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SIMULATION OF TRANSVERSE COMBINING OF SPACE-CHARGE DOMINATED BEAMS*

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SIMULATION OF TRANSVERSE COMBINING OF SPACE-CHARGE DOMINATED BEAMS*

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ABSTRACT

Rms emittance growth in the transverse plane due to the transverse combining of four identical elliptical beams of uniform density has been investigated. The emittance growth can be related by conservation of energy to the change in the electrostatic field energy. Its dependence on initial beam positions and radii has been calculated analytically for round beams and by computer simulation for elliptical beams.

INTRODUCTION

Limitations on the charge per unit length which can be focused suggest that the ion beam for heavy ion fusion be transported as separately focused multiple beams at the low energy end of the accelerator. When the energy of the beams is large enough, it may be more cost-effective to combine several of them. In this paper the transverse combining of four such beams is considered, with the goal of selecting initial conditions which will minimize transverse rms emittance growth. Only transverse dynamics are considered -- longitudinal fields are ignored -- and the beam is assumed to be coasting, with no spread in longitudinal energy.

A system for combining beams might consist of a matching section, where the shape of the beams is tailored to minimize emittance growth during the combining process; a bending section, where the beams' centroid trajectories are deflected to decrease the distance between beams; and a merging section, where the beams "see" each other and form a single beam. In this paper we consider only the merging section. The merging is assumed to occur in a focusing lattice which may be alternating-gradient or constant focusing.

We will measure the growth of the one dimensional rms emittance using the ratio of the final emittance to the total initial phase space area in that dimension, i.e., $\epsilon_{fx}/(2\epsilon_{ix})$, where ϵ_{ix} and ϵ_{fx} are the initial single beam emittance and the final emittance in the x direction. This quantity can be written as

$$H_x = \frac{\epsilon_{fx}}{2\epsilon_{ix}} = \left(\frac{\epsilon_{ox}}{2\epsilon_{ix}} \right) \left(\frac{\epsilon_{fx}}{\epsilon_{ox}} \right), \quad (1)$$

where ϵ_0 is the emittance of the composite beam when the initial beams first see each other. The first factor in Eq. (1) is due to the fact that empty spaces between the beams fill with particles, adding to the phase

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space occupied by the beam. This "geometric" emittance dilution would be the only increase in emittance for low intensity beams. The second factor is the phase space dilution caused by space charge (conversion of electrostatic field energy to transverse kinetic energy) and, if the rms radii of the beam change, by work done by the focusing fields. Simulation has shown that because space charge couples the transverse dimensions, the final state of the beam tends to have equal emittance in the two planes. This has also been observed experimentally in the SBTE at Lawrence Berkeley Laboratory for a single space-charge-dominated beam.¹ Then if the rms beam radii are not changed during the merging, conservation of energy gives the result:

$$\frac{\epsilon_{fx}}{\epsilon_{ox}} = \sqrt{\frac{\langle x^2 \rangle \epsilon_{oy}/\epsilon_{ox}^2 + \langle y^2 \rangle - \langle y^2 \rangle \Delta U/T_{ox}}{\langle x^2 \rangle + \langle y^2 \rangle}} \quad (2)$$

and similarly for y, where U is the total electrostatic field energy per unit length (beam and surrounding vacuum), $\Delta U = U_f - U_i$, and T_{ox} is the total x kinetic energy per unit length for the four beams. This is a general result for the change in emittance when the spatial configuration of a beam is changed, and reduces to the result of Wangler² for the case of azimuthal symmetry. If the rms beam radii change during the process, the work done by the focusing fields can be added to Eq. (2). For constant focusing, for instance, with focusing force $F_x = -k_x x$, ΔU in Eq. (2) would then become $\Delta U + Nk_x \Delta a^2/2$, where "a" is the beam radius and N is the number of particles per unit length.

This paper will consider the combining of four identical beams which are upright ellipses in configuration space, and have uniform density and gaussian velocity distributions before the merge. The initial rms x and y momenta for each beam are spatially uniform, and the x and y emittances are equal. Initially (when the beams first "see" each other) the beam centroid positions are $(x,y) = (\delta_x, \delta_y)$, $(-\delta_x, \delta_y)$, $(\delta_x, -\delta_y)$, and $(-\delta_x, -\delta_y)$, with the origin at the vacuum chamber axis and the quadrupoles on the x and y axes in the case of AG focusing (see Fig. 1). The boundary condition is a perfectly conducting pipe of radius R. We consider first the simple case of four round beams of radius "a" with $\delta_x = \delta_y = \delta/\sqrt{2}$.

COMBINING ROUND BEAMS

Wangler² has indicated that the equilibrium state for a space-charge-dominated beam is likely to have a uniform density. Anderson³ has shown this profile to be a minimum of the electrostatic field energy for a one dimensional (sheet) beam. This neglects the sheath at the beam edge, but the sheath width is of the order of the Debye length, λ_D , and $\lambda_D \ll a$ for space-charge-dominated beams. We will assume then that the beam evolves after the merge to a state with uniform density. Further, since the initial configuration and assumed final beam state both follow the same rms envelope equation, we set the initial rms radius equal to its matched value, and assume the final beam will then be matched, with the same rms radius. Use of a particle-in-cell simulation code shows both

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Fig. 1. Transverse plane geometry showing four beams when they first "see" each other.

assumptions to be true only for small δ/a , $\delta/a < 2$. For larger δ/a , the rms radius of the beam oscillates and particle loss occurs. With these assumptions,

$$\Delta U = \frac{N^2 q^2}{16} \left[3 - 4 \ln \frac{1-g^8}{4} \left(\frac{a}{\delta} \right)^3 \left(1 + 2 \frac{\delta^2}{a^2} \right)^2 \right], \quad (3)$$

where $g = \delta/R$. Note that U_i , U_f , and therefore ΔU , will always be proportional to the total charge per unit length squared, regardless of the beam shape, orientation, or density profile. This is the only dependence of M on Nq . Since g^8 is generally much less than unity, ΔU essentially depends only on δ/a and Nq . M_x can now be calculated from Eqs. (1), (2), and (3), since $\epsilon_{0x}/(2\epsilon_{1x}) = 0.5 \sqrt{1+2\delta^2/a^2}$.

We can use these results to determine desirable qualities for beam merging geometries. We will keep constant the emittance of the original beams, so that $T_{0x} \propto a^{-2}$, and also the charge per unit length, in order to study scaling with geometrical factors. We might wish to hold δ/a , $\delta-a$, or δ constant for practical experimental reasons. If δ/a is kept constant, M_x increases monotonically with "a". This reflects the fact

that ΔU is unchanging, but the additional transverse kinetic energy it supplies makes a larger relative change in the total when the beam is colder. The results when the clearance between beams is held constant, to allow, for instance, for a septum in the bending section, are shown in Fig. 2

Fig. 2. One dimensional emittance growth for round beams with constant clearance, $\delta - a$, and varying "a". $(\delta - a)/R = 0.284$. $F_2 = ((\sigma^2 - \sigma_0^2)/\sigma_0^2)(\delta - a)^2/a^2$, where σ and σ_0 are the betatron phase advance per lattice period with and without space charge for a uniform beam with the same charge per unit length and x and y kinetic energy as the initial configuration.

for a typical set of parameters. Note that the monotonic decrease of M_x in Fig. 2 is not representative of its behavior at all values of the clearance, as can be seen from Eqs. (1)-(3). M_x decreases with increasing "a" if δ is held constant and "a" is increased (Fig. 3). In both of these cases, as "a" increases, ΔU and T_{ox} both decrease. Of course for constant beam radius, M_x monotonically increases with δ/a and Nq . Finally, we might wish to fix the radius of the final beam. Then if δ increases, "a" must decrease. In this case M_x increases monotonically with decreasing "a", as the space charge dilution factor decreases, while the area dilution increases.

Fig. 3. One dimensional emittance growth for round beams with constant δ . $\delta/R = 0.506$. $F_1 = ((\sigma_0^2 - \sigma^2)/\sigma^2)(\delta^2/a^2)$. See Fig. 1 for definitions of σ and σ_0 .

COMBINING FOUR ELLIPTICAL BEAMS

The two dimensional particle-in-cell simulation code SHIFTXY has been used to compute the quantity ϵ_f/ϵ_0 for the case of four elliptical beams described in the Introduction. Alternating gradient focusing was produced using a thin lens approximation. For all runs a focusing system which provided a single particle phase advance of 60° per lattice period was used. Emittance of the initial single beams was correct to give a space charge depressed phase advance of 20° per lattice period in a 60° lattice upstream of the merge. Again the attempt was made to rms-match the beam. The rms radii of the initial configuration were set equal to those of a matched uniform beam with the same charge per unit length, since this was the assumed final state. Therefore the initial $\langle x^2 \rangle / \langle y^2 \rangle$ was 2.79 for all runs. The initial value of x_{rms}/R was 0.35.

For $\delta_x/a = \delta_y/b$, where a and b are the initial beams' major and minor radii, simulation results for M_x and M_y had the values which would be calculated using the value of ΔU given in Eq. (3) for round beams, with $\delta = \sqrt{2} \delta_x$. (Runs were done for $1 \leq \delta_x/a \leq 2.5$.) Therefore the quantity $\zeta = (\delta_x/a)/(\delta_y/b)$ would seem to be of interest, as a "shape

factor" characterizing the departure of the behavior of the system from that of round beams ($\zeta = 1$). The geometry of the initial configuration is then completely specified by ζ , δ_x/a , and (x_{rms}/y_{rms}) . A brief investigation of the dependence of M_x and M_y on the first two of these parameters was done, but much remains to be studied. The results are shown in Figs. 4-6. The initial rms radii of the configuration, single beam emittances, and focusing system were held constant for all runs. Therefore as δ_x/a increased, "a" decreased, and T_{ox} increased.

As shown in Fig. 4, matching was again a problem for configurations where the beams were not almost touching. For these cases the final beam was also not uniform after ~ 100 lattice periods, even though the emittance growth seemed to have ceased. $\langle x^4 \rangle / \langle x^2 \rangle^2$ was larger than its value for a uniform beam, indicating probably some hollowing of the beam. Beam loss also occurred for $\delta_x/a \geq 2$, and increased with δ_x/a . For the case of Figs. 4 and 5, 5% of the beam was lost in 150 lattice periods. For $\delta_x/a = 2.5$, $\zeta = 1$, 13% was lost in 100 periods. Figure 5 demonstrates that, as shown computationally by Wangler² and analytically by Anderson,³ most of the emittance growth due to the change in the electrostatic field energy occurs within a fraction of a plasma period, though subsequently the x and y emittances equilibrate.

Though the range of parameters spanned was limited by computer time constraints (the small hot beams for large δ_x/a require extremely good spatial and temporal resolution) the results given in Fig. 6 indicate that, as expected, emittance growth increases with δ_x/a -- the initial

Fig. 4. x_{rms} vs. z for $\zeta = 1$, $\delta_x/a = 2$. Other parameters are given in the text.

Fig. 5. $\epsilon_{fy}/\epsilon_{oy}$ vs. z for $\zeta = 1$, $\delta_x/a = 2$. Other parameters are given in the text.

configuration should be as close as possible to a uniform beam to minimize emittance growth. $\zeta = 2$ gives lower emittance growth than $\zeta = 1$ for the same δ_x/a . As seen from a comparison of the dashed and solid curves in Fig. 6, for the x dimension this is because the space charge dilution factor is lower, while for y the area dilution factor is lower than for $\zeta = 1$. Note that $M_x = M_y$ because $\epsilon_{fx} = \epsilon_{fy}$.

MATCHING

As described above, an attempt was made to rms-match the merging beams using the rms envelope equation :

$$\frac{d^2 x_{rms}}{dz^2} + k_x x_{rms} - \frac{\epsilon_x^2}{x_{rms}^2} - \frac{q}{m\gamma^3 v_z^2} \frac{\langle x E_x \rangle}{x_{rms}} = 0 . \quad (4)$$

For the case of four round beams with $\delta_x = \delta_y$, the last term in Eq. (4) has the same value as for a uniform round beam, since in both cases $\langle x E_x \rangle = Nq/2$. Therefore, if the initial configuration were rms-matched, and the final beam were uniform, one might hope that the beam would stay matched throughout the merging process. However, for $\delta/a \geq 2$, oscillations of the rms radii were seen at all z , with an amplitude which decreased with increasing z . This demonstrates a limitation on the use of the rms envelope equation for matching during time development of the

Fig. 6. One dimensional rms emittance growth vs. δ_x/a for $\zeta = 1$ and $\zeta = 2$, computed by SHIFTXY. Circles and triangles are simulation results for $\zeta = 1$ and $\zeta = 2$ respectively. Solid lines are total emittance growth, $\epsilon_{fx}/(2\epsilon_{ix})$, while dashed lines are the space charge dilution factor only.

distribution function. Note that as the merging progressed, the emittance term in the envelope equation remained negligible compared to the space charge term, so that the changing emittance was not important in matching the beam.

For elliptical beams in an AG focusing system a similar result was found. The matched rms radii appeared to be a function of z , changing as the distribution function evolved.

CONCLUSIONS

Transverse rms emittance growth due to the transverse combining of identical uniform elliptical beams with gaussian velocity distribution and uniform x and y rms momenta has been investigated. For round beams the "space charge dilution factor" depends on δ/a and the initial beam kinetic energy, with negligible dependence on the vacuum chamber radius. For elliptical beams, simulations show that round beam results can be used to calculate emittance growth for $\delta_x/a = \delta_y/b$, i.e., $\zeta = 1$. Some results for other cases ($\zeta = 2$) have been given. The one dimensional relative emittance growth, $\epsilon_{fx}/(2\epsilon_{ix})$, for experimentally reasonable

conditions can be made ≈ 2 . For cases where the beams were not almost touching each other, matching using the rms envelope equation was shown to present a problem, since $\langle xE_x \rangle$ and $\langle yE_y \rangle$ vary with time. In such situations beam loss occurred, and the beam did not settle down to a uniform final density profile within the 100-150 lattice periods for which the simulation code was run.

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