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THE EFFECT OF PRESSURE ON
THE TRANSIENT SWELLING RATE
OF OXIDE FUEL

by

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The Effect of Pressure on the Transient Swelling Rate of Oxide Fuel

E. E. Gruber

Abstract

An analysis of the transient swelling rate of oxide fuel, based on fission-gas bubble conditions calculated with the FRAS3 code, has been developed and implemented in the code. The need for this capability arises in the coupling of the FRAS3 fission-gas analysis code to the FPIN fuel-pin mechanics code. An efficient means of closely coupling the calculations of swelling strains and stresses between the modules is required. The present analysis provides parameters that allow the FPIN calculation to proceed through a fairly large time step, using estimated swelling rates, to calculate the stresses. These stress values can then be applied in the FRAS3 detailed calculation to refine the swelling calculation, and to provide new values for the parameters to estimate the swelling in the next time step.

The swelling rates were calculated for two representative transients and used to estimate swelling over a short time period for various stress levels. Comparison of these estimates to the results of detailed swelling calculations for the same stresses shows that the analysis should provide the desired capability with acceptable accuracy over a broad range of transient conditions. A simplified form of the results may be useful in more approximate models of fission-gas behavior.

1. Introduction

A major objective in modeling fuel-pin response to a hypothetical fast-reactor accident transient is to predict the timing and location of cladding failure. This objective can be met only by a careful thermal-mechanical analysis of the strains that develop within the pin. A major component of volumetric strain in irradiated oxide fuel is the fission-gas swelling. In fact, the incremental strain may be much more dependent on fission-gas behavior than on temperature changes in the regime of interest -- that is, when pin failure occurs.

The FPIN code is being developed by T. H. Hughes at Argonne National Laboratory to analyze the complex phenomena that determine transient fuel-pin behavior [1]. The current version of this code does not effectively treat the

contribution of fission-gas swelling; such a treatment requires an extension of the code by coupling it to a fission-gas behavior modeling code, such as FRAS3 [2]. Preliminary work has been done to perform a "loose" coupling of the FRAS3 code to the FPIN code [3]. Stresses and temperatures calculated in the FPIN code are used to drive the FRAS3 calculation of fission-gas redistribution and fuel swelling, but the strain component arising from fuel swelling is not included in the FPIN calculation of stresses.

Implementation of a more realistic coupling of the fission-gas behavior code and the pin-mechanics modeling code requires an efficient means of communicating fission-gas behavior data to the FPIN modules. A first approach might be to calculate the stresses in FPIN, then use these stresses in the FRAS3 analysis of swelling. However, the swelling values returned by the FRAS3 code could cause a large change in the stresses subsequently calculated by the FPIN code. Thus very short time steps, or an iterative approach, might be required to obtain satisfactory convergence.

To eliminate the need for such inefficient procedures, the FRAS3 analysis has been extended to include calculation of the swelling rate and its sensitivity to pressure. This information can be used within the FPIN analysis to accurately estimate the swelling change during a given time step. This estimated swelling can be used with other components of strain, within the FPIN analysis, to determine the stresses before invoking the FRAS3 module.

The purpose of this paper is to describe the factors that contribute to fission-gas swelling, as modeled in the FRAS3 code, and to present an analysis of the swelling rate based on that description. It is helpful to preface this analysis with a broader discussion of transient fission-gas behavior in oxide fuels, along with a description of the approach used to model this physical behavior.

II. Modeling of Transient Fission-Gas Behavior

A physically realistic modeling effort must take into account a variety of phenomena related to fission-gas interactions with the host material. Fission gas is generated randomly throughout the fuel. Because saturation levels are quickly exceeded, the gas tends to precipitate into bubbles. This tendency is offset by dynamic re-resolution effects that tend to destroy the bubbles during normal operation of the reactor, particularly in the cooler

parts of the fuel. When the temperature rises during a transient, however, the fission gas precipitates rapidly into bubbles.

Transient behavior of fission gas is therefore characterized by the interaction of discrete bubbles with the complex environment dictated by the material structure and the transient conditions. Much of the gas generated in the hotter, restructured fuel escapes from the fuel prior to the transient. The gas retained in the cooler regions of the fuel is initially present as isolated gas atoms and in small bubbles, mostly within grains. This gas begins to migrate within the grains as the mobility increases with increasing temperature. Some of the gas migrates to grain boundaries, where it is temporarily trapped. As the transient progresses, the gas that builds up on boundaries finds its way to grain edges; when sufficient edge porosity develops, this edge gas can move to cracks and to the central void, and eventually can escape from the fuel.

A variety of techniques has been applied to model this transient redistribution of fission gas and the associated changes in morphology and swelling. The approach represented by the FRAS3 code applies a number of these techniques in a consistent framework. The first step in modeling transient fission-gas behavior in the FRAS3 code is to consider the intragranular fission gas.

Swelling due to intragranular fission gas (grain swelling) is modeled by calculating the evolution of the bubble-size distribution that results from migration, coalescence, and expansion of the bubbles within the grains. A large number of size classes (about 65) is used in the calculations to reflect accurately the effects of various parameters on the calculated distribution. Volume is conserved in the analysis of bubble coalescence; as the transient progresses, the gas pressure in the bubbles quickly exceeds the pressure that would be in equilibrium with the local instantaneous temperature and stress. The net bubble volume is calculated to change only by bulk diffusion of thermal vacancies, driven by the excess pressure in the bubbles. Grain swelling is then determined by summing the volumes of the bubbles in the various size classes.

Grain swelling is also affected by transfer of bubbles to the grain boundaries. There is no immediate change in total swelling: the volume associated with these bubbles is merely transferred from the category of grain

swelling to that of boundary swelling, since volume is again conserved. However, it is necessary in the overall modeling effort to consider these categories separately. The gas transfer to boundaries is calculated in the FRAS3 code to occur by both random bubble migration and by thermally biased migration.

Grain-boundary swelling is characterized by a rather complex system. Generally, during a transient, gas bubbles migrate from the grains to become trapped on the boundaries. Coalescence between grain-boundary bubbles, and with additional bubbles from the grains, results in a population of bubbles that are pressurized with fission gas above the equilibrium pressure. These bubbles will expand, primarily by grain-boundary diffusion. This accumulation and expansion lead very quickly to saturation of the grain faces.

Under these conditions the actual behavior of the boundary-bubble system is difficult to model in detail. For smaller bubbles, surface diffusion is rapid enough to allow the bubble shape to adjust very quickly to the equilibrium lenticular shape. As bubbles get larger, and coalescence occurs more frequently, shape adjustment becomes incomplete. Bubbles will then take on rod-like shapes, eventually joining to form channels that permit venting of the gas to edge porosity. This venting should lead to a cyclic swelling behavior on a given grain face, so that swelling increases to a level corresponding to saturation, then rapidly decreases to a low value, from which bubble accumulation and growth start once again.

Modeling efforts, however, must be directed to the average behavior over a region that can be considered to comprise a large number of grain faces. In some of our modeling calculations, it is appropriate to characterize the behavior of the system by the most probable situation in a small region; an example is in the bubble migration and coalescence calculations. In others, however, we must modify the model so that it is still mechanistic and physically based, but not so detailed as to give non-realistic results. Since the expected cyclic behavior on a given grain face would not be in phase with that on nearby faces, the overall local swelling behavior will not be cyclical.

Thus the grain-boundary swelling is modeled as an idealized accumulation, growth, and loss of discrete bubbles. Bubbles are added to the grain-boundary population as gas is released from the grains. This gas is characterized by bubbles with volume equal to the average size of the grain bubbles. These

bubbles are either added directly to the grain-boundary distribution, as individual bubbles, or modify the boundary distribution by coalescence with existing grain-boundary bubbles. In fact, the coalescence probability is quite high (~98%) when the boundary is saturated.

Saturation is characterized by a limiting fractional coverage of the grain-boundary area by bubbles; it is modeled as a steady-state condition, in which any calculated supersaturation is relieved by arbitrarily removing bubbles from the grain-boundary population. The gas associated with these excess bubbles is transferred to the interconnected porosity, associated with grain-edge swelling. The resulting model is similar in effect to the "zoom-lens" model [4].

This grain-edge component of swelling need not concern us here, but a brief discussion will serve to complete the description of swelling as modeled in the FRAS3 code. Again, the real behavior of fission gas is complex. Bubbles will tend to accumulate at grain edges and especially at grain-edge intersections (grain "corners"). During a transient, these edges will rapidly accumulate sufficient gas to stabilize a system of (at least partially) interconnected porosity. This porosity is assumed to be associated with any remaining as-fabricated porosity.

The current FRAS3 model of grain-edge porosity is a "linear tunnel" model. Straight, cylindrical tunnels are assumed to exist on grain edges, and the tunnel diameter is taken as the equilibrium value corresponding to the pressure exerted by the grain-edge gas. When the tunnel volume reaches a level of 5% swelling, the tunnels are considered stably interconnected, and their diameter is "frozen". Subsequent calculations consider only the pressure of the gas in the tunnels, for application in later considerations of gas redistribution or fuel dispersal. It should be noted that FRAS3 transient calculations usually predict stably interlinked edge porosity relatively early in the transient. Thus more detailed models, such as the toroid model [5], which could easily be implemented in the analysis, do not appear necessary at present.

The hydraulic behavior of the gas within this system of tunnels is beyond the scope of the single-node analysis of the FRAS3 code. Instead, this behavior must be considered in the context of a multi-node calculation, such as that provided in the FPIN code. The FRAS3 analysis can provide the source

term for fission gas, and a prediction of the grain-edge swelling, which can be applied to estimate the local fuel permeability. The rate at which gas is transferred to the tunnel network can then be combined with temperature and tunnel volume estimates to provide information on the gas pressure that drives gas redistribution within the fuel pin.

III. Analysis

The objective of this analysis is to estimate the fuel swelling at the end of a time increment Δt at an arbitrary pressure p , based on calculations with the FRAS3 code at the beginning of the time increment (i.e., at time t_0) at a somewhat different pressure p_0 . The formalism adapted for this purpose is to express the estimated swelling increment ΔS in terms of partial derivatives, in the form

$$\Delta S \cong (\dot{S} + \dot{S}' \Delta p) \Delta t, \quad (1)$$

where

$$\dot{S} = \left(\frac{\partial S}{\partial t} \right)_{p_0},$$

$$\dot{S}' = \frac{\partial^2 S}{\partial t \partial p},$$

and $\Delta p = p - p_0$.

The problem addressed in this section is the derivation of expressions for $\dot{S}$ and $\dot{S}'$ that can be evaluated in the FRAS3 code. It is helpful to consider separately the contributions to $\dot{S}$ and $\dot{S}'$ from gas within grains (intragranular, or "matrix", gas) and from gas trapped on boundaries (boundary gas).

A. Swelling Rates Due to Gas Within Grains

Intragranular swelling results from the growth of bubbles by vacancy diffusion, driven by a gas pressure within the bubbles that is greater than the equilibrium pressure. The expression used in FRAS3 calculations for the rate of volume increase of a bubble of radius r is [6]

$$\frac{dV}{dt} = 4\pi r D_u \frac{\phi \Omega}{kT}, \quad (2)$$

where D_u is the volume-diffusion coefficient, ϕ is the excess gas pressure, Ω is the molecular volume, k is the Boltzmann constant, and T is the absolute temperature. Since the swelling rate varies for different bubble sizes, the swelling is obtained by summing the contributions from each size class i . The component of the swelling rate due to growth of bubbles within grains is therefore

$$\dot{S}_1 = \frac{4\pi D_u \Omega}{kT} (1 - f_g) \sum g_i r_i \phi_i, \quad (3)$$

where $(1-f_g)g_i$ is the number of matrix bubbles per unit volume in size class i . The summation sign is used to symbolize summation over the appropriate distribution for all size classes, which are signified by the subscript i , and the superior dot represents differentiation with respect to time, as in Eq. (1). Gas transfer from the grains to grain boundaries is calculated in the FRAS3 code as the fraction f_g of the grain volume that is free of gas because of bubble migration to the boundaries. The overpressure ϕ is

$$\phi_i = p_{g,i} - p - 2\gamma/r_i, \quad (4)$$

where $p_{g,i}$ is the gas pressure in the bubble, calculated from the reduced Van der Waals equation for xenon, p is the hydrostatic compressive stress, and γ is the specific surface free energy.

The rate at which gas is transferred to grain boundaries reduces the swelling rate. If the fractional release rate is $\dot{f}_g/(1-f_g)$, the reduction in the swelling rate due to gas transfer to boundaries is

$$\dot{S}_2 = S_g^0 \dot{f}_g/(1 - f_g), \quad (5)$$

where S_g^0 is the intragranular swelling at time t_0 . The net swelling rate within the grains is therefore

$$\dot{S}_g = \dot{S}_1 - \dot{S}_2. \quad (6)$$

B. Pressure Sensitivity of Intragranular Swelling Rate

The partial derivative of $\dot{S}$ with respect to pressure, $\dot{S}'$, is obtained by differentiating the result obtained in the preceding subsection. Although $\dot{S}_2$ is a function of pressure, because $\dot{f}_g$ varies slightly with pressure, analytic calculation of $\dot{S}_2'$ is difficult. Estimates of the magnitudes of these parameters have been obtained from the results of calculations with the FRAS3 code, however; the results indicate that $\dot{S}_2'$ is much smaller than $\dot{S}_1'$. We therefore approximate the intragranular swelling-rate pressure sensitivity by the derivative with respect to pressure of the swelling rate $\dot{S}_1$, given by Eq. (3):

$$\dot{S}'_g \cong \dot{S}'_1 = - \frac{4\pi D_u \Omega}{kT} (1 - f_g) \sum g_i r_i . \quad (7)$$

We have used the result $\partial \phi_i / \partial p = -1$, which follows from Eq. (4). This result requires that instantaneous changes in $p_{g,i}$, γ , and r_i with changes in pressure are small; this is a reasonable approximation.

C. Grain Boundary Swelling Rate

The grain-boundary swelling is defined in terms of the bubble distribution by

$$S_b = \sum h_i V_i , \quad (8)$$

where V_i is the volume associated with a single bubble in the size-class i , and h_i is the bubble density (number in size class i per unit initial volume of fuel).

It follows that the swelling rate is given by

$$\dot{S}_b \cong \sum h_i \dot{V}_i + \sum \dot{h}_i V_i . \quad (9)$$

The situation of most interest is that of grain-boundary saturation with fission-gas bubbles. The gas bubble grows, either by diffusion or by coalescence with bubbles transported from the grains; this growth is represented by the first sum in Eq. (9). It is assumed for simplicity that all of the gas bubbles transported to the boundaries coalesce with pre-existing boundary bubbles, so that h_i is not altered by this process.

Under saturation conditions, bubble growth requires an equivalent reduction in bubble density. This reduction must be apportioned over the size distribution. The method used in the FRAS3 code preserves the character of the distribution by making the reduction in each size class proportional to the density in that class, so that

$$\dot{h}_i = \beta h_i \quad (10)$$

where β is a constant. Of course, if the boundary is not saturated, $\beta = 0$. The saturation condition can be expressed by the requirement that the total grain-boundary area A occupied by bubbles is constant, so that

$$\dot{A} = 0 = \sum \dot{h}_i a_i + \sum h_i \dot{a}_i, \quad (11)$$

where $a_i = \pi r_i^2$ is the grain-boundary area associated with a single bubble in size-class i . It follows from Eqs. (10) and (11) that

$$\beta = -2 \frac{\sum h_i r_i \dot{r}_i}{\sum h_i r_i^2}. \quad (12)$$

Equation (9) can be simplified to

$$\dot{S}_b = \sum h_i (\dot{V}_i + \beta V_i). \quad (13)$$

It is convenient to consider separately the contributions to $\dot{S}_b$ from diffusive growth and from gas transported to the boundaries from the grains.

1. Grain-boundary Bubble Growth by Diffusion

The rate of change of bubble volume is given by [7]

$$\dot{V}_i = C_1 \phi_i, \quad (14)$$

where

$$C_1 = 2\pi\Omega D'/kTL,$$

in which D' is the product of the grain-boundary diffusion coefficient and the effective thickness of the boundary, and

$$L = \alpha - (\alpha^2 + \ln \alpha^2 + 3)/4 .$$

The parameter α is the fractional coverage of grain-boundary area by bubbles.

The grain-boundary morphology is assumed to be lenticular, with a bubble half-angle at the grain boundary of $\theta = 50$ deg. The volume of a bubble is then given by

$$V_i = 4\pi\rho_i^3\eta/3 , \quad (15)$$

where $\rho_i = r_i/\sin\theta$ and $\eta = 1 - (3 \cos\theta - \cos^3\theta)/2$. Equation (15) can be written as

$$V_i = C_2 r_i^3 \quad (16)$$

where $C_2 = 1.57112$ for $\theta = 50$ deg. Differentiation of Eq. (16) with respect to time, and comparison to Eq. (14), gives the result

$$\dot{r}_i = C_3 \phi_i / r_i^2 , \quad (17)$$

where $C_3 = C_1/3C_2$.

Equation (13) can be used to determine the component of the swelling rate due to diffusive growth of boundary bubbles: substituting in terms of r_i , we obtain

$$\dot{S}_3 = C_1 \sum h_i \phi_i \left(1 - \frac{2}{3} \frac{\sum h_i \phi_i / r_i}{\sum h_i \phi_i} \frac{\sum h_i r_i^3}{\sum h_i r_i^2} \right) . \quad (18)$$

Again, the second term should not be included if the grain-boundary occupation has not reached the saturation level. The sums needed for evaluation of $\dot{S}_3$ in Eq. (18) are all calculated quite easily within the FRAS3 computer code. The equation is written in this form for purposes of later discussion.

2. Grain-boundary Bubble Growth Due to Gas Release from Grains

The rate at which gas volume is transferred from grains to grain boundaries, $\dot{S}_2$, was described in a preceding section. If the grain boundaries are saturated, this contribution to the grain-boundary swelling rate is, from Eq. (13),

$$\dot{S}_4 = \dot{S}_2 + \beta \sum h_i V_i . \quad (19)$$

In order to evaluate β for this process, we must consider the manner in which the bubble volume entering the boundary is apportioned among the existing boundary bubbles.

The coalescence probability for bubbles entering a saturated boundary is very nearly unity, as noted earlier. This probability is proportional to the collisional cross section defined by the circle of radius $r_i + r_0$, where r_0 is the average radius in the grain boundary of bubbles transported from the grains. The relative probability of coalescence with a particular bubble can be written

$$P_i = \frac{\dot{V}_i}{\dot{S}_2} = \frac{(r_i + r_0)^2}{\sum h_i (r_i + r_0)^2} . \quad (20)$$

Differentiation of Eq. (16) gives the result

$$\dot{V}_i = 3C_2 r_i^2 \dot{r}_i , \quad (21)$$

and substitution in Eq. (20) leads to

$$\dot{r}_i = \frac{\dot{S}_2}{3C_2} \frac{(r_i + r_0)^2 / r_i^2}{\sum h_i (r_i + r_0)^2} . \quad (22)$$

Substituting this result in Eq. (12), to determine β , leads to the result from Eq. (19)

$$\dot{S}_4 = \dot{S}_2 \left(1 - \frac{2}{3} \frac{\sum h_i (r_i + r_o)^2 / r_i}{\sum h_i (r_i + r_o)^2} \frac{\sum h_i r_i^3}{\sum h_i r_i^2} \right) . \quad (23)$$

Again, if boundary coverage has not reached saturation, the second term should be omitted.

The net boundary-swelling rate is obtained by summing the contributions defined by Eqs. (18) and (23):

$$\dot{S}_b = \dot{S}_3 + \dot{S}_4 . \quad (24)$$

D. Pressure Sensitivity of the Boundary Swelling Rate

The effects of pressure are evident in several aspects of boundary swelling, including the rate at which gas is transferred to boundaries from grains. Most of these effects are difficult to calculate, although they can be quantified from analysis of results calculated with the FRAS3 code. The dominant effect, however, is on the rate of bubble growth by diffusion. This effect can be calculated directly by differentiating Eq. (18), which leads to the result

$$\dot{S}'_b \equiv \dot{S}'_3 = -C_1 \sum h_i \left(1 - \frac{2}{3} \frac{\sum h_i / r_i}{\sum h_i} \frac{\sum h_i r_i^3}{\sum h_i r_i^2} \right) . \quad (25)$$

This equation has been written in this form for consistency with Eqs. (18) and (23). The significance of the sums in the parentheses will be discussed in the next section.

IV. Results

Detailed calculations were carried out with the FRAS3 code for two cases that had been analyzed previously for other purposes. These test cases were used for convenience because they represent a contrast in conditions that provides an interesting test for the results of this analysis. The calculations were carried out by using the restart feature of the FRAS3 code. Calculations were completed initially at a nominal pressure p_0 , up to a point in each

transient near the maximum temperature for the fuel node. Some pertinent results at the restart points for the two cases are compared in Table 1.

Table 1. Selected results at restart points for the two studies

Parameter	Case 1	Case 2
Temperature, K	2430	2700
Heating Rate, K/s	480	120
Thermal Gradient, K/mm	1500	1000
Mean Grain-Bubble Radius, nm	43	150
Mean Boundary-Bubble Radius, nm	104	300
Gas Retained Within Grains, percent	83	20
Initial Pressure, MPa	0.25	3.0
Matrix Swelling, percent	8.5	1.5
Boundary Swelling, percent	0.4	1.2

The major differences in these two cases are the higher temperature and slower heating rate of Case 2, which lead to larger fission-gas bubbles, both within grains and on boundaries, and to much less gas retained within the grains. As a result, Case 1 shows large matrix swelling with limited swelling on grain boundaries. Case 2 shows less overall swelling, but swelling is more balanced between the grains and the boundaries. These differences provide a good basis for evaluating the analysis of swelling rates outlined in the preceding section.

The parameters of interest were determined with the FRAS3 code at the restart point. The results are presented in Table 2.

Table 2. Values of swelling-rate parameters calculated with the FRAS3 code

	Case 1	Case 2	
$\dot{S}_g$	6.67	-1.14	%/s
$\dot{S}_b$	1.36	0.56	%/s
$\dot{S}'_g$	-0.274	-0.037	%/s/MPa
$\dot{S}'_b$	-0.022	-0.012	%/s/MPa

A series of restart calculations was carried out for each case, with pressure ranging from the initial pressure shown in Table 1 to a maximum of 50 MPa. These calculations were extended for several short history steps to permit evaluation of the effect of the length of the time step on the accuracy of the estimated swelling. The results are plotted in Figs. 1 and 2 for the

Fig. 1. Dependence of swelling on transient time during a portion of the Case 1 thermal transient, at both high and low pressure. Solid lines: code calculations. Dashed lines: estimates based on parameters calculated at 9.95s.

Fig. 2. Dependence of swelling on transient time during a portion of the Case 2 thermal transient, at both high and low pressure. Solid lines: code calculations. Dashed lines: estimates based on parameters calculated at 2.6s.

initial and maximum pressures. The solid curves represent the results calculated with the FRAS3 code, for both matrix and total (i.e., matrix plus grain-boundary) swelling. The straight, dashed lines indicate the estimated swelling results, based on the parameters listed in Table 2. The cross-hatched regions represent the discrepancy between the estimated and calculated results.

Two features are apparent from the plotted results. The first is that the estimated results are usually more accurate for shorter times. This result should be expected, since the derivatives are calculated from data corresponding to the initial times in the plots. The second feature is that the estimated swelling is not consistently in error in the same direction for the two cases considered. That is, it is too high for the total swelling in Case 1 at .25 MPa, but slightly low at 50 MPa; and it is somewhat high in Case 2 for both pressures.

The effect of pressure is shown in greater detail in Figs. 3 and 4, which show the swelling at specific times as functions of the pressure. Again, the solid curves represent the total swelling calculated with the FRAS3 code. The straight, dashed lines show the estimated results. These figures include the data plotted in Figs. 1 and 2, along with data for intermediate pressures.

The overall results indicate that the estimated swelling is accurate enough for the intended application, even for fairly large time steps (say, 0.1 s in Case 1, or even 0.3 s in Case 2). The slopes, in fact, agree very well with the calculated slopes for very short times. Errors appear to develop for longer times because of several factors that are not included in the analysis. A brief discussion of these factors is appropriate here.

One factor that should be considered is the size of the time step, Δt . The swelling S at time $t = t_0 + \Delta t$ could be written as

$$S = S_0 + \Delta S \cong (N_0 + \dot{N}\Delta t)(V_0 + \dot{V}\Delta t), \quad (26)$$

where $S_0 = N_0 V_0$ is the initial swelling, characterized by N_0 bubbles each containing volume V_0 of fission gas. $\dot{N}$ and $\dot{V}$ represent the rate of change of bubble density and volume, respectively. This expression can be generalized to the sums used previously, but this form is useful to show that the swelling rate can be written as

Fig. 3. Variation with pressure of the swelling during a portion of the Case 1 thermal transient, at several transient times. Solid lines: code calculations. Dashed lines: estimates based on parameters calculated at 9.95s.

Fig. 4. Variation with pressure of the swelling during a portion of the Case 2 thermal transient, at several transient times. Solid lines: code calculations. Dashed lines: estimates based on parameters calculated at 2.6s.

$$\dot{S} \approx \frac{\Delta S}{\Delta t} \approx N\dot{V} + V\dot{N} + \dot{N}\dot{V}\Delta t. \quad (27)$$

The last term is generally neglected; for large Δt , however, it can be significant. Test calculations indicate that better agreement can be obtained for the estimated boundary-swelling rate when this effect is included; but the improvement is not justified by the additional calculations required.

Another factor that should be recognized is that Eq. (1) is an approximation. A number of other factors should be considered in addition to the pressure sensitivity of the swelling rate. The temperature sensitivity is probably the most important additional factor, but the changing thermal gradient could also affect the calculated results. In general, however, it appears that the present result is sufficiently accurate that its relative simplicity should be retained.

The analysis of grain-boundary swelling rates is complicated by the need to consider the details of the bubble-size distribution. The trend in fission-gas analysis seems to be toward more approximate models, such as the NEFIG model for transient intragranular fission-gas behavior [8]. Such models are particularly useful in combination with multi-node pin-mechanics codes because of their greater economy. The premise of these models is that the fission-gas bubble distribution can be approximated by an average bubble size. Such an assumption was applied in preliminary analysis of the grain-boundary swelling rate, with the result that the combinations of sums in Eqs. (18), (23), and (25) were all simplified to unity. This result can be seen quite easily if it is assumed that h_i is non-zero for only one value of i .

The resulting estimates of swelling, however, were clearly inadequate when compared to the calculated swelling. Values of the combinations of sums for the two test cases were computed, to determine the error associated with the approximate analysis. The results are shown in Table 3, where E_1 , E_2 , and E_3 represent the values of the combinations of sums in Eqs. (18), (23), and (25), respectively. Although these values differ from unity by less than 25 percent in all cases, the error that results in the calculated parameters by their omission is considerably greater; the error is magnified because the results depend on relatively small differences between larger numbers. Values of the parameters $\dot{S}_3$, $\dot{S}_4$, and $\dot{S}_b$ were evaluated according to the approximate

analysis, and the errors were determined by comparison to the more accurate results. These errors are also tabulated in Table 3. The relatively large errors make the more complicated analysis necessary.

Table 3. Values of combinations of sums and the error associated with their omission in Eqs. (18), (23), and (25)

	Case 1	Case 2
E_1 [Eq. (18)]	1.1453	1.0938
Error in $\dot{S}_3$	-41%	-23%
E_2 [Eq. (23)]	1.1264	1.1123
Error in $\dot{S}_4$	-34%	-29%
E_3 [Eq. (25)]	1.2477	1.2453
Error in $\dot{S}_b'$	-98%	-96%

As faster, more economical models are developed, this effect must be considered. An approximate correction factor, based on these results, might prove useful in such models.

V. Summary and Conclusion

An analysis of transient swelling rates due to fission gas within grains and on grain boundaries, based on conditions calculated with the FRAS3 code, has been developed and implemented in the code. The results of this analysis are in the form of parameters that can be used to express the swelling increment as a function of the time and pressure increments. This analysis was developed to improve the interfacing of the FRAS3 code with the FPIN pin-mechanics code, by providing a capability for the FPIN code to calculate swelling strains without iterations with the fission-gas analysis, and with convenient selections of time steps.

The results were tested by comparison to detailed swelling calculations for two diverse transient situations, for a range of stresses. Both of these comparisons were made near the peaks of the transients, to provide relatively severe tests of the results. The conclusion from these comparisons is that the predicted swelling rates satisfy all expectations with regard to general applicability and accuracy.

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