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Evidence for Quarks and Gluons

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EVIDENCE FOR QUARKS AND GLUONS

by

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ABSTRACT

The evidence from the experiments of deep inelastic electron and muon scattering and of μ pair production at high energy that the neutron and proton are made of quarks and gluons is given.

I. INTRODUCTION

My talk is about quarks and gluons, but I want to begin with a warning, Hungarian-style; the statements I'm about to make may turn out to be true. The evidence seems to point that way; it's the evidence for quarks and gluons that this talk is all about.

I hold in my hand a piece of chalk. It is made entirely of quarks and gluons, aside from the usual electrons and photons. There are other things like gravitons, other leptons, different flavors of quarks, but these are embellishments, like icing on the cake. Common, everyday matter, everywhere in the world, is made of ordinary quarks and gluons, ordinary electrons and photons.

Since quarks are so important I thought it would be a good idea to begin by looking the word up in a dictionary. Here is what I found in the Random House Unabridged Dictionary:

Quark (kwork), n. Any one of three types of elementary particles that are believed by some physicists to form the basis of all matter in the universe. (applied by M. Gell-Mann after a coinage in "Finnegans Wake" by James Joyce).

I like to think that the object of my lecture is to get future dictionaries to remove the word "some"; then perhaps to remove the word "physicists". Who knows, it might one day be appropriate to insert the word "congressmen".

As many of you know, the quarks were proposed in 1964 by Murray Gell-Mann¹ and also George Zweig², both of Cal Tech. Zweig called them "aces", but that name didn't stick. Murray Gell-Mann, who possibly had a better sense of their importance, wanted a more striking name, and the passage from Finnigan's Wake³, "Three quarks for Muster Mark", is where he got it. That passage is almost prophetic, because the number three is a magic number, and in Murray Gell-Mann's theory, all baryons are made of three quarks, and each quark, in our present view, comes in three colors.

As you probably know, Murray Gell-Mann's theory called, "The Eight-Fold Way"⁴, was enormously successful. He was able to show that every known baryon could be described as a different combination of three basic flavors of quarks, now referred to as, "up", "down", and "strange", and all the known bosons could be described by different combinations of quark, antiquark pairs. But despite the overwhelming success of this scheme, including the discovery of new particles that the scheme inspired, for example the omega minus which he needed to complete the picture, Gell-Mann himself didn't insist that the quarks were real. He preferred to say that the scheme was mathematical, and if the scheme helped visualize the mathematics, that was fine, but the quarks need not be real. This point of view was reinforced when, year after year, the attempts to discover the free quark continued to yield negative results. And although it became a popular sport in high energy physics during the 1960's to explore the consequences of the symmetry principles, elaborated in the Gell-Mann scheme, it is probably correct to say that, except for these quark searches which invariably failed, no experiment was carried out with the explicit object of studying quark dynamics. This was reflected by the fact, and again my research may be a little incomplete, that the term "gluon", and I believe Chris Llewellyn-Smith⁵ was the first to introduce the term, didn't appear until 1972. That was a little too late for the dictionary, which doesn't say anything about gluons.

A major theoretical breakthrough came in 1973, when Gross and Wilczek⁶ on the one hand and, on the other hand, David Politzer⁷ wrote their famous papers setting forth their asymptotically free gauge theory which is called AFGT. Then, subsequently, carefully shepherded by Murray Gell-Mann⁸, this theory became known as Quantum Chromodynamics, QCD, to emphasize the close analogy with the highly successful Quantum Electrodynamics and to make evident the important differences that the color introduces. I asked Murray the last time I saw him here whether he had coined the term Quantum Chromodynamics, and he said, "No, that's all due to Nambu." Then I asked Nambu whether he had something to do with it and he said that he had written a little paper about color, but he had no idea about Quantum Chromodynamics. These great theorists find it becoming to appear a bit modest.

Today I want to review some of the more substantial evidence that supports the idea of quarks and, in particular, the gluons because these are less well established. I also want to emphasize that for the gluons, in every case, the evidence that came out about them was unexpected and unanticipated. We now believe that quarks and gluons play the role for nucleons and nuclei that electrons and photons do for atoms and molecules.

II. RUTHERFORD SCATTERING

We begin with Fig. 1 which illustrates the classical way of studying small distances. This is the principle of Rutherford scattering. The idea is very simple: a charged particle scatters against another charged particle of mass M , and you measure the incident energy E , the outgoing energy E' , and the angle of scattering θ . That's the Rutherford experiment; the important thing is that the interaction between the incident particle and the target particle is known. It is the Coulomb interaction.

Rutherford derived this formula, and although I've written it [Eq. (1)] slightly differently, it really is the Rutherford formula.

$$\frac{d\sigma}{d\Omega} = \frac{4\alpha^2 (E')^2}{Q^4} \frac{E'}{E} \cos^2 \frac{\theta}{2} \quad (1)$$

Fig. 1.

Rutherford scattering: The quantities E , E' and θ are measured for each event and the cross section compared with the point-charge formula.

A factor E'/E is added to take into account the recoil. This factor is close to one in the Rutherford experiment, because in that case the target was gold, much heavier than the alpha particles that were scattered. I've also introduced the term Q^2 instead of the scattering angle θ . The two quantities are related by,

$$Q^2 = 4EE' \sin^2 (\theta/2) . \quad (2)$$

The quantity Q^2 is very important in our discussion. It is the square of the momentum transfer in the collision. The rule for elastic scattering is that the energy loss,

$$E - E' = Q^2/2M . \quad (3)$$

I'll refer to this relation again and again.

The essential point is that the law of force is accurately known. Thus, in carrying out the experiment the measured cross section can be compared to the Rutherford formula. Deviations indicate that the scattering is not pointlike. In Fig. 2 we plot the ratio of observed low values of Q^2 to the Rutherford cross sections. At low Q^2 the ratio should be one.

However, as Q^2 increases, a deviation is seen from which the size of the scattering object can be determined. An estimate may be obtained from the uncertainty principle. The size of the scatterer is roughly,

$$[d \approx \hbar c (Q^2)^{-1/2}] . \quad (4)$$

Since $\hbar c = 0.2 \text{ GeV-fermi}$, the distance resolved with $(Q^2)^{1/2} = 0.2 \text{ GeV}$ is about 10^{-13} cm . But with $(Q^2)^{1/2} \approx 5 \text{ GeV}$, $d \approx 4 \times 10^{-15} \text{ cm}$. Rutherford's alpha particles took him to the 10^{-13} cm region. He looked at large-angle scatters with momentum transfers around 0.4 GeV^2 . By noting where his observed angular distribution began to deviate from his formula he obtained a measure of the nuclear size

Fig. 2.

Deviations from Rutherford scattering provide an estimate of the size of the scatterer.

III. FORM FACTORS

Deviations from point scattering are expressed by the form factor $F(Q^2)$ as given in Eq. (5); the idea is illustrated in Fig. 3.

$$\frac{d\sigma}{d\Omega} = \frac{4\alpha^2 (E')^2 \cos^2 \frac{\theta}{2}}{Q^4} \frac{E'}{E} F(Q^2) . \quad (5)$$

A plot of the ratio of the observed and the point cross sections is a plot of the form factor. Then, using the uncertainty relation, or more precisely, by taking its Fourier transform, the spatial extent of the scattering object can be obtained.

Equation (6) is the relativistic point scattering formula, a la Dirac.

$$\frac{d\sigma}{d\Omega} = \frac{4\alpha^2 (E')^2}{Q^4} \frac{E'}{E} \left(\cos^2 \frac{\theta}{2} + \frac{Q^2}{2M^2} \sin^2 \frac{\theta}{2} \right) . \quad (6)$$

The particles now have spin = 1/2. They still have point charges, but Rutherford's expression changes; there's a second term, $(Q^2/2M^2) \sin^2 (\theta/2)$, because there are two possible orientations of spin 1/2 particles.

It's not hard to see how the formula comes about. Many of you are familiar with Fermi's Golden Rule,

$$\sigma = \frac{1}{v_{rel}} \frac{2\pi}{\hbar} |m|^2 \rho(E) . \quad (7)$$

After all, it was taught here in Los Alamos by Fermi himself in the early days. It has been in nuclear physics textbooks ever since. The cross section is the product of one over the relative velocity of the two objects, a factor involving

Fig. 3.

The form factor is a measure of the deviation from Rutherford scattering.

Planck's constant, the square of the matrix element, and the density of final states. The mysterious thing is always the square of the matrix element, but you can get that using Feynman's rules. For point scattering Feynman's rules are particularly simple. Refer to Fig. 4. In four momentum notation, the muon has incoming energy momentum p , outgoing p' . For the proton, taken to be a point charge, the momenta are P , P' . The matrix element is simply a factor e from the first vertex, a second factor e from the other vertex, and for the propagator, a division by Q^2 . When you square that, you get the factor e^4 over Q^4 . In units, $\hbar = c = 1$, the e^4 can be replaced by α^2 . That is the origin of the α^2/Q^4 that appears in all the formulas that I'll show you today. That is all you really need to know about the theory, except for what is in Eq. (8).

Fig. 4.

Feynman diagram for the Dirac scattering between two spin 1/2 point charges.

IV. INELASTIC SCATTERING

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{4\alpha^2(E')^2}{Q^4} \frac{E'}{E} [W_2(\nu, Q^2) \cos^2 \frac{\theta}{2} + 2W_1(\nu, Q^2) \sin^2 \frac{\theta}{2}] ,$$

where

(8)

$$\nu = E - E' ,$$

$$Q^2 = 4EE' \sin^2 \frac{\theta}{2} ,$$

$$x = Q^2/2M\nu .$$

Note: $x = 1$ is the condition for elastic scattering.

Here we go to inelastic scattering. It's the same idea, except that we introduce the double derivative because the cross section is now a function of both Q^2 and E' (or ν). There are two form factors, W_1 and W_2 , and you can see that if we set for form factor for a point charge,

$$W_1^{pt}(\nu, Q^2) = \frac{1}{M} \delta\left(\frac{Q^2}{2M\nu} - 1\right), \quad W_2^{pt}(\nu, Q^2) = \frac{1}{\nu} \delta\left(\frac{Q^2}{2M\nu} - 1\right) ,$$

(9)

then for elastic scattering, $Q^2 = 2M\nu$. Taking the integral over ν reduces the expression to the one for elastic scattering we had before. The delta function is a nice device for converting the expression for elastic scattering over to one for inelastic scattering. We now have two variables, and we still have the two form factors, but they're now called structure functions instead of form factors.

V. SLAC-SLAC/MIT EXPERIMENTS

We are now ready to discuss the first experiment, the one shown in Fig. 5. This is the one done at the Stanford Linear Accelerator Center (SLAC) by SLAC and the Massachusetts Institute of Technology (MIT) groups during the 1960's; they started around 1962 or so, and they kept working for over 10 years, perhaps even a little more than that. Some of the latest Ph.D. theses were just finished a year or so ago, and I am still receiving papers about the results that were obtained. It's a scattering experiment, and here you see clearly the incident electron beam from the big SLAC two mile linear accelerator that went up to 20 GeV. There is a liquid hydrogen target and the angle of scattering is measured by the position of the spectrometer, and the final momentum (or energy) of the scattered electron is measured by the amount of magnetic deflection in the spectrometer. The incident energy E is known from the beam, and we see how E' and θ can be measured. This 8 GeV spectrometer, and another, the 2 GeV spectrometer, were used in the long series of measurements at SLAC that I want to discuss. Some of the results that were reported in the early days of the work are shown in Fig. 6. One report was given by Panofsky¹⁰ in 1968 at the so-called, "Rochester" Conference in

Fig. 5.

Deep inelastic electron scattering at SLAC using the 8-GeV spectrometer.

Vienna. What is shown is a comparison of the form factor for inelastic scattering with that for the elastic scattering form factors which had been obtained earlier by Hofstadter at Stanford, with lower energy electrons. The elastic form factor falls sharply, by a factor of 10^4 at $Q^2 = 7 \text{ GeV}^2$ and is used to show that the size of the proton is about 0.8 fermi (10^{-13} cm).

VI. SCALING

The inelastic scattering experiments had the purpose of measuring the form factors of the excited nucleon states - the 1.236 GeV resonance, the 1.525 GeV resonance, the 1.688 GeV resonance, etc. The idea was to see how these differed from the proton. They found that if you plotted the form factor for a given value of the mass W , of the final hadron system, the Q^2 dependence turned out to be surprisingly small. That immediately suggested, as Feynman was quick to point out, that if you have no Q^2 dependence, whatever is doing the scattering must be smaller than the corresponding value of Q^2 . When this was plotted with the proper variable¹¹ $\omega (= 1/x)$ held constant (instead of W), and with the structure function, νW_2 (instead of $\sigma/\sigma_{\text{mott}}$), as a function of Q^2 , you can see (Fig. 7) how flat it is.¹² The flatness is the indication that something unresolved and very small is doing the scattering. It also indicates that a scaling principle applies here. The structure function is a function of one variable, ω (or x), the same whatever the value of Q^2 , provided Q^2 is large enough. This is called Bjorken scaling and the variable, the Bjorken x , after J. D. Bjorken who was the first to point out that this behavior would be a consequence of scattering from pointlike objects.¹¹

Fig. 6.

$(\sigma/\sigma_{\text{mott}})^{\text{YP}} = W_2 + 2W_1 \tan^2 \theta/2$ plotted against Q^2 at fixed missing mass W .

Fig. 7.

Plot of νW_2 vs Q^2 from SLAC, MIT-SLAC deep inelastic electron scattering.

VII. QUARK-PARTON MODEL

In the beginning Feynman wasn't ready to identify his partons with Gell-Mann's quarks, but he is now. I've written quarks. The idea that the quarks in the proton account for the behavior of the structure function is the following: first you assume that the quarks carry a fraction x of the energy momentum of the target proton, and are free (impulse approximation). Note that the four momentum P of the target proton is just its mass; $P^2 = M^2$ in any Lorentz frame. To make the derivation simple, neglect any momentum the quark might have transverse to the collision axis. Then assume that the distribution $f_i(x)$ is different for each quark flavor, i . When you do that, you get the same Feynman diagram we had before, as seen in Fig. 8. The electron or the muon that does the scattering has momentum p incoming and p' outgoing. It interacts by emitting a photon which is absorbed in turn by the quark it hits. If the scattering is elastic and the invariant mass of the quark is unchanged by the scattering, we have

$$(xP)^2 = (xP + Q)^2 , \quad (10)$$

from which it follows that

$$-Q^2 = 2xP \cdot Q$$

in any frame. In the lab this leads to

$$Q^2 = 2Mx \quad . \quad (11)$$

The structure function is given by

$$\nu W_2(x, Q^2) = \sum_i e_i^2 x f_i(x) = F_2(x) , \quad (12)$$

where e_i^2 is the square of the charge of the quark of flavor i . The two really important points are: 1) the structure function measures the fraction of the momentum carried by the quarks, weighted by their charge squared, and 2) the x is a measurable quantity. You measure Q^2 , because E , E' and θ are measured, you know the mass of the proton, and the energy loss $\nu = E - E'$. Thus, you measure the energy momentum of the quark that is struck. That's the way you find out about quarks and their momentum distribution. Later on, I'll refer to the fact that the integral, $\int F_2(x) dx$ is equal to the total energy momentum carried by the quarks, weighted by the charge squared, in units of M .

Fig. 8.
Elastic point charge scattering of quarks.

VIII. μp SCATTERING

This brings me to the next experiment¹³, the one I worked on. The people and universities involved are shown in Fig. 9. Notice the names of Howard Matis and Wayne Kinnison, both now at LASL. Four universities were involved and more than 12 Ph.D. theses came from that experiment.¹⁴

Figure 10 shows the Fermilab accelerator at which the experiment was done. The machine has a radius of 1 km and produces protons at 400 GeV. We used muons instead of the electrons of the SLAC experiment. The muons were a byproduct of the neutrino beam. They have some rather nice properties, about which I'll say more later, but basically, they are point charges, and behave in this regard just like electrons.

The experimental setup is shown in Fig. 11. The incoming muons are measured by a set of scintillation counters and multiwire proportional chambers with a magnet to measure their momenta. Then there is a liquid hydrogen target, following which you look for the scattered muon. Its momentum is measured in a

CHICAGO	HARVARD	ILLINOIS	OXFORD
H. L. Anderson	B. A. Gordon	W. R. Francis	V. K. Bhargadwaj
R. M. Fine	W. A. Loomis	R. G. Hicks	N. E. Booth
R. H. Heisterberg	F. M. Pipkin	T. B. W. Kirk	G. I. Kirkbride
W. W. Kinnison	S. H. Pordes		J. Proudfoot
H. S. Matis	A. L. Sessoms		T. W. Quirk
L. W. Mo	W. D. Shambroom		A. Skuja
L. C. Myrianthopoulos	C. Tao		M. A. Staton
S. C. Wright	L. J. Verhey		W. S. C. Williams
	Richard Wilson		

Fig. 9.
Members of the CHIO collaboration in Fermilab experiment E98/398, deep inelastic muon scattering of hydrogen and deuterium.

Fig. 10.

Sketch of the Fermilab accelerator showing the neutrino area in which the experiment was done.

big magnet, which was the old Chicago cyclotron magnet. We had taken it to Fermilab and turned it into a spectrometer, a very fine spectrometer. Every time I think of that magnet I think of John Marshall, because he designed it. After 30 years it continues to be used in particle physics.

It's easy to pick out the scattered muon, because they are extremely penetrating. They go through 8 feet of steel with very little loss and make a signal in the hodoscope counter M and a track in the spark chambers S4. All the other particles, the hadrons, pions, photons, and kaons which might come out of the collision don't get through. So you can trigger the experiment by detecting the muon that penetrates this thick shield. For that reason, as well as others, it's better to use muons than electrons.

Figure 12 shows a reconstructed event. Here you see an incoming muon and the scattered muon. In this particular case the scattering was by an electron.

Fig. 11.

Experimental arrangement for deep inelastic muon scattering at Fermilab.

Fig. 12.

Reconstruction of an event in which the muon is elastically scattered by an electron in the target.

There were many such events due to the electrons in the hydrogen of the target. Note that the electron doesn't get through the hadron shield, but the muons do. If you look carefully in one of these spark chambers, the electron makes a lot of sparks, because of three inches of iron (not shown) that converts the electron into a shower of gamma rays and electron-positron pairs.

Figure 13 shows an actual reconstruction of the kind of event we're talking about. There is the incoming muon, and several hadrons emerging from the target, there are three shown here, and then the muon plowing through the steel and getting into the muon detector behind it, where it is detected and measured. It's easy to see that with such an arrangement you can measure the incoming energy, and because of the observed deflection of the magnet, the outgoing energy as well. You measure the angle, because you reconstruct the whole trajectory.

IX. DEVIATIONS FROM SCALING

In Fig. 14 I show the results. I show F_2 as a function of Q^2 for various bins in ω , just the way it was done at SLAC. Note the box with $3 < \omega < 5$, or $\langle \omega \rangle = 4$. That's the same value of ω that was shown on the plot I showed you earlier. Notice how flat it is. The x's represent the SLAC points, and the circles represent the Fermilab points. Notice the Fermilab data goes to $Q^2 = 40 \text{ GeV}^2$ and it's still flat. This is a beautiful confirmation of the SLAC result. However, except for that, for other values of ω (or x) the trend of the data is either rising or falling. That is, the Q^2 dependence is no longer flat; for low ω (x near one) it falls, for high ω (x small) it rises. That's an important characteristic. In terms of x it is flat at $x = 1/4$; falling for $x > 0.4$ and rising for $x < 0.1$.

Figure 15 shows essentially the same effect, from an experiment done with neutrinos at CERN by Steinberger and the group CDHS.¹⁵ The neutrinos are point particles like muons but they interact, not with the electric charge, but with the weak charge. They exchange a charged or neutral boson instead of a photon, but since they are interacting with a proton the weak charge feels the momentum

Fig. 13.

Reconstruction of a deep inelastic muon scattering event showing the accompanying hadronic shower.

Fig. 14.

F_2 for hydrogen as a function of Q^2 for various ω bins, using $R = 0.52$, showing the combined Fermilab data (circles), the MIT-SLAC data (triangles), and the SLAC data (crosses). The errors shown are statistical only. The horizontal bars indicate the effects of changing R to 0.69 and 0.37. Note the difference in scale and suppressed zeros on the ordinates.

distribution of the quarks inside. Thus, you measure the same structure function, instead of the square of the electric charge, you have the square of the weak charge. There is a difference. For the up quark, the square of the electric charge is $4/9$; for the down quark it is $1/9$, $1/4$ as much. The weak charge is the same for up and down quarks so the structure function is a different linear combination of the quark flavors for neutrinos than for muons. Nevertheless, the same effect is evident. For $x > 0.4$, F_2 falls; for $x < 0.1$, it rises with Q^2 . For $x \approx 0.25$ it is flat. This is the so-called deviation from scaling.

I have to show you Fig. 16 which comes from MIT-SLAC experiments with deuterium.¹⁶ At $x = 0.25$, the Q^2 dependence is flat; at $x \geq 0.4$, it is falling; $x \leq 0.2$, it is rising. The scale breaking behavior is beautifully displayed in these experiments done much earlier than our Fermilab experiments. However, at that time it was believed that scaling deviations could be accounted for by suitable choice of the scaling variable.

X. QUANTUM CHROMODYNAMICS

Now there's another way of plotting. You take the same data and plot the structure function as a function of x for different bins in Q^2 . This is shown in Fig. 17. The quantity x varies from zero to one. The value $x = 1$ corresponds to elastic scattering. Notice that the function $F_2(x, Q^2)$ gets steeper and steeper as you go up in Q^2 , or if you will, the average momentum is going down. In quark terms, the whole curve moves toward smaller x and shows that as Q^2 increases there are more quarks at low x . This is the pattern of quantum chromodynamics.¹⁷ The curve with $15 < Q^2 < 30 \text{ GeV}^2$ is much steeper than the curve with $1 < Q^2 < 2 \text{ GeV}^2$. A theory to account for this behavior was already available; it had been worked

Fig. 15.

The structure function F_2 obtained from the deep inelastic scattering experiment of the CDHS collaboration at CERN.

Fig. 16.

The structure function F_2 obtained by the MIT-SLAC deep inelastic electron-deuteron scattering experiment.

out in 1973-1974 by Politzer⁷ and by Gross and Wilczek⁸, as I mentioned earlier. According to the theory the scaling violations are due to gluons. Because of the way quarks can emit and absorb gluons, the momentum distribution of the quarks is not a function of x alone, but shows a Q^2 dependence like that indicated in Fig. 17. We are familiar with the mechanism that causes the Q^2 dependence from quantum electrodynamics. According to QED an electron can create and absorb photons (from the vacuum). This process, shown in Fig. 18(a) is fundamental. That's how electrons interact electromagnetically. Conversely, the one photon exchange between two charges that we used in deriving our scattering formulas is just this mechanism. A photon can produce an electron-positron pair, as in Fig. 18(b). In particular, an electron that's just moving along is not just an electron: it can emit a photon and then reabsorb it [Fig. 18(c)]; or it can emit a photon which can make an electron-positron pair, which then annihilates and produces a photon again which is finally reabsorbed, as in Fig. 18(d). That's the way one charge interacts with another; either by emitting and absorbing a photon, or in a more complicated way, emitting a photon which becomes an electron-positron pair, and then coming back to photon and being absorbed again, or doing it many times, as shown in Fig. 18(e). The process in which electron-positron pairs are made, albeit for a very short time, is called vacuum polarization. It has a lot to do with the magnitude of the charge the electron is observed to have. The familiar Coulomb law is a consequence of the one photon exchange term only. When the interacting objects come close enough together, more complicated exchanges can occur. The creation of an electron-positron pair violates energy conservation, but the uncertainty principle allows this if the time involved is short enough or if the distance involved is short enough.

Fig. 17.
Structure function $F_2(x)$ for various values of Q^2 .

In quantum chromodynamics, a similar situation prevails; a quark can emit a gluon, as in Fig. 18(f), or a gluon can make a quark-antiquark pair as in Fig. 18(g). The process shown in Fig. 18(h) has no analogue in QED; one gluon makes two, or conversely two gluons make one. This is because of the nature of strong force. The gamma ray doesn't have a charge, but the gluons have a "color" charge which allows gluons to make gluons. In QCD a quark can emit a gluon and reabsorb it, or a quark can emit a gluon which can make a gluon-antigluon pair and then become a gluon again which may finally be reabsorbed. Quarks can interact with other quarks within the proton, either by emitting and absorbing gluons, or by emitting and then making quark-antiquark pairs, making a gluon, etc. So that by the emission and absorption of gluons and quark-antiquark pairs you have the processes shown in Fig. 18(k).

I now point out that in the process, Fig. 18(j), for example, if you hit things with high enough Q^2 , the distances involved are short, and the corresponding

Fig. 18.
Radiative processes in QED and QCD.

times are short, and you may catch the quark in a state in which it has just emitted a gluon but has not yet reabsorbed it. If you look at just this instant of time, you'll find the quark with somewhat less momentum than it had. Or, as in Fig. 18(j), you may find not one quark but three. In any case, the momentum of the quark that was there at low Q^2 will show up differently at high Q^2 , with its momentum being shared by these other quarks and by gluons. That is why at high Q^2 , the average momentum is shifted lower, just as the experiments show. Figure 18(k) indicates how quarks interact with quarks via gluons in a manner analogous to the interaction between electric charges in QED [Fig. 18(e)].

XI. ENERGY-MOMENTUM SUM RULE

We can say something more precise about the quarks and the gluons by discussing the integral, $\int F_2(x, Q^2) dx$. In parton theory this gives the total energy-momentum carried by the quarks, weighted by their charge squared, in units of the proton mass.

$$\int_0^1 F_2(x, Q^2) dx = \sum_i e_i^2 \int_0^1 [x f_i(x, Q^2) dx] . \quad (13)$$

Actually, this rule, also known as the Callan-Gross sum rule¹⁸, has a more general validity. Now consider what to expect if the proton is made of three quarks, two

"up" and one "down", as Gell-Mann would have had it originally. In this case, the value would be one-third. That is, take the square of the charge of the first up quark, which would be 4/9, add 4/9 for the second and 1/9 for the down quark, take the average, and obtain 1/3.

Let me now show you the result of the measurement given in Fig. 19. The values of $F_2(x, Q^2)$ are plotted for $Q^2 = 9 \text{ GeV}^2$. In these plots the Fermilab μp and the MIT-SLAC ep measurements are combined and show quite good agreement; the data are complete enough to obtain the integral numerically. Figure 20 shows the data at $Q^2 = 12.5 \text{ GeV}^2$. When we calculate the integral, we find not 0.33, but a value close to 0.18. That number, 0.18, the fact that it's less than 0.33, is a clear evidence that there's something neutral inside that's carrying a large fraction, almost 50%, of the momentum. In the light of QCD these are the gluons.

XII. GLUONS

In fact, the pattern exhibited in Fig. 17 of the Q^2 variation of F_2 is attributed to the gluons.

To be more quantitative we introduce the idea of the quark moments. The integral over x times of the number distribution $f_i(x, Q^2)$ weighted by x^{n-1} is the n^{th} moment of the quarks with flavor i . The $n = 2$ moment,

$$\langle q_i(Q^2) \rangle = \int_0^1 x f_i(x, Q^2) dx \quad (14)$$

Fig. 19.

Structure function $F_2(x)$ at $Q^2 = 9(\text{GeV})^2$ from Fermilab and SLAC data combined. The value of R was taken to be 0.52 for μp , 0.21 for ep.

is the total momentum carried by the quarks. Correspondingly, a similar expression shown in Eq. (15) gives the total momentum carried by the gluons:

$$\langle G(Q^2) \rangle = \int_0^1 x G(x, Q^2) dx . \quad (15)$$

In these terms the energy momentum sum rule requires that the sum of the momenta carried by the quarks and the gluons be set equal to one (in units of the proton mass).

$$\sum_i \langle q_i(Q^2) \rangle + \langle G(Q^2) \rangle = 1 . \quad (16)$$

Thus, the gluons were introduced to account for the missing energy momentum just as in earlier times the neutrino was introduced to account for the missing energy in beta decay. In QCD you can find out what to expect from the sum rule.^{5,7} In the limit of large Q^2 the formula is almost intuitive. I give the formula in Eq. (17). What enters is the average charge squared, the number of different quarks, the product of number of colors and number of flavors, and the number of gluons suitably weighted.

Fig. 20.

$F_2(x)$ at $Q^2 = 12.5 \text{ GeV}^2$, FNAL and SLAC data combined. The value of R was taken to be 0.52 for μp , 0.21 for $e p$.

$$\int_0^1 F_2(x, Q^2) dx \xrightarrow{Q^2 \rightarrow \infty} \overline{e_i^2} \frac{cq}{cq + 2g} . \quad (17)$$

q = number of quark flavors = 4
 c = number of quark colors = 3
 g = number of gluons = 8

The formula gives the result expected if there were an equipartition of the quarks and the gluons. The quarks, of course, are the only things the experiments can see directly, because it's the electric charge of the quarks that does the scattering. But since you can measure the momentum carried by them you find that they carry only a fraction of what's available. If there are 4 quarks, the number accessible to present experiments, this number would be 0.119.

Figure 21 shows what happens if you try to fit the measurements, according to the prescription of QCD. The theory prescribes how the moments vary with Q^2 . We calculate the moments for various values of Q^2 . We calculate not only the second moment but the fourth and sixth as well.¹⁹ In fact, we calculated

$$M_2(n, Q^2) = \int_0^1 x^{n-2} F_2(x, Q^2) dx , \quad (18)$$

with

$n = 2, 4, 6,$

being careful to include the Nachtmann and the elastic scattering corrections. The fit is very good. For the second moment, which is the energy momentum sum rule, we find a value ~ 0.18 decreasing slowly with Q^2 . The plot shows how nicely the asymptotic limit can be connected with the data. The fit is good but $Q^2 \rightarrow \infty$ is a long way off. This is for hydrogen. Figure 22 shows deuterium. Since it is deuterium, a neutron and a proton, this number is twice what it was in the case of the hydrogen. Notice that this is done for colored vector gluons and colored quarks, and for four flavors.

XIII. OTHER MODELS

Now let's see what happens if you try some of the other possible models.²⁰ Figure 23 shows the fit for noncolored vector gluons and colored quarks. The theory doesn't fit at all. The theory would like to have the second moment rise with Q^2 but it actually falls. This observation supports the idea that the gluons are more likely colored than not.

A theory based on colored scalar gluons and colored quarks leads to a rise in the second moment, but the data shown in Fig. 24, falls and doesn't fit at all; so this theory can be ruled out.

The case of a theory based on noncolored scalar gluons and colored quarks is shown in Fig. 25. Again it does not fit. Although most theorists in the

Fig. 21.

Fit to the $n = 2, 4, 6$ moments of F_2 for the 4 flavor, 3 color quark model with colored vector gluons; for hydrogen.

Fig. 22.

Fit to the $n = 2, 4, 6$ moments of F_2 for the 4 flavor, 3 color quark, colored vector gluon model; for deuterium.

business wouldn't be surprised at this result, it's good to have an experiment that supports the idea that the gluons are vector, not scalar, and that, like the photon, they have an angular momentum 1.

XIV. QUARK AND GLUON MOMENTUM DENSITY DISTRIBUTIONS

Another thing we try to do, when we analyze the data, is to obtain the actual quark and gluon momentum distributions which are consistent with the moments. This requires a few ad hoc assumptions, but even if the results cannot be certified they are at least plausible.²⁰ Figure 26 shows the momentum distribution of the "up" quarks, the "down" quarks, and the gluons, at low Q^2 . You will be surprised to notice that the gluons are very important. They carry, after all, almost 50% of the energy momentum, and so they play a very prominent role. On the other hand, it's hard to arrange a direct encounter with a gluon. The gluon has a color charge; it interacts strongly with quarks and other gluons, but it has no electric charge. So it's hard to detect by the usual means. We all await a clever way to detect a gluon directly.

Figure 27 shows what happens when you go up in Q^2 . Notice that all the curves are shifting toward lower x , and the gluons in particular are rising very rapidly at low x .

The next figures (Fig. 28, 29, and 30) show the evolution as Q^2 increases. Notice how the gluons become more and more prominent at low x . At $Q^2 = 50 \text{ GeV}^2$, even the "up's" and "down's", are noticeably rising. The rise is due to the

Fig. 23.

Attempt to fit the $n = 2, 4, 6$ moments of Γ_2 of the proton for the model with non-colored vector gluons.

Fig. 24.

Attempt to fit the $n = 2, 4, 6$ moments of F_2 of the proton with colored scalar gluons.

quark-antiquark pairs that are called the sea quarks. These will be important in what I want to talk about next. Earlier I indicated that the valence quarks, the original triplet of quarks, will occasionally convert into additional quark-antiquark pairs. The extra quarks are only seen when you go to very large Q^2 . When you get close enough, you see that the quark is not so simple; it is dressed up. It has a cloud of gluons, quarks and antiquarks that show up depending on how close you get when you probe it.

XV. DRELL-YAN PROCESS

In Fig. 31 I try to emphasize how closely the inelastic scattering is related to other processes that have been studied. Figure 31(a) shows the inelastic scattering. It shows a muon coming in, giving off its photon, which makes a collision with one of the quarks, changing its momentum. The quarks do not come out free by themselves, they metamorphose, and appear as hadrons, pions, kaons, and all kinds of things. That's deep inelastic scattering. The same diagram, turned around, as in Fig. 31(b), is what is called the Drell-Yan process.²² A quark and antiquark annihilate and produce a photon. The photon in its turn transforms into a $\mu^+\mu^-$ pair. The Feynman diagram is the same, except that the square of the four momentum carried by the virtual photon is M^2 , a positive quantity, instead of the negative quantity Q^2 in deep inelastic scattering. The two cross sections are closely related. Now consider Fig. 31(c) and consider, for example, pions on carbon. The antiquark in the pion, and one of the quarks in carbon annihilate and

Fig. 25.
Attempt to fit the $n = 2, 4, 6$ moments of F_2 of the proton for a model with non-colored scalar gluons.

Fig. 26.
Quark and gluon distributions at $Q^2 = 4(\text{GeV}^2)$.

Fig. 27.
Quark and gluon distributions at $Q^2 = 10 (\text{GeV}^2)$.

Fig. 28.
Quark and gluon distributions at $Q^2 = 30 (\text{GeV}^2)$.

Fig. 29.

Quark and gluon distributions at $Q^2 = 50 \text{ (GeV)}^2$.

Fig. 30.

Quark and gluon distributions at $Q^2 = 100 \text{ (GeV)}^2$.

produce a massive relative of the photon, in this case the J/ψ . The J/ψ turns into a photon, which, in turn, becomes a $\mu^+\mu^-$ pair. The J/ψ is a vector meson and the process is called vector meson production. This process, in reverse, is the one Panofsky talked about last Friday. If you have an e^+e^- colliding beam you can make, via an intermediate virtual photon, a vector meson; a J/ψ , a T , a ϕ or a ρ , depending on the energy available.

XVI. $\mu\mu$ PAIR EXPERIMENT

The experiment looking for $\mu\mu$ pairs was done by a group from Chicago (some of my colleagues; Jim Pilcher and Eli Rosenberg) and another group from Princeton, Stu Smith and some of his collaborators.²³ Gary Sanders, now at LASL, was in that experiment. They used the same Chicago Cyclotron Magnet spectrometer, but arranged things in a different way. Instead of muons, they used either protons or pions coming into a target, which was not hydrogen, but carbon or something much heavier. To absorb the shower of hadrons that came out but to let the muons through, they used the steel absorber seen in Fig. 32 and then tracked the two muons through the whole spectrometer, making sure they were muons, by requiring that they penetrate the eight feet of steel which was our muon filter.

Figure 33 shows some of their results. By measuring the momentum and the opening angle of the two muons, they could deduce the invariant mass, $M_{\mu\mu}$. When they plot the number of events as a function of $M_{\mu\mu}$, they obtained a remarkable curve. There is an enormous peak, which is the J/ψ , presumably made by the third mechanism I described. Even the ψ' shows up. What was a great discovery in 1974 had become a background for the part of the experiment I want to discuss here.

Fig. 31.

Feynman diagrams for the processes: a) $\mu p \rightarrow \mu X$ (spacelike), b) $\pi N \rightarrow \mu^+ \mu^-$ (timelike), and c) $\pi^+ C \rightarrow J/\psi X$ (timelike).

Figure 34 shows Leon Lederman's great discovery, the upsilon, Υ .²⁴ After he discovered the upsilon he continued the curve to larger values of $M_{\mu\mu}$. He was, of course, looking for bumps, but he didn't find any. What he did find was just as interesting. He was seeing the Drell-Yan process. The cross-section for this process is given by,

Fig. 32.

Experimental arrangement for $\pi N \rightarrow \mu^+ \mu^- X$ using the Chicago Cyclotron spectrometer; the Chicago-Princeton experiment. The hadron absorber is the shaded black following the target. The muon filter is the shaded block between the F and P hodoscopes.

Fig. 33.

μ pairs from π -nucleon collisions; the Chicago-Princeton experiment at Fermilab.

$$\frac{d^2\sigma}{dMdx} = \frac{8\pi\alpha^2}{9M^3} \sum_i \frac{e_i^2}{x_1 + x_2} [x_1 f_i^B(x_1) x_2 f_i^T(x_2) + x_1 f_i^B(x_1) x_2 f_i^T(x_2)] ,$$

where

(19)

$$M = (x_1 x_2 s)^{1/2}$$

$$x_{1,2} = 1/2(\sqrt{x^2 + 4M^2/s} \pm x)$$

$$x = 2P_L^*/s \quad \text{Note: } (x_1/x_2) = e^{2y}$$

and P_L^* is the longitudinal momentum of the mu-pair in the overall center of mass; s is the center of mass energy squared of the hadron-hadron collision.

We have the usual α^2 for electromagnetic processes with two vertices; then instead of Q^2 you have M^2 . The quark from the target is from one of the nucleons, the antiquark from the beam is from a proton or a pion, whichever is used. These annihilate, forming a virtual photon, which in turn converts into the now familiar $\mu^+\mu^-$ pair.

The quantities $f_i(x)[\bar{f}_i(x)]$ refer to the quark [antiquark] momentum distributions for beam B or target T, respectively. A factor three has already been incorporated in the denominator of the expression to take into account the fact that the quarks come in three colors and the annihilation can occur only if the quark, antiquark pair have the same color. The factor three is needed for agreement with the experiments and adds to the evidence that the quarks are colored.

The quantities e_i^2 refer to the fractional charge carried by the quarks. The quantity x_1 is the fraction of the momentum carried by the beam antiquark (quark). x_2 is the fraction of the momentum carried by the target quark (antiquark). Because two quarks annihilate we have a product, $x_1 f_i(x_1) \cdot x_2 f_{\bar{i}}(x_2)$ and $x_1 f_{\bar{i}}(x_1) \cdot x_2 f_i(x_2)$.

What would you expect? If you try a nuclear target there will be very few antiquarks either in the beam or in the target. What antiquarks there would be would have low x . On the other hand, with a pion beam, the pion already has an antiquark. A π^+ , for example, is made of an "up" quark and an "antidown" quark, $u\bar{d}$. The antiquark is already there. High values of x_1 come when M is high and x is high. Thus, with x and M high, we expect few antiquarks in the beam proton, but a large fraction in the beam pion. Thus we expect many more μ pairs with π beams than with p beams at large M . Indeed, the experiments show (Fig. 35) a 100-fold effect, emphasizing that pions have antiquarks, not only in the way Murray Gell-Mann thought because of the symmetry, but in a way that can make μ^- pairs.

Suppose you compare the π^+ and the π^- , an easy thing to do when you have a pion beam; just reverse the currents in the magnets. In the π^+ you have $u\bar{d}$; in π^- it's $\bar{u}d$. The antiquark is the important one when the target is a nucleon and is almost all quarks. As we've seen, the cross section depends on e^2 , the square of the charge. We have $e_u^2 = 4/9$ for $\bar{u}u$, $e_d^2 = 1/9$ for $\bar{d}d$ so the cross section for π^+ and π^- should approach a ratio 1/4 when $M_{\mu\mu}$ is large enough. Lo and behold, as Fig. 36 shows, the measurements do give the ratio 1/4 for $M_{\mu\mu} \approx 8$ GeV, a remarkable confirmation of the quark structure of pions.

XVII. PION STRUCTURE FUNCTION

One more important result out of this experiment. In Eq. (20) the Drell-Yan formula for pion-nucleon collisions has been rewritten in a way that factorizes the quark distribution in the pion and in the nucleon as a product.

$$\frac{d^2\sigma}{dx_1 dx_2} = \frac{4\pi\alpha^2 s}{9} x_1 f^\pi(x_1) \left[\frac{4}{9} x_2 f_u^N(x_2) + \frac{1}{9} x_2 f_d^N(x_2) \right] . \quad (20)$$

This uses the fact that for the pion,

$$f_u^\pi(x) = f_{\bar{u}}^\pi(x) = f_d^\pi(x) = f_{\bar{d}}^\pi(x) \quad (21)$$

by isospin symmetry and charge conjugation. Thus, there is one factor that depends on the distribution of quarks in the pion, and another factor that depends on the distribution of quarks in the nucleon. The last is known from deep inelastic scattering. With this relation it becomes possible to deduce something about the quark momentum distribution in the pion.

Figure 37 shows the fit obtained. It's a good fit, and it came as a surprise to the experimenters who hadn't anticipated they would be able to obtain the pion structure function when they proposed their experiment. It came out because of the development of theory, and because those guys are on their toes; they get everything there is to get out of their experiments.

Fig. 34.
The CFS dimuon data taken at Fermilab.

Fig. 35.
The ratio of π^- -induced to proton-induced μ -pair cross section at $y_{c.m.} = 0.2$ as a function of mass. Proton data at 225 GeV/c has been calculated from the scaling observed in 200-, 300-, and 400-GeV/c data of the CFS collaboration.

Fig. 36.
The ratio $\sigma(\pi^+C \rightarrow \mu^+\mu^- + X) / \sigma(\pi^-C \rightarrow \mu^+\mu^- + X)$.

Fig. 37.
The pion structure function $f^\pi(x_1) = x_1 u^{\pi-}(x_1)$.

If you then take the energy momentum integral for the pions, you find

$$\int_0^1 x_1 F^\pi(x_1) dx_1 = 0.20 \pm 0.13 \quad . \quad (22)$$

This result should be multiplied by two because there are two quarks in the pion. This gives 0.40. Again the energy momentum sum rule says that there must be something missing. This suggests gluons again, taking up about 60% of the energy momentum. The other nice check is to ask how many quarks there are in the pion. This is given by

$$2 \int_0^1 f^\pi(x_1) dx_1 = 2.18 \pm 0.32 \quad , \quad (23)$$

close to the value two expected.

XVIII. CONCLUSION

That's the kind of evidence that has been coming out of recent high-energy experiments. The evidence is that there are quarks and gluons in nucleons and other hadrons. Their behavior seems to be governed by quantum chromodynamics, a theory that parallels quantum electrodynamics in many ways. We see the quarks and gluons as the basis of the force that nuclear physicists have been looking for more than 40 years. It is the force that binds the nucleus together. We had pions playing this role but now we see that the pions are just one of many forms through

which the underlying quarks and gluons act. We now have a deeper understanding which promises a new set of insights. It will be exceedingly interesting to see how much quantum chromodynamics with its quarks and gluons will be able to tell us about the nucleus.

REFERENCES

1. M. Gell-Mann, "A Schematic Model of Baryons and Mesons," Phys. Letters 8, 214 (1964).
2. G. Zweig, "An SU_3 Model for Strong Interaction Symmetry and its Breaking," CERN Preprints TH 401, 412 (1964).
3. James Joyce, *Finnegan's Wake* (Viking Press, New York, 1939) p. 383.
4. M. Gell-Mann and Y. Ne'eman, "The Eightfold Way," W. A. Benjamin, Reading, Mass., 1972.
6. D. Gross and F. Wilczek, "Ultraviolet Behavior of Non-Abelian Gauge Theories," Phys. Rev. Letters 30, 1343 (1973).
D. Gross and F. Wilczek, "Asymptotically Free Gauge Theories, I," Phys. Rev. D8, 3633 (1973).
7. H. D. Politzer, "Reliable Perturbative Results for Strong Interactions?" Phys. Rev. Letters 30, 1346 (1973).
H. Georgi and H. D. Politzer, "Electroproduction Scaling in an Asymptotically Free Theory of Strong Interactions," Phys. Rev. D9, 416 (1974).
H. D. Politzer, "Asymptotic Freedom: An Approach to Strong Interactions," Phys. Reports 14C, 130 (1974).
8. H. Fritsch, M. Gell-Mann and H. Leutwyler, "Advantages of the Color Octet Gluon Picture," Phys. Lett. 47B, 365 (1972). This paper introduced the octet of colored vector gluons that underlies QCD.
9. G. Miller, "Inelastic Electron Scattering at Large Angles," SLAC-129 Thesis (1971); J. S. Poucher, et. al., "High-Energy Single-Arm Inelastic e-p and e-d Scattering at 6° and 10° ," Phys. Rev. Lett. 32, 118 (1974) and "High-Energy Single Arm Inelastic e-p and e-d Scattering at 6° and 10° ," SLAC-1309 (1973); A. Bodek, "Inelastic Electron-Deuteron Scattering and the Structure of the Neutron," Technical Report 93, MIT Thesis (1972); E. M. Riordan, et. al., "Extraction of Structure Functions and σ_L/σ_T from Deep Inelastic e-p and e-d Cross-Sections," SLAC-PUB 1634 (1975); W. B. Atwood, et. al., "Inelastic Electron Scattering from Hydrogen at 50° and 60° ," Phys. Lett. 64B, 479 (1976); W. B. Atwood, "Electron Scattering of Hydrogen and Deuterium at 50° and 60° ," SLAC-185 Thesis (1978); M. D. Mestayer, "A Measurement of the Proton Structure Functions Using Inelastic Electron Scattering," SLAC-214 Thesis (1978).
10. W. Panofsky, "Low Q^2 Electrodynamics, Elastic and Inelastic Electron (and Muon) Scattering," Proc. 14th Int. Conf. on High Energy Physics, Vienna (Geneva, CERN) (1968).

11. J. D. Bjorken, "Asymptotic Sum Rules at Infinite Momentum," Phys. Rev. 179, 1547 (1969).
12. J. I. Friedman and H. W. Kendall, "Deep Inelastic Electron Scattering," Ann. Rev. Nucl. Sci. 22, 203 (1972).
13. H. L. Anderson, et. al., "Measurement of Nucleon Structure Functions in Muon Scattering at 147 GeV/c," Phys. Rev. Lett. 37, 4 (1976); "Measurement of the Proton Structure Function from Muon Scattering," 38, 1450 (1977); B. A. Gordon, et. al., "Muon Scattering at 219 GeV and the Proton Structure Function," Phys. Rev. Lett. 41, 615 (1978).
14. S. H. Pordes, "Inelastic Muon Scattering from Hydrogen at 147 GeV/c," PhD Thesis, Harvard Univ. (1975); G. I. Kirkbride, "Muon-Nucleon Inclusive Interactions at 15° GeV," D. Phil. Thesis, Univ. of Oxford (1975); H. S. Matis, "Inclusive Hadron Production in Inelastic Muon-Proton Scattering at 15° GeV," PhD Thesis, Univ. of Chicago (1975); R. H. Heisterberg, "Hadron Distributions from μ D Scattering at 147 GeV," PhD Thesis, Univ. of Chicago (1976); V. K. Bharadwaj, "Deep Inelastic Muon-Nucleon Scattering," PhD Thesis, Univ. of Oxford (1976); B. A. Gordon, "Deep Inelastic Muon-Proton Scattering," PhD Thesis, Harvard Univ. (1978); J. Proudfoot, "Hadron Production in Deep-Inelastic Muon-Proton Scattering at 219 GeV," D. Phil. Thesis, Univ. of Oxford (1978); L. C. Myrianthopoulos, "Diffractive Production of Rho Mesons by 147 GeV Muons Incident on a Deuterium Target," PhD Thesis, Univ. of Chicago (1978); R. G. Hicks, "Hadron Production in High Energy Muon Scattering," PhD Thesis, Univ. of Illinois (1978); R. M. Fine, "The Nucleon Structure Function νW_2 and Photoproduction Cross Sections for Deuterium Measured with Virtual Photons at 150 GeV," PhD Thesis, Univ. of Chicago (1978); C. Tau, "The Structure and Formation of Hadrons: An Experiment with a Muon Beam of 219 GeV on a Hydrogen Target," PhD Thesis, Harvard Univ. (1979).
15. J. G. H. de Groot, et. al., "Inclusive Interactions of High Energy Neutrinos and Antineutrinos in Iron," Z. Physik C1, 143 (1979).
16. A. Bodek, M. Breidenbach, D. I. Dubin, J. E. Elias, J. I. Friedman, H. W. Kendall, J. S. Poucher, E. M. Riordan, and S. M. Sogard, D. H. Coward, and D. J. Sherden, "Experimental Studies of the Neutron and Proton Electromagnetic Structure Functions," Phys. Rev. D20, 1471 (1979).
17. H. Kogut and L. Susskind, "Parton Models and Asymptotic Freedom," Phys. Rev. D9, 697, 3391 (1974); Wu-ki Tung, "Patterns of Scaling Violation Implied by Field Theories," Phys. Rev. 12D, 3613 (1968).
18. C. G. Callan, Jr. and D. J. Gross, "Crucial Test of a Theory of Current," Phys. Rev. Lett. 21, 311 (1968).
19. H. L. Anderson, H. S. Matis, L. C. Myrianthopoulos, "Determination of the Quark and Gluon Moments of the Nucleon," Phys. Rev. Lett. 40, 1061 (1978).
20. M. Gluck and E. Reya, "Discriminative Deep Inelastic Tests of Strong Interaction Field Theories," Nucl. Phys. B156, 456 (1979). I thank R. J. McKee for the analysis given here.

21. H. L. Anderson, H. S. Matis and L. C. Myriantopoulos, "Quarks and Gluons in Deep Inelastic Scattering," Proc. XIX Int. Conf. on High Energy Physics, Tokyo, Japan (1978). I thank R. J. McKee for the analysis shown here.
22. S. D. Drell and T-M. Yan, "Massive Lepton-Pair Production in Hadron-Hadron Collisions at High Energies," Phys. Rev. Lett. 25, 316, 902(E) 1970; "Partons and Their Applications at High Energies," Ann. Phys. 66, 578 (1971).
23. K. J. Anderson, R. N. Coleman, G. E. Hogan, K. P. Karhi, K. T. McDonald, C. B. Newman, J. E. Pilcher, E. I. Rosenberg, G. H. Sanders, A. J. S. Smith and J. J. Thaler, "Production of Muon Pairs by 225-GeV/c $\pi^\pm$, $K^\pm$, and $p^\pm$ Beams on Nuclear Targets," Phys. Rev. Lett. 42, 944 (1979); G. E. Hogan, et. al., "Comparison of Muon-Pair Production to the Quark-Antiquark Annihilation Model," Phys. Rev. Lett. 42, 948 (1979); C. B. Newman, et. al., "Determination of the Pion Structure Function from Muon-Pair Productions," Phys. Rev. Lett. 42, 951 (1979).
24. L. M. Lederman, "Prompt Dilepton Production in Hadron Collisions," Proc. XIX Int. Conf. High Energy Physics, Tokyo, Japan (1978).