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NUMERICAL SIMULATIONS OF
COLLISIONLESS DRIFT INSTABILITIES
FOR LOW-DENSITY PLASMAS

BY

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AND

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PLASMA PHYSICS
LABORATORY

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EARB

Numerical Simulations of Collisionless Drift Instabilities
for Low-Density Plasmas

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Nonlinear behavior of the collisionless drift instabilities are studied for the universal and current driven modes by means of electrostatic particle simulations in two-and-a-half dimensions. Realistic mass ratios of the electrons to the ions are used in the simulations, where the guiding-center approximation for the electrons and the exact dynamics for the ions are employed. Several nonlinear effects including the quasilinear diffusion of the particle density, the frequency shift due to the ambipolar field, the mode competition among the unstable waves, and the quasilinear diffusion in the velocity space are found to be the dominant mechanisms for the saturation. The stabilization of the collisionless drift instabilities by the magnetic shear has also been studied.

I. INTRODUCTION

The importance of low-frequency drift instabilities on plasma confinement has long been recognized in connection with controlled thermonuclear research.¹ It is because that drift waves can become unstable under a variety of experimental conditions and result in enhanced plasma transport across magnetic field. Needless to say, a great deal of theoretical effort has been performed on the linear stability of drift waves and their nonlinear consequences on plasma confinement.¹ Experimentally, identification of the drift instabilities in Q-devices confirmed the presence of the instability and its importance on plasma diffusion.² More recently, the presence of drift-type instabilities and the strong correlation with anomalous plasma diffusion have been verified in toroidal confinement devices as well.³

However, with the availability of these experimental results, it becomes apparent that our present theoretical understanding of the nonlinear development of the instabilities and their consequences on plasma confinement is not entirely satisfactory. While pioneering work on strong plasma turbulence due to drift instabilities has already been carried out,⁴ a more comprehensive nonlinear theory is needed in order to interpret the experimental results and to give an adequate estimate of the plasma transport.

Recently, it has been demonstrated that particle code simulations can be an effective tool in providing much needed insight on the nonlinear

development of the drift instabilities and the accompanied plasma transport.^{5, 6} Moreover, the simulation approach becomes even more attractive due to the recent advances in computers and our techniques in numerical modeling of plasmas, which allow us to use rather realistic plasma parameters in both two-and-a-half and full three-dimensional models.^{7, 8}

It is the purpose of this paper to present the simulation results on the universal and current-driven drift instabilities in a collisionless plasma using an electrostatic two-and-a-half-dimensional model (two coordinate space and three velocity space). While the present model does not allow full three-dimensional turbulence,^{6, 8} several nonlinear effects are identified which are inherent to drift instabilities. Furthermore, realistic parameters such as real mass ratio are used without any difficulty in the present model. The model is described in Sec. II. The simulation results are presented in Sec. III along with the theoretical considerations. Concluding remarks are given in Sec. IV.

II. MODEL

We shall describe the two-and-a-half-dimensional simulation model briefly here together with the plasma parameters used. Figure 1 shows a sketch of the model,⁷ in which the plasma is confined between two conducting plates in the x-direction where the electrostatic potentials are zero. In the y-direction, the system is periodic. Particles are reflected from the plates in such a way that no sheath current would be produced. A strong confining

magnetic field is imposed externally with

$$\vec{B} = B_0 \hat{z} + B_y \hat{y}.$$

For shearless cases, B_y is constant and for sheared cases,

$$B_y(x) = B_0 (x/L_s),$$

where L_s is the shear length. Since the model is two dimensional, only k_x and k_y are allowed in the system ($k_z \equiv 0$). The wavenumber along the magnetic field, $k_{||}$, is given by

$$k_{||} = \frac{\vec{k} \cdot \vec{B}}{B} = \frac{k_y B_y}{B},$$

which is constant for shearless case and, for sheared case, varies linearly with x . The particular simple choice of the boundary conditions and the magnetic field configurations allows us to compare the numerical results with the theoretical predictions in both linear and nonlinear regimes of the instabilities.⁶ As for the particle motion, the guiding-center approximation for the electrons and the exact dynamics for the ions have been used in the present model. The guiding-center approximation removes the high frequency oscillations associated with the electron gyromotion from the system, and relaxes the restrictions on the mass ratios and time increments used in the simulation.⁷

The plasma is assumed initially to have an exponential density profile,

$$n(x) = n_0 \exp(-\kappa x),$$

and a spatially constant temperature. Typically, a 64×64 grid has been used in the simulation. However, in some instances, 64×32 and 64×128 grids were also used. The simulation parameters are: mass ratio $m_i/m_e = 1837$, temperature ratio $T_e/T_i = 4 \sim 9$, magnetic field $\Omega_e/\omega_{pe} = 5 \sim 10$ and the ion gyroradius in terms of grid size $\rho_i/\Delta \simeq 2$. These parameters may be considered as corresponding to a plasma in a Q-machine or a low-density toroidal confinement device. The time step for the integration is usually $\omega_{pe} \Delta t = 10 \sim 20$.⁷

The diagnostics in the code include the measurements of the field energy, particle energy, mode structures, velocity distributions, particle and energy diffusion and power spectra of the fluctuations. They are essential for monitoring the development of the instabilities and the resultant anomalous diffusion.

It should be pointed out that there exists a basic difference between the simulation work reported here and the real machine experiments. In the present simulation, these instabilities have been treated as an initial value problem in which the plasma, in response to the instability, is allowed to evolve from the initial unstable state to a final stable configuration through nonlinear interactions. On the other hand, in the real experimental situations in devices like tokamaks, a stationary state is usually established by the balance of the energy input and the particle recycling with the anomalous plasma transport induced by the instabilities. Although the time-dependent turbulence may be somewhat different from a steady state one, we believe that the present simulation should still provide us clues for the mechanisms

of anomalous diffusion. The simulation for steady state turbulence is rather complicated and should be considered as a task for the future.

A brief account of the present work using a small mass ratio, $m_i/m_e = 25$, and with an emphasis on the quasilinear density diffusion has been reported earlier.⁵

III. RESULTS OF SIMULATIONS

A. Simulation of Universal Instability

Let us first discuss the solution of the linear dispersion relation for the drift instabilities appropriate for our simulation model. For the choice of exponential density profile, the diamagnetic drift frequency $\omega_e^* \equiv k_y c T_e \kappa / e B$ is constant everywhere where $\kappa = -d \ln n / dx$. The normal modes for such a system with the conducting wall boundary condition are given by⁵

$$\phi = \sin (m \pi x / L_x), \quad m = 1, 2, 3 \dots \quad (1)$$

The corresponding dispersion relation is¹

$$\sum_{\alpha} \frac{1}{T_{\alpha}} \left[1 + \frac{\omega - \omega_{\alpha}^*}{\sqrt{2} k_{\parallel} v_{t\alpha}} \Gamma_{\alpha} Z \left(\frac{\omega}{\sqrt{2} k_{\parallel} v_{t\alpha}} \right) \right] \phi = 0 \quad (2)$$

where $\Gamma_{\alpha} = I_0(b_{\alpha}) \exp(-b_{\alpha}^2)$, $b_{\alpha}^2 = \rho_{\alpha}^2 k_{\perp}^2 = (v_{t\alpha} / \Omega_{\alpha})^2 k_{\perp}^2$, Ω_{α} is the gyrofrequency of the species α , $k_{\perp}^2 = k_x^2 + k_y^2$, $\omega_e^* = \omega^*$, $\omega_i^* = -\omega^* T_i / T_e$ and Z is the plasma dispersion function.⁹

In the long-wavelength limit, $k_{\perp} \rho_i \ll 1$, the solution of Eq. (2) is given by

$$\omega = \omega_e^* \Gamma_i / (2 - \Gamma_i)$$

and

$$\gamma / \omega_e^* = 2 (\pi)^{1/2} (\omega_e^* / k_{\parallel} v_{te}) \Gamma_i (1 - \Gamma_i) / (2 - \Gamma_i)^3.$$

However, for larger values of $k_{\perp} \rho_i$, Eq. (2) can only be solved numerically. Figure 2 shows the solution of the frequency and the growth rate as a function of $k_y \rho_i$ for the case where $m_i/m_e = 1837$, $T_e/T_i = 9$, $\Omega_e/\omega_{pe} = 10$, $\kappa \rho_i = 0.2$, $k_x \rho_i = 0.14$ and $B_y/B_0 = 190$ (shearless case). As we can see, drift waves become highly dispersive for $k_y \rho_i \gtrsim 0.1$ and both the frequency and the growth rate reach their maxima at $k_y \rho_i \simeq 0.3$. It should be noted that the results here are for the two-and-a-half-dimensional model where the ratio of $k_{\parallel}/k_y$ is fixed for every value of k_y .

1. Single Mode Unstable Case (Case 1).

Let us first describe the simulation results using the exponential density profile without shear. The first example is the case where only one unstable mode is present in the system (Case 1). The system size is $64(x) \times 32(y)$ and the ion gyroradius is $\rho_i/\Delta = 2.8$, so that the first mode in y has $k_y \rho_i = 0.56$. Higher k_y modes have smaller growth rates as shown in Fig. 2, and saturate very quickly by nonlinear effects and essentially do not show any growth in the simulation.

Figure 3 shows the time development of the density fluctuations $\delta n_{k_y} / n_0$ associated with the first mode in y averaged over x . After the exponentiation with the predicted growth rate, the instability saturates at around $\omega_{pe} t = 14000$ or $\Omega_i t = 76$. The decrease in amplitude of the density fluctuation after the saturation is caused by the nonlinear excitation of other modes with different k_x in the system. The measured frequency also agrees well with that of the linear theory.

The corresponding mode structure near the time of saturation is shown in Fig. 4. We see that the mode extends over the entire plasma column peaking near the center as predicted by the linear theory. The maximum $e\phi_k / T_e$ reaches 0.14 for the most unstable mode, which is in close agreement with the measurements in the linear devices.² In general, the saturation amplitude observed in toroidal devices³ is smaller presumably due to the presence of strong shear.

Several nonlinear effects can be clearly identified from Figs. 5 and 6 which show the particle density and the electron parallel temperature along the field lines, $T_{e\parallel}$, at the time of saturation. From the electron and ion density profiles, one can see the strong local diffusion and formation of plateaus which incidentally are not right on top of each other. This charge separation generates strong ambipolar electric field reaching $e\phi_k / T_e \lesssim 0.4$ at the maximum which in turn is found responsible for the nonlinear saturation due to the frequency shift associated with the cE_x / B drift. This will be discussed more later.

Note also for the present model, particle diffusion is strongly prohibited due to the imposed boundary conditions on the potential at the

wall and, in fact, the average density gradient is maintained throughout the calculations. If the wall potentials are not held fixed, we find the density gradient is substantially reduced.⁶ In real experimental devices however, it is usually true that the density gradient is maintained because of the balance of particle input and output. Obviously there are certain choices for the boundary condition and the present model resembles to the steady state experiment in that respect.

As we can see from Fig. 6, the electron temperature profile, which is uniform initially, has undergone a considerable change during the course of the instability. It is due to the inverse electron Landau damping which extracts thermal energy from the electrons and gives it to the wave. Here by temperature, we mean the total kinetic energy including both the random thermal motion and the coherent motion associated with the waves. This is done because it is not clear how to distinguish the coherent and random motions in the presence of large amplitude waves interacting nonlinearly with each other. The coherent motions are eventually thermalized by the dissipative processes either by classical collisions or by plasma turbulence and at that stage total kinetic energy may be called the temperature.

Using the local energy conservation between the electrons and the waves, we have

$$\frac{1}{2} \frac{\partial n T_{e\parallel}(x)}{\partial t} = - \frac{\sum_k 2\gamma_k W_k}{L_x L_y}, \quad (3)$$

where W_k is the wave energy associated with the drift instability and the ambipolar potential ($k_y = 0$) generated as a result of the difference between ion and electron diffusion. For drift waves, W_k is found to be¹⁰

$$W_k = \frac{|E_k|^2}{8\pi} \omega \frac{\partial \epsilon}{\partial \omega} = \frac{|E_k|^2}{8\pi} \left(1 + \frac{1}{k^2 \lambda_e^2} + \frac{\omega_{pi}^2}{\Omega_i^2} \right) \quad (4)$$

where ϵ is the dielectric constant of the drift wave given by

$$\epsilon = 1 + \frac{1}{k^2 \lambda_e^2} + \frac{1 - \Gamma_i}{k^2 \lambda_i^2} - \frac{\Gamma_i}{k^2 \lambda_e^2} \frac{\omega_e^*}{\omega}$$

The three terms on the right-hand side of Eq. (4) are electric field, electron shielding, and ion $c \mathbf{E} \times \mathbf{B} / B^2$ energies, respectively. For our present parameters, ion drift energy is the largest among the three terms for drift waves and the ambipolar potential. Integrating Eq. (3) in time, we find

$$\begin{aligned} \frac{T_{e||}(x, t) - T_{e||}(x, 0)}{T_{e||}(x, 0)} &\approx \frac{\sum_k |E_k|^2(x, t)}{2\pi L_x L_y n(x) T_{e||}(x, 0)} \frac{\omega_{pi}^2}{\Omega_i^2} \\ &= \frac{\omega_{pi}^2}{2\pi L_x L_y n(x) T_{e||}(x, 0) \Omega_i^2} \left[|E_{k_y=0}(x, t)|^2 \right. \\ &\quad \left. + |E_{k_y}(t)|^2 \sin^2 \pi x / L_x \right] \end{aligned} \quad (5)$$

where $|E_{k_y=0}|$ is the ambipolar field, and $|E_{k_y}|$ is the unstable drift wave whose mode structure is assumed to be $\sin \pi x / L_x$. Using the

amplitude and the mode structure measured in the simulation for the ambipolar field to be discussed later, one can estimate Eq. (5) and the result is plotted in Fig. 6. The overall agreement is remarkably good considering the simple theory used for the model. However, Eq. (5) does not agree with the measurement with regard to the detailed structure of $T_{e\parallel}$. In particular, the measurement indicates the substantial increase in temperature for $0 \lesssim x/L_x \lesssim 0.25$ while the theory predicts only the decrease of temperature in the region. It appears that this increase of electron temperature is caused by particle diffusion due to drift waves which carries the low-energy electrons preferentially leaving the high-energy electrons behind. This results in the increase of temperature as seen in Fig. 6. Note also in Eq. (3), energy transport due to wave propagation and heat conduction is neglected because the group velocity of the drift waves is rather slow as will be discussed later.

Considerable steepening of the electron parallel velocity distribution through nonlinear interaction has also been observed in the simulation. The spatially averaged velocity space distribution is shown in Fig. 7. This quasilinear steepening, which occurs in a region near the phase velocity of the wave, is certainly one of the nonlinear effects leading to the saturation through the increase in Landau damping.¹¹ However, for all the cases we have studied here for the universal modes, the steepening alone is not sufficient for the total stabilization of the instability, despite the fact that velocity space gradient has increased by a factor of two, or so, near the resonant region as shown in Fig. 7. Near the marginal stability, the linear growth

rate can be written as¹¹

$$\gamma_k \cong -\frac{\pi T_e \omega_k}{2|k_{\parallel}| m_e} \left(k_{\parallel} \frac{\partial}{\partial v_{\parallel}} + \frac{k_y}{\Omega_e} \frac{\partial}{\partial x} \right) f_e(v_{\parallel}, x) \Big|_{v_{\parallel} = \omega/k_{\parallel}},$$

where the first term in the bracket is the stabilizing Landau damping term and the second the destabilizing density gradient term which, in our choice of simulation parameters is always larger than the damping term by a factor of 4 or 5. Therefore, doubling the Landau damping rate does not cause the total stabilization unless the density gradient is also modified. Furthermore, the instabilities studied here are far from the marginal stability and other nonlinear effects will certainly come into play as well. One such effect is the buildup in charge separation between electrons and ions which generates a strong dc electric field which, in turn, produces a local $v_d = cE_x/B\hat{y}$ drift, as shown in Fig. 8. As mentioned earlier, the charge separation is caused by the quasilinear diffusion in real space shown in Fig. 5. Since the induced drift speed in some local region becomes comparable to the diamagnetic drift v_d^* , one expects that it should have strong effects on the drift instabilities because of the change in the dispersion relation due to the Doppler frequency shift $\Delta\omega (=k_y v_d) \gtrsim \omega^* (=k_y v_d^*)$. The relative magnitude of the drift can be written as

$$\frac{v_d}{v_d^*} = \frac{cE_x}{B} \frac{eB}{T_e k c} = \frac{e\phi}{T_e} \frac{k_x}{k} \approx \frac{e\phi}{T_e}$$

where k_x is the wave number characterizing the spatial structure of the

generated ambipolar field ($k_x > \kappa$). One sees when $(e\phi/T_e)(k_x/\kappa) > 1$, v_d can easily exceed v_d^* giving rise to the large change in the dispersion relation. Note that the observed drift in the simulation is not uniform along the x-direction and, therefore, the Doppler frequency shift is actually spatially dependent. If the drift velocity, shown in Fig. 8, is included in the linear dispersion relation Eq. (1), one finds that the growth rate becomes much smaller than the frequency as the drift speed v_d approaches v_d^* and any small nonlinear effects will quench the instabilities. We, therefore, conclude that the ambipolar drift is the dominant nonlinear saturation mechanism for our case. The condition for such an effect to take place is

$$v_d > v_d^* \quad \text{or} \quad e\phi/T_e \gtrsim \kappa/k_x. \quad (6)$$

Finally, vortex-type plasma motion has been observed, as shown in Fig. 9, where a set of test ions initially located at $x=20$ in a thin strip along y were followed in time. Clearly, the vortex structure in (x, y) plane is due to the $\tilde{E} \times \tilde{B}/B^2$ drift where $\tilde{E}_x$ is the ambipolar field, and $\tilde{E}_y$ is the drift wave field. In this two-and-a-half-dimensional model, no zero frequency modes were generated as observed in three-dimensional simulations.⁶ This is because the present model does not allow the full three-dimensional turbulence which prohibited the nonlinear excitation of convective cells.¹²

2. More than One Mode Unstable Case (Case 2, 3).

We shall now look at the second example (case 2) in which all the parameters are the same as case 1, except that the system length in y is doubled so that a 64×64 grid is now used. There are two dominant modes in the system, $k_y \rho_i = 0.28$ and $k_y \rho_i = 0.56$, as seen from Fig. 2. The frequency for both modes is about the same, $\omega / \Omega_i = 0.12$, and the growth rate for the first mode ($k_y \rho_i = 0.28$) is $\gamma / \Omega_i = 0.1$ and 0.09 for the second mode ($k_y \rho_i = 0.56$).

The results of the simulations show an interesting mode competition which was absent in the previous case. Figure 10 shows the time development of the density perturbation $\delta n_{k_y} / n_o$ averaged over x for the first, second and the third mode. We see that the first mode grows to large amplitude $\delta n_{k_y} / n_o \approx 0.2$ with the expected growth rate while the growth of the second and third modes is reduced and they saturate at quite a low level of $\delta n_{k_y} / n_o \lesssim 0.03$. Clearly, the first mode is quenching the second mode which grew to a larger amplitude previously (Case 1). In fact, the following argument appears to explain the observed results. At first, mode 1 and 2 grow from the initial noise level. When their amplitude becomes finite, the wave electric fields perturb the particle orbit through the $c \underline{E} \times \underline{B} / B^2$ drift. The Doppler frequency shift of the mode 2 due to the electric field of mode 1 may be given by

$$\Delta \omega_2 = k_{y2} \frac{c E_{x1}}{B} = k_{x1} k_{y2} \frac{c}{B} \phi_{k1} \quad (7)$$

Similarly, the Doppler shift of mode 1 due to mode 2 is given by

$$\Delta \omega_1 = k_{x2} k_{y1} \frac{c}{B} \phi_{k2} \quad (8)$$

It is natural to expect that the region of the wave particle resonance is significantly broadened when $\Delta \omega \approx \omega$ and the growth rate is reduced since the waves now see an averaged velocity space gradient. For the present case $\omega_1 \approx \omega_2$ but $\Delta \omega_2 > \Delta \omega_1$ because

$$\frac{\Delta \omega_2}{\Delta \omega_1} = \frac{k_{x2}}{k_{x1}} \frac{k_{y2}}{k_{y1}} \frac{\phi_{k1}}{\phi_{k2}} \approx 2 \exp(\gamma_1 - \gamma_2) t > 2 \quad (9)$$

where $k_{x1} = k_{x2} = \pi/64$ and $\gamma_1 > \gamma_2$. Taking $k_{y2} = 4\pi/64 = 0.2$ and $e\phi_{k1}/T_e = 0.1$ at $t = 6000 \omega_{pe}^{-1}$, we find

$$\Delta \omega_2 = 4 \times 10^{-4}$$

which is comparable to $\omega_2 = 6.5 \times 10^{-4}$ so that the second mode should have significant change in the growth rate at this time. In fact, this is the case as seen in Fig. 10. It is clear that, after the second and the third modes are quenched, the first mode should grow like in the single mode unstable case.

After the first mode grows to a large amplitude, the nonlinear effects are essentially the same as in case 1. The dominant stabilization mechanism again comes from the Doppler shift due to the ambipolar field.

When the system size is further increased to a 64×128 grid (case 3), we have observed that, unlike case 2, several modes $k_y \rho_i = 0.14, 0.28, 0.42, \text{ and } 0.56$ attain finite amplitude as shown in Fig. 11. The reason is as follows. As the system length becomes larger, not only the number of unstable modes increases, but the wave number separation Δk among the modes also decreases. Consequently, the Doppler shift $\Delta \omega$ for each mode becomes comparable to that for the other modes. Therefore, there would be no dominant single mode present when there are several linearly unstable modes with the comparable frequency and growth rate. The saturation for the first three modes are $\delta n_k / n_0 = 0.17, 0.1 \text{ and } 0.075$. Similar results are observed for three-dimensional simulations where more than 10 modes are unstable.^{6, 12} Near saturation of the instability, the plasma is more turbulent due to the presence of several large amplitude modes, and one can again observe vortex-type structure in test particle orbits, shown in Fig. 12, where the vortex is dominated by the first mode which has the largest amplitude.

The dominant saturation mechanism in this case is still the $c \underline{E} \times \underline{B} / B^2$ drift due to the ambipolar electric field, although mode coupling (competition) and other nonlinear interaction also contribute to the final stabilization.

B. Universal Instability in the Presence of Shear (Case 4)

The effect of magnetic shear has been studied previously using a mass ratio of $m_i / m_e = 25$,⁵ and the simulation results have been found to

be consistent with the theoretical stabilization criteria.¹³ Here, we will present a few examples of shear stabilization using a realistic mass ratio. Magnetic shear is introduced by employing an external magnetic field $\vec{B}(x) = B_0 [\hat{z} + (x/L_s) \hat{y}]$ as stated previously where L_s is the shear length. According to the stabilization criterion in a slab model with the exponential density,

$$L_s/L_n \lesssim (m_i/m_e)^{1/3} \approx 12$$

is required for a hydrogen plasma for total stabilization. The mode structure, using the outgoing energy boundary condition, is given by¹³

$$\phi_n(x) = H_n [\exp(i\pi/4) \sigma^{1/2} x] \exp(-i\sigma x^2/4)$$

where H_n is the Hermite polynomial and

$$\sigma = 2k_y v_i / L_s \omega \rho_i.$$

Let us now examine whether the outgoing energy boundary condition is valid in the simulation by estimating the time it takes for wall reflections to interfere with the central region. To do so, let us consider the eigenfunction for $n=0$ given by

$$\phi(x, t) = \exp(-i\sigma x^2/4 - i\omega t).$$

The phase and the group velocity of the wave packet in this case is

$v_p = -4\omega/\sigma x$ and $v_g = 4\omega/\sigma x = -v_p$ respectively. Therefore, the time for a wavepacket to reflect from the wall and come back to the central region

can be estimated by

$$2 \int_0^{L/2} \frac{dx}{|v_g|} = \frac{1}{16} \frac{\sigma L^2}{\omega} \approx 2 \times 10^4 \omega_{pe}^{-1}$$

for $L_s = 1000$. For smaller L_s , the time for reflection is even longer. In general, our simulation is within this limit and, therefore, the outgoing energy boundary condition may be justified.

Figure 13 shows the growth of the average density perturbation for the first k_y mode in a 64×32 system for different shear lengths. The simulation parameters are the same as in case 1 (except that the initial preparation of the plasma is slightly different). Note that the density scale length is $L_n/\Delta = 14$ in all cases. For the case with $L_s/L_n = 150$, shear has little effect on the growth of the drift instability except for the final saturation level which is somewhat smaller than that of the shearless case (case 1). The time lag in growth between the two cases is due to the initial noise level corresponding to the different normal mode structures. As the shear increases ($L_s/L_n = 75, 37.5$), the saturation level of the instability decreases accordingly. However, the growth rates are harder to determine because of the fluctuations of the normal modes. At any rate, the plasma clearly remains unstable. A typical mode structure for the sheared case is shown in Fig. 14, which, as expected, deviates considerably from the one for the shearless case in Fig. 4. However, it is also different from the theory, because the fact that the region where $k_{||} = 0$ does not contribute

to the wave-particle resonance, has been ignored in the theoretical formulation.¹³

The effect of shear on the local particle diffusion has been studied in terms of the change in the density profile due to the instabilities. The time development of the quantity $\delta n_{k_x}/n_o$, is shown in Fig. 15, where δn_{k_x} is the first mode of the density profile in the x-direction averaged over y. The stabilization due to shear is evident. The corresponding diffusion coefficients $D_{\perp}$ have been obtained by using

$$\frac{\partial \delta n/n_o}{\partial t} = -k_x^2 D_{\perp} \delta n/n_o ,$$

and plotted in Fig. 16. The measured $D_{\perp}$ for the shearless case is about 30% of the value given by the turbulent diffusion coefficient $\gamma/k_{\perp}^2$.¹ By extrapolating the results in Figs. 15 and 16, one can safely conclude that the required shear for total stabilization is consistent with the theoretical predicted values.¹³

C. Simulation of the Current-Driven Instability (Case 5)

We shall now discuss the current-driven drift instabilities in a collisionless plasma.¹ The dispersion relation is similar to Eq. (2) except for the Doppler shift of the frequency $\omega - k_{\parallel} v_o$ in the electron Z function where v_o is the electron drift relative to the ions along the magnetic field. The frequency and the growth rate is given by for $k_y \rho_i \ll 1$

$$\omega = \omega_e^*$$

and

$$\gamma/\omega_e^* = (\pi)^{1/2} v_o/v_{te}.$$

Note that there is no threshold of the current for this instability. The numerical solution for the case of $v_o/v_{te} = 0.3$ is shown in Fig. 17 together with the universal instability for comparison. As we can see, the frequency is almost the same for both cases, but the growth rate is about 50% higher for the current driven instability. Note the instability does not require the finite gyroradius effect here.

Figure 18 shows the time development of the density fluctuations δn_{k_y} for the first $k_y = 2\pi/L_y$ mode averaged over x . Here a 64×64 grid is used. After the exponential growth with a value which agrees well with the theory, $\delta n_{k_y}/n_o$ saturates at around 7% for the most unstable mode and then decreases slowly. The observed frequency spectrum measured locally at $x=36$ also gives excellent agreement with the frequency from the linear theory, shown in Fig. 19. The saturation level is substantially lower here compared with the universal mode and we found that the particle and energy diffusion is considerably less compared with the previous cases for the universal instabilities. However, we have found considerable modification of the electron velocity distribution along the field lines, as shown in Fig. 20, where the initial drift is wiped out considerably and the distribution approaches a stationary Maxwellian at saturation. This modification of velocity distribution

appears to be the dominant saturation mechanism and releases less free energy for the instability than for the case of the universal instabilities. The fact that the strong universal instability does not develop after the current-driven mode is stabilized by velocity space diffusion is partly because the growth rate for the universal instability is smaller for the present simulation parameters. The $cE \times B/B^2$ drift due to the ambipolar electric field is also considerably smaller. In a real laboratory experiment such as a tokamak, the electron current is usually continuously fed in or driven by the external circuits and, therefore, the saturation level may be higher.

IV. CONCLUSION

We have presented several examples of numerical simulations of collisionless drift instabilities using a two-and-a-half-dimensional slab model. The real mass ratio for a hydrogen plasma was used throughout the simulation. It was found that several nonlinear effects including quasilinear diffusion in real space, Doppler shift due to strong local ambipolar field, competition among the unstable modes and the velocity space steepening are the dominant saturation mechanisms. While the present simulation results do not allow full three-dimensional turbulence,^{6, 12} they reveal several important nonlinear effects which must occur in full three-dimensional cases. Shear stabilization has been investigated and found that the simulation results agree well with the theoretical predictions.

As far as the numerical techniques are concerned, the present work, has proved that realistic plasma parameters including the real mass ratio can be used without difficulty for simulating low-frequency drift instabilities. While the present work treated an isolated system, effects due to energy input and particle recycling such as seen in laboratory experiments may be added to the present model.

Theoretical analysis on the nonlinear saturation based on the quasi-linear theory is being worked out and will be published elsewhere.¹⁴

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REFERENCES

* Present address: TRW Systems, Redondo Beach, California.

- ¹ B. B. Kadomtsev, Plasma Turbulence (Academic Press, 1965), p. 78;
B. B. Kadomtsev and O. P. Pogutse, Nucl. Fusion 11, 67 (1971).
- ² P. A. Politzer, Phys. Fluids 14, 2410 (1971); P. E. Stott, P. F. Little, and J. Burt, Phys. Rev. Lett. 25, 996 (1970).
- ³ E. Mazzucato, Phys. Rev. Lett. 36, 792 (1976); C. M. Surko and R. E. Slusher, Phys. Rev. Lett. 37, 1747 (1976); S. M. Hamberger, L. E. Sharp, J. B. Lister, and S. Mrowka, Phys. Rev. Lett. 37, 1345 (1976); M. Okabayashi and V. Arunasalam, Nucl. Fusion (to be published).
- ⁴ T. H. Dupree, Phys. Fluids 9, 1773 (1966); 10, 1049 (1967); and 11, 2680 (1968).
- ⁵ W. W. Lee and H. Okuda, Phys. Rev. Lett. 36, 870 (1976).
- ⁶ C. Z. Cheng and H. Okuda, Phys. Rev. Lett. 38, 708 (1977).
- ⁷ W. W. Lee and H. Okuda, J. Com. Phys. (to be published).
- ⁸ C. Z. Cheng and H. Okuda, J. Com. Phys. (to be published).
- ⁹ B. D. Fried and S. D. Conte, The Plasma Dispersion Function (Academic Press, New York, 1961).
- ¹⁰ C. S. Liu, M. N. Rosenbluth, and W. M. Tang, Phys. Fluids 19, 1040 (1976).
- ¹¹ R. Z. Sagdeev and A. A. Galeev, in Nuclear Plasma Theory edited by T. M. O'Neil and D. L. Book (Benjamin, New York, 1969) p. 73.

¹². H. Okuda and C. Z. Cheng, Phys. Rev. Letters (submitted for publication).

¹³ D. Pearlstein and H. L. Berk, Phys. Rev. Lett. 23, 220 (1969).

¹⁴ Y. Y. Kuo-Yau, W. W. Lee, and H. Okuda (in preparation).

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Fig. 1. A sketch of the 2 1/2-dimensional model. Plasma is confined between two conducting walls at $x=0$ and $x=L$ and is periodic in y . Strong magnetic field B is imposed externally to confine the plasma.

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Fig. 2. Numerical solution of the collisionless universal mode given by Eq. (2) for a 2 1/2-dimensional model, $k_{\parallel}/k_y = 1/190$.

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Fig. 3. Time development of the density fluctuation associated with the universal mode with only a single unstable mode. Case 1.

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Fig. 4. Mode structure of the most unstable mode near saturation. Case 1.

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Fig. 5. Particle density of the ions and electrons for the initial and final stages. Note the charge separations between ions and electrons. Case 1.

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Fig. 6. Electron parallel temperature $T_{e\parallel}$ for the initial and final stages compared with Eq. (3). Case 1.

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Fig. 7. Initial and final electron velocity distribution along the magnetic field. Note the steepening of the distribution at the wave phase velocity. Case 1.

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Fig. 8. Ion and electron drift due to ambipolar drift cE_x/B resulting from the local charge imbalance. Case 1.

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Fig. 9. Test ion orbit in (x, y) plane at $\omega_{pe} t = 17000$. Note the coherent vortex type structure of the convective flow associated with the $c\vec{E} \times \vec{B}/B^2$ motion. Case 1.

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Fig. 10. Time development of the density fluctuation and the electric field energy for modes $k_{yi} = 0.28, 0.56$ and 0.84 . A 64×64 grid is used. Case 2.

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Fig. 11. Time development of the average density fluctuation for modes $k_{yi} = 0.14, 0.28, 0.42$ and 0.56 . A 64×128 grid is used. Case 3.

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Fig. 12. Vortex-type structure of the test ion orbit associated with the $c\tilde{E} \times \tilde{B}/B^2$ plasma convection. Case 3.

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 Fig. 13. Time development of the average density perturbation for different strength of shear. Case 4.

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Fig. 14. A typical mode structure for the sheared case with $L_s/L_n = 150$. Case 4.

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Fig. 15. Time development of the average density profile for the first mode $k_x = 2\pi/L_x$ for different shear strengths. Case 4.

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Fig. 16. Density diffusion coefficient D_1 vs. L_n/L_s .
Case 4.

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Fig. 17. Frequency and the growth rate from the linear theory for the universal and current-driven instabilities. Case 5.

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Fig. 18. Time development of the average density perturbation for the first mode $k_y = 2\pi/L_y$. Case 5.

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Fig. 19. Frequency spectrum measured at $x = 36$ which agrees with the linear theory. Arbitrary ordinate unit. Case 5.

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Fig. 20. Velocity distribution of the electron along the magnetic field for the current driven drift instability. Case 5.