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PLASMA LASER ACCELERATOR: LONGITUDINAL DYNAMICS, THE PLASMA/LASER
INTERACTION, AND A QUALITATIVE DESIGN

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ABSTRACT

In this paper we present our studies on a plasma laser accelerator. First we look at the longitudinal dynamics and the trapping of particles in the potential well due to the longitudinal electric field in a plasma density wave. Next we study the plasma/laser interaction to obtain power requirements. Lastly, we qualitatively design a plasma/laser accelerator with parameters somewhat more modest than existing suggestions.

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Introduction

There has been recent interest in particle acceleration with a combination of a medium (plasma) and a laser^{1,2}. The laser is used to generate a plasma wave of a particular phase velocity, v_p . The longitudinal electric field of the plasma wave is then used for acceleration.

In this context many questions arise. First, if we are given a plasma wave, we need to understand the longitudinal dynamics of a test particle in the electric field generated by the wave. This leads to questions concerning particle trapping, small oscillations and "enclosed" phase space area. Secondly, we must understand how the plasma wave is generated by the E.M. wave of the laser. What is the growth rate of the wave, and what are the power requirements on the laser to yield a plasma wave of a given amplitude? Lastly, we need to identify the parameter range of interest for high energy accelerator (say e^-) applications.

In the first section we treat longitudinal dynamics (ignoring transverse effects), in the second section we study the plasma/laser interaction, and in the last section we attempt a qualitative accelerator design. In all sections the test particle is an electron.

1) Longitudinal Dynamics

a.) The Equations of Motion

Consider a plasma density wave (ions stationary) given by

$$n_e = n_0 + n_1 \cos(kz - \omega_p t) \quad (1)$$

(n_0 & n_1 are constants, $n_1 > 0$)

From this form the phase velocity is given by

$$v_p = \omega_p / k \quad (2)$$

It is also obvious that, in this model, since $n_e > 0$

$$n_1 \max = n_0 \quad (3)$$

The electric field due to such a wave is found from Maxwell's Eq. to be

$$\vec{E} = \frac{4\pi n_1 e}{k} \sin(kz - \omega_p t) \hat{z} \quad (4)$$

We can also write the potential energy of a test particle in such a wave as

$$e\phi = \frac{4\pi n_1 e^2}{k^2} \cos(kz - \omega_p t) \quad (5)$$

The motion of a particle in this potential is obtained from Hamilton's Equations with the Hamiltonian:

$$H = e\phi(z - v_p t) + c \sqrt{m^2 c^2 + p_z^2} \quad (6)$$

where motion in the transverse degrees of freedom has been neglected. One of our tasks is to find constants of the motion which lead to curves in phase space that close and so, thereby, trap particles in the interior.

It is obvious that H is not a constant of the motion. However, it is possible to go to a new coordinate system in which the new Hamiltonian will be a constant of the motion. Consider the canonical transformation given by the generating function:

$$(z, p_z) \rightarrow (z', p_z') \\ F_2(z, p_z', t) = (z - v_p t) p_z' \quad (7)$$

which yields

$$\begin{aligned} z' &= z - v_p t \\ p_z' &= p_z \\ H' &= H - v_p p_z' \end{aligned} \quad (8)$$

This is simply a coordinate system which moves with the wave. The new Hamiltonian is

$$H' = e\phi(z') + c \sqrt{m^2 c^2 + p_z'^2} - v_p p_z' \quad (9)$$

and is a constant of the motion. Defining

$$\alpha = \frac{4\pi n_1 e^2}{m c^2 k^2} \quad (10)$$

we have

$$H' = \alpha m c^2 \cos k z' + c \sqrt{m^2 c^2 + p_z'^2} - v_p p_z' \quad (11)$$

Note that at $n_1 \text{ max} = n_0$ we have

$$\alpha_{\text{max}} = (v_p/c)^2 \approx 1 \quad (12)$$

In Eq. (11) we have not expanded the p_z' terms into a power series to simplify the problem since we anticipate the application of this Hamiltonian for particles with non-relativistic to ultra-relativistic velocities.

Comparing Eq. (11) with the Hamiltonian for a pendulum or for a particle in an accelerator, we know that part of the phase space will be enclosed and correspond to trapped motion (oscillations) while another part will not be; these are separated by a separatrix. To find the value of H' on the separatrix, simply note that the fixed points of H' lie at

$$k z' = n \pi$$

and

$$\frac{p_z'}{m\gamma} = v = v_p \quad (13)$$

Evaluating H' at the unstable fixed point yields the value

$$H'_s = m c^2 (\alpha + 1/\gamma_p) \left[\gamma_p^2 \equiv 1/(1 - v_p^2/c^2) \right] \quad (14)$$

which is the value of H' on the separatrix.

So given the initial conditions of an individual particle we can calculate H' . If $H' < H'_s$, the particle is trapped; if $H' > H'_s$, it is not trapped.

In Fig. 1 we sketch several sets of separatrices for differing values of α . The peculiar shape of the "buckets" is due to relativistic effects. Ultra relativistic particles tend to be inside the separatrix since they travel with $v \sim v_p \sim c$, while non relativistic particles tend to drop out of the bucket due to their slower velocities.

b) Emittances

It is interesting to calculate the phase space area enclosed in the trapped region. This gives us the maximum possible value of the trapped longitudinal emittance.

The area inside the separatrix is given by

$$\text{Area} = \epsilon = \oint_{\text{separatrix}} P_z(z) dz = \int_0^{2\pi/k} dz (P_+(z) - P_-(z)) \quad (15)$$

where $P_{\pm}(z)$, the upper and lower branches of the curve in phase space corresponding to the separatrix, is easily seen to be

$$\begin{aligned} \frac{P_{\pm}(z)}{m \gamma_p v_p} &= (2 \gamma_p \alpha \sin^2(kz/2) + 1) \pm \\ &\quad \frac{\gamma_p c}{v_p} \left[4 \alpha^2 \sin^4(kz/2) + \frac{4 \alpha \sin^2(kz/2)}{\gamma_p} \right]^{1/2} \end{aligned} \quad (16)$$

The integral above can be simplified considerably with the result

$$\epsilon = \frac{16 \gamma_p^2 m c \alpha}{k} \int_0^1 \left(1 + \frac{1}{\alpha \gamma_p} - x^2 \right)^{1/2} dx. \quad (17)$$

If for cases of interest α is not too much smaller than unity and $\gamma_p \gg 1$ ($v_p \sim c$), then we find

$$\epsilon \approx \frac{4\pi \gamma_p^2 \alpha m c}{k} \left(1 + \frac{1}{\alpha \gamma_p} + \dots \right) \quad (18)$$

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Fig. 1. Separatrices in phase space for different α and γ_p . Note that the wave breaking limit is $\alpha_{\max} = 1 - 1/\gamma_p^2$, and also that the nested curves correspond to different peak electric field strengths ($eE_z = \alpha mc^2/k_p$).

c) Trapping

Electrons in the plasma will be trapped by the travelling plasma wave if the initial conditions of the electrons are inside of the separatrix. In a cold plasma, the initial distribution in phase space is that of a line distribution:

$$f(p) = \frac{kN}{2\pi} \delta(p) \quad (19)$$

$$\int_0^{2\pi/k} dz \int_{-\infty}^{\infty} dp f(p) = N$$

Whether cold electrons will be trapped by the plasma wave depends on whether the separatrix touches (and crosses) the line $p = 0$; the condition for this to happen is $p_-(z) \leq 0$ at $kz = \pi$, where $p_-(z)$ is given by Eq. (16). In other words, no cold electrons will be trapped by the plasma wave if

$$p_-(\pi/k) \geq 0 \quad (20)$$

or equivalently,

$$2\alpha + \frac{1}{\gamma_p} \leq 1. \quad (21)$$

If (21) is not satisfied, the trapping fraction of electrons by the plasma wave is determined by the fraction of the line $p = 0$ which is inside of the separatrix. Thus the trapping fraction is given by

$$T_f = \frac{\pi - kz_0}{\pi} \quad \text{where } p_-(z_0) = 0 \quad (22)$$

or after some algebra,

$$T_f = 1 - \frac{2}{\pi} \sin^{-1} \sqrt{\frac{1}{2\alpha} \left(1 - \frac{1}{\gamma_p}\right)} \quad (23)$$

As α increases beyond the value in Eq. (21), the trapping fraction starts to increase from zero; as α reaches the maximum value $(v_p/c)^2 \approx 1$, the trapping fraction reaches $\approx 50\%$.

Note that the plasma density in Eq. (1) assumes that the trapped particles do not perturb the electrostatic potential well significantly. In this sense,

our calculation is only self-consistent if the trapped fraction of particles is sufficiently small. Note also that the trapping fraction applies to the longitudinal dimension only. When transverse motions are included, the actual trapping could be lower.

d) Small Amplitude Oscillations

If we inject a test particle into the plasma wave near the velocity $v \sim v_p$ and near the position $z' = \pi/k$, it will execute small oscillations at some frequency, ω_s , with no average acceleration. It is easy to show that if we let

$$x = z' - \pi/k \quad (24)$$

$$p_x = p_z' - m\gamma_p v_p$$

where x, p_x are small, then we obtain the new Hamiltonian

$$H = \text{const} + \frac{p_x^2}{2 m \gamma_p^3} + \frac{\alpha m c^2 k^2 x^2}{2} \quad (25)$$

which yields a small oscillation frequency of

$$\omega_s = \frac{\sqrt{\alpha} \omega_p}{\beta_p \gamma_p^{3/2}} \quad (26)$$

(Notice the $\gamma_p^{3/2}$ due to the relativistic longitudinal mass)*.

If we anticipate using this as an accelerator, then we, of course, should not inject particles at the stable fixed point. Rather we should inject in such a way that at the end of acceleration, the injected beam sits close to the top of the separatrix. This necessarily involves injection away from the stable fixed point.

2) The Plasma/Laser Interaction

a) The "ponderomotive force" of the EM wave

Consider a laser wave packet striking a plasma or, for simplicity, let us follow Ref. 1 and use 2 frequencies beating. Let the 2 frequencies be

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(*) Pointed out by Phil Morton

such that

$$\omega_1 - \omega_2 = \omega_p \quad (27)$$

and $\omega_1, \omega_2 \gg \omega_p$. ($\omega_1, \omega_2 \approx \omega$)

Then a plasma wave will be generated with a wave number given by

$$k_p = k_1 - k_2 \quad (28)$$

and using the dispersion relation for an EM wave in a plasma ($\omega^2 = k^2 c^2 + \omega_p^2$)

we find

$$v_p \equiv \frac{\omega_p}{k_p} \approx c \left[1 - \frac{1}{2} \left(\frac{\omega_p}{\omega} \right)^2 \right] \quad (29)$$

So the phase velocity of the plasma wave matches the group velocity of the EM wave. Because of this matching the laser can drive the plasma on resonance at its natural frequency.

To calculate the force exerted by the laser, consider a superposition of 2 plane waves such that

$$\vec{E} = \frac{E_0}{\sqrt{2}} [\sin(k_1 z - \omega_1 t) + \sin(k_2 z - \omega_2 t)] \quad (30)$$

Using the transverse gauge ($\vec{\nabla} \cdot \vec{A} = 0$) we find the vector potential

$$\vec{A} = - \frac{c E_0}{\sqrt{2}} \left(\frac{\cos}{\omega_1} (k_1 z - \omega_1 t) + \frac{\cos}{\omega_2} (k_2 z - \omega_2 t) \right) \quad (31)$$

with B given as usual by $\vec{B} = \vec{\nabla} \times \vec{A}$. The intensity of the laser is simply given by

$$I = \frac{E_0^2}{8\pi} c \quad (\epsilon \sim 1) \quad (32)$$

The force on a particle due to this field can be calculated with the Hamiltonian

$$H = \frac{(\vec{p} - e/c \vec{A})^2}{2m} \quad (33)$$

Note that this is non relativistic; this approximation is good so long as $eE_0/\omega \ll mc$ and not too bad if $eE_0/\omega \approx mc$.

To find the average force for times $\gg \frac{1}{\omega}$, we simply average over a time $2\pi/\omega$ to obtain

$$\bar{H} = \frac{\vec{p}^2}{2m} + \frac{e^2}{2} \left(\frac{\vec{A}^2}{2} \right) \quad (34)$$

or using Eq. (31) we find.

$$\bar{H} = \frac{\vec{p}^2}{2m} + \frac{e^2 E_0^2}{4 m \omega^2} \cos(k_p z - \omega_p t) \quad (35)$$

where we have retained the interference term since $\omega \gg \omega_p$, and we have dropped constant terms. The 2nd term in this Hamiltonian is just the "ponderomotive" potential for this problem. The force in z direction is thus given by

$$F_z^L = \frac{k_p e^2 E_0^2}{4 m \omega^2} \sin(k_p z - \omega_p t) \quad (36)$$

b) The Plasma Response

The simplest approach in this section is to consider the response of a cold plasma with stationary ions. In the one dimensional linear* regime it is easy to see that the perturbed electron density obeys the equation

$$\frac{\partial^2 n}{\partial t^2}(z, t) + n_0 \frac{\partial}{\partial z} \left[\frac{F_z^L}{m} + \frac{e E_1}{m} \right] = 0 \quad (37)$$

where E_1 is the field due to n_1 and n_0 is the background electron density.

Evaluating E_1 and using Eq. (36) we find

$$\frac{\partial^2 n_1}{\partial t^2} + \frac{n_0 k_p^2 e^2 E_0^2}{4 m^2 \omega^2} \cos(k_p z - \omega_p t) + \omega_p^2 n_1 = 0 \quad (38)$$

Now search for a solution of the form

$$n_1(z, t) = f(k_p z - \omega_p t) \sin(k_p z - \omega_p t) \quad (39)$$

The plasma wave is thus represented as a simple harmonic oscillator driven on resonance by the laser. Requiring $n_1 = 0$ ahead of the laser pulse, we obtain the solution

$$f = \frac{-k_p^2 e^2 E_0^2 n_0}{8 m^2 \omega^2 \omega_p^2} (k_p z - \omega_p t) \quad (40)$$

(*) Note that for the large amplitude waves we need, the actual physical situation will be neither linear nor one dimensional.

which yields a growth rate for a given z of

$$\frac{\dot{f}}{n_o} = \frac{k_p^2 e^2 E_o^2}{8 m^2 \omega \omega_p} \quad (41)$$

The time, τ , to reach a longitudinal electric field of

$$E_z = \alpha m c \omega_p / \beta_p \quad (42)$$

is then given by

$$\tau = \frac{8 m^2 \omega^2 c^2 \alpha}{e^2 E_o^2 \omega_p} \quad (43)$$

Let us summarize the picture by looking at a particular point, z , along the accelerator. We see the laser beat wave and the plasma wave going by. The plasma wave is shifted by π from the beating electric field and is growing at a rate given in Eq. (41). After the laser pulse has passed, the plasma wave remains at the amplitude in Eq. (42) provided the pulse length and coherence length of the laser is given by Eq. (43). Note that since the plasma is cold the group velocity of the plasma wave is zero, and there is no damping.

To write Eq. (43) in terms of power/energy requirements note that

$$\frac{E_o^2 c}{8 \pi} = W / \frac{\pi}{2} \sigma^2, \quad (44)$$

where W is the laser power and σ is the gaussian spot size radius. Then we find for a laser pulse length, τ , the energy requirement

$$E_{\text{Laser}} = W \tau = \frac{\alpha m^2 c^2 \omega^2}{2 e^2 \omega_p} \sigma^2 \quad (45)$$

and we simply divide by τ to obtain the power. Since it is the intensity ($\sim E_o^2$) which enters into the growth rate, the spot size is a critical factor. In section 3b we discuss spot size requirements and obtain the final formula for laser power.

3) Discussion: A Plasma/Laser Accelerator

In this section we use the results of the previous sections to put forward a qualitative design of a high energy plasma/laser accelerator. First, however, we need to discuss a few other aspects of acceleration.

a) Phase Slippage

Since the emphasis in this paper is on high energy electron acceleration, we avoid the problem of changes in particle velocity. However, since the plasma wave will travel slower than, for example, an injected high energy electron bunch, we must consider phase slippage. We certainly don't want the plasma wave to trap the electron bunch and decelerate it!

Consider a plasma wave with a velocity

$$v_p \approx \left(1 - \frac{1}{2\gamma_p^2}\right) c \quad (46)$$

where γ_p is much less than the γ of the accelerated particle. Then the velocity difference between wave and particle is just

$$\frac{\Delta v}{c} = \frac{1}{2\gamma_p^2} = \frac{1}{2} \left(\frac{\omega_p}{\omega}\right)^2 \quad (47)$$

which yields a phase slippage of

$$\delta = \left(\frac{\omega_p}{\omega}\right)^2 \frac{Lk}{2} \quad (48)$$

where L is the length of the accelerator or acceleration stage. Solving for the plasma frequency we find

$$\omega_p = \left(\frac{2\delta c \omega^2}{L}\right)^{1/3} \quad (49)$$

The plasma frequency is at our disposal; however, we don't want the phase to slip too much during acceleration. Is there an optimal phase shift δ which maximizes the acceleration for a given length L ? The acceleration gradient is given by

$$e E_z = \alpha m c \omega_p \quad (\beta_p \approx 1); \quad (50)$$

however, this must be modified by averaging over the phase slip. The best we can do is therefore

$$\begin{aligned}
 e E_z &= \alpha m c \omega_p \frac{1}{\delta} \int_{-\delta/2}^{\delta/2} \cos \theta d\theta \\
 &= \alpha m c \omega_p \frac{2 \sin(\delta/2)}{\delta}
 \end{aligned} \quad (51)$$

then using Eq. (49) we find

$$e E_z = \frac{\sin \delta/2}{\delta^{2/3}} \quad (52)$$

Due to Eq. (52) there is an optimum phase shift given approximately by

$$\delta = \frac{5\pi}{8} \quad (53)$$

which, in turn, determines the plasma frequency for a given length L.

$$\omega_p = \left(\frac{5\pi}{4} \frac{c \omega^2}{L} \right)^{1/3} \quad (54)$$

and the effective accelerating electric field:

$$e E_z = \alpha m c \omega_p \quad (.85) \quad (55)$$

b) Scaling Laws

From the previous section we know the plasma frequency. What is the final energy for an acceleration stage of length L? This is simply given by

$$E_{\max} = e E_z L = \alpha m c \omega_p L \quad (.85) \quad (56)$$

Solving Eq. (54) for the length L and using Eq. (56) we find

$$\gamma_m = E_{\max}/mc^2 = \alpha \frac{5\pi}{4} \left(\frac{\omega}{\omega_p} \right)^2 \quad (.85) \quad (57)$$

In order for Eq. (56) and Eq. (57) to be valid, the intensity of the laser must be about constant over the length of an acceleration stage. With this in mind we set the Rayleigh length,

$$R = L/2 \quad (58)$$

which yields a spot size

$$\begin{aligned}
 \sigma^2 &= R\lambda/\pi \\
 &= L\lambda/2\pi \\
 &= \frac{5\pi c^2 \omega}{4 \omega_p^3}
 \end{aligned} \quad (59)$$

where, again, Eq. (54), has been used to eliminate L. So using Eq. (45) we find the laser energy to be

$$W \tau = \frac{5\pi\alpha}{8} \frac{m^2 c^5}{e^2 \omega_p} \left(\frac{\omega}{\omega_p} \right)^3 \quad (60)$$

We have ignored depletion effects in the model used to calculate the laser/plasma interaction. To see if this is consistent let's calculate the total energy deposited in the plasma. This is simply given by the energy density times the volume, or

$$P. E. = \frac{E_z^2}{8\pi} \frac{\pi \omega^2}{2} L \quad (61)$$

Substituting from Eq.'s (50), (54) and (59) we find

$$P. E. = \frac{\alpha^2 25\pi^2}{256} \frac{m^2 c^5}{e^2 \omega_p} \left(\frac{\omega}{\omega_p} \right)^3 \quad (62)$$

So using Eq. (60) we find the fraction of energy deposited by the laser

$$\frac{P.E.}{W \tau} = \frac{\alpha}{2} \quad (63)$$

Thus for a somewhat small α , the depletion effects are moderate.

Lastly, we would like to calculate the number of particles, N, which can be accelerated. Conservation of energy yields

$$N E_{\max} = \epsilon P.E. \quad (64)$$

where $E_{\max}$, P. E. are evaluated for one stage of the accelerator and ϵ is an efficiency factor.

Then using Eq.'s (57) and (62), we find

$$N = \frac{\epsilon \alpha 5}{(.85) 128} \frac{\lambda_p}{r_e} \left(\frac{\omega}{\omega_p} \right), \quad \left(r_e = \frac{e^2}{mc^2} \right). \quad (65)$$

It is interesting to note that as we decrease the length of an acceleration stage, ω_p increases. This means higher acceleration gradients but lower energy requirements on the laser pulse! (This also decreases the number of particles

which can be accelerated). The decrease in the laser energy is due to two effects. The first is due to the resonant effect driving the plasma. From Eq. (43) the growth time

$$\tau \propto \frac{1}{E_o^2 \omega_p} \quad (66)$$

For larger ω_p , a smaller laser intensity yields the same growth time.

The second effect is due to the spot size;

$$\sigma^2 \propto \frac{1}{\omega_p^3} \quad (67)$$

For a larger ω_p , we need much less power to yield the same intensity because the spot size decreases rapidly.

c) The Basic Design

In this section we will use the results of the previous sections to put forward two qualitative designs for a high energy plasma/laser accelerator. In our view the plasma should be used as an acceleration element only and not as the source of the beam; thus, an injected beam is necessary. In addition the preceding sections suggest that a higher laser frequency is better for high energy applications. With this in mind we fix the laser to be

Nd: Glass Laser

$$\lambda = 1.06 \text{ } \mu\text{m} \quad (68)$$

$$\omega = 1.78 \times 10^{15} / \text{sec}$$

The relations derived in the preceding sections are very restrictive; if we now specify the length and number of acceleration sections, everything else is determined. So consider 2 possible e^- accelerators:

Case 1: 1 km in length, one acceleration stage

Case 2: 1 km in length, 100 acceleration stages.

$$\text{injection energy} > 10 \text{ GeV} \quad (69)$$

Note that in the multistage case we assume that optimal phasing can be achieved at each stage

The next step is to restrict α . In our case we would like to avoid trapping since this would contaminate the accelerated phase space. For a cold plasma this is accomplished if

$$\alpha \leq \frac{1}{2} \left(1 - \frac{1}{\gamma_p} \right) \quad (70)$$

or since we anticipate that $\gamma_p \gg 1$ we adopt

$$\alpha = 1/2 \quad (\text{case 1 \& 2}) \quad (71)$$

The optimal plasma frequency can be determined by Eq. (54):

$$\begin{aligned} \text{Case 1)} \quad \omega_p &= 1.55 \times 10^{12} / \text{sec}, \lambda_p = 1.2 \text{ mm} \\ \text{Case 2)} \quad \omega_p &= 7.20 \times 10^{12} / \text{sec}, \lambda_p = .26 \text{ mm} \end{aligned} \quad (72)$$

This in turn sets the density of our plasma to be:

$$\begin{aligned} \text{Case 1)} \quad n_0 &= 7.6 \times 10^{14} / \text{cm}^3 \quad (2.2 \times 10^{-2} \text{ torr}) \\ \text{Case 2)} \quad n_0 &= 1.6 \times 10^{16} / \text{cm}^3 \quad (4.7 \times 10^{-1} \text{ torr}) \end{aligned} \quad (73)$$

To drive the plasma wave above we must use the beating between 2 frequencies as suggested in Ref. 1. It is this resonance effect which forces the wave to grow throughout the entire coherence length, τ .

It is interesting to note at this point that a SLAC size bunch is just about the right size for Case 1 and only a bit too long for Case 2.

The accelerating field in each case can now be calculated from Eq. (55):

$$\begin{aligned} \text{Case 1)} \quad e E_z &= 1.1 \text{ GeV/m}, \\ \text{Case 2)} \quad e E_z &= 5.1 \text{ GeV/m} \end{aligned} \quad (74)$$

which yields a final energy for a 1 km accelerator of

$$\begin{aligned} \text{Case 1)} \quad E_f &= 1.1 \text{ TeV}, \\ \text{Case 2)} \quad E_f &= 5.1 \text{ TeV}. \end{aligned} \quad (75)$$

Now we come to the question of spot size. Applying Eq. (59) we find

$$\begin{aligned} \text{Case 1: } \sigma &= 1.3 \text{ cm} \\ \text{Case 2: } \sigma &= .13 \text{ cm} \end{aligned} \quad (76)$$

Note that these spot sizes are much larger than the optical wavelength due to the long Rayleigh length necessary for the accelerator.

Before addressing pulse length and power lets look at the total laser energy necessary to drive the plasma wave. From Eq. (60) we find

$$\begin{aligned} \text{Case 1.) } W \tau &= 8.4 \times 10^6 \text{ joules} \times 1 \text{ stage} \\ \text{Case 2.) } W \tau &= 1.7 \times 10^4 \text{ joules} \times 100 \text{ stages} \end{aligned} \quad (77)$$

Again, recall that the difference in energy is mostly due to the much smaller volume in Case 2 due to the smaller spot size.

The energy in Case 1 is large. However, if long pulse and coherence lengths can be achieved, the instantaneous power required can be reduced considerably. Of course the coherence is particularly important since the plasma wave growth is due to resonance. So we suggest to scale the coherence length with the acceleration stage length:

$$\begin{aligned} \text{Case 1.) } W &= 8.4 \times 10^{13} \text{ watts} \times 1 \text{ stage} \\ \tau &= 100 \text{ ns} \end{aligned} \quad (78)$$

and

$$\begin{aligned} \text{Case 2.) } W &= 1.7 \times 10^{13} \text{ watts} \times 100 \text{ stages} \\ \tau &= 1 \text{ ns} \end{aligned} \quad (79)$$

Of course, either of the above two cases is subject to trade offs between coherence length and power.

Lastly, lets calculate the number of particles which can be accelerated in each case. Using Eq. (65) we find

$$\begin{aligned} \text{Case 1) } N &= \epsilon \times 1.1 \times 10^{13} \\ \text{Case 2) } N &= \epsilon \times 5.3 \times 10^{11} \end{aligned}$$

The efficiency ϵ is hard to estimate; however, since the beam size must be somewhat less than the plasma wave size ϵ would probably be less than 10%.

Given the above designs the accelerators work as follows: The laser pulse is injected into the plasma with the electron bunch at its tail at the proper phase. In practice one might have to inject a long bunch and only extract the accelerated part.

The bunch is accelerated through one stage and either reinjected into the next stage (Case 2) or extracted (Case 1).

The purpose of this exercise in designing a high energy plasma wave accelerator has been to emphasize that this approach yields a "reasonable" accelerator without stretching present technology too far. The numbers seem to fall into place somewhat naturally. On the other hand there are many questions to be answered before such a scheme could be realistically contemplated.

The most important of these is the excitation of plasma waves by EM waves. Of course, much work has been done on this subject;³ however, the emphasis in past work has been mostly on the heating of already hot plasmas. We would like the wave to persist rather than "thermalize", so past results need to be reapplied to the case at hand.

There are many other unanswered questions in this type of scheme⁴; however, this simple study suggests that the plasma-laser accelerator is interesting. We therefore suggest further studies along the lines and within the parameters outlined in this brief discourse.

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