TH}$
- Engineering power density, $P_{ETh}/V_c = 2(N + 1/4) I_w r_w / (r_c + 4/2)^2$
- Module size, maintenance scheme, accessibility (coil-coverage factor), operational layout
- Mass Utilization, $(M_{BS} + M_c)/P_{TH}$
- Capital and energy costs

Fig. 1. Summary of coil geometry, notation, and essential elements and interrelationships of the MSR parametric systems analysis.
The basic models and assumptions embodied in Fig. 1 and the parametric systems analysis (PSA) are described in Sec. II. Parametric results and comparison between a high-beta and a low-beta MSR design are given in Sec. III. Key conclusions and future directions for the modular-coil approach to S/T/H reactors are presented in Sec. IV.

II. PARAMETRIC SYSTEMS MODEL

A. Plasma Model

The plasma model adopted for this study follows closely that described in Ref. 4, with the exception of not explicitly stating the stability/equilibrium beta limits as functions of aspect ratio, magnetics, coil, and other parameters. For present purposes, beta is specified parametrically, thereby decoupling the plasma performance from specific magnetics requirements. As assumed in Ref. 4, the particle/energy confinement time, $\tau_E(s)$, is described by empirical Alcator transport.

$$\tau_E = 3.0(10)^{-21} \langle n \rangle r_p^2,$$

where $\langle n \rangle (m^{-3})$ is the average ion density, $r_p(m)$ is an average plasma radius, and mks units are used. This expression for $\tau_E$ is a factor of $\sim 3/5$ less than that usually assumed for tokamak reactor scaling.\textsuperscript{6} If $\langle \beta \rangle \equiv \langle p \rangle/(B_o^2/2u_o)$ is the average plasma beta, where $\langle p \rangle$ is the average plasma pressure and $B_o$ designates the average confining toroidal field strength measured at the magnetic axis, plasma pressure balance dictates

$$\langle n \rangle \langle T \rangle f_{Z1} = 1.24(10)^{21} (I_{1/2}^2/I_1) \langle \beta \rangle B_o^2,$$

where $\langle T \rangle (keV)$ is the average ion and electron temperature, assumed here to be equal. The profile factors, $I_y$, are given by\textsuperscript{4}
where \( v \) is the plasma pressure profile index. In this formalism, the temperature and density profiles are assumed equal. Application of Eqs. (1) and (2) to a global energy balance that requires alpha-particle heating to equal radiation and transport losses gives the following ignition condition.

\[
\frac{[\langle B \rangle B^2_{\Omega p}]}{1.35(10)^{24}} = \frac{f_{21}^2 <T>^3}{f_{32} <\phi> (I_2/I_1) - 5.13 f_{21} Z_{\text{eff}} [I_{5/4}(I_{3/2}/I_1^2)] <T>^{1/2}}
\]

In Eqs. (2) and (4), \( f_{21} = [1 + (Z_{\text{eff}} - 1)/2Z_{\text{eff}}]^2 \), and \( f_{32} = f_{21}^{1/2} (1 + \frac{1}{2} T_a <T>) \).

The ratio of alpha-particle temperature, \( T_a \), to plasma temperature is taken to be \( \sim 10 \). The alpha-particle energy trapping fraction, \( f_a \), has been estimated\(^{10} \) to be a function of the plasma aspect ratio, \( A = R_T/r_p \), where \( R_T \) is the plasma major radius. The energy trapping fraction used in Eq. (4) is approximated here by the following expression.

\[
f_a = 1 - e^{-A/5.22}
\]

In a sense, the parameter \( \langle B \rangle B^2_{\Omega p} \) represents an "engineering Lawson criterion" for ignition and is based on pressure balance and a specific form (i.e., Alcator scaling) for plasma transport.

The ignition condition, Eq. (4), is evaluated in Fig. 2 as a function of \( <T> \) for a range of aspect ratios (or \( f_a \) values), pressure profiles, and \( Z_{\text{EFF}} \) values. In addition to a flat pressure profile \( (\nu = \infty) \), a cubic pressure profile \( (\nu = 3) \) corresponding to the Ref. 4 design point \((f_a = 0.88, A = 11)\) is shown. Specific points for the reactor designs given in Table I are also shown. It should be noted that the EBTR point in Fig. 2 has been included only for comparison, the neo-classical, bumpy-torus transport \((<n>T_E = A^2 T^{3/2})\) having no relationship to the empirical transport \((<n>T_E = (<n>r_p)^2)\) used for most of the
Fig. 2. Combined ignition and pressure balance conditions as a function of average plasma temperature for a $\nu = 3$ (dashed line) and a $\nu + \infty$ (solid line) pressure profile for the $Z_{\text{EFF}}$ values indicated. Typical points used for the range of reactor design points given in Table I are also shown, reflecting a certain degree of optimism for the T-1, Heliotron-H, and UWTOR-M designs, relative to the MSR design. The EBTR point is included only for completeness and to compare the implication of bumpy-torus neoclassical theory with empirical Alcator scaling. The STARFIRE point corresponds to a fairly high-$Z_{\text{EFF}}$, driven plasma, whereas all other designs have assumed ignition.

reactor designs cited in Fig. 2. It is seen that minimum values of $\langle B \rangle B_{0r\rho}^2$ occur in the plasma temperature range 7-10 keV, this ignition parameter equaling 2.0-3.0 $T^2 m$ for a wide range of aspect ratios, $Z_{\text{eff}}$ values, and assumed pressure profiles. A nominal value of $\langle B \rangle B_{0r\rho}^2 = 3.0 T^2 m$ is selected for the remainder of this study and depends primarily on the assumed transport (i.e.,
the value of \( \tau_E/\langle n \rangle r_p^2 \). It is noted that optimism in the assumed ignition condition for any given design is reflected by lower values of \( \langle \beta \rangle B_0^2 r_p \).

The majority of the physics subelements given in Fig. 1 are either treated parametrically (i.e., \( \langle \beta \rangle \)) or are left unspecified. An alternative to a purely parametric treatment of \( \langle \beta \rangle \) would vary the rotational transform, \( \tau \), with \( \langle \beta \rangle \), for example, being given by a relationship of the following form.\(^{11}\)

\[
\langle \beta \rangle = \tau^2 / A . \tag{6}
\]

Such an assumption, however, would lead the present analysis toward the more restrictive beta-coupled approach used in Ref. 4, and for this reason is not emphasized here. A few results are given, however, where \( \tau \) instead of \( \langle \beta \rangle \) is specified parametrically, and Eq. (6) is used to determine \( \langle \beta \rangle \) for a given reactor geometry. It is noted that \( \tau \) in Eq. (6) corresponds to the total rotational transform (i.e., vacuum plus finite beta components).

**B. Overall Plant Performance**

Key subelements that characterize the overall plant performance are also listed in Fig. 1. The total thermal power, \( P_{TH}(W) \), generated by a reactor operating with a blanket neutron-energy multiplication, \( M_N \), is given by

\[
P_{TH} = 1.11(10)^{-11} (I_2/f_{Z2}^2 I_{1/2}^4 \langle n \rangle)^2 \langle \sigma_v \rangle (M_N + 1/4) R_T r_p^2 .
\]

\[
= 1.71(10)^{31} (I_2/f_{Z2}^2 I_{1/2}^2) (\langle \sigma_v \rangle / \langle T \rangle^2) (M_N + 1/4) (\langle \beta \rangle B_0^2 r_p)^2 R_T
\]

\[
= 1.65(10)^7 (M_N + 1/4) (\langle \beta \rangle B_0^2 r_p)^2 R_T . \tag{7}
\]

In developing Eq. (7), pressure balance [Eq. (2)] and the approximation that \( \langle \sigma_v \rangle / \langle T \rangle^2 \approx 1.1(10)^{-24} \text{ m}^3/\text{s keV}^2 \), which is accurate to within 10% in the temperature range \( 8 < T < 20 \text{ keV} \), could be used. To calibrate the present approach more carefully against past work,\(^4\) however, the actual value at 8 keV \( \langle \sigma_v \rangle / \langle T \rangle^2 = 9.64(10)^{-25} \text{ m}^3/\text{s keV}^2 \) was used. The latter expression for \( P_{TH} \) sets \( I_2/f_{Z2}^2 I_{1/2}^2 \) to unity, which corresponds to a flat pressure profile with \( Z_{EFF} = 1.0 \) or a cubic profile with \( Z_{EFF} = 1.05 \); either assumption is consistent with

8
\( \langle \beta \rangle B_{o}^{2}r_{p} = 3.0 \ T^{2} \ m \) at 8 keV (Fig. 2). Taking a typical value of \( M_{N} = 1.1 \) and the ignition condition as \( \langle \beta \rangle B_{o}^{2}r_{p} = 3.0 \ T^{2} \ m \), Eq. (7) reduces to the following expression.

\[
P_{TH}(MWt) = 200.5 \ R_{T} \ . \tag{8}
\]

This highly condensed relationship represents the first of three key "working equations" used in this study to cast the overall plant characterization in terms of geometry.

The recirculating power fraction, \( \varepsilon \), for ignited operation is expected to be below 0.1, which for a thermal conversion efficiency of \( \eta_{TH} = 0.35 \) gives a net plant efficiency of \( (1-\varepsilon)\eta_{TH} = 0.32 \). For the purposes of this parametric systems analysis, however, only the total thermal power, \( P_{TH} \), is monitored. The net electrical output, \( P_{E}(MWe) \), is given by \( (1-\varepsilon)\eta_{TH}P_{TH} = P_{TH}/3 \).

Additional parameters can be identified as "figures-of-merit" in evaluating overall power-plant performance without performing a detailed cost analysis. If \( V_{C} \) is defined as the total engineered toroidal volume enclosed by and including the magnet-coil annulus, the engineering or system power density, \( P_{TH}/V_{C}(MWt/m^{3}) \), can serve as one such figure-of-merit.

\[
P_{TH}/V_{C} = \frac{P_{TH}}{2\pi^{2}R_{T}(r_{c} + \Delta b/2)^{2}}
= \frac{7.51(M_{N} + 1/4)}{(r_{p}/x + \Delta b + \delta/2)^{2}} \tag{9}
\]

In Eq. (9) \( r_{c} = r_{p}/x + \Delta b + \delta/2 \) is the coil current-center radius, \( \Delta b \) is the FW/B/S thickness, \( r_{w} = r_{p}/x \) is a nominal first-wall radius, and \( \delta \) is the overall coil thickness, including a square cross-section conductor with thickness, \( \delta_{c} \), and a surrounding structural/cryogenic shell.

A more direct economic measure of overall plant performance is the total fusion-power-core mass, taken here to be the combined mass of FW/B/S, \( M_{B/S} \), and coils, \( M_{C} \), divided by the total thermal power, \( P_{TH} \). A strong correlation
between the unit direct costs, UDC($/kWe), given in Table IB and this "mass-utilization" (i.e., inverse specific power) factor, \((M_{B/S} + M_c)/P_{TH}\), is identified in Fig. 3 for a range of fusion reactor designs as well as for commercial Light-Water (fission) Reactors (LWRs). Given an average FW/B/S density, \(\rho_{B/S}\) (typically, 5-6 tonne/m\(^3\)), and an average coil density, \(\rho_c\) (typically 6-8 tonne/m\(^3\)), the mass-utilization factor can be expressed as follows, where \(M = M_{B/S} + M_c\).

\[
\frac{M}{P_{TH}} = \left(\frac{\pi}{10}\right)^2 \left[\rho_{B/S} \Delta b^2 (1 + 2 \delta w/\Delta b) + \rho_c (N/\pi) \delta^2 r_c g_c/R_t \right].
\]  

Equation (8) has been used, and \(N\) is the total number of modular coils. The coil form factor, \(g_c\), is the ratio of the current-center circumference of a deformed coil, with toroidal deformation (Fig. 1) given by

\[
y = d \sin \angle 0,
\]

to the circumference at the current center of an undeformed toroidal coil (i.e., \(2\pi r_c\)). If \(k\) is defined by the expression

\[
k^2 = \frac{(\angle d)^2}{(\angle d)^2 + r_c^2},
\]

then \(g_c\) is given by

\[
g_c = \frac{(2/\pi)E(k, \pi/2)}{(1 - k^2)^{1/2}} \\
= 1 + 0.21 \frac{\angle d}{r_c},
\]

\[\text{(13)}\]
where \( E(k, \pi/2) \) is the complete elliptic integral of the second kind and the approximation to \( \bar{g}_c \) is accurate to within a few percent over the practical range of \( \ell d/r_c \) values. It is noted from Eqs. (10) and (13) that MSRs with higher polarity (i.e., \( \ell \) values) will operate with poorer (higher) mass utilization and greater cost. Equation (10) is used to monitor an important aspect of system

![Diagram](image_url)

**Fig. 3.** Correlation of mass utilization factor, \( M/P_{TH} \) (tonne/MWt), with unit direct cost (UDC). The UDC values given here do not include indirect costs (typically 23%), interest during construction (IDC), or escalation during construction (EDC). Reactor design points shown are: BWR/PWR, Ref. 12; Compact Reversed-Field Pinch Reactor (CRFPR), Ref. 13; Reference Reversed-Field Pinch Reactor, RFPR, Ref. 8, 14; STARFIRE tokamak reactor, Ref. 6; NUWMAK tokamak reactor, Ref. 16; Modular Stellarator Reactor (MSR), Ref. 3, 4; Elmo Bumpy Torus Reactor (EBTR), Ref. 7. Costs have been adjusted to 1980 dollars.
performance. Typically, most of the conventional fusion reactor designs summarized on Table IB and Fig. 3 predict mass utilizations in the 5-10 tonne/MWt range, compared to values of 0.2-0.4 tonne/MWt for LWRs. For the purposes of this study a goal of \( M/\dot{P}_{\text{TH}} \leq 5 \) tonne/MWt is established for the MSR, in order to give a MSR performance that is comparable to the best tokamak projections (Fig. 3). Detailed design and cost estimates, however, must be performed in order to substantiate the desirability and magnitude of this constraint.

C. First-Wall/Blanket/Shield (FW/B/S) Model

A simplified FW/B/S model is used for the purposes of this parametric systems analysis. Specifically, the blanket/shield thickness for systems using superconducting coils is specified to have a nominal thickness \( \Delta b \approx 1.3 \) m, and the blanket energy multiplication is specified to be \( \text{M}_N \approx 1.1 \). These values of \( \Delta b \) and \( \text{M}_N \) are considered typical for most DT fusion systems. \(^4\) Introducing the 14.1-MeV neutron first-wall loading, \( I_w(MW/m^2) \), leads to a second "working equation" used in this model.

\[
I_w(r_p/x) = \frac{\dot{P}_{\text{TH}}/(\text{M}_N + 1/4)}{(2\pi)^2 R_T} 
\]

\[= 3.76\quad (14)\]

where the first expression is simply a definition and the last expression results from use of Eq. (8). Although the plasma filling fraction, \( x = r_p/r_w \), can be estimated from the magnetics, \( x \) is optimistically fixed to 0.7 for the purposes of this study. The scaling of \( x \) considered in Ref. 4 gives \( x = 0.66(m/A)^{1/3} \), where \( m \) is the number of toroidal field periods and \( A = R_T/r_p \) is the plasma aspect ratio for elliptic plasma cross sections with \( \ell = 2 \). Indeed, this scaling could be a handicap for systems with higher beta values and/or low values of \( m \) (i.e., \( m \leq 5 \)) with adequately high values of rotational transform, \( \tau \). This scaling applies to plasmas filling the last closed flux surface \( (r_s = r_w) \) consistent with divertor impurity control. Larger values of \( x \) by a factor \( (1-\Delta b/r_c)^{-2/3} \) are expected if a limiter is used to define the plasma.
boundary \((r_s = r_c)\) as noted in Ref. 4. In addition, use of noncircular first-wall configurations, which could place the first wall near the plasma surface, and which are molded to whatever shape the plasma might have, could reduce the adverse impact of the present modelistic assumption of circular first-wall cross section; effective \(s\) values as high as 0.7 could result.

It is interesting to note that by specifying the ignition condition, transport scaling, and reactor geometry \((r_p, R_T)\), both \(P_{TH}\) and \(I_w\) are determined. Because the ignition condition combines with pressure balance to give \(\langle \beta \rangle B_0^2 r_p = 3.0 \ T^2 \ m \) [Eq. (4)] for a plasma described by Alcator transport, and because \(\langle \beta \rangle\) is either specified parametrically or related to \(\tau\) through geometry [e.g., Eq. (6)], the ignition condition can also be expressed conveniently in a geometric "\(r_p - R_T\) space" once the on-axis field, \(B_0\), is related to a limiting coil condition (field, current density, or stress limits) through an appropriate geometric relationship. This missing link, as described in terms of the coil-set model in the next section, completes the parametric systems model, which becomes dependent only on geometry once a specific ignition condition and \(\langle \beta \rangle\) are specified. Although \(\langle \beta \rangle\) is treated parametrically in this study, in principle by relating \(\langle \beta \rangle\) to \(\tau, \tau'\), or \(\psi\), through a given coil (magnetics) configuration, \(\langle \beta \rangle\) also becomes a function of geometry. Reactor design studies of S/T/H configurations would benefit immensely by the availability of such a relationship, particularly if the relationship shared more acceptance than the one originally evaluated in the course of the Ref. 4 study.

D. Coil-Set Model

The "beta-decoupled" parametric systems model described to this point can in principle be applied to any ignited toroidal system that is characterized by Alcator transport scaling. Features that are unique to the modular S/T/H configuration are injected into this analysis primarily through the models and constraints applied to the magnetics topology \((\langle \beta \rangle, x)\) and coil design \((\delta, \delta_c, d, r_c, R_T)\). The third "working equation" that completes this parametric systems model uses the pressure-balance/transport/ignition relationship [i.e., Eq. (4), \(T = 8 \ \text{keV}, \ \langle \beta \rangle B_0^2 r_p = 3.0 \ T^2 \ m\) and relates the on-axis toroidal magnetic field, \(B_0\), to an estimate of the peak (inboard) field at the coil winding, \(B_{CM}\), through the following approximate expression (characterizing only the toroidal field).
Figure 1 depicts the geometry used, and $B_{CM}$ is evaluated at the inboard side of the torus at the conductor surface. By specifying $B_{CM}$, the on-axis field, $B_0$, can be eliminated from the ignition condition, yielding a set of relationships that depend solely on geometry, once $\langle \beta \rangle$ is specified.

Equation (15) along with the ignition condition, $\langle \beta \rangle B_0^2 r_p = 3.0 \, T^2 \, m$, provides a means by which a trajectory in $r_p-R_T$ space can be traced if $\delta_c$, $N$, $B_{CM}$, and $\langle \beta \rangle$ [or $\tau$, Eq. (6)] are specified. In the spirit of a "best case" parametric systems study, variables are chosen to exhibit physical upper limits. Although $\langle \beta \rangle$ and $B_{CM}$ can be so specified, $N$ and $\delta_c$ are not as conveniently posed. For this reason, the (maximum) allowable current density in the coil conductor, $j_c$, the linear inboard coil "filling fraction", $f_c$ (Fig. 1), and the coil interference parameter, $f_c^*$, are introduced. In this way, variations of $N$ and $\delta_c$ can be recast in terms of variation in $j_c$, $f_c$, and $f_c^*$. The following expression is used to relate $\delta_c$ and $N$ to $j_c$.

$$\delta_c^2 = \frac{2 \pi R_T B_0}{\mu_0 N j_c \delta_I}$$

where the current form factor, $\delta_I$, is the ratio of effective poloidal current to total current in a coil with conductor cross-sectional area, $\delta_c^2$, and (maximum) lateral distortion $d$. For a modular coil described by Eq. (11) and using $k$ as defined in Eq. (12), the current form factor is given by

$$\delta_I = \frac{2/\pi}{(1 - k^2)^{1/2}} K(k, \pi/2)$$

where $K(k, \pi/2)$ is the complete elliptic integral of the first kind. In effect, $\delta_I \leq 1$ gives the increased coil cross section needed for a limiting current.
density to provide for a given on-axis toroidal field. The actual rotational transform produced for a given winding law [i.e., Eq. (11)], distortion, \(d/r_c\), and number of poloidal-field periods, \(\lambda\), must be determined by a three-dimensional vacuum magnetics calculation. Nevertheless, for the purposes of this model incorporation of the stellarator-like character into the otherwise purely toroidal-field coil model through \(g_1\) is adequate.

The linear inboard filling fraction is given by

\[
f_c = \frac{N\delta}{2\pi(R_T - r_c + \delta/2)} ,
\]

where \(\delta > \delta_c\) is the radial thickness of the coil, \(\delta_c\) being the conductor thickness. The space required around the inboard torus circumference for a coil with toroidal distortion, \(d/r_c\), can be introduced into the parametric systems analysis through the parameter \(f_c\). Separate, more detailed magnetics calculations again relate \(f_c\) (i.e., \(d/r_c\), \(R_T\), etc.) to the desired magnetics parameters (i.e., \(\tau\), \(d\tau/dr\), \(V^n\), etc.), which in turn must be related to \(\langle\beta\rangle\) by a yet-to-be-specified stability/equilibrium condition. Experience derived from detailed magnetics models indicates that \(f_c\) values much above \(\sim 0.3\) will not be allowed if rotational transforms above \(\sim 0.7\) are desired from realistically distorted (i.e., \(d/r_c \sim 0.5\)) coils operating with supportable forces.

An approximate coil interference constraint can be generated that specifies the condition, \(f_c^* = 1\), where a coil set with distortion \(d/r_c\) will require intersecting coil envelopes, as indicated by the dashed lines in Fig. 1. This condition is given by the following expression:

\[
f_c^* = \frac{\delta + 2d}{2(R_T - r - \delta/2) \tan(\pi/N)} \leq 1
\]

\[
= [\frac{\pi/N}{\tan(\pi/N)}] (1 + 2d/\delta) f_c
\]

where Eq. (18) is used to express the basic noninterference constraint, \(f_c \leq 1\), in terms of the linear, inboard filling fraction, \(f_c\). It is noted that if \(f_c^*\) exceeds unity, then a given coil may not be removed from the torus by a simple
horizontal (outward-radial) translation. Generally, for \( d/r_c \leq 0.5 \), \( f_c^* \) as high as \( \sim 1.2 \) may be allowed before in-place coils physically interfere; this situation, however, will require more complex maneuvering in order to extricate a given coil from the torus.

Equation (10) can be evaluated to give the mass utilization, \( M/P_{TH} \), for the \( f_c^* = 1 \) condition; this condition represents the best mass usage and least accessibility, while still requiring only simple horizontal (outward-radial) coil movement for replacement. A case where actual coil interference is expected to occur (\( f_c^* \leq 1.2 \)) is also examined. Typically, it is expected\(^b\) that \( d/r_c \geq 0.4 \) will be required to achieve high rotational transforms (\( \tau \geq 1.0 \)) in a modular-coil system. In any event, specifying either \( f_c^* \) or \( f_c \) will give an added relationship that supplants the need to specify either \( N \) or \( \delta \). In the spirit of the "best case" parameter search, \( f_c^* = 1 \) is used in Eq. (18) to relate \( N \) and \( \delta \) to the \( r_p-R_T \) geometry space.

E. Summary of Systems Model

Figure 4 gives a logic diagram for the MSR parametric systems model. Three working equations have been evolved that relate three systems quantities (\( P_{TH}; L_w \), ignition condition) solely in terms of geometric quantities \( (r_p, r_w, \Delta b, r_c, d, R_T, \delta, \delta_c, l, f_c^*) \) and a minimum number of physics and physical constraints \( \langle \beta \rangle, \langle \beta \rangle, B_{CM}^2, \langle \beta \rangle \). These quantities and constraints can be expressed as curves or surfaces in \( r_p-R_T \) space and are summarized below.

SYSTEMS QUANTITIES:

- Total thermal power, \( P_{TH} \) [Eqs. (7) or (8)].
- Neutron wall loading, \( L_w \) [Eq. (14)].
- Pressure-balance/transport/ignition, \( \langle \beta \rangle B_{CM}^2 r_p \) [Eqs. (4) and (15) with \( \langle \beta \rangle B_{CM}^2 r_p \leq 3.0 \ T^2 \ m \)].

PHYSICS AND PHYSICAL CONSTRAINTS (parametrically varied):

- Average plasma beta, \( \langle \beta \rangle \) (0.04-0.12).
- Coil filling fraction, \( f_c^* \) (> 0.3-0.4), \( f_c^* \leq 1 \) (coil envelope interference) for a given \( d/r_c \), or \( f_c = 1.2 \) (coil interference).
A PARAMETRIC APPROACH TO MFE REACTOR CHARACTERIZATION

CHARACTERIZE PLASMA

PRESSURE BALANCE

GEOMETRY

CHARACTERIZE FW/B/S

CHARACTERIZE COILS

IMPURITY CONTROL

CHARACTERIZE OVERALL PLANT PERFORMANCE

COST FACTOR

3D MAGNETIC MODEL

Fig. 4. Logic diagram for parametric systems analysis model.
• System power density [Eq. (9)] and mass utilization [Eq. (10)].

Quantities that are generally held fixed throughout this analysis are summarized below.

• Peak field at coil conductor, $B_{CM}$ (12 T).

• Conductor current density, $j_c$ (usually ≈ 20 MA/m²), equal to the overall coil current density, $j(= 13$ MA/m²), when $\delta_c = \delta$.

• Blanket/shield thickness, $\Delta b = 1.3$ m.

• Blanket neutron energy multiplication, $M_N = 1.1$.

• Aleator transport scaling, $\tau_e/\langle n > r_p^2 = 3(10)^{-21}$ s m.

• Ignition condition, $\langle B \rangle r_p = 3.0$ T² m at $\langle T \rangle = 8$ keV.

• Plasma-to-wall radius ratio, $x = r_p/r_w = 0.7$.

• Average mass densities of FW/B/S and coils, $\rho_{B/S} = 5.5$ tonne/m³ and $\rho_C = 7.0$ tonne/m³.

The results given in the following section define ignited, DT reactors in terms of a range of $\langle B \rangle$ and $d/r_c(\gamma_c^* = 1)$ values for otherwise optimally (maximum) selected parameters (i.e., $j_c$, $j$, $B_{CM}$, minimum ignition conditions, maximum $x = r_p/r_w$, tight coil packing $\gamma_c^* = 1$, etc.). In a sense, these results depict the "best" reactor performance for the assumed transport. It remains for more detailed support computation to relate the desired (prescribed) parameters [$\langle B \rangle$ and $d/r_c(\gamma_c^* = 1)$] to required magnetics ($\tau$, $d\tau/dr$, $V''$, etc.), coil ($d/r_c$, $R_T$, forces, etc.), and stability/equilibrium (i.e., $\langle B \rangle$ versus $\tau$, $d\tau/dr$, $V''$, etc.) parameters and/or constraints. In a sense, these results dictate the minimum range of $\langle B \rangle$ or the magnitude of a $\langle B \rangle = \tau^2/A$ scaling needed to achieve the optimally specified performance. Furthermore, the continual monitoring of the mass utilization, $M/P_{TH}$, should give a fairly accurate indication of expected UDCs on the basis of the correlation shown in Fig. 3. It is recognized that the best-case reactors presented here can in fact be made "better" for a given $\langle B \rangle$ primary by reducing the ignition value for $\langle B \rangle B_{OTP}^2$ [Fig. 2], although the value of 3 T² m was shown to encompass a range of assumed plasma conditions and is considered "typical".
III. RESULTS

A. General Parametric Results

All parametric results are expressed as curves in $r_p$-$R_T$ space. As indicated by Eq. (8), the total power, $P_{TH}$, is related to the major radius by a scaling parameter ($P_{TH}/R_T = 200 \text{ MWt/m}$), and the neutron wall loading is related to the minor radius by Eq. (14). Figure 5 shows the ignition curves in $r_p$-$R_T$ space for $d/r_c = 0.4$ and a range of $\langle \beta \rangle$ values. For each value of $\langle \beta \rangle$, the constraints that $x = 0.7$, $B_{CM} = 12 \text{ T}$, $j_c = j = 13 \text{ MA/m}^2$, $\delta = \delta_c$, etc. are enforced. Generally, increasing $R_T$ for a toroidal array of coils will allow

![Sample ignition curves](image.png)

Fig. 5. Sample ignition curves for the $\langle \beta \rangle = \text{ constant and } \langle \beta \rangle = r^2/A$ cases for the $d/r_c = 0.4$ case.
higher on-axis fields, $B_0$, for a given limit imposed on the peak coil field, $B_{CM}$. The ignition condition, $\langle \beta \rangle B_0^2 r_p = \text{constant}$, then allows $r_p$ to decrease with increasing $R_T$. Also, as expected from Eq. (9), increasing $\langle \beta \rangle$ allows higher system power density for a given total power. Figure 5 also gives the ignition curves for the case where $\langle \beta \rangle = \pi^2/\lambda$ and a range of fixed values of rotational transform. In this case the behavior of these system-constrained ignition curves is somewhat more complex. The ignition condition now requires $\langle \beta \rangle B_0^2 r_p = (\pi B_0 r_p)^2/R_T$ to be constant. As before, increasing $R_T$ generally allows higher on-axis fields for a given field constraint at the coil, $B_{CM}$, and $r_p$ decreases initially as $R_T$ is increased. A point is reached, however, where further increases in $R_T$ must require $r_p$ to increase again in order to maintain the ignition condition. The minimum shown in Fig. 5 for the $\langle \beta \rangle = \pi^2/\lambda$ curves results from this interplay between $R_T$, $r_p$, and $B_0$ for constant $\langle \beta \rangle B_0^2 r_p$. For both the $\langle \beta \rangle = \text{constant}$ and $\langle \beta \rangle = \pi^2/\lambda$ ignition curves displayed in Fig. 5, the dependence of $f[a]$ on $A$, and hence that of $\langle \beta \rangle B_0^2 r_p$ on $A$, has been ignored (Fig. 2). A more consistent application of Eqs. (4) and (5) would give curves that rise more rapidly at lower values of $R_T$ ($A \lesssim 11$) and fall more rapidly at higher values of $R_T$ ($A \gtrsim 11$).

The ignition curves shown in Fig. 5 depend on $d/r_c$. Generally, decreasing $d/r_c$ will allow higher system power densities, as measured by decreased $r_p$ or increased $I_w (\text{MW/m}^2)$ for the same total power. Decreasing $d/r_c$ simply allows a higher on-axis field for a given $R_T$ (and $P_{TH}$), and the ignition condition for a given $\langle \beta \rangle$ allows $r_p$ to decrease and $I_w$ [Eq. (14)] to increase. Decreasing $d/r_c$, however, will lead to reduced rotational transform, an effect that ultimately must be reconciled with the specified value of $\langle \beta \rangle$ through detailed magnetics and stability/equilibrium computations.

Figures 6 depict three sets of ignition curves in $r_p-R_T$ space for $d/r_c = 0.3$, 0.4, and 0.5. In addition to the $I_w$ and $P_{TH}$ values, lines of constant number of coils, $N$, and mass utilization, $M/P_{TH}$, are shown. Generally, as $d/r_c$ is increased from 0.3 to 0.5 in Fig. 6A through 6C, the values of $r_p$ required for ignition, using either beta-scaling relationship, shift upward. The interpretation of this behavior is that for a fixed $r_p-R_T$ geometric configuration, as $d/r_c$ increases and the on-axis field strength decreases relative to the fixed peak field at the coil as a result of the Eq. (17) correction, the required value of $\langle \beta \rangle$ needed to satisfy the ignition condition must increase. The constant-$N$ curves, which are shown in Figs. 6,
simultaneously shift to the right as fewer coils fit into a fixed $r_p - R_T$ coil array as $d/r_c$ increases. A slight shift downward in the lines of constant $M/P_{TH}$, which are also shown in Figs. 6, also occurs as $d/r_c$ increases. This behavior is a result of the increase in coil mass required to retrieve a given $B_0$ because of the Eq. (17) correction for a given combination of $r_p$ and $\langle \beta \rangle$ satisfying the ignition condition. Within the limits of this model, this adverse effect is expected to be greater for $\ell = 3$ than for the $\ell = 2$ case examined in Figs. 6. Systems with mass utilization below $\sim 5$ tonne/MWt will require $\langle \beta \rangle$ above $\sim 0.06$ if $d/r_c$ can be held at or below 0.3, or $\langle \beta \rangle$ in excess of $\sim 0.08$ will be needed if the more likely values of $d/r_c$ in the 0.4-0.5 range prove necessary to achieve the $\tau$ values suggested for stable, higher-beta operation.
Fig. 6B. Ignition curves for the \( d/r_c = 0.4 \) case, superposed onto a grid of constant \( N \) and \( M/P_{TH} \) lines with other fixed parameters as given in Fig. 5.

Figures 7 give a distillation of key results from the preceding Figs. 5 and 6, for a low-beta (\( \langle \beta \rangle = 0.04 \), Fig. 7A) case and a high-beta (\( \langle \beta \rangle = 0.08 \) Fig. 7B) case. Again, if \( d/r_c \geq 0.4 \), then \( \langle \beta \rangle \geq 0.08 \) will be required if \( M/P_{TH} \) is to be maintained at or below 5 tonne/MWt. If the \( \langle \beta \rangle = t^2/A \) scaling is used, then \( t \) in excess of 1.0 would be required unless \( d/r_c = 0.3 \). These requirements for \( \langle \beta \rangle \), \( t \), or \( d/r_c \) are significantly relaxed if higher values of mass utilization can be tolerated on the basis of projected cost. Figures 7C-D specifically use the approximate UDC scaling derived from Fig. 3 [i.e., \( c_D = 850 + 255(M/P_{TH})^{1/2} \)]. It is emphasized that small differences in these base UDCs are magnified by time-related costs (i.e., IDC and EDC), which in turn are related to plant complexity and total mass through the construction time. None of these effects are included in the UDCs given in Figs. 7C and 7D.
Fig. 6C. Ignition curves for the \( d/r_c = 0.5 \) case, superposed onto a grid of constant \( N \) and \( M/P_{TH} \) lines with other fixed parameters as given in Fig. 5.

Additionally, optimism in the fusion reactor cost data base (i.e., nominal fusion-power-core cost of \(~ 24 \$ / \text{kg}, \) Fig. 3) may give way to higher unit costs for fabricated material, in which case the cost sensitivity MSR and other fusion reactors to the mass utilization figure-of-merit will become even more pronounced.

Given economic constraints that limit the allowable mass utilization, however, the composite results given in Fig. 7 must be viewed as limiting in terms of the optimism associated with the specified values of \( x(0.7), B_{CM}(12 \ T), j(13 \text{ MA/m}^2), \Delta b(1.3 \text{ m}), \) and \( \lambda(2) \). It is difficult to envisage refinements in the model used here that would give more promising results while maintaining or even decreasing these fixed constraints.
In Fig. 8, \( f_c^* \) is set equal to 1.2 for the \( d/r_c = 0.4 \) case with other parameters being held consistent with the Fig. 5–6 results. As noted previously, this \( f_c^* = 1.2 \) case corresponds to a tight-packing of the modular-coil set, which would be expected to reduce the toroidal-field ripple but could complicate the maintenance/access question. Compared with Fig. 6B, the ignition curves shift upward, since \( f_c^* > 1 \) is consistent with greater-coil distortion, all other parameters/constraints being equal. The constant-N curves, however, shift to the left, allowing more coils to be accommodated in the same \( r_p-R_T \) configuration. The curves of constant \( M/P_{TH} \) shift upward, allowing equivalent or better (lower) mass utilization in a more tightly packed configuration with somewhat lower \( \langle \beta \rangle \) for a given \( r_p-R_T \) configuration.
Fig. 7B. Synthesis of key results from Figs. 5 and 6 for $\langle \beta \rangle = 0.08$ and various values of $d/r_c$ emphasizing mode/magnitude tradeoffs in achieving systems at lower mass utilization.

B. Focused Parametric Results

In order to focus onto the benefits of higher-beta performance, the results of Figs. 6A through 6C are recalled. For a nominal total power output of $P_{TH} = 4.0$ GWt, corresponding to $R_T = 20$ m, the mass utilization figure-of-merit, $M/P_{TH}$, plotted in Fig. 9, is a function of $\langle \beta \rangle$ for various values of the modular-coil distortion parameter, $d/r_c$. For low values of $\langle \beta \rangle$, $M/P_{TH}$ (and cost) can become unattractively large. As $\langle \beta \rangle$ increases, however, $M/P_{TH}$ decreases to an asymptotic value near $\sim 4-5$ tonne/MWt. The incremental improvement in $M/P_{TH}$ as $\langle \beta \rangle$ exceeds $\sim 0.08$ is small. Systems with lower values of $d/r_c$ are capable of achieving the required on-axis field with lower values of $M/P_{TH}$ because less current is used to generate poloidal or radial fields (i.e.,
Fig. 7C. Same as Fig. 7A except the lines of constant mass utilization have been replaced with lines of constant UDC obtained from Fig. 3.

$g_T$ in Eq. (16) approaches unity). As will be seen below, however, these cases with lower $d/r_c$ may not provide the levels of rotational transform expected to be required by the assumed beta values. For the specific case of $d/r_c = 0.4$, which is thought to be adequate from a magnetics viewpoint, unit direct costs are plotted in Fig. 10 as a function of $<\beta>$, again fixing $P_{TH}$ to 4.0 Gt at $R_T = 20$ m. Curve A in Fig. 10 uses the cost scaling as a function of $M/P_{TH}$ presented in Fig. 3 [i.e., $c_D = 850 + 255(M/P_{TH})^{1/2}$] and used in Fig. 7B, while curve B indicates the penalty incurred by assuming that fusion-power-core unit costs are nominally double those used in recent fusion reactor assessments [i.e., $c_D = 850 + 510(M/P_{TH})^{1/2}$]. Again, the incremental benefit of $<\beta>$ values in excess of 0.08 is small.
Fig. 7D. Same as Fig. 7B except the lines of constant mass utilization have been replaced with lines of constant UDC obtained from Fig. 3.

The construction time, $\tau_c(y)$, of the MSR power plant may be a function of total mass, $M_T = M + M_{BOP}$, where the balance-of-plant (BOP) mass utilization, $M_{BOP}/P_{TH} = 9.5$ tonne/MWt, is such that larger plants take somewhat longer to construct. This behavior is consistent with contemporary fission-industry experience, as represented by the following relationship.

$$\tau_c = 1.1(10)^{-b} M_T + 3.125 \quad (20)$$

The results in Ref. 18 give $\tau_c$ as a function of plant capacity, $P_{TH}$, and the assumption was made that $M_T/P_{TH}$ was nominally constant over the narrow range of interest. The time required to build the various plants of Fig. 10 can be estimated and plotted in Fig. 11 as a function of $\langle \beta \rangle$. Again, the total thermal
Fig. 8. Ignition curves for the $d/r_c = 0.4$ case, superposed onto a grid of constant $N$ and $M/P_{TH}$ lines with a tightly packed ($f_c^* = 1.2$ modular-coil set). These results should be compared to the results given in Fig. 6B ($d/r_c = 0.4$, but $f_c^* = 1.0$).

output power is $P_{TH} = 4.0$ GWT and $d/r_c = 0.4$. The construction time decreases slowly from 12 to 9 years as $\langle \beta \rangle$ increases from 0.04 to 0.12 and is consistent with the nominal ($\tau_c = 10$ y) estimate used in earlier cost estimates for the MSR (Table IB). Although the ten-year construction time is also consistent with present fission reactor experience, two of the fusion reactor designs summarized in Table IB assumed construction time of 5-6 years, assuming the benefit of mass production, standardization, and 10th-plant learning experience.

The unit direct costs of Fig. 10, together with construction time scaling of Fig. 11, can be used to estimate the total cost as a function of $\langle \beta \rangle$ for the nominal ($P_{TH} = 4.0$ GWT, $d/r_c = 0.4$) MSR. These total unit costs are given in Fig. 12. Again, the lower curves (A) represent the cost scaling developed in
Fig. 9. Dependence of mass utilization, $M/P_{TH}$, on average beta, $<\beta>$, for a fixed thermal output power, $P_{TH} = 4.0 \text{ GWt}$, corresponding to $R_T \approx 20 \text{ m}$ for various values of modular-coil distortion, $d/r_c$.

Fig. 3, the upper curves (B) reflect the penalty of doubled unit costs for the fusion-power-core components. The solid curves for each case use the construction time scaling of Fig. 11. Also shown are curves for a nominal 10-year construction time and an optimistic 5-year construction time for each case. Indirect costs are taken to be 23% of the unit direct costs. Time-related interest (10%/y) and escalation (5%/y) charges for the construction period are assumed using the then-current-dollar mode of Ref. 15. The advantage of higher-beta operation is magnified by the inclusion of time-related costs and the potential for mass-related construction times. Again, the incremental improvement of pushing $<\beta>$ beyond 0.08-0.10 is small for $P_{TH} = 4.0 \text{ GWt}$. Higher beta, however, will be required for power plants of lower capacity.
Fig. 10. MSR costs as a function of $\langle \beta \rangle$ for $P_{TH} = 4.0$ GWt and $d/r_c = 0.4$ assuming (curve A) that $c_D = 850 + 255(M/P_{TH})^{1/2}$ and assuming (curve B) that $c_D = 850 + 510(M/P_{TH})^{1/2}$, following the Fig. 3 results.

A missing link in the discussion thus far is the relation between the modular-coil configuration as characterized by coil aspect ratio, $A_c = R_T/r_c$, coil number, $N$, number of toroidal-field periods, $m$, and lateral coil distortion, $d/r_c$, on the one hand and resultant rotational transform, $\tau$, on the other. Detailed resolution of this issue is the subject of ongoing magnetics computations and optimization. A simplified, analytic picture is available, however, using the models of Ref. 4. The dependence of on-axis rotational transform, $\tau(0)$, as a function of lateral coil distortion, $d/r_c$, for various plasma aspect ratios, $A = R_T/r_p$, in the range 10-30 is shown in Fig. 13 for an MSR with the indicated fixed parameters. A plasma aspect ratio of $A = 10$ is consistent with $P_{TH} = 4.0$ GWt and $\langle \beta \rangle = 0.04$; $A = 30$ would be required for
Fig. 11. Construction time, $T_c(y)$, as a function of $\langle \beta \rangle$ using an empirical fission-industry mass scaling for a nominal MSR with $P_{TH} = 4.0$ GWt and $d/r_c = 0.4$.

$P_{TH} = 4.0$ GWt at $\langle \beta \rangle = 0.08$, as can be seen from Fig. 6B. For the nominally fixed number of coils, $N = 24$, only those coil configurations to left of the dashed curves representing the coil-interference constraint of Eq. (19) are allowed. The tighter-packed constraint ($f_c^* = 1.2$) allows more coil distortion than the envelope constraint ($f_c^* = 1.0$) and hence higher on-axis transform values. The larger values of plasma aspect ratio allow the system to achieve the higher values of rotational transform required of the $\langle \beta \rangle = r^2/A$ scaling. Generally, however, the coil configuration is always pushing the $f_c^*$ geometric constraint, and more sophisticated coil winding laws than the sinusoidal deformations considered here may be required to make the overall design of the MSR fully self-consistent. In the spirit of the goals of the parametric systems model, therefore, guidance is provided to assure that the more time-consuming
Fig. 12. Total MSR reactor cost as a function of \(<\beta>\) for fixed values of
\(P_{TH} = 4.0\) GWt and \(d/r_c = 0.4\). Solid curves represent the
construction-time scaling of Fig. 11, while fixed 10-year and 5-year
times are also shown. The lower curves (A) represent the cost scaling
of Fig. 3 while the upper curves (B) reflect the doubling of assumed
fusion-power-core unit costs.

The results of the previous parametric studies can be further condensed
into a useful set of comprehensive design curves for purposes of identifying
self-consistent MSR design points. For a nominal total thermal output power
\(P_{TH} = 4.0\) GWt and the conservative coil envelope interference condition,
\(f_c^* = 1.0\), Fig. 14 displays the dependence of plasma aspect ratio,
\(A = R_T/r_p\), number of coils, \(N\), coil radius, \(r_c\), and conductor thickness, \(\delta_c\), on the
modular-coil distortion parameter, \(d/r_c\). Recall that fixing \(P_{TH}\) determines \(R_T\)
by Eq. (8) to be \(\sim 20\) m. Results for a low-beta (\(<\beta> = 0.04\) and a high-beta
Fig. 13. Approximate dependence of the on-axis rotational transform, $\tau(0)$, on the modular-coil lateral distortion, $d/r_C$, for various plasma aspect ratios, $A = R_T/r_p$, and the indicated fixed parameters. Geometric coil interference constraints using Eq. (19) for the base case ($f_C^* = 1.0$) and the tighter-packed case ($f_C^* = 1.2$) are indicated. 

($\langle B \rangle = 0.08$) MSR are presented in Figs. 14A and 14B, respectively. As indicated by Figs. 6A-C, MSR reactors with $\langle B \rangle = 0.04$ and $P_{TH} = 4.0$ GWt are inaccessible for $d/r_C \geq 0.3$. The on-axis rotational transform, $\tau(0)$, is also plotted for the assumed number of toroidal field periods, $m = 4$. Higher values of $m$ would tend to result in lower values of $\tau(0)$ performance. Only integer values of $N/m$ greater than or equal to three would be allowed in candidate design points. Selection of a candidate design point with $\langle B \rangle = 0.08$ and $M/P_{TH} \leq 5.0$ tonne/MWt leads directly to the choice of $N = 24$ for which $N/m = 6$ and $d/r_C = 0.41$. All other parameters follow by inspection of Fig. 14 or by straightforward calculation. The parameter list of Table II results. A similar process for the low-beta case leads to the selection of a design point with the rotational-
Fig. 14. MSR design-point prescription using the parametric results of Figs. 6A-C for $\beta = 0.04$ (top) and $\beta = 0.08$ (bottom) as a function of coil distortion, $d/r_c$, and the indicated fixed parameters. Rotational transforms are calculated assuming $m = 4$ toroidal-field periods.
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<th>(&lt;\beta&gt; = 0.04)</th>
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<th>(&lt;\beta&gt; = 0.08)</th>
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<td>Ref. 4a</td>
<td>((0.0067, 0.012))</td>
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<td>Number of poloidal-field periods, (l)</td>
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<td>Number of coils, (N)</td>
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<td>On-axis rotational transform, (\tau(0))</td>
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<td>0.28(m=4)</td>
<td>1.36(m=4)</td>
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<td>Major radius, (R_T(m))</td>
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<td>Coil aspect ratio, (A_c = R_T/r_C)</td>
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<td>4.49</td>
<td>6.45</td>
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<td>Coil minor radius, (r_C(m))</td>
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<td>4.45</td>
<td>3.1</td>
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<td>Plasma aspect ratio, (A = R_T/r_p)</td>
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<td>11.6</td>
<td>27.5</td>
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<td>Plasma minor radius, (r_p(m))</td>
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<td>1.72</td>
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<td>Lateral coil distortion, (d/r_C)</td>
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<td>1.51</td>
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<td>Blanket/shield thickness, (\Delta b(m))</td>
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<td>Interference parameter, (f_C^*)</td>
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<td>Inboard filling fraction, (f_C)</td>
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<td>Mass utilization, (M/P_{TH}(\text{tonne/MWt}))</td>
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<td>7.2</td>
<td>4.9</td>
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<td>Plasma power density, (P_{TH}/V_p(\text{MWt/m}^3))</td>
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<td>System power density, (P_{TH}/V_c(\text{MWt/m}^3))</td>
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<td>Total thermal power, (P_{TH}(\text{GWt}))</td>
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<td>First-wall neutron loading, (I_w(\text{MW/m}^2))</td>
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<td>Radial plasma filling fraction, (x)</td>
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<td>0.7(0.61)^c</td>
<td>0.7(0.54)^c</td>
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<td>On-axis field, (B_0(T))</td>
<td>6.37</td>
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<td>Peak field at coil, (B_{CM}(T))</td>
<td>~11.5</td>
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<td>Coil current density, (j(\text{MA/m}^2))</td>
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<td>13.0</td>
<td>13.0</td>
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<tr>
<td>Total coil current, (I(\text{MA}))</td>
<td>28.5</td>
<td>29.64</td>
<td>34.12</td>
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</table>

^aNumber of toroidal-field periods, \(m = 8\); ignition condition, \(<\beta>B_0^2 r_p = 3.15 T_2 m\).

^bBeta values obtained from expression \(<\beta> = \tau^2(0)/A\) for number of toroidal-field periods \(m = 4\) and \(3\), respectively.

^cObtained from the \(x = 0.066(m/A)^{1/3}(1-\Delta b/r_C)^{-2/3}\) formalism described in Ref. 4 for \(m = 4\) and \(r_s \approx r_C\), consistent with limiter impurity control.
transform performance that may be too low as noted in Ref. 4. Table II also includes the Ref. 4 design point with $<\beta>$ = 0.04 for comparison. The relatively low value of coil distortion (i.e., $d/r_c$ = 0.25-0.30) and low coil aspect ratio (i.e., $A_c$ = 4.5) allowed in the low-beta case (i.e., $<\beta>$ = 0.04) with $N$ = 24 leads to poor rotational-transform performance (i.e., $\tau(0) < 0.5$) in contrast to the high-beta (i.e., $<\beta>$ = 0.08) case. Figure 14 can be used to determine quickly the sensitivity of design-point performance to changes in parameter choices and to guide the application of more elaborate computations.

The results in Fig. 8-14 have focused on MSR systems with $P_{TH} = 4.0$ GWt at $R_T = 20$ m. Such systems lead to total unit cost estimates of 3000-4000 $/kWe for $<\beta>$ = 0.08 (Fig. 12); depending on the degree of optimism used in the cost database. For a nominal net output power $P_E = 1250$ MWe, these unit costs would imply total capital costs of 3.8-5.4 B$. The ability of utilities to raise such vast sums in financial markets as are seen currently may force consideration of smaller plants in order to keep the total capital cost at more manageable levels, despite a penalty incurred by sacrificing the unit-cost economies of scale of the larger plant. The consequences of such a strategy can be seen from Fig. 6B for the $<\beta>$ = 0.08 case of Table II. Lower-power (smaller plant) systems are achieved at lower values of $R_T$. The plasma radius, $r_p$, and hence the first-wall loading, $I_w$, may be held fixed as $R_T$ is decreased. In this case, the mass utilization decreases slowly, but this decrease occurs at the expense of higher required values of $<\beta>$ (e.g., $<\beta>$ = 0.12 for $R_T$ = 10 m and $P_{TH}$ = 2.0 GWt). A smaller reactor system or a demonstration device, therefore, would require even more favorable physics performance in terms of $<\beta>$ and $\tau$. Alternatively, for a fixed $<\beta>$ = 0.08 and $d/r_c$ = 0.4, as $R_T$ = 10 m for $P_{TH} = 2.0$ GWt, the plasma radius, $r_p$, and the mass utilization, $M/P_{TH}$, can grow to unacceptable levels at the asymptotic limit, necessitating a lower value of coil distortion (Fig. 6A). A third approach is to fix the mass utilization and examine the trade-off between $P_{TH}$ and $<\beta>$. Figure 15 depicts the trade-off for coil distortion values in the range 0.3-0.5, consistent with Figs. 6A-C, for $M/P_{TH} = 5$ tonne/MWt. The thermal power is a strong function of beta for systems with more than fifteen coils. Below fifteen coils, the curves in Fig. 15 change behavior reflecting the crowding of the constant-$<\beta>$ curves at the leftward asymptotic limits shown in Figs. 6A-C and such that values of $P_{TH}$ less than $\sim 1.5$ GWt are generally inaccessible. Lower values of $d/r_c$ give insufficient rotational transform. However, as $N$ falls below 15, the toroidal-field ripple
Fig. 15. Thermal power output, $P_{TH}$, as a function of $\langle \beta \rangle$ for fixed mass utilization, $M/P_{TH} = 5$ tonne/MWt and various values of coil distortion, $d/r_c$, and coil number, $N$, assuming $f_c^* = 1.0$. Systems with $N < 15$ are probably inaccessible.

increases and the magnetics requirement that $N/m \geq 3$ can be met only for very low $m$ values. The curves in Fig. 15 for $N < 15$ are therefore dashed to indicate that this attractive low-power regime is probably not accessible. Determination of the optimum MSR configuration that reduces total cost and maintains favorable mass utilization while meeting the set of imposed physics and engineering constraints is the subject of ongoing work.
IV. CONCLUSIONS

A parametric systems model has been developed, described, and evaluated for the Modular Stellarator Reactor (MSR). On the basis of optimistic (i.e., "best-case") assumptions and constraints, a plasma beta of 0.08 or greater appears to be necessary if the MSR is to be competitive with other approaches to magnetic fusion of the $P_{TH} = 4.0$ GWt class; higher beta values will be required for lower values of $P_{TH}$. In achieving this goal of mass utilization, $M/P_{TH}$, near or below 5 tonne/MWt, the desired value of $\langle \beta \rangle$ must be achieved with a magnetics topology that can be created by a modular-coil set of distortion $d/r_c \leq 0.4$. If a $\langle \beta \rangle \approx \tau^2/A$ scaling applies, this constraint (i.e., $\langle \beta \rangle \geq 0.08$, $M/P_{TH} \leq 5$ tonne/MWt) implies rotational transforms on the order of unity must be obtained for $d/r_c = 0.4$. Additional optimism needed to alleviate these conditions can be injected into these results by:

- Increasing the assumed Alcator transport coefficient (i.e., $\tau_B/\langle n \rangle r_p^2 > 3(10)^{-21}$ s m) or otherwise "fine tune" other plasma parameters (i.e., profiles, $f_a$, $Z_{EFF}$, etc.) to reduce the $\langle \beta \rangle B_0^2 r_p$ value needed for ignition.

- Increasing $f^*$ beyond unity, with the attendant complexity of "interlocked" coils and reduced torus access.

- Accounting for differences between vacuum and beta-related rotational transforms in the $\langle \beta \rangle \approx \tau^2/A$ scaling.

- Decreasing the nominal thickness of blanket/shield, $\Delta b$.

It is recalled, however, that optimism built into the constraints that $B_{CM} = 12$ T, $j = 13$ MA/m$^2$, $x = 0.7$, $\Delta b = 1.3$ m, $\ell = 2$, etc. will more than likely not be fully attained by a detailed engineering design and that the parametric results presented here are judged to be close to a "best case". These findings, however, must be tested against more exact analyses, although they can be considered indicative of the requirement that the MSR should provide $\langle \beta \rangle$ values in the range 0.08-0.10 if a truly interesting alternative fusion system is ultimately to emerge from this approach.

The analytic rotational transform results displayed in Fig. 13 must be verified by three-dimensional magnetics computations before a self-consistent MSR design point can be selected.
These conclusions, however, must be moderated by obvious modelistic shortcomings. This parametric survey does not treat accurately several important but subtle influences on the ultimate reactor desirability of the MSR. For example, systems with attractively low values of \( M/P_{TH} \) and higher values of \( \langle B \rangle \) also must operate at higher values of first-wall loading; therefore, for a fixed first-wall life (10-20 MWy/m\(^2\)) these higher-power-density systems may operate with higher rate of FW/B change out, lower plant availability, and higher operating costs. Second, the dependence of plasma filling fraction, \( x = r_p/r_w \), on the number of toroidal-field periods, \( m \), and plasma aspect ratio, \( A = R_T/r_p \), has been ignored for present purposes, as was noted previously. Inclusion of this dependence may make it difficult to achieve the desired values of lower mass utilization in systems with high beta values and correspondingly large values of \( A \), particularly if \( m \) must be sufficiently low to yield appropriately high values of rotational transform, \( \tau \). The desire to maximize \( x \) suggests that pumped-limiter impurity control may be preferable to divertor impurity control, in the absence of overriding technical considerations, such as heat removal and maintenance. Again, three-dimensional magnetics computations will be required to determine the final MSR design point.

Lower output power than the nominal \( P_{TH} = 4.0 \text{ GWe} \) for a fixed mass utilization requires higher beta values, but offers the prospect of reduced total cost. Clearly, an optimum power density, mass utilization, and cost exists for the MSR. This optimum probably lies in the \( \langle B \rangle = 0.08-0.10 \), \( I_w = 3-4 \text{ MW/m}^2 \) range rather than the \( \langle B \rangle = 0.04-0.05 \), \( I_w = 1-2 \text{ MW/m}^2 \) range projected by past MSR studies.\(^2,4\) Ongoing work in this area is expected to resolve more clearly this important issue.

REFERENCES

1. 3rd IAEA Technical Committee Meeting and Workshop on Fusion Reactor Design and Technology, Tokyo, Japan (October 5-16, 1981).


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