Support-Electrode Torque

on a

Spherical Superconducting Gyroscope*

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by

Louis B. Holdeman

→ Electrical Measurements and Standards Division ✓

National Bureau of Standards

Washington, DC 20234

and

Jonas T. Holdeman, Jr.

Computer Sciences at Oak Ridge National Laboratory**

Oak Ridge, TN 37830

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I. INTRODUCTION

In 1960, L.I. Schiff observed that precise measurement of the precession of a spherical gyroscope orbiting the earth could provide a test of general relativity.¹ The current effort to implement this experiment was initiated shortly thereafter by W.M. Fairbank.² The gyroscope will be a fused-quartz sphere coated with superconductive niobium. The spinning superconducting coating generates a small magnetic field (the London field) which outside the rotor is that of a magnetic dipole and inside is uniform and parallel to the spin axis.³ The magnetic flux that this field produces in superconducting loops encompassing the rotor will change as the gyroscope precesses; the precession of the gyroscope will be measured by measuring the change in flux.⁴ Because the anticipated relativistic precession is extremely small, it is essential that no significant torques be coupled to the gyroscope through its London field.

The torque on a superconducting sphere rotating in an arbitrary magnetic field can be expressed in terms of the $\ell=1$ coefficients of the expansion of the field in spherical harmonic functions.^{5,6} In general, a boundary-value problem must be solved in order to obtain these coefficients. In a recent publication,⁶ we solved the boundary-value problems and calculated the torques for a number of configurations pertinent to the gyroscope experiment. However, an important torque that was not calculated is the diamagnetic torque produced by superconducting support electrodes. We calculate that torque here.

We consider the simplest case of a single electrode. The mixed boundary conditions for the potential in this problem lead to dual series equations of a form that was studied by W.D. Collins.^{7,8} Collins reduced the problem of determining the coefficients in the dual series equations to that of solving a Fredholm integral equation of the second kind. We transform Collins'

integral equation into an infinite set of linear equations from which the coefficients in the dual series equations can be calculated using standard techniques.

In section II we derive the dual series equations for the electrode problem. In section III we summarize Collins's approach and convert his integral equation into the set of linear equations. In section IV we present our results.

II. DERIVATION OF THE DUAL SERIES EQUATIONS

The origin of the coordinate system is the center of the gyroscope and the z-axis is the symmetry axis of the electrode as shown in Fig. 1. The angle between the spin axis $\vec{\omega}$ and the z-axis is Ω , and the x-axis is chosen to lie in the plane defined by $\vec{\omega}$ and the z-axis. The support electrode is approximated as a spherical cap of radius R' and half-angle α that is concentric with the gyroscope. The torque will be calculated in the limit of vanishingly small superconducting penetration depths, so that the boundary conditions are $\vec{B} \cdot \hat{r} = 0$ at the surface of the electrode and $(\vec{B} - \vec{B}_L) \cdot \hat{r} = 0$ at the surface of the gyroscope, where $\vec{B}$ is the magnetic field and $\vec{B}_L$ is the external London field of the gyroscope. The magnetic field must be continuous and vanish at infinity. Thus the magnetic scalar potential $\phi_<$ for $R < r < R'$ must have the form

$$\begin{aligned} \phi_< = & Mr^{-2}(\sin\Omega \sin\theta \cos\phi + \cos\Omega \cos\theta) \\ & + \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} \alpha_{\ell m} \left[r^{\ell} + \left(\frac{\ell}{\ell+1}\right) R^{2\ell+1} r^{-(\ell+1)} \right] Y_{\ell m}(\theta, \phi), \end{aligned} \quad (1a)$$

and the potential $\phi_>$ for $r > R'$ must have the form

$$\phi_> = Mr^{-2}(\sin\Omega \sin\theta \cos\phi + \cos\Omega \cos\theta)$$

where $M = -mc\omega R^3/e$, m and $e = -|e|$ are the mass and charge of an electron, R and ω are the radius and angular velocity of the gyroscope, c is the speed of light (the units are cgs Gaussian), and $Mr^{-2}(\sin\Omega \sin\theta \cos\phi + \cos\Omega \cos\theta)$ is the scalar potential of the London Field. The functions $Y_{\ell m}(\theta, \phi)$ are the spherical harmonic functions defined by

$$Y_{\ell m}(\theta, \phi) = \left[\frac{(2\ell+1)\Gamma(\ell-m+1)}{4\pi \Gamma(\ell+m+1)} \right]^{1/2} P_{\ell}^m(\cos\theta) e^{im\phi}, \quad (2)$$

where $P_{\ell}^m(\cos\theta)$ is the associated Legendre function of degree ℓ and order m . The associated Legendre functions are defined as usual by

$$P_{\ell}^m(x) = (-1)^m (1-x^2)^{m/2} \frac{d^m}{dx^m} P_{\ell}(x) \quad (-1 < x < 1), \quad (3a)$$

$$P_{\ell}^{-m}(x) = (-1)^m \frac{\Gamma(\ell-m+1)}{\Gamma(\ell+m+1)} P_{\ell}^m(x), \quad (3b)$$

where ℓ and m are positive integers and $P_{\ell}(x)$ is the Legendre polynomial of degree ℓ .

The boundary condition at the electrode gives $\partial\phi_{>}/\partial r = \partial\phi_{<}/\partial r = 0$ at $r = R'$, $0 < \theta < \alpha$, and since the magnetic field is continuous, $\phi_{>} = \phi_{<}$ at $r = R'$, $\alpha < \theta < \pi$. These boundary conditions yield the dual series equations

$$\sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} \alpha_{\ell m} R'^{\ell-1} \left[1 - \left(\frac{R}{R'} \right)^{2\ell+1} \right] Y_{\ell m}(\theta, \phi) = 2MR'^{-3} (\sin\Omega \sin\theta \cos\phi + \cos\Omega \cos\theta), \quad (0 < \theta < \alpha), \quad (4a)$$

$$\sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} \alpha_{\ell m} R'^{\ell} \left(\frac{2\ell+1}{\ell+1} \right) Y_{\ell m}(\theta, \phi) = 0 \quad (\alpha < \theta < \pi). \quad (4b)$$

Since the associated Legendre functions of each order are linearly independent functions on the interval $0 < \theta < \alpha$, it follows from the orthogonality of the functions $e^{im\phi}$ on the interval $0 < \phi < 2\pi$ that $\alpha_{\ell m}$ is real and that $\alpha_{\ell m} = 0$ for $|m| > 1$. Furthermore, equations (4) decouple into two sets of dual series equations:

$$\sum_{\ell=1}^{\infty} \alpha_{\ell 0} R^{\ell-1} \left(\frac{2\ell+1}{4\pi}\right)^{\frac{1}{2}} \left[1 - \left(\frac{R}{R'}\right)^{2\ell+1}\right] P_{\ell}(\cos\theta) = 2MR'^{-3} \cos\Omega \cos\theta \quad (0 < \theta < \alpha) \quad (5a)$$

$$\sum_{\ell=1}^{\infty} \alpha_{\ell 0} R^{\ell} \left(\frac{2\ell+1}{\ell+1}\right) \left(\frac{2\ell+1}{4\pi}\right)^{\frac{1}{2}} P_{\ell}(\cos\theta) = 0 \quad (\alpha < \theta < \pi) \quad (5b)$$

and

$$\sum_{\ell=1}^{\infty} \alpha_{\ell 1} R^{\ell-1} \left[1 - \left(\frac{R}{R'}\right)^{2\ell+1}\right] \left[\frac{(2\ell+1)(\ell-1)!}{4\pi(\ell+1)!}\right]^{\frac{1}{2}} P_{\ell}^1(\cos\theta) = MR'^{-3} \sin\Omega \sin\theta \quad (0 < \theta < \alpha) \quad (6a)$$

$$\sum_{\ell=1}^{\infty} \alpha_{\ell 1} R^{\ell} \left(\frac{2\ell+1}{\ell+1}\right) \left[\frac{(2\ell+1)(\ell-1)!}{4\pi(\ell+1)!}\right]^{\frac{1}{2}} P_{\ell}^1(\cos\theta) = 0 \quad (\alpha < \theta < \pi). \quad (6b)$$

As will be shown in section IV, both sets of equations can be put into the form of the dual series equations that were studied by Collins.

III. SOLUTION OF A SET OF DUAL SERIES EQUATIONS

Collins has developed a method for determining the unknown coefficients A_n in dual series equations of the form

$$\sum_{n=0}^{\infty} (1+H_n) A_n P_{n+m}^{-m}(\cos\theta) = f(\theta) \quad (0 < \theta < \alpha) \quad (7a)$$

$$\sum_{n=0}^{\infty} (2n+2m+1) A_n P_{n+m}^{-m}(\cos\theta) = g(\theta) \quad (\alpha < \theta < \pi) \quad (7b)$$

where H_n is a known coefficient, $f(\theta)$ and $g(\theta)$ are given functions of the variable θ , and $P_{n+m}^{-m}(\cos\theta)$ is the associated Legendre function of degree $n+m$ and order $-m$. For our purposes, we need only consider equations (7) for the case $m = 1$ and $g(\theta) = 0$. In order to solve this set of equations, Collins first solves the auxiliary equations obtained by setting $H_n = 0$, which for our case are:

$$\sum_{n=0}^{\infty} A_n^0 P_{n+1}^{-1}(\cos\theta) = f(\theta) \quad (0 < \theta < \alpha), \quad (8a)$$

$$\sum_{n=0}^{\infty} (2n+3) A_n^0 P_{n+1}^{-1}(\cos\theta) = 0 \quad (\alpha < \theta < \pi). \quad (8b)$$

This is done by adding to equation (8b) the equation

$$\sum_{n=0}^{\infty} (2n+3) A_n^0 P_{n+1}^{-1}(\cos\theta) = h(\theta) \quad (0 < \theta < \alpha), \quad (8c)$$

where $h(\theta)$ is to be determined. The sum of (8b) and (8c) gives a series that is valid over the whole range of the variable θ , and thus can be formally inverted for the coefficients A_n^0 using the orthogonality of the Legendre functions. These are then substituted into (8a) and mathematical manipulations lead to an integral equation for $h(\theta)$ which Collins solves to obtain

$$h(\theta) = -2 \tan(\theta/2) \csc\theta \frac{d}{d\theta} \int_{\theta}^{\alpha} \frac{H^0(u) \sin u \, du}{(\cos\theta - \cos u)^{1/2}} \quad (0 < \theta < \alpha), \quad (9)$$

where

$$H^0(u) = \frac{\cot^2(u/2)}{\pi} \frac{d}{du} \int_0^u \frac{f(x) \tan(x/2) \sin x \, dx}{(\cos x - \cos u)^{1/2}} \quad (0 < u < \alpha). \quad (10)$$

It is unnecessary to actually calculate $h(\theta)$ since the coefficients A_n^0 can be obtained from $H^0(u)$:

$$A_n^0 = \frac{(2\pi)^{1/2} \Gamma(n+3)}{\Gamma(n+\frac{3}{2})} \int_0^{\alpha} H^0(u) \sin^2(u/2) \cos(u/2) P_n^{(\frac{1}{2}, \frac{3}{2})}(\cos u) \, du, \quad (11)$$

where $P_n^{(\frac{1}{2}, \frac{3}{2})}(\cos u)$ is the Jacobi polynomial

$$P_n^{(\frac{1}{2}, \frac{3}{2})}(\cos u) = \frac{-2}{\sqrt{\pi}} \frac{\Gamma(n+\frac{3}{2})}{\Gamma(n+3)} \frac{1}{\sin u} \frac{d}{du} \left[\frac{\cos(n+\frac{3}{2})u}{\cos(u/2)} \right]. \quad (12)$$

A similar treatment of equations (7) [for $g(\theta) = 0$ and $m = 1$] yields the Fredholm integral equation

$$H(u) + \cot(u/2) \int_0^{\alpha} K(u, v) \tan(v/2) H(v) \, dv = H^0(u) \quad (0 < u < \alpha), \quad (13)$$

where $H^0(u)$ is given by equation (10) and the symmetric kernel $K(u, v)$ is given by

$$(14) \quad K(u, v) = \frac{1}{2} \sin u \sin v \cos \frac{u}{2} \cos \frac{v}{2} \sum_{n=0}^{\infty} \frac{\Gamma(n+3)\Gamma(n+1)H_n}{[\Gamma(n+\frac{3}{2})]^2} P_n^{(\frac{1}{2}, \frac{3}{2})}(\cos u) P_n^{(\frac{1}{2}, \frac{3}{2})}(\cos v)$$

Once $H(u)$ is determined from (13), the coefficients A_n are calculated from $H(u)$ in the same way that the coefficients A_n^0 were calculated from $H^0(u)$; i.e., by replacing $H^0(u)$ with $H(u)$ in equation (11).

We note that

$$\begin{aligned} \int_0^\alpha K(u, v) \tan \frac{v}{2} H(v) dv &= \sin u \cos \frac{u}{2} \sum_{n=0}^{\infty} \frac{\Gamma(n+1)}{\Gamma(n+\frac{3}{2})} \frac{H_n}{(2\pi)^{\frac{1}{2}}} P_n^{(\frac{1}{2}, \frac{3}{2})}(\cos u) \\ &\times \left\{ \frac{(2\pi)^{\frac{1}{2}}}{2} \frac{\Gamma(n+3)}{\Gamma(n+\frac{3}{2})} \int_0^\alpha \sin v \cos \frac{v}{2} \tan \frac{v}{2} H(v) P_n^{(\frac{1}{2}, \frac{3}{2})}(\cos v) dv \right\}, \end{aligned}$$

and comparison with equation (11) shows that the expression in the braces is just A_n . Thus if we multiply through equation (13) by

$$(2\pi)^{\frac{1}{2}} \frac{\Gamma(\ell+3)}{\Gamma(\ell+\frac{3}{2})} \sin^2(u/2) \cos(u/2) P_\ell^{(\frac{1}{2}, \frac{3}{2})}(\cos u) du$$

and integrate from $u = 0$ to $u = \alpha$, we obtain

$$A_\ell + \sum_{n=0}^{\infty} K_{\ell n} A_n = A_\ell^0 \quad (\ell = 0, 1, \dots), \quad (15)$$

where

$$K_{\ell n} = \frac{H_n}{2\pi(n+1)(n+2)} \quad (16)$$

$$\begin{aligned} &\times \{ 2 \tan \frac{\alpha}{2} [(\ell+1) \sin(\ell+2)\alpha + (\ell+2) \sin(\ell+1)\alpha] [(n+1) \sin(n+2)\alpha + (n+2) \sin(n+1)\alpha] \\ &+ (\ell+1)(n+1) \sin(\ell+n+4)\alpha - (\ell+2)(n+2) \sin(\ell+n+2)\alpha + (\ell+2)(n+1) \sin(\ell-n-1)\alpha \\ &- (\ell+1)(n+2) \sin(\ell-n+1)\alpha + [(\ell+1)(\ell+2) + (n+1)(n+2)] \left[\frac{\sin(\ell+n+3)\alpha}{\ell+n+3} + \frac{\sin(\ell-n)\alpha}{\ell-n} \right] \} \end{aligned}$$

For $\ell=n$, the above expression is valid with $[\sin(\ell-n)\alpha/(\ell-n)]$ replaced by α ; i.e., in the limit as $\ell \rightarrow n$.

$K_{\ell n}$ is not symmetric. If we multiply through equation (15) by $N_\ell \equiv \{|H_\ell|/[(\ell+1)(\ell+2)]\}^{\frac{1}{2}}$ and define $\tilde{A}_\ell \equiv N_\ell A_\ell$, $\tilde{A}_\ell^0 \equiv N_\ell A_\ell^0$, and $\tilde{K}_{\ell n} \equiv (N_\ell/N_n) K_{\ell n}$, we obtain

$$\tilde{A}_\ell + \sum_{n=0}^{\infty} \tilde{K}_{\ell n} \tilde{A}_n = \tilde{A}_\ell^0, \quad (\ell = 0, 1, \dots), \quad (17)$$

where $\tilde{K}_{\ell n}$ is symmetric. Equations (17) can be truncated and solved using standard techniques.

In previous publications,^{5,6} we showed that the torque components are given by

$$N_x = -iM(3/8\pi)^{1/2}(\alpha_{11} - \alpha_{11}^*) \cos\Omega, \quad (18a)$$

$$N_y = M(3/8\pi)^{1/2}(\alpha_{11} + \alpha_{11}^*) \cos\Omega + M(3/4\pi)^{1/2} \alpha_{10} \sin\Omega, \quad (18b)$$

$$N_z = iM(3/8\pi)^{1/2}(\alpha_{11} - \alpha_{11}^*) \sin\Omega. \quad (18c)$$

Thus after α_{11} and α_{10} are determined using the above techniques, N_y can be calculated from equation (18b); $N_x = N_z = 0$ in the coordinate system of Fig. 1 since the coefficients $\alpha_{\ell m}$ are real.

IV. RESULTS

Using the differential equation for $P_{n+1}^{-1}(\cos\theta)$,

$$(n+1)(n+2)\sin^2\theta P_{n+1}^{-1}(\cos\theta) = -\sin\theta \frac{d}{d\theta} \left[\sin\theta \frac{d}{d\theta} P_{n+1}^{-1}(\cos\theta) \right] + P_{n+1}^{-1}(\cos\theta). \quad (19)$$

the two sets of dual series equations that were derived in section II can be put into the form of equations (7). We first transform equations (5). Let

$B_\ell = \{\alpha_{\ell 0} R'^{\ell-1} [(2\ell+1)/4\pi]^{1/2} / \{(\ell+1)MR'^{-3} \cos\Omega\}$ and let $\ell = n+1$:

$$\sum_{n=0}^{\infty} (n+1)(n+2) B_{n+1} [1 - (R/R')^{2n+3}] P_{n+1}(\cos\theta) = 2 \cos\theta, \quad (0 < \theta < \alpha), \quad (20a)$$

$$\sum_{n=0}^{\infty} (2n+3) B_{n+1} P_{n+1}(\cos\theta) = 0, \quad (\alpha < \theta < \pi). \quad (20b)$$

From equations (3) we have

$$\frac{d}{d\theta} P_{n+1}(\cos\theta) = -(n+1)(n+2) P_{n+1}^{-1}(\cos\theta). \quad (21)$$

From (21) and the differential equation for $P_{n+1}^{-1}(\cos\theta)$ it follows that

$$\sin \theta P_{n+1}^{-1}(\cos \theta) = \int_0^{\theta} \sin \chi P_{n+1}(\cos \chi) d\chi. \quad (22)$$

Using (22) in (20a) and (21) in (20b) gives

$$\sum_{n=0}^{\infty} \left[1 - \left(\frac{R}{R'}\right)^{2n+3}\right] A_n P_{n+1}^{-1}(\cos \theta) = \sin \theta, \quad (0 < \theta < \alpha), \quad (23a)$$

$$\sum_{n=0}^{\infty} (2n+3) A_n P_{n+1}^{-1}(\cos \theta) = 0, \quad (\alpha < \theta < \pi), \quad (23b)$$

where $A_n = (n+1)(n+2)B_{n+1}$. These equations now have the desired form, with $H_n = - (R/R')^{2n+3}$.

We can now solve the auxiliary equations for equations (23).

Substituting $\sin \chi$ for $f(\chi)$ in equation (10) and integrating gives

$$H^0(u) = \frac{4\sqrt{2}}{\pi} \cos^3(u/2). \quad (24)$$

Substitution of $H^0(u)$ from (24) into (11) and integration gives

$$A_n^0 = \frac{1}{\pi} \left\{ \frac{n+2}{n} \sin(n\alpha) + \sin(n+1)\alpha - \sin(n+2)\alpha - \frac{n+1}{n+3} \sin(n+3)\alpha \right\}. \quad (25)$$

Next we transform equations (6). Let $C_{\ell-1} =$

$\{\ell \alpha_{\ell 1} R'^{\ell-1} [(2\ell+1)(\ell-1)!/4\pi(\ell+1)!]^{1/2} / MR'^{-3} \sin \Omega; \text{ from equation (3b),}$

$$\sum_{n=0}^{\infty} (n+1)(n+2) \left[1 - \left(\frac{R}{R'}\right)^{2n+3}\right] C_n P_{n+1}^{-1}(\cos \theta) = -\sin \theta \quad (0 < \theta < \alpha) \quad (26a)$$

$$\sum_{n=0}^{\infty} (2n+3) C_n P_{n+1}^{-1}(\cos \theta) = 0 \quad (\alpha < \theta < \pi). \quad (26b)$$

Using equation (19), equation (26a) can be replaced by

$$\sum_{n=0}^{\infty} \left[1 - \left(\frac{R}{R'}\right)^{2n+3}\right] C_n P_{n+1}^{-1}(\cos \theta) = -\frac{1}{2} \sin \theta + k_0 \tan \frac{\theta}{2} \quad (0 < \theta < \alpha) \quad (26c)$$

where $-\frac{1}{2} \sin \theta$ is a particular solution to the differential equation

$$\sin\theta \frac{d}{d\theta} [\sin\theta \frac{d}{d\theta} F(\theta)] - F(\theta) = \sin^3\theta \quad (27)$$

and $k_0 \tan(\theta/2)$ is a solution to its complimentary homogeneous equation.

The constant k_0 is determined from the auxiliary dual series equation through the requirement that $H^0(\alpha) = 0$. Substitution of $-\frac{1}{2}\sin\chi + k_0 \tan(\chi/2)$ for $f(\chi)$ in equation (10) and integration gives

$$H^0(u) = -\frac{2\sqrt{2}}{\pi} \cos^3(u/2) + \frac{k_0}{\pi\sqrt{2}} \csc(u/2) (u + \sin u). \quad (28)$$

Requiring $H^0(\alpha) = 0$ gives $k_0 = 2 \sin\alpha \cos^2(\alpha/2)/(\alpha + \sin\alpha)$. Substitution of (28) into (11) and integration gives

$$C_n^0 = -\frac{1}{2\pi} \left\{ \frac{n+2}{n} \sin(n\alpha) + \sin(n+1)\alpha - \sin(n+2)\alpha - \frac{n+1}{n+3} \sin(n+3)\alpha \right\} \quad (29)$$

$$+ \frac{2\sin\alpha \cos^2(\alpha/2)}{\pi(\alpha + \sin\alpha)} \left\{ \frac{\sin(n+1)\alpha}{n+1} + \frac{\sin(n+2)\alpha}{n+2} - \frac{(\alpha + \sin\alpha)}{\cos(\alpha/2)} \cos(n + \frac{3}{2})\alpha \right\}.$$

Using equations (25) and (29), the torque can now be calculated from equations (17) and (18b). The results can be written in the form

$$N_y = (2\pi)^{-1} M^2 R'^{-3} \sin 2\Omega G(\alpha, R/R'), \quad (30)$$

where $G(\alpha, R/R') = \pi(A_0^0 + 2C_0^0)$. An analytic expression for $G(\alpha, R/R')$ is readily obtained only for the limit $R/R' = 0$: We note that if $R = 0$, equations (1) transform into the auxiliary equations above, so the coefficients A_0^0 and C_0^0 can be substituted into equation (18b). The torque is given by equation (30) with $G(\alpha, 0) = \pi(A_0^0 + 2C_0^0) \equiv F(\alpha)$, where

$$F(\alpha) = 4 \sin^3\alpha - \frac{(2\alpha - \sin 2\alpha)(2\sin\alpha + \sin 2\alpha)}{2(\alpha + \sin\alpha)}. \quad (31)$$

(This result gives the diamagnetic torque that would be exerted on a point dipole of magnetic moment M located at the center of curvature of a spherical superconducting shell of radius R' .)⁶ The function $F(\alpha)$ is plotted in Fig. 2. For other values of R/R' , $G(\alpha, R/R')$ was calculated numerically; the results

of these calculations are presented graphically in Figs. 3 & 4.

Because of the closeness of the electrodes to the gyroscope, the torque due to an actual six-electrode support system will be given to a very good approximation by the vector sum of the torques calculated as above for each electrode independently. It is planned that the spin axis of the gyroscope be aligned through a corner of the cube of symmetry defined by the six electrodes, in which case the net torque for a perfect electrode system would be zero.

For the planned alignment, $\sin 2\Omega = 2\sqrt{2}/3$. For $\alpha = 0.5236$ radian, specific numerical values for G are 6.99 for $R/R' = 0.996$ and 13.37 for $R/R' = 0.998$.[†] For $R = 1.9$ cm and $R' - R = 38$ μ m, $R/R' = 0.998$. Assuming $\omega = 400\pi$ s⁻¹ (200 Hz), equation (30) gives $N_y = 7 \times 10^{-15}$ Nm for the magnitude of the torque due to each electrode, a value that is 3500 times larger than the design goal for the residual torque (2×10^{-18} Nm). This result, when considered in conjunction with the estimated magnitude of torques due to magnetic flux that might be trapped in superconducting electrodes,⁹ suggests that it is not unreasonable to consider normal-metal electrodes as a possible alternative.

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[†]These values were obtained by truncating equations (17) at 36, 72, and 144 terms and extrapolating quadratically to the infinite limit.

REFERENCES

1. L.I. Schiff, Proc. Natl. Acad. Sci. U.S.A. 46, 871 (1960).
2. C.W.F. Everitt, W.M. Fairbank, and L.I. Schiff, in The Significance of Space Research for Fundamental Physics (European Space Research Organization, Interlaken, Switzerland, 1969), ESRO Special Publication SP-52, p. 33.
3. F. London, Superfluids, Vol. I: Macroscopic Theory of Superconductivity (Dover, New York, 1961), section 12.
4. J.T. Anderson and C.W.F. Everitt, IEEE Trans. Magn. MAG-13, 377 (1977); J.B. Hendricks, IEEE Trans. Magn. MAG-11, 712 (1975).
5. L.B. Holdeman, Magnetic Torque on a Rotating Superconducting Sphere, NASA Technical Report TR-443 (NASA, Washington DC, 1975).
6. L.B. Holdeman and J.T. Holdeman, Jr., J. Appl. Phys. 47, 4943 (1976).
7. I.N. Sneddon, Mixed Boundary Value Problems in Potential Theory (Wiley, New York, 1966), Chapt. 5.
8. W.D. Collins, Proc. Camb. Phil. Soc. 57, 367 (1961).
9. L.B. Holdeman, NBS Interagency Communication, NASA G.O. #H-27908B, 20 Feb. 1980, p. 11 (unpublished).

FIGURE CAPTIONS

Figure 1: Coordinate system for calculating the diamagnetic torque on the gyroscope due to a superconducting support electrode.

Figure 2: Plot of the function $F(\alpha) = 4\sin^3\alpha - \frac{(2\alpha - \sin 2\alpha)(2\sin\alpha + \sin 2\alpha)}{2(\alpha + \sin\alpha)}$.

The torque on a point magnetic dipole of moment $\vec{M}$ at the center of a spherical superconducting shell of radius R' and half-angle α is given by $N = (2\pi)^{-1} M^2 R'^{-3} \sin 2\Omega F(\alpha)$, where Ω is the angle between $\vec{M}$ and the symmetry axis of the shell.

Figure 3: Plot of the function $G(\alpha, R/R')$ as a function of α for different values of R/R' .

Figure 4: Plot of the function $G(\alpha, R/R')$ as a function of α for different values of R/R' .