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PREDICTION OF THE ENTROPY
PRODUCTION AND PRESSURE LOSSES
IN TWO-PHASE FLOW FROM THE
MIXING LENGTH THEORY

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Prediction of the Entropy Production and
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by

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This report examines the case of vertical two-phase flow under the light of the mixing-length theory which was successfully applied to turbulent flows. The purpose of the analysis is to obtain the pressure distribution along the geothermal wells. The well is modeled as a vertical pipe carrying a fluid of variable density; the density distribution is described by two parameters. The conservation equations are written in boundary layer coordinates and the pressure losses are computed through the entropy production function. The two-phase friction factor is defined and calculated and through this the pressure distribution of the geothermal well is obtained. Finally a comparison is made of the results emanating from this model to other theoretical predictions and known experimental data.

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1. Geothermal well flow.

Geothermal power, although a curiosity twenty years ago, is advocated now as one of the viable alternative energy sources for the future. The number of geothermal power plants is increasing at a fast rate and it is estimated that by the year 1985 there will be 3500 MW of installed capacity in the United States alone.

The geothermal plants utilize the high temperature, high pressure hot water resources enclosed in pockets of permeable rock under the earth's surface at depths of 1-3 km. Under these conditions the water substance exists as a solution of various salts (NaCl , KCl , CaCl_2 , CaCO_3) and gases (CO_2 , H_2S , N_2 , CH_4) at a pressure higher than the saturation pressure of the solution. As a result, the early stages of the well flow are characterized by a one-phase flow regime, up to the "flashing level", where the prevailing static pressure becomes equal to the saturation pressure of the solution and flashing occurs, thus forming a two-phase flow regime, which is characterized by bubble formation (bubbly flow). At the upper stage of the well the flow pattern usually changes to slug, froth and eventually to annular two-phase flow.

For the estimation of the available power from a geothermal well, it is necessary to know the mass flow rate of the geothermal fluid and its condition at the wellhead, which is characterized by the temperature and the saturation pressure (two-phase mixture). The wellhead conditions could be determined satisfactorily if the bottomhole conditions were known accurately and, in addition, there were reliable analytical methods for the calculation of the variation of pressure in the one- and two-phase flow regimes. One-phase flow is well understood physically and existing theories provide accurate determination for the pressure varia-

tion, but this is not the case in the two-phase flow regimes, where the results obtained by various theories differ by as much as 80% and no reliable quantitative prediction is possible. At the upper stages of the well choking occurs often and places a limit on the amount of fluid that can be drawn out of the well. The power produced by the geothermal plant is directly proportional to the mass flow rate of the brine. Given that the choking conditions depend on the thermodynamic state of the fluid, it follows that a sufficiently accurate determination of this state is imperative for the prediction of the power plant output. This requires adequate knowledge of the two-phase flow phenomena and the development of reliable flow models. Some of the two-phase flow theories will be briefly presented in the next section together with a new one, proposed by the authors.

2. Two-phase flow theories.

The first reliable theory of two-phase flow was developed by Martinelli in 1944, who proposed a correlation for the pressure drop in horizontal pipes for a two-component (air-water) fluid. He made use of similarity laws to bypass the difficulties encountered by the existence of various flow regimes, and did not consider the flow structure or the local variation of quantities. The correlation was successful in engineering applications and was used under conditions beyond the range of its applicability (vertical flows) with various degrees of accuracy.

The fact that the Martinelli correlation is inaccurate in many cases of two-phase flow gave rise to the development of other models, less empirical. Additional data provided more information about two-phase flows and the second generation of models appeared, which can be characterized as separated flow models because they take an average

velocity for each phase. Levy, who contributed his own model to this category, calls them lumped systems; the "slip" is introduced in these theories to explain the difference in the velocities of the two phases.

The lumped models are not accurate either, because of their averaging procedure. Although they give good prediction for some cases, they fail in others since their deviations from experiment exceed 80% in some cases. They also require additional assumptions for the two-phase flow pressure loss, which must come from experimental data. Their major deficiency though, lies in the fact that they do not describe the local variation of density or velocity and even if they successfully predict the pressure distribution for some length of pipe, they fail to account for phenomena which depend on the local quantities.

Accounting for local variations is the characteristic of the flow pattern models, where the flow equations for the two phases are solved for the case of predetermined flow pattern structures (bubbly, slug, annular). However, the resulting equations for two-phase flow are very complicated and require the adoption of special assumptions which are difficult to justify. Another serious setback of these models is that they require an additional assumption for the pressure drop over the pipe length. Models in this category include Griffith and Wallis's for slug flow, Levy's for annular flow and Bankoff's for variable density. A model for geothermal well flow is currently being developed at Brown University, based on these flow pattern solutions of the flow equations and empirical correlations for the void fraction of the flow.

The mixing length theory model for the two-phase flow appeared in 1962 by Levy. He applied the theory which was so successful in one-phase turbulent flows developed by Prandtl. He takes the shear stress

to be

$$\tau = \ell_u^2 \frac{d\bar{u}}{dy} \quad (1)$$

with

$$\ell_u = K_1 y [1 - \exp(-y/F_1)] \quad (2)$$

and a density distribution characterized by a density mixing length (equal to ℓ_u) and the density distribution equation

$$\bar{\rho} = A \bar{u} + B \quad (3)$$

This theory is a significant contribution to two-phase flows and to the understanding of the localized phenomena, and gives qualitatively accurate results. Quantitative accuracy is believed to be achieved with a better choice of velocity and density distributions, a task which requires extensive experimentation.

The application of the above model is restricted to horizontal pipes only, since no provision for gravity effects were made. For this reason it is inapplicable to the case of geothermal wells, where the gravity effects play an important role. A new approach to the subject of two-phase vertical flow is proposed in the following sections based on the mixing length theory and introducing entropy as the main variable, instead of pressure. This model will give the localized time average conditions in the well and its special features are further believed to give answers to two of the major problems encountered in the geothermal wells: the pumping of the well and the deposition of salts at the wall of the well.

3. Model of the geothermal well.

The geothermal well is assumed here as a long vertical pipe carrying pure water substance; this assumption about the geothermal fluid is justified because in the majority of the

wells the fluid is a very dilute solution. All the flow quantities of interest are taken as time averaged quantities, an assumption which is also made in the lumped models. The flow conditions at the bottom of the well are known by geophysical methods and the wellhead conditions need to be predicted. Figure (1) shows a short element of the well between states (1) and (2), and the relevant coordinate axes r and z . Here, P is the prevailing static pressure, r_0 the radius of the pipe and τ_0 the frictional stress at the wall. The well is furthermore assumed adiabatic and unperforated.

Fig. 1. An element of the geothermal well.

The height of the well is much greater than the diameter ($L \gg r_0$). The flow in the well can be assumed one dimensional in the z direction only ($u_r=0$).

Application of these two facts in the boundary layer equations for turbulent pipe flow will yield the following equation:

$$-\frac{dP}{dz} + \frac{\partial}{\partial r} \left[\nu \frac{\partial u_z}{\partial r} - u'_r u'_z \right] + \bar{\rho} g = 0 \quad (4)$$

and

$$P = P(z) . \quad (5)$$

where u_z and u_r are the time-average vertical and radial components of velocity, the bar (-) denotes time averages and the prime (') turbulent fluctuations, ν is the kinematic viscosity and g the gravitational acceleration. The physical significance of the above equation lies in the fact that the radial pressure distribution is constant and the static pressure becomes a function of the height only; the acceleration terms of the momentum equation disappear in eqn. (4) because the continuity equation is applied in the one-dimensional flow and the term $\frac{\partial \rho}{\partial z}$ is taken to be negligible, given the size of the pipe length.

It is always assumed throughout the discussion that the turbulent flow regime is established in the pipe and we can take time averages of the quantities involved to determine the spatial distribution of density and velocity. Under these conditions, the conservation equations for mass and momentum (in terms of stresses) take the following forms:

$$\int_0^{r_0} \rho u r dr = \dot{m} = \text{const.} , \quad (6)$$

$$r\tau + \frac{1}{2} r^2 \frac{dP}{dz} + g \int_0^r \rho r dr = 0 , \quad (7)$$

where u is now the time averaged velocity in the z direction, ρ the time averaged density, τ the stress and g the gravitational acceleration. The second equation is in effect a mechanical equilibrium equation with the absence of the acceleration terms and may be transformed into the following form:

$$\tau = \frac{r}{r_0} \left[\tau_0 + \frac{1}{2} g r_0 \rho_0 \left(G(1) - G\left(\frac{r}{r_0}\right) \right) \right] , \quad (8)$$

where

$$\tau_o = -\frac{r_o}{2} \frac{dP}{dz} - \frac{g}{r_o} \int_0^{r_o} \rho r dr \quad , \quad (9)$$

and

$$G \left(\frac{r}{r_o} \right) = 2 \left(\frac{r_o}{r} \right)^2 \int_0^{r/r_o} \frac{\rho}{\rho_\ell} r dr \quad , \quad (10)$$

It is evident that for horizontal (absence of gravity term) or constant density flow the above equation gives a linear shear stress variation across the pipe.

Now τ_o is the stress at the wall ($r=r_o$) and ρ_ℓ the density of the liquid phase (in contrast to ρ which is the local time averaged density of the two-phase mixture)*. It will be seen that this form of the momentum equation, which corresponds to the usual assumptions in the analysis of frictional pipe flows, makes the computations easier compared to other forms containing the acceleration term.

4. The choice of the density function.

It was observed from experimental data that the average density in two-phase flows is not constant across and along the pipe. Evaporation of the liquid phase accounts for the differences of the density profiles along the pipe; three dimensional effects and forces of aerodynamic nature are responsible for the higher concentration of bubbles near the center of the pipe and the apparent minimum of the density function there. As a result of this, the walls of the pipe appear always wetted, while the area near the center of the pipe carries most of the vapor phase with some entrained droplets. This phenomenon

*It should be noted that henceforth for simplicity the bar (-) is dropped from the time averaged quantities. It must not be forgotten though that now P , u and ρ represent such averages.

suggests that the function ρ for the time averaged density distribution must be monotonically increasing with r and attain the value ρ_ℓ at the wall ($r=r_0$), the shape of the curve being deduced from experimentation. Available experimental results suggest a power variation of the density function and the following expression is taken for this study as the most suitable:

$$\rho = \rho_\ell [1 + \beta(x^n - 1)] \quad , \quad (11)$$

where

$$x = \frac{r}{r_0} \quad . \quad (12)$$

At $x = 0$ (center of the pipe), the density function attains its lowest value $\rho_{\min} = (1 - \beta)\rho_\ell$. Near the flashing level β is small and $\rho_{\min}$ is close to ρ_ℓ . At the upper stages of the pipe though β becomes higher and the minimum density is closer to the vapor density. The highest value β may attain is $(1 - \rho_v/\rho_\ell)$ when the density at the center of the pipe becomes exactly equal to the density of the vapor at the prevailing temperature. In fact, β does not attain this value because of the presence of droplets at the center of the pipe (entrainment). In this case the maximum value β may attain is determined by the local dryness fraction of the steam x^* . The density at the center of the pipe is given by the following equation:

$$\frac{\rho_c}{\rho_\ell} = \frac{1}{1 - x^* + x^* \frac{\rho_\ell}{\rho_v}} = 1 - \beta_{\max} \quad ,$$

and

$$\beta_{\max} = \frac{1 - \frac{\rho_v}{\rho_\ell}}{1 + \frac{1 - x^*}{x^*} \frac{\rho_v}{\rho_\ell}} = \left(1 - \frac{\rho_v}{\rho_\ell}\right) \phi^* \quad , \quad (13)$$

where ϕ^* is the local void fraction and is less than unity.

The other parameter n in eqn. (11) is a positive number for which available experimental data suggest a value greater than 5. One of the two parameters β and n must be determined from experiment and the other can be calculated from the continuity equation. The resulting function for the density distribution is depicted in fig. 2 where curves of ρ/ρ_l are shown as functions of position, varying the parameter n from 1 to 15 and keeping β constant at 0.8.

The void fraction ϕ is defined as the fraction of the elemental volume occupied by the vapor phase and is an important parameter in the two-phase flow calculations. According to the choice of density function described above, the local void fraction is given as:

$$\phi = \beta \frac{1-x^n}{1-\rho_v/\rho_l} \quad , \quad (14)$$

and

$$\rho = (1-\phi) \rho_l + \phi \rho_v \quad . \quad (15)$$

Then eqn. (10) can be written as:

$$G(x) = 1-\beta \left[1 - \frac{2}{n+2} x^n \right] \quad , \quad (16)$$

$$\text{and } G(0) = 1-\beta \quad , \quad (17-a)$$

$$G(1) = 1-\beta \frac{n}{n+2} \quad . \quad (17-b)$$

The spatially averaged density ρ_s is given by the following equation:

$$\rho_s = \int_0^1 \rho x dx$$

or

$$\frac{\rho_s}{\rho_l} = G(1) = 1-\beta \frac{n}{n+2} \quad . \quad (18)$$

Because of the different average velocities of the two phases, this quantity differs from the discharge density $\bar{\rho}$ of the pipe which will be

Fig. 2. Density profiles.

defined later.

The above equations can now be substituted into the mechanical equilibrium equation (7) to give an expression for the rate of change of pressure along the pipe,

$$-\frac{dP}{dz} = 2 \frac{\tau_0}{r_0} + g \rho_L G(1) = 2 \frac{\tau_0}{r_0} + g \rho_S ; \quad (19)$$

this will be related to the entropy production or dissipation, which is the primary target of the study (section 8).

5. The mixing length theory.

Prandtl developed the original mixing length hypothesis under the assumption of rectangular flow with velocity u parallel to the x axis and zero time average transverse components v and w ,

$$u = \bar{u}(y) , v = \bar{v} = 0 , w = \bar{w} = 0. \quad (20)$$

Although the time average transverse velocities vanish, their fluctuations v' and w' are different than zero and the Reynold's stresses

$$\tau_t = -\rho u'v' , \quad (21)$$

remain different than zero.

Someone may visualize aggregates of liquid particles (lumps) moving from one lamina to the other transversely in the flow field and carrying with them their original momentum. Figure 3 shows two such lumps of fluid coming from distances $y-l$ and $y+l$ which coalesce at the point y . The lump from point $y-l$ has velocity $u(y-l)$ and after transverse movement through length l reaches the streamline at y with $u' > 0$, where the velocity is $u(y)$.

Fig. 3 Velocity distribution near the pipe wall.

The difference of the two velocities is:

$$\Delta u_1 = u(y) - u(y-l) \approx l \left(\frac{du}{dy} \right)_y . \quad (22)$$

Similarly, another lump of fluid traverses from $y+l$ to y with $u' < 0$ and the difference of the two velocities is approximately

$$\Delta u_2 = u(y+l) - u(y) \approx l \left(\frac{du}{dy} \right)_y . \quad (23)$$

The velocity differences caused by this transverse motion can be regarded as the turbulent fluctuation of the flow. Hence, the time average of this fluctuation in absolute value is:

$$|u| = \frac{1}{2} \left(|\Delta u_1| + |\Delta u_2| \right) = l \left| \left(\frac{du}{dy} \right)_y \right| . \quad (24)$$

In light of eqn. (24) the mixing length may be interpreted as the distance in the transverse direction, which must be covered by an agglomeration of fluid particles traveling with its original mean velocity, in order to make the difference between its velocity and the velocity

in the new lamina equal to the mean transverse fluctuation in turbulent flow.

The two lumps of fluid at the lamina y will either coalesce (if the slower precedes the faster) or have a relative velocity equal to $2|u'|$ (if the faster precedes). In both cases they give rise to a transverse component v' of velocity which must be of the same order of magnitude but of different sign (because lumps with the higher velocity $u(y+\ell)$ move with negative transverse velocity u' towards the wall and lumps with lower velocity $u(y-\ell)$ move with positive v').

Hence we can write for the time averages:

$$|v'| = c \cdot |u'| = c \cdot \ell \left(\frac{du}{dy} \right), \quad (25)$$

where c is a negative constant. We can incorporate this constant in the mixing length parameter since this is unknown also and write for the Reynold's stress

$$\tau = -\rho (u'v') = \rho \ell^2 \left| \frac{du}{dy} \right| \frac{du}{dy}, \quad (26)$$

taking into account that the sign of τ must change if the flow is reversed.

This mixing length hypothesis (Prandtl, 1925) has served in establishing a relationship for turbulent flow similar to Newton's law for laminar flow, between stress and velocity gradient. Thus, in laminar flow we have:

$$\tau_\ell = \mu \frac{\partial u}{\partial y}, \quad (27-a)$$

and in turbulent

$$\tau = A_t \frac{du}{dy}, \text{ where } A_t = \rho \ell^2 \left| \frac{du}{dy} \right|. \quad (27-b)$$

In many cases it is possible to establish a relation between the mixing length ℓ and a characteristic length of the pipe. One working

hypothesis is to take the mixing length ℓ proportional to the distance from the wall:

$$\ell = \chi y . \quad (28)$$

This is the simplest form of a relationship between ℓ and y given that ℓ vanishes at the wall of the pipe. Some other function $\ell = f(y)$ with $f(0) = 0$ could have been taken, but this would have introduced unnecessary complications.

This transforms eqn. (26) into:

$$\tau_t = \rho \chi^2 y^2 \left| \frac{du}{dy} \right| \cdot \frac{du}{dy} , \quad (29)$$

or

$$\tau_t = \rho \chi^2 \left| \frac{du}{d\eta} \right| \cdot \frac{du}{d\eta} \quad (30)$$

where

$$\eta = \ln y . \quad (31)$$

Experimentation is needed to determine the parameter χ ; it was found through many experiments that it has a constant value 0.4 in one-phase flows.

Equation (3) was applied very successfully to the study of one-phase turbulent flow along walls and free turbulent flow. The results were very favorable as to the utility and the validity of the mixing length theory and by now there is no hesitation in applying the theory for predictions.

It is believed that the application of the same principles in two-phase flows will yield equally useful results, once the parameter χ^* for the entrainment is determined experimentally.

6. Transformation of the equations into boundary layer coordinates.

It is seen that in eqn.(31) the variable $\eta = \ln y (d\eta = dy/y)$ appears as the independent variable of the equation instead of y . It is, there-

fore, advantageous in the development of the theory to transform all the equations into the so-called "boundary layer" system of coordinates which is written for the case of pipe flow:

$$\eta = \ln y / y_0 \quad (y = r_0 - r) \quad , \quad (32)$$

with

$$\eta_0 = \ln \frac{r_0}{y_0} \quad , \quad (33)$$

where y_0 is determined by the laminar sublayer and is a function of the Reynolds number and the roughness of the pipe. It was said before that the pipe wall is always wetted. Therefore, the laminar sublayer is composed of the liquid phase only and hence, one may use the experimental results of the one-phase flow studies to determine the quantity η_0 or y_0 . These results, modified for our purposes, lead to the following correlation for η_0 which was used extensively in one-phase flow and is proposed for the present calculation as well:

$$\eta_0 = \ln \frac{r_0}{k} - \ln 0.111 - \ln \left(0.3 + \frac{1}{R^*} \right) \quad . \quad (34)$$

The derivation of this form of eqn. (34) will be given in Appendix I. Here, k is the roughness of the pipe and R^* the shear Reynolds number at the boundary layer. The shear velocity V^* for this dimensionless number is defined by the shear stress at the wall as follows:

$$V^* |V^*| = \tau_0 / \rho \quad . \quad (35)$$

The characteristic length for this Reynolds number is taken as the roughness k of the pipe. With the viscosity of the liquid phase μ_l this Reynolds number is defined as:

$$R^* = \frac{V^* \rho_l k}{\mu_l} \quad . \quad (36)$$

It is seen that the shear Reynolds number is in fact the ratio of the two important lengths in the boundary layer: the roughness k and the viscous length $\mu/\rho V^*$. This quantity is a dimensionless group characterizing the flow at the turbulent boundary layer only and does not contain information about the flow outside this layer. Furthermore, it does not represent the ratio of the inertia to viscous forces as the usual definition of Reynolds number implies. The flow in the pipe is characterized by another Reynolds number Re which is defined in terms of the diameter D , the cross-sectional average velocity $\bar{u}$, and density ρ , as follows:

$$Re = \frac{\bar{u} \cdot \bar{\rho} D}{\mu} = \frac{4\dot{m}}{\pi D \mu} \quad (37)$$

In all one-phase flow applications the Reynolds number of the average flow is related to the shear Reynolds number through the friction factor f of the flow. A similar relation exists for the two-phase flow conditions: we can define the two-phase friction factor for the frictional pressure loss in the two-phase flow as follows:

$$\frac{dP}{dz} - \bar{\rho}_s g = f_{TP} \frac{\bar{\rho} \bar{u}^2}{2D} \quad (38)$$

in analogy with the single phase flow. Then eqn. (19) yields:

$$\tau_o = \frac{f_{TP}}{8} \bar{\rho} \bar{u}^2 \quad (38-a)$$

where f_{TP} is the two-phase friction factor.

Then the use of eqns. (35, 36 & 37) with the above equation yields:

$$\left(\frac{\mu_\ell R^*}{k \rho_\ell} \right)^2 = \frac{1}{8} \frac{\rho_a}{\rho_\ell} f_{TP} \left(\frac{\rho_\ell RE}{2r_o \bar{\rho}} \right)^2$$

or

$$R^* = \sqrt{\frac{f_{TP}}{32} \cdot \frac{\rho_\ell}{\bar{\rho}} RE \frac{k}{r_o}} \quad (39)$$

This equation for the two-phase flow is exactly analogous to one-phase flow equations, after taking into account the density variation in the pipe.

Under the light of the new equations the expression for η_0 becomes:

$$\eta_0 = 3.4 - \log \left[\frac{k}{r_0} + \frac{18.9}{RE \sqrt{\frac{f_{TP}}{32} \frac{\rho_L}{\rho}}} \right] , \quad (40)$$

or

$$\eta_0 = 0.46 - \log \left[\frac{0.053k}{r_0} + \frac{1}{RE \sqrt{\frac{f_{TP}}{32} \frac{\rho_L}{\rho}}} \right] . \quad (41)$$

Equation (32) can now be written in another form which will be used later, in the computational section:

$$\eta - \eta_0 = \ln \frac{y}{r_0} = \ln(1-x) , \quad (42-a)$$

or

$$x = 1 - \exp(\eta - \eta_0) . \quad (42-b)$$

In connection with vertical pipe flow another velocity, the gravitational velocity V_g , can be defined and a dimensionless number F whose significance will be apparent later:

$$V_g = \sqrt{g \frac{r_0}{2}} = \sqrt{g \frac{D}{4}} , \quad (43-a)$$

$$F = \frac{V}{V_g} . \quad (43-b)$$

The definition of the gravitational velocity is reminiscent of a unit Froude number for this flow; another Froude number based on the average velocity will appear later (eqn. 66). All the previously obtained quantities are used to simplify the expression for the stress to the following form:

$$\begin{aligned}\tau_t &= x \rho_\ell v_g^2 \left[\frac{1}{|F|F} - G(x) + G(1) \right] \\ &= \rho_\ell v_g^2 \left(1 - e^{\eta - \eta_0} \right) \left[\frac{1}{|F|F} - G(\eta) + G(0) \right]\end{aligned}\quad (44)$$

The function G can be transformed into boundary layer coordinates to make all the equations consistent in the same independent variables:

$$G(\eta) = 1 - \beta \left[1 - \frac{2}{n+2} \left(1 - e^{\eta - \eta_0} \right)^n \right]. \quad (45)$$

The quantity η_0 is an order of magnitude greater than unity, and hence $e^{-\eta_0} \approx 0$.

Then

$$G(0) \approx 1 - \beta \left(1 - \frac{2}{n+2} \right), \quad (46)$$

and

$$G(\eta) - G(0) = \frac{2\beta}{n+2} \left[\left(1 - e^{\eta - \eta_0} \right)^n - 1 \right]. \quad (46-a)$$

7. The equations for two-phase flow.

The mixing length theory and the transformation of the independent variable y onto η yields an ordinary differential equation for the velocity of the flow. The local velocity u can be nondimensionalized with reference to either the gravitational or shear velocity and the combination of eqns. (30), (44) and (46) will yield in the two cases:

$$\begin{aligned}x^2 \frac{du^+}{d\eta} \left| \frac{du^+}{d\eta} \right| &= \frac{\rho_\ell}{\rho} \left(1 - e^{\eta - \eta_0} \right) \left[\frac{v^*}{|v^*|} - F^2 \left[G(\eta) - G(0) \right] \right], \\ \text{with } u^+ &= \frac{u}{|v^*|}\end{aligned}\quad (47)$$

or

$$\begin{aligned}x^2 \frac{du_f}{d\eta} \left| \frac{du_f}{d\eta} \right| &= \frac{\rho_\ell}{\rho} \left(1 - e^{\eta - \eta_0} \right) \left[\frac{1}{|F|F} - G(\eta) + G(0) \right], \\ \text{with } u_f &= \frac{u}{v_g}.\end{aligned}\quad (48)$$

These differential equations can be solved in principle in order to give the velocity distribution function across the pipe. The boundary condition is that the velocity vanishes at $\eta=0$. The fact that the equations are nonlinear suggests that an analytic solution may not exist, but a numerical solution is, nevertheless, feasible. Once the velocity distribution is obtained, all other quantities of interest can be determined.

For the case of the pipe flow there are two average physical quantities of interest, namely the volumetric flow rate $\dot{V}$ (or equivalently the cross-sectional average velocity $\bar{u}$) and the mass flow rate $\dot{m}$ (or the mass flux j). These quantities are related through the following equations:

$$\bar{u} = \frac{\dot{V}}{A}, \quad (49)$$

$$j = \frac{\dot{m}}{A}. \quad (50)$$

The ratio of the two is the discharge density of the fluid, an expression of which was obtained in eqn. (18)

$$\bar{\rho} = \frac{\dot{m}}{\dot{V}} = \frac{u}{j}. \quad (51)$$

The average velocity u is obtained by integration across the pipe as follows:

$$\bar{u} = 2 \int_0^1 x u(x) dx, \quad (52)$$

or in the light of the boundary layer coordinates,

$$\bar{u} = \int_0^{\eta_0} \frac{du_f}{d\eta} \left(1 - e^{-\frac{n-n_0}{2}}\right)^2 d\eta = \frac{V_g}{\chi} I_1. \quad (53)$$

The average flux is given similarly:

$$j = 2\rho \int_0^1 \frac{\rho}{\rho_\ell} u \cdot x dx =$$

$$\begin{aligned}
 &= 2\rho_l V_g \int_0^{\eta_0} u_f \frac{\rho}{\rho_l} \left(1 - e^{\eta - \eta_0}\right) e^{\eta - \eta_0} d\eta = \\
 &= \rho_l V_g \int_0^{\eta_0} \frac{du_f}{d\eta} G(\eta) \left(1 - e^{\eta - \eta_0}\right)^2 d\eta = \frac{\rho_l V_g}{\chi} I_2 .
 \end{aligned} \tag{54}$$

The integrals I_1 and I_2 defined above are functions of η_0 , β , F and n ; out of these, only η_0 is known through equation (34). A relation of these integrals is then made to other quantities of the flow, such as the average discharge density and the friction factor for the flow in order to obtain a closed set of equations and determine the unknowns. Of course, the solutions of the equations is through numerical integration and involves an iteration. The relations which will help determine the unknowns are:

$$\frac{\bar{\rho}}{\rho_l} = \frac{I_2}{I_1} , \tag{55}$$

and

$$f_{TP} = \frac{8\chi^2}{F|F|} \frac{1}{I_1 I_2} . \tag{56}$$

In view of the above integrals I_1 and I_2 , the ratio of the spatially averaged to the discharge density can be written as:

$$\frac{\bar{\rho}_s}{\bar{\rho}} = \frac{I_1}{I_2} \left(1 - \beta \frac{n}{n+2}\right) . \tag{57}$$

It must be noted that the two phases move with different average velocities because of buoyancy effects. This is the cause of the difference in the two densities, the one of which ($\bar{\rho}_s$) is the average over the cross-sectional area and the other, the density obtained from the mass flux. In the case of single-phase flow there is no difference between the two and the above ratio is unity. In this case $\beta=0$ and $I_1=I_2$.

$$I_1 = I_2 = \int_0^{\eta_0} \frac{1}{F} \left(1 - e^{-\eta_0}\right)^{5/2} d\eta \approx \frac{1}{F} (\eta_0 - 1.67) , \quad (58)$$

and the friction factor for the single-phase flow is

$$f_{SP} = \frac{8X^2}{(\eta_0 - 1.67)^2} . \quad (59)$$

Now the ratio M of the two-phase to the single-phase friction factor can be given as follows:

$$M = \frac{f_{TP}}{f_{SP}} = \frac{(\eta_0 - 1.67)^2}{F|F| \cdot I_1 \cdot I_2} . \quad (60)$$

It can be seen from the above eqns. (55 through 60) that the two integrals I_1 and I_2 play an important role in the computations; all the quantities of interest can be written in terms of them and this fact is enormously helpful in the calculations.

8. Entropy production in the pipe.

It was mentioned in section 2 that the connection to the equilibrium thermodynamic quantities is going to be made through the concept of entropy production rather than the traditional method of calculating the "pressure drop". This approach is thought advantageous in a situation where equilibrium thermodynamic quantities, such as temperature and entropy, play an important role because the final objective of the geothermal projects is the determination of exergy* in geothermal fluids and its subsequent conversion into available work. The direct determination of the entropy production in the well is going to give a detailed picture of the available work losses which occur inside the well and their causes. Furthermore, questions related to

*Exergy is used to denote the maximum available work of an open system against the atmosphere.

pumping in the well or the use of a stepped diameter will be answered directly from the entropy production function. The approach to the flow parameters through entropy production will also yield a clear indication of the onset of critical flow, because the entropy is then maximized.

There are two causes of entropy production in the geothermal well: the dissipation due to the wall friction and that due to the difference of velocities between the two phases. The first is the usual type of shear at the wall met in all flows (single and two-phase). The second effect is due to buoyancy (and hence to gravity) forces acting on the lighter phase which then acquires higher velocity and lifts the heavier phase through slippage. Thus, there is an unbalanced force created between the two phases which creates the entropy production; this phenomenon is called holdup.

Once the entropy is determined at the well the pressure distribution can be deduced through the Gibbs' equations. Throughout these calculations an energy conservation equation is taken for the adiabatic well case, since the heat transfer from the well to the surroundings is negligible:

$$\frac{dh^\infty}{dz} = 0 \quad . \quad (61)$$

where $h^\infty = h + \frac{1}{2} \bar{u}^2 + \bar{\rho} z$ is the static enthalpy of the fluid at rest in the reservoir. Under these conditions the rate of change of entropy is derived as follows:

$$\frac{T d\dot{S}}{dz} = T \dot{m} \frac{ds}{dz} = -2\pi \int_0^{r_0} r \tau \frac{du}{dr} dr \quad . \quad (62)$$

Integrating the last equation by parts:

$$T_m \frac{ds}{dz} = 2\pi \int_0^{r_o} u \frac{d(\tau r)}{dr} dr \quad (62-a)$$

Using eqn. (7)

$$T_m \frac{ds}{dz} = -2\pi \int_0^{r_o} r u \frac{dp}{dz} dr - 2n \int_0^{r_o} g \rho u r dr \quad (63)$$

Substituting eqn. (19)

$$T \frac{ds}{dz} = \frac{2\tau_o}{\bar{\rho} r_o} + g \left(\frac{\bar{\rho}_s}{\bar{\rho}} - 1 \right) \quad (64)$$

It is observed that in the last equation the entropy production is composed of two terms, the first due to the friction at the wall and the second to the holdup, as explained above. This equation may be written in terms of the friction factor as follows:

$$T \frac{ds}{dz} = f_{TP} \frac{u^2}{2D} + g \left(\frac{\bar{\rho}_s}{\bar{\rho}} - 1 \right) \quad (65)$$

We can anticipate a Froude number for the two-phase flow defined as follows:

$$FR = \frac{\sqrt{gD}}{u} \quad (66)$$

and then eqn. (65) is written in its final form:

$$T \frac{ds}{dz} = g \left[\frac{f_{TP}}{2(FR)^2} + \left(\frac{\bar{\rho}_s}{\bar{\rho}} - 1 \right) \right] \quad (67)$$

9. The solution of equations.

It is observed that the evaluation of the entropy production rate involves the knowledge of the Froude number for the flow and the parameter β . Alternatively, the Froude number can be calculated from the dimensionless parameter F defined in equation (43-b) as follows:

$$FR^2 = f_{TP} \cdot \frac{F|F| \cdot \rho_l}{\bar{\rho}} \quad (68)$$

Now F and β are calculated from the integrals I_1 and I_2 and their relationships to the mass flow rate $\dot{m}$ and the density ratio $\bar{\rho}/\rho_1$ as given by equations (54) and (55). The calculation involves the numerical solution of two simultaneous nonlinear ordinary integral equations and may be accomplished with an electronic computer. The final equation for the entropy production rate is given below as a consequence of (67) and (68):

$$\frac{T}{g} \frac{d\dot{s}}{dz} = \frac{1}{F|F|} \frac{\rho_l}{\bar{\rho}} + \frac{\bar{\rho}_s}{\bar{\rho}} - 1, \quad (69)$$

with the first term giving the entropy production at the wall due to friction and the second two terms to the production due to the hold-up.

The pressure loss resulting from the entropy production, the gravitational effect and the acceleration of the component phases can now be calculated starting from the condition that the total enthalpy of the fluid is conserved:

$$h^\infty = h + gz + \frac{\bar{u}^2}{2} = \text{const}, \quad (70)$$

with $\bar{u}^2/2$ being the kinetic energy term and h the static enthalpy of the fluid. An implicit assumption is made here, that $\overline{u^2} \approx \bar{u}^2$, or that the variance of the velocity distribution is negligible compared to the square of the average velocity.

Then

$$0 = Tds + \frac{1}{\bar{\rho}} dp + g dz + c dc, \quad (71)$$

$$-\frac{dp}{dz} = \bar{\rho} \left[T \frac{ds}{dz} + g + c dc \right] \quad (72)$$

Substitution from (63) yields the final expression for the pressure change:

$$-\frac{dp}{dz} = \frac{g\rho'_l}{F|F|} + g\bar{\rho}_s + \bar{\rho}_l cdc \quad (73)$$

This expression provides a measure of the pressure gradient at every station along the geothermal well in terms of quantities that can be evaluated. In this case the static pressure may be computed numerically at any point of the well and the condition of thermodynamic equilibrium will yield all the quantities of interest such as temperature, quality, enthalpy of the fluid, etc.

The temperature of the fluid is uniquely determined by the pressure. The quality (dryness fraction) of the two-phase mixture is thus computed from the initial stagnation enthalpy h^∞ and the static enthalpy at the given point from the equation:

$$x = \frac{h^\infty - gz - \frac{\bar{u}^2}{2} - h_l(T_s)}{h_{fg}(T_s)} \quad (74)$$

where $h_l(T_s)$ and $h_{fg}(T_s)$ are the enthalpy of the liquid at the saturation temperature T_s of the given point and the heat of vaporization at the same temperature. Then any specific static extensive property ϕ of the two-phase mixture at the saturation temperature T_s is given as:

$$\phi(T_s) = \phi_l(T_s) [1-x] + \phi_v(T_s) \cdot x \quad (75)$$

where the symbols l and v denote the liquid and vapor phase correspondingly.

It was indicated above that the main key to obtain the rate of entropy production is the simultaneous solution of the two nonlinear, integral equations (54) and (55) for the values of β and F . For that matter, an iterative numerical procedure was adopted which is based on the "trial and error method". A first pair of the values of β and F is selected (F_0, β_0) and the integrals I_1 and I_2 are evaluated

with the value of n being taken constant and equal to 7. The average density of the fluid $\bar{\rho}$ is computed according to the void fraction definition

$$\bar{\rho} = \rho_L (1-\phi) + \rho_V \phi \quad , \quad (76)$$

where ϕ is estimated from the non-slip condition:

$$\phi = \frac{1}{1 + \frac{1-x}{x} \frac{\rho_V}{\rho_L}} \quad . \quad (77)$$

The values of $\bar{\rho}$ and j are computed according to equations (54) and (55) and they are compared to the values obtained from the mass flow rate in ($j = \dot{m}/A$) and equation (76). The comparison yields another pair of values for β and F (β_1, F_1) and the procedure is repeated again until the values of j and $\bar{\rho}$ obtained by the integral equations and those obtained by the mass flow rate and equation (76) agree within 2%. This error margin gave values of β and F which agreed with their actual values within 0.8%. The whole procedure is programmed on an IBM-370 computer as a sub-routine of a main program, which is designed to calculate the properties of the water substance and to compute stepwise the pressure and the other equilibrium thermodynamic quantities along the geothermal well; a flow diagram of the program is given in fig. 4 and the whole program in Appendix 2.

At high elevations the pressure of the two-phase mixture drops considerably and high dryness fraction and void fractions appear; consequently, the value of β rises and may reach its upper limit $(1-\rho_V/\rho_L)$. The above computational procedure is physically meaningless after that point and for this reason, the following method was adopted: The value of β is computed as above until it reaches a predesignated

limit $\beta_{\max}$ which may be less than $(1-\rho_v/\rho_\ell)$ due to the fact that some droplets are always present in the center of the pipe and increase the average density there, above the density of the pure vapor. When the step by step procedure reaches this limit the method is altered and keeps β constant at the $\beta_{\max}$ value while it changes n in order to solve the equations. Thus, the outcome of the procedure is now n and F and these are used for the calculation of the entropy production and the other thermodynamic quantities. The whole change in procedure corresponds physically to the fact, that due to the increased amount of steam present in the pipe the liquid phase is "pushed" towards the wall, thus being limited to a thinner region.

The velocity of the mixture is taken according to the specified mass flow rate and the discharge density:

$$\bar{u} = \frac{\dot{m}}{\bar{\rho}A} \quad (78)$$

The elevation from the flashing level is calculated from the inverse of the ratio dP/dz . The procedure takes a step of the static pressure ΔP (1 bar) and the finite change of elevation Δz is estimated from equation (73). This method has the advantage, over the alternative of choosing a step of Δz , that the variation of the thermodynamic quantities between two steps is smaller along the well for a given number of steps, and hence, the accuracy obtained for the wellhead pressure is higher. The program ends when the elevation from the flashing level is equal to the estimated depth of the two-phase region in a given geothermal well or when the flow becomes critical and the ratio dP/dz is higher than 0.5 bars/m. At this stage the specific entropy of the fluid approaches a maximum value (turning point at the Fanno line).

Fig. 4. Flow chart of the computer program.

10. Comparison of results.

The validity of the described hypotheses was tested against known results obtained experimentally at the IID-1 well (Imperial irrigation district). Furthermore, the results were checked against the two models known to exist developed by the Denver Research Institute and the Brown University group for energy studies. Both of these last models are based on the various flow patterns that appear in two-phase flow, namely bubbly, slug, froth and annular flow and are, therefore, markedly different from the assumptions made in the present work.

The prevailing bottomhole pressure and temperature at the above well is 14.9 MPa and 295°C, correspondingly. The one-phase flow (liquid) regime is calculated according to well-known equations for the friction factor of the regime and the flashing point is determined when the pressure is equal to the saturation pressure of the substance at the prevailing temperature. There are estimates for the wellhead pressure for two values of the roughness for the pipe (equation 34), namely $0.002 r_0$ and 0.00 (smooth pipe). The mass flow rate was taken 50 kg/s and a value of the void fraction at the center $\phi^* = 0.9$ was chosen.

The results are depicted in figure 5, where the two dotted curves represent the results from the two-flow pattern models (Denver and Brown) and the solid curves represent the above described two cases for the roughness. The Denver curve was estimated for the given flash point conditions from a series of prepared graphs. The encircled point represents the experimentally determined wellhead pressure from field observation. It is seen that the smooth pipe model ($k=0.00$)

agrees very well with the Brown model and both predict fairly well the wellhead pressure at the experimental well (elevation discrepancy 3.5% and 4.3% at $P_{wh}=34$ bar, or pressure discrepancy 8% at $z=820$ m).

The next figure (6) shows the two-phase flow regimes predicted from the mixing length model ($k=0.00$) and the Brown model under three different mass flow rates, namely 30, 50 and 65 kg/s. The dotted lines represent again the flow pattern model and the solid lines the mixing length model. It is seen that the agreement is fairly good with the experimental results for both the models, the pattern model having a slight advantage at high mass flow rates and the mixing length at low mass flow rates.

The last figure (7) is a cross plot of the previous one and depicts the mass flow rate against the wellhead pressure for the given depth of the well (1520 m). The three curves given are the experimental curve and the ones from the two previous models. It shows again the good agreement of the two models with the experimental values, a fact that suggests the good prediction power of both the models and confirms the usefulness of the mixing length theory model.

11. Conclusions.

It is seen from the comparison of the predictive power of the model that the mixing length hypotheses give quantitatively good agreement with actual well cases especially at medium-range mass flow rates when critical conditions are not established in the pipe yet. It must be pointed out though that the measurements in geothermal wells are subject to a high degree of uncertainty and the values obtained with this model may be inside the uncertainty limits.

The predictive power of the hypotheses is believed to increase

with the accurate knowledge of the parameter x^* , the quality of the steam at the center of the pipe as mentioned in section 4. If this parameter is known with sufficient accuracy then all other computations are more accurate and the same holds for the final result. This requires experimentation with actual one-component, two-flow cases. Another quantity whose knowledge is still obscure is the density distribution across the well. It is believed that two parameters (β and n) are sufficient to describe this distribution and that one of them must be supplied by experimental evidence. The experimental determination of one of these parameters will yield an accurate expression for the shape of the density distribution and this in turn of the velocity distribution, two quantities extremely important in the analysis of two-phase flow. In this field studies of three-dimensional flow in the wells must be conducted in order to understand better the liquid film which always surrounds the walls.

Fig. 5. Theoretical predictions of the pressure distribution for mass flow rate ($\dot{m}$) 50 kg/s.

Fig. 6. Pressure distributions for the pattern and the mixing length theory model.

Fig. 7. Wellhead pressure vs. mass flow rate.

List of Symbols

Latin

A_t	mixing coefficient	RE	Reynolds number
A	constant (Levy's eqn.)	s	entropy
B	constant (Levy's eqn.)	T	temperature
D	diameter	U	velocity
F	dimensionless number	V^*	friction velocity
F_1	constant (Van Driest eqn.)	V_g	gravitational velocity
f	friction factor	v	velocity
FR	Froude number	$\dot{V}$	volume flow rate
G	function defined in eqn. (10)	w	velocity
g	gravitational acceleration	x	dimensionless coordinate
h	enthalpy	x^*	dryness fraction
I_1	integral defined in eqn. (53)	y	coordinate (cartesian)
I_2	integral defined in eqn. (59)	z	coordinate (cartesian)
j	mass flux		
k_1	constant (Van Driest eqn.)		
k	roughness		
L	depth of well		
ℓ	mixing length		
M	ratio of friction factors		
$\dot{m}$	mass flow rate		
n	exponent in density function		
p	pressure		
R^*	shear Reynolds number		
r	radius		

Greek

β	coefficient for the density
γ	constant in eqn. (13) ^{function}
Δ	difference
η	b.l. coordinate
ϕ	dynamic viscosity
ν	kinematic viscosity
ρ	density
τ	shear stress
χ	constant

List of Symbols (cont'd.)

Superscripts

- ∞ conditions at reservoir
- $'$ turbulent fluctuations
- $-$ spatial average
- $\sim$ time average
- $+$ dimensionless velocity

Subscripts

- f dimensional velocity
- g gravitational
- l laminar
- l liquid (for properties)
- o at wall
- r radial component
- s discharge
- SP single phase
- t turbulent
- TP two-phase
- v vapor
- z axial component

Appendix I.

The velocity of the fluid adjacent to the wall is zero because of intermolecular forces (no slip condition). Therefore, the velocity function attains small values and the flow regime is laminar. At some distance from the wall the laminar contribution to the shear stress is considerably higher than the turbulent contribution and the region is called the laminar sub-layer; here the velocity profile is linear and obeys the equations:

$$u^+ = \frac{yV^*}{v_\ell} \quad (A-1)$$

At further distances a transition region develops where the turbulent effects are of the same order of magnitude with the laminar effects and even further, purely turbulent friction predominates in the flow. For the latter the velocity distribution was found from experiments to be given by the following equations:

$$u^+ = 2.5 \ln \frac{V^*y}{v_\ell} + 5.5 \quad \text{for smooth pipes,} \quad (A-2)$$

and

$$u^+ = 2.5 \ln \frac{y}{k} + 8.5 \quad \text{for rough pipes.} \quad (A-3)$$

The onset of turbulence is characterized by the values η_0 and y_0 of eqn. (33), and the two formulae above yield for y_0 :

$$y_0 = \frac{v_\ell}{V^*} \cdot 0.111 \quad (\text{smooth}) \quad (A-4)$$

$$y_0 = k \cdot 0.0334 \quad (\text{rough}) \quad (A-5)$$

The two expressions when added give an expression for commercial pipes:

$$y_0 = 0.111 k \left[\frac{1}{R^*} + 0.3 \right] \quad (A-6)$$

where

$$R^* = \frac{kV^*}{v_1}$$

It can be seen in the above equation that if $1/R^* < 0.3$, the roughness effects are predominant in the pipe which can be characterized as rough, while in the inverse case the pipe is a smooth pipe. This value of y_o when substituted in eqn. (33) will yield the appropriate equation for y_o :

$$\eta_o = \ln \frac{r_o}{y_o} = \ln \frac{r_o}{k} - \ln 0.111 - \ln \left(\frac{1}{R^*} + 0.3 \right) \quad . \quad (A-7)$$

APPENDIX II

FORTTRAN IV PROGRAM FOR THE COMPUTATIONS

11 72 *this program is composed of three main parts:the
first is the master program,contains information about the
flow,the properties of the fluid and the characteristics of
the well; the second is a subroutine designated to perform
the itteretional calculations for the determination of the
variables f and beta;the third part is a subroutine also
called by the previous one and its function is to perform
the numerical integrations for the solution of the
equations.*

REAL L1

COMMON K,BE,FR,X0,L1,RM0,A0,M

V1(T) = ((-0.6608E-10*T+0.4557E-7)*T-10.69048E-6)*T+973E-6

H1(T) = ((0.12453E-4*T-0.45137E-2)*T+4.81155)*T-29.578

HFG(T) = ((-0.3797E-4*T+0.11799E-1)*T-4.04181)*T+2582

R2(T) = P*100./((-0.1296E-2*T+0.6325)*T+121.05)

R1(T) = (-2.47524E-3*T-0.2030)*T+1003.8

RMF=50.

T=270.36

P=55.93

ACC1=0.

C0=0.

A0=1.

C=0.0

D=0.178

G=9.81

DEPTH=700.

Z=478.2

L1=7.

DS=0.1

DP=1.

ACC=0.0

H0=1316.3

RU=0.000

FR=4.63

FTP=0.02

N=0

X1=0.07754

PHI=0.6955

M=2

AR=3.14159*(D**2.)/4.

9 C=RMF*A0/(R1(T)*AR)

RD=R2(T)/R1(T)

10 RM0=RMF/(R1(T)*AR*(2.*G*D)**0.5)

RE=4.*RMF/(3.14159*D*V1(T))

A0=1./(1.-PHI+RD*PHI)

X0=3.4-ALOG(RU+18.9/(RE*(FTP*AR)**0.5))

CALL ITTE (SG,SW,A1,RM,FTP)

IF (L1.GT.100.) GO TO 120

IF (BE.GT.(0.8)) GO TO 112

11 DZ1=(DP*100000)-(R1(T)*(C-C0)/A0)

DZ2=G*R1(T)*((1.-BE*L1/(L1+2.))+1./(FR**2.))

DZ=DZ1/DZ2

IF (DZ.LT.4.) GO TO 120

Z=Z+DZ

P=P-DP

DT=-DP*1E2*(T+273.15)*(1./R2(T)-1./R1(T))/HFG(T)

T=T+DT

X1=(H0-H1(T)-G*Z/1000.)/HFG(T)

C0=C

PHI=1./((1.-X1)/X1*RD+1.)

WRITE (2,70) Z,DZ,SW,SG,RE,X0,RD,PHI

70 FORMAT (8E14.6)

WRITE (2,80) P,DP,T,DT,X1,RM,A1

80 FORMAT (7E14.6,/,)

IF (Z.GT.DEPTH) GO TO 110

GO TO 9

110 WRITE (2,176)

176 FORMAT (3X,'WELL ENDS')

GO TO 120

112 WRITE (2,86)

86 FORMAT (3X,'BMAX EXCEEDED')

M=2

GO TO 11

120 CONTINUE

STOP

END

SUBROUTINE ITTE

SUBROUTINE ITTE (SG,SW,A1,RM,FTP)

REAL L1

COMMON K,BE,FR,X0,L1,RM0,A0,M

GO TO (1,130),M

1 CONTINUE

N=0

L1=7.

AF=6.

BE=(1.-(1./A0))*(L1+2.)/L1

20 CALL SIMP (S1,S2)

RM=S2*2.5/8.**0.5

A1=S1/S2

65 FORMAT (3X,4E12.6)

IF (N.GT.20) GO TO 100

IF (ABS(A1-A0).GT.ABS(A1P-A0)) GO TO 30

A1P=A1

DA=ABS((A1-A0)/A0)

BE1=BE

IF (DA.LT.0.04) GO TO 30

25 BE=BE+(1.-BE)*(A0-A1)/(AF-A1)

GO TO 20

30 FR=FR*(RM/RM0)**0.5

CALL SIMP (S1,S2)

RM=S2*2.5/8**0.5

A1=S1/S2

IF (ABS(RM-RM0).GT.ABS(RM1-RM0)) GO TO 25

```
RM1=RM
DRM=ABS((RM-RM0)/RM0)
FR1=FR
IF (DRM.LT.0.04) GO TO 40
GO TO 30
40  DA=ABS((A1-A0)/A0)
    IF (DA.LT.0.02) GO TO 50
    GO TO 25
130  BE=0.8
140  L1=L1+1.
    CALL SIMP (S1,S2)
    RM=S2*2.5/8.**0.5
    A1=S1/S2
    IF (ABS(A1-A0).GT.ABS(A1P-A0)) GO TO 145
    A1P=A1
    GO TO 140
145  L1=L1-1.
    FR=FR*(RM/RM0)**0.5
    CALL SIMP (S1,S2)
    RM=S2*2.5/8.**0.5
    A1=S1/S2
    IF (ABS(RM0-RM).GT.ABS(RM1-RM0)) GO TO 50
    RM1=RM
    DRM=ABS((RM-RM0)/RM0)
    FR1=FR
    IF (DRM.LT.0.02) GO TO 50
    GO TO 145
```

```
50  WRITE (2,70)
70  FORMAT (3X,'SUCCESSFULL ITTERATION')
    WRITE (2,80) FR,BE,RM,A1,L1
80  FORMAT (3X,'F=',E12.6,3X,'BE=',E12.6,3X,'MR=',E12.6,3X,'A='
C,E12.6,3X,'N=',E12.6)
    SW=A1/FR**2.
    SG=A1*(1.-(BE*L1/(L1+2.)))-1.
    FTP=1.28/(FR**2.*S1*S2)
    WRITE (2,90) SW,SG,FTP,S1,S2
90  FORMAT (5X,'SW=',E12.6,3X,'SG=',E12.6,3X,'FTP=',E12.6,2E15.6,/)
100  CONTINUE
    RETURN
    END
```

SUBROUTINE SIMP

SUBROUTINE SIMP (S1,S2)

REAL L1

COMMON K,BE,FR,X0,L1,RM0,A0,M

H1(X)=1.-EXP(X-X0)

H2(X)=1./ (1.-BE* (1.-EXP(X-X0)) **L1-1.))

H3(X)=1.-BE* (1.-2./ (L1+2.) * (1.-EXP(X-X0)) **L1)

H4(X)=1./ (FR**2.) - (2.*BE/ (L1+2.)) * ((1.-EXP(X-X0)) **L1-1.)

H5(X)=(ABS(H2(X)*H4(X)*H1(X))) **0.5

H6(X)=SIGN(1.,H2(X)*H4(X))

F1(X)=H6(X)*H5(X)*H1(X)**2.

F2(X)=H6(X)*H5(X)*H1(X)**2.*H3(X)

2 CONTINUE

X=0.

S1=0.

S2=0.

5 DX=0.2

G1=DX/6.* (F1(X)+4.*F1(X+DX/2.)+F1(X+DX))

G2=DX/6.* (F2(X)+4.*F2(X+DX/2.)+F2(X+DX))

84 FORMAT (3E15.7)

A1=H1(X)

A2=H2(X)

A3=H3(X)

A4=H4(X)

A5=H5(X)

A6=H6(X)

A7=F1(X)

```
A8=F2(X)
95  FORMAT (8E14.6,/)
85  FORMAT (3X,3E15.6)
S1=S1+G1
S2=S2+G2
X=X+DX
IF (X.GE.(X0-DX-0.1)) GO TO 8
GO TO 5
8   CONTINUE
RETURN
END
```

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