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A THEORETICAL STUDY OF THE TUBE
FLOW OF ONE COMPONENT STICKY FOAM
SOLUTIONS

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A THEORETICAL STUDY OF THE TUBE FLOW OF
ONE COMPONENT STICKY FOAM SOLUTIONS

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ABSTRACT

A theoretical model that describes the tube flow of one-component sticky foam solutions was developed to provide a rational explanation of previous experimental observations. These polymer solution systems have been formulated to serve as access deterrents in the Safeguard Program. Below the foaming pressure the solution expands to form a low density, tacky foam. If foam forms inside the tubing of a delivery system, the flow can be highly restricted. Experiments have also shown that the dispensing rate of solution goes through a maximum with temperature in the application range. The model which is developed assumes fully-developed flow of solution followed by a foam phase in the tube. Both phases are considered to be incompressible power law fluids. Instantaneous expansion of the solution is assumed to occur at the foaming pressure. The relationship between pressure drop and flow rate, and the fraction of foam in the tube are calculated. The model predicts that a small amount of foam in the tube can cause a large reduction in the flow rate relative to the foam-free case. The larger viscosity and smaller density of the foam phase relative to the original solution cause this effect. When the temperature-dependence of the foaming pressure and the viscosities are incorporated into the calculations, a maximum in flow rate versus temperature is predicted. Increased foam formation in the tube at higher temperatures can cause a decrease in flow rate despite the fact that the solution and foam viscosities may decrease with increasing temperature. An appreciation of this effect causes concern about the performance of a sticky foam delivery system at moderately high temperatures. It is felt that the model correctly describes the major features of foam flow but quantitative predictions cannot be expected because of several simplifying assumptions that have been incorporated.

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Introduction

The Safeguard Program has fostered the development of Stickvel foam solution (SF-283), which is liquid under pressure and a high expansion, tacky foam under ambient conditions. This so-called one component sticky foam consists of a multicomponent polymeric resin dissolved in dichlorodifluoromethane (Freon 12). Because of the poor low temperature performance of SF-283, P. B. Rand (5813) is currently developing a sticky foam system that is based upon a thermoplastic elastomeric resin system. An understanding of the dispensing characteristics of these one component foams is essential to the development of fast and reliable delivery systems required by the Safeguard Program.

A current delivery system utilizes a pressurized reservoir for storage and tubing for deployment. In the flow situation, the pressure decreases from the reservoir to the exit of the tube. If the foaming pressure exceeds atmospheric pressure and the kinetics of foaming are sufficiently fast, foam will form prior to the tube exit. Under these conditions, flow rates calculated on the basis of the solution viscosity alone may not compare favorably with reality.

P. B. Rand (5813) and K. C. Goettsche (1714)¹ have measured the effect of temperature on the tube flow of Stickvel foam solution and reported a maximum in flow rate at 4°C with all other conditions held constant. The eventual decrease in flow rate with increasing temperature seems anomalous in view of the fact that the viscosity of the solution and foam would be expected to decrease with increasing temperature. The simplified flow model which is developed in this report provides a rational basis for interpreting this and other foam flow phenomena.

The Flow Model

When dealing with one isothermal incompressible fluid phase, knowledge of the viscosity function alone allows one to calculate pressure drops for fully-developed tube flow. The power law model is one constitutive equation which is frequently used to approximate the shear rate dependent viscosity that is characteristic of most polymer solutions.

$$\mu = m\dot{\gamma}^{n-1}$$

where $\dot{\gamma}$ is the shear rate and m and n are constants that represent the viscosity at unit shear rate and the power law index respectively. If n equals one, the material is Newtonian. In most polymer systems the viscosity decreases with increasing shear rate and thus n is less than one.

Consider the fully-developed isothermal flow of an incompressible power law fluid through a tube of length L and radius R . Neglecting entrance and exit effects, the mass flow rate is given by

$$M = \left(\frac{p_o - p_a}{2mL} \right)^{\frac{1}{n}} \frac{\rho n \pi R^{\frac{3n+1}{n}}}{3n+1} \quad (1)$$

where $p_o - p_a$ is the total pressure drop and ρ the constant fluid density. If the solution foams before leaving the tube, this formula is only valid for that portion of the tube which contains the original solution and thus p_a and L must be replaced by the foaming pressure p_f and the "solution length" L_s as described in Figure 1.

Figure 1

$$M_s = \left(\frac{p_o - p_f}{2m_s L_s} \right) \frac{1}{n_s} \frac{\rho_s n_s \pi R \frac{3n_s + 1}{n_s}}{3n_s + 1} \quad (2)$$

The subscript s refers to those parameters that are related to the solution phase. The flow is assumed to be fully-developed up to the foaming inter-phase. An accurate prediction of the effects of the foaming process on the flow would require detailed knowledge of foaming kinetics and two additional constitutive models, one representing the pressure-volume-temperature behavior of the foam and another describing its flow characteristics. The numerical solution of a system of coupled partial differential equations would be required.

The adoption of several simplifying assumptions allows algebraic solutions for relevant parameters. First, neglect the kinetics and assume that foaming occurs instantaneously when the solution pressure is reduced to p_f . Although knowledge of the constitutive models for the foam phase is limited, two important facts can be established. First, the density of the foam is much smaller than that of the solution. Also, a general discussion of the mechanics of foams by Bikerman² suggests that their viscosity is much larger than that of the original solution. This is also true of the foams produced from SF-283. For simplicity, assume that the foam can also be modeled as an incompressible power law fluid. It will become clear that the increased viscosity and not the compressibility of the foam phase dominates the physics of flow. The mass flow rate of foam is then given by

$$M_f = \left(\frac{p' - p_a}{2m_f L_f} \right) \frac{1}{n_f} \frac{\rho_f n_f \pi R \frac{3n_f + 1}{n_f}}{3n_f + 1} \quad (3)$$

Here p' represents the downstream foaming interface pressure and L_f , the length of tube containing foam, as in Figure 1. The subscript f refers to the foam phase. The flow in the foam phase is also considered to be fully-developed throughout. The pressure drop across the foaming interface is a consequence of the instantaneous fluid expansion that occurs there. One can show from macroscopic mass and momentum balances that

$$p_f - p' = \frac{3n_f + 1}{2n_f + 1} \rho_f \langle v_f \rangle^2 \left\{ 1 - \frac{(3n_s + 1)(2n_f + 1)}{(2n_s + 1)(3n_f + 1)} \frac{\rho_f}{\rho_s} \right\} \quad (4)$$

$\langle v_f \rangle$ represents the average velocity in the foam phase.

At steady state, conservation of mass dictates that the mass flux of solution must equal that of the foam ($M_s = M_f$). Also, each portion of the tube contains either solution or foam ($L = L_s + L_f$). Using these relationships and assuming that $n_s = n_f = n$; (2), (3) and (4) can be combined to yield an expression for the volumetric flowrate of solution dispensed.

$$Q_s = \left(\frac{p_o - p_a}{2m_s L} \right)^{\frac{1}{n}} \frac{n\pi R}{3n + 1} \frac{3n + 1}{n} \left\{ \frac{\theta + a}{\theta + 1} \right\}^{\frac{1}{n}} = Q_{so} \left\{ \frac{\theta + a}{\theta + 1} \right\}^{\frac{1}{n}} \quad (5)$$

where

$$\theta \equiv \frac{p_o - p_f}{p_f - p_a}$$

$$D \equiv \frac{p_f - p'}{p_f - p_a}$$

and

$$a \equiv (1 - D) \frac{m_s}{m_f} \left(\frac{\rho_f}{\rho_s} \right)^n$$

θ represents the over-pressure ratio and D the dimensionless interfacial dissipation term which is calculated from (4). Ratios of material parameters and the dissipation term are included in a which is expected to be

much smaller than 1 on physical grounds. This flow rate expression is only valid when $p_o \geq p_f \geq p_a$. The formation of foam in the tube is responsible for the term in braces which is always less than unity since $0 < \theta < \infty$ and $a \ll 1$. The remaining term, Q_{so} , is equal to the volumetric flow rate of solution in the absence of foaming. Thus, the formation of foams always decreases the solution dispensing rate. The increased volumetric flow rate and viscosity of the foam phase relative to the original solution is responsible for the large flow resistance. The fraction of the tube that is filled with foam is given by

$$\frac{L_f}{L} = \frac{a}{a + \theta} \quad (6)$$

In Table 1, (5) and (6) are used to calculate the reduction in the solution dispensing rate relative to the foam-free case, and the tube foam content. a was set equal to 10^{-2} and 10^{-3} , which represent reasonable orders of magnitude since $\frac{\rho_s}{\rho_f}$ is usually greater than 10 and the viscosity ratio $\frac{\eta_f}{\eta_s}$ can be at least that large.

These calculations show that when the overpressure ratio is small, foam fills a significant portion of the tube and the flow is highly restricted. Whereas when θ is large, there is very little foam in the tube and the effect of foaming on the flow is negligible. It is interesting to note that if the foaming pressure is equal to the average of the reservoir and exit pressures ($\theta = 1$), there is relatively little foam in the tube ($\frac{L_f}{L} < 10^{-2}$ for $a < 10^{-2}$) yet the dispensing rate is reduced by more than half relative to the foam-free case. Thus, a little foaming can have significant consequences with respect to flow.

Table 1

Foam Fill and Flow Rate Reduction as a Function of the
Overpressure Ratio

θ	$\frac{L_f}{L}$		$\left(\frac{Q_s}{Q_{so}}\right)^n = \left\{\frac{\theta + a}{\theta + 1}\right\}^*$	
	$a = 10^{-2}$	$a = 10^{-3}$	$a = 10^{-2}$	$a = 10^{-3}$
10^{-3}	0.9091	0.5000	0.0110	0.0020
10^{-2}	0.5000	0.0909	0.0198	0.0109
10^{-1}	0.0909	0.0099	0.1000	0.0918
1	0.0099	0.001	0.5050	0.5005
10	0.001	0.0001	0.9100	0.9092
10^2	0.0001	0.00001	0.9902	0.9901

*Note that n (which is approximately one) must be known to calculate the actual reduction in the flow rate relative to the foam-free case.

Definition of Terms for Table 1

$\frac{L_f}{L}$ is equal to the fraction of foam in the tube and $\left(\frac{Q_s}{Q_{so}}\right)^n$ represents the reduction in the volumetric flow rate of solution relative to the foam-free case. The overpressure ratio, θ , is defined as $\frac{P_o - P_f}{P_f - P_a}$ and introduces the importance of the foaming pressure. a is a collection of material parameters and a foaming interface dissipation term. If a were equal to 1, foaming would have no effect on the flow rate.

The Temperature Effect on Flow Rate

Temperature-dependent physical properties can be introduced into (5) to determine the relationship between ambient thermal conditions and flow rate. The foaming pressure is related to the partial pressure of the most volatile component, Freon 12 in the case of SF-283. For simplicity, assume that the foaming pressure equals the vapor pressure of Freon 12. Then, the temperature dependence of p_f is readily obtained from the literature.³ The vapor pressure data for Freon 12 are well-represented by the Clausius-Clapeyron equation

$$p_f = p_{fo} e^{-\frac{B}{T}} \quad (7)$$

with $p_{fo} = 2168$ MPa (314500 psi) and $B = 2450^\circ\text{K}$. Considering the temperature dependence of the foaming pressure alone, (5) predicts that the flow rate decreases with increasing temperature since the overpressure ratio would decrease. But this does not preclude a maximum in the flow rate. Recall that (5) is only valid when $p_o \geq p_f \geq p_a$ which implies that the tube is partially filled with foam. If the temperature is low enough that the foaming pressure is less than atmospheric pressure, no foaming will occur and the relationship between pressure drop and flow rate is determined by the solution viscosity alone. Thus when $p_f \leq p_a$, the term in braces in (5) can be set equal to one. Under these conditions, if m_s decreases with increasing temperature and n_s remains constant, the flow rate will increase. A maximum in flow rate will occur when $p_f = p_a$ and foam just starts to form at the tube exit.

A highly temperature-dependent viscosity can lead to a maximum in the flow rate when the tube is partially full of foam. The temperature dependence of m_s was assumed to follow

$$m_s = m_{so} e^{\frac{A}{T}} \quad (8)$$

where m_{so} and A are constants. For analytical simplicity assume that n and a are insensitive to the temperature. This does not preclude that m_f also decreases with increasing temperature. Substituting (7) and (8) into (5) and rearranging

$$Q_s = C_1 e^{-\frac{A}{Tn}} \left\{ 1 - C_2 e^{-\frac{B}{T}} \right\}^{\frac{1}{n}} \quad (9)$$

where

$$C_1 = \frac{n\pi R}{3n+1} \left\{ \frac{p_o - ap_a}{2Im_{so}} \right\}^{\frac{1}{n}}$$

and

$$C_2 = \frac{p_{fo} (1 - a)}{p_o - ap_a}$$

C_1 and C_2 do not depend on temperature. The extremum temperature (which corresponds to a maximum in Q_s) is shown to be

$$T^* = \frac{B}{\ln \left\{ C_2 \left(1 + \frac{B}{A} \right) \right\}} \quad (10)$$

If $a \ll 1$, T^* only depends upon p_o , p_{fo} , A and B , the reservoir pressure.

and three parameters that describe the temperature dependence of the foaming pressure or solution viscosity. It is interesting to note that there is no dependence upon other material or geometric parameters if a is very small. The constraints on (5) require that

$$\frac{B}{\ln(P_{fo}/P_a)} < T^* < \frac{B}{\ln(P_{fo}/P_o)} \quad (11)$$

for (9) to be a valid maximum.

Whether the volumetric flow rate decreases or increases with temperature depends upon the competition between two effects:

- (1) the decreasing viscosity of the solution phase,
- (2) the increasing importance of foaming in the tube.

Note that Q_s can decrease with increasing temperature regardless of the fact that m_s and m_f also decrease. Once again, foaming in the tube dominates the physics of the dispensing operation.

The calculations in this section show that a maximum in flow rate can occur if the tube is not completely filled with foam at T^* $\left(T^* < \frac{B}{\ln \frac{P_{fo}}{P_o}} \right)$ and the maximum temperature is given by

$$T_{MAX} = \text{MAX} \left(T^*, \frac{B}{\ln \frac{P_{fo}}{P_a}} \right) \quad (12)$$

The agreement between this result and related experiments will now be considered. Although a rational prediction of a maximum in flow rate is obtained, this simple model does not yield quantitative agreement.

Comparison Between Theory and Experiment

The data of Rand and Goettsche are included in Table 2. The reservoir of SF-283 was pressurized to $p_o = 1.10$ MPa (160 psig). A $\frac{1}{4}$ -D orifice was attached to the end of a long ($L/D = 60$) 0.187-inch I.D. tube. The flow rate was then measured at different temperatures. Although orifice effects are not included in the previous model, it is believed that the qualitative results are unchanged.

Assume that p_{fo} and B are equal to 2168 MPa and 2450°K, respectively. If $a \ll 1$, A is the only remaining parameter that is necessary to determine T^* from (10). The temperature sensitivity of SF-283 is not known at this time. Rheological measurements on Stickvel P resin 224R (which does not contain Freon 12 but is otherwise similar to the 283 resin) by G. K. Baker⁴ of Bendix, Kansas City, show that A is approximately 16000°K in the temperature range of interest. These parameter values yield a T^* of 314°K (41°C), which corresponds to a valid maximum by the criteria in (11). If the foaming pressure is equal to the vapor pressure of Freon 12, foam cannot form below 246°K (-27°C) where $p_f = p_a$. Experimentally, the maximum temperature is 277°K (4°C). On the absolute temperature scale, there is about a 12% discrepancy between theory and experiment, which is reasonable in view of the assumptions used to derive (9) and the uncertainty in the evaluation of p_{fo} , B and A. Several realistic adjustments in the parameters which would improve upon the agreement could be proposed, but the absence of nozzle effects in the model still prohibit exact calculations. Thus, quantitative agreement could be fortuitous. Rand and Goettsche suggested a connection between the temperature dependence of the Freon 12 vapor pressure, foam formation and the maximum in flow rate that they observed. Their suppositions are supported by the calculations presented here.

Table 2

Flow and Density of Stickvel Sticky Foam (SF-283)
at Selected Temperatures

Temperature °C (°F)	Flow ¹ kg/min(lbs/min)	Density ² kg/m ³ (lbs/ft ³)
24.0 (75.0)	1.48 (3.27)	27.0 (1.70)
10.0 (50.0)	1.71 (3.77)	29.7 (1.86)
4.0 (40.0)	2.05 (4.52)	31.9 (2.00)
-1.0 (30.0)	0.99 (2.19)	29.5 (1.84)
-5.0 (23.0)	0.43 (0.95)	28.4 (1.77)

¹Dispensed through 0.187" I.D. tube 60 D long with
1/4 D orifice

²Per SS-323208

Some deficiencies in these calculations include:

- (1) The assumption of instantaneous expansion during the foaming process overestimates the importance of foaming.
- (2) The partial pressure of Freon 12 and thus the foaming pressure in SF-283 is probably smaller than the vapor pressure of the pure component.
- (3) The viscosity of SF-283 probably shows less temperature dependence than that of the pure resin system.
- (4) The compressibility of the foam was not considered.
- (5) Exit effects, due to the nozzle, are absent.
- (6) The constitutive assumptions that were chosen to describe the solution and foam phase are certainly oversimplifications.

Summary

A theoretical model describing the flow of a solution that foams during tube flow was developed. The larger viscosity and smaller density of the foam phase relative to the original solution are responsible for the large resistance to flow that occurs when foaming takes place. The experimental observations of P. B. Rand (5813) and K. C. Goettsche (1714) that describe a maximum in flow rate as the temperature is increased are explained by the model. The temperature-dependence of the Freon 12 vapor pressure causes the decrease in flow rate at higher temperatures. Though the viscosities of the foam and solution may decrease with increasing temperature, the higher foaming pressure causes a larger proportion of the tube to be filled with the more viscous foam. The result is a reduced mass flow rate. The practical implications of this result on the high-temperature performance of a foam delivery system must be considered by the design engineer since foaming has the greatest detrimental effect under those conditions.

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