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PAIRING INTERACTION EFFECTS IN EXCITON LEVEL DENSITIES†

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ABSTRACT

Recent progress in pairing corrections for exciton state-density formulas used in precompound nuclear reaction theories is reviewed. These correction factors are, strictly speaking, dependent on the nuclear excitation energy U and the exciton number n. A simple formula for (U, n)-dependent pairing corrections has been derived, based on the BCS pairing equations for constant single-particle spacing, for the exciton state-density formula for one kind of Fermion. It has been shown that the constant-pairing-energy correction used in standard state-density formulas, such as U_0 in Gilbert and Cameron, is a limiting case of the general (U, n)-dependent results. Spin cutoff factors with pairing effects were also obtained using the same theory and parameterized into an explicit (U, n)-dependent function, thereby defining a simple exciton level-density formula for applications in quantum mechanical precompound theories. Preliminary results from extending such simple pairing-interaction representations to level-density formulas for two kinds of Fermions are summarized. The results show that the ratios in the exciton level densities in the one-Fermion and two-Fermion approaches vary with both U and n, thus likely leading to differences in calculated compound to precompound ratios. However, the differences in the spin cutoff factors in the two cases are found to be rather small.

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1. INTRODUCTION

Total and exciton level density formulas are used in modern Hauser-Feshbach codes to calculate compound and precompound reaction cross sections. Pairing interaction effects are reasonably understood for the former but not for the latter. The purpose of this paper is to review the progress in the (U,n)-dependent pairing correction for the exciton level-density formula for one-kind of Fermion (Sections 2 and 3) and report on some preliminary work by the author for two kinds of Fermions (Sections 4 to 8). It is important to note in the very beginning that the standard Fermi-gas level density formula used in Hauser-Feshbach model codes and used for extracting the level-spacing values from neutron resonance data is for two kinds of Fermions. To be consistent, the two-Fermion formulas should also be used for calculating the precompound effects in such codes.

2. PAIRING CORRECTIONS IN THE ONE-FERMION FORMULAS

The total state density in the one-Fermion formulation is given by[1]

$$\omega_1(U) = \frac{\exp[2(aU^*)^{1/2}]}{\sqrt{48}\,U^*} \ , \tag{1}$$

where the subscript "1" indicates the one-Fermion formulation in which neutrons and protons are not distinguished and occupy the same set of single particle states; the parameter a is related to the single-particle state density g by $a = \pi^2 g/6$; and the effective excitation energy U^* is given by $U^* = U - U_0$, where U is the excitation energy and U_0 is a constant-pairing-energy correction, such as tabulated by Gilbert and Cameron.[2]

The corresponding exciton state density has the same form as the Williams formula[3] and is given by

$$\omega_1(p,h,U) = \frac{g^n(U - A_1 - P_1)^{n-1}}{p!h!(n-1)!} \ , \tag{2}$$

where p is the particle number, h is the hole number, and n is the exciton number equaling $p+h$. The quantity A_1 is a correction factor for the Pauli exclusion principle, modified for pairing effects.[4,5] The pairing correction $P_1(U,n)$ is that suggested by Ignatyuk and Sokolov[6] and is given by

$$P_1(U,n) = \frac{g}{4}[\Delta_0^2 - \Delta^2(U,n)] \ , \tag{3}$$

where Δ_0 is the ground-state pairing gap. In Eq. (3) the quantity $\Delta(U,n)$ is the excited-state pairing gap calculated from the pairing theory[7] using Δ_0 and g as input parameters. The values of $\Delta(U,n)$ have been obtained and parameterized[4] for everyday use as a simple function of U, n, Δ_0, and g. Depending upon the values of U and n, the pairing correction $P_1(U,n)$ has values lying between 0 and U_0. Eq. (2) with the new (U,n)-dependent pairing correction was validated in Ref. 4 by comparing with the exciton state densities calculated directly from the BCS equations by Morreto[7]. The conventional correction factor for the

Pauli exclusion principle was modified in Ref. 4 to account for the pairing effects and was further improved by Kalbach[5].

It has been shown[4] that the pairing correction U_0 for the total state density is equal to the value of $P_1(U,n)$ evaluated along the most probable exciton number, $\hat{n}$, for $U \geq 3.15U_0$. For a fixed U, $\Delta(U,n) = 0$ if $n \geq \hat{n}$, therefore,

$$U_0 = P_1(U,\hat{n}) = \frac{1}{4} g \Delta_0^2 \ . \tag{4}$$

This relation provides a means of calculating Δ_0 from a given U_0, which is well understood. It has further been shown that even for $U < 3.15U_0$, Eq. (4) is still adequate because this low excitation-energy range has mostly discrete levels. Therefore, the pairing correction $P_1(U,n)$ is completely defined by the two parameters g and U_0, and the pairing correction U_0 used in Eq. (1) and the pairing correction $P_1(U,n)$ used in Eq. (2) are consistent.

3. SPIN CUTOFF FACTORS IN THE ONE-FERMION FORMULATION

The spin distribution formula corresponding to Eq. (1) is given by[2,6]

$$R(U,J) = \frac{(2J+1)}{2(2\pi)^{1/2}\sigma^3(U)} \exp[\frac{-(J+1/2)^2}{2\sigma^2(U)}] \ , \tag{5}$$

where J is the total spin quantum number and $\sigma^2(U)$ is the spin cutoff factor given by:

$$\sigma^2(U) = g \langle m^2 \rangle [(U - U_0)/a]^{1/2} \ , \tag{6}$$

where $\langle m^2 \rangle$ is the mean square of the projections of spins of the single particles and holes on the z axis of the nucleus. The pairing effects enter into the spin distributions through the constant-pairing-energy correction U_0 in Eq. (6).

The particle-hole spin distribution for one kind of Fermion is given by[6]

$$R(n,U,J) = \frac{(2J+1)}{2(2\pi)^{1/2}\sigma^3(U,n)} \exp[\frac{-(J+1/2)^2}{2\sigma^2(U,n)}] \ , \tag{7}$$

where the spin cutoff factor, through which the pairing effects enter, is given by

$$\sigma^2(U,n) = 2g\langle m^2 \rangle \int_0^\infty f(\epsilon)[1 - f(\epsilon)] \, d\epsilon \ , \tag{8}$$

and

$$f(\epsilon) = \frac{1}{1 + \exp[(E - \xi)/T]} \ , \tag{9}$$

with

$$E = (\epsilon^2 + \Delta^2)^{1/2} \ . \tag{10}$$

This spin cutoff factor was derived on the basis of the uniform pairing model[6] for an even-even nucleus; thus $p = h = n/2$. The quantity ϵ is the single-particle energy, ξ is

the Lagrange multiplier constraining the excited system to a fixed value of n, and T is the thermodynamical temperature. The values of Δ, ξ, and T can be obtained numerically from the pairing equations for each combination of g, Δ_0, U, and n. Thus the (U,n)-dependence in $\sigma^2(U,n)$ of Eq. (8) is implicit in the functions Δ, ξ, and T. The pairing equations used are those used previously for obtaining the one-Fermion pairing correction[4] and the numerical results for $\sigma^2(U,n)$ have been calculated and parameterized[8] as an explicit function of (U,n) for easy application of this advanced formula.

The product of Eqs. (1) and (5) defines the total level-density formula in the one-Fermion approach:

$$\rho_1(U,J) = \omega_1(U)\, R(U,J)\ . \tag{11}$$

Similarly, the product of Eqs. (2) and (7) defines the exciton level-density formula for one kind of Fermion:

$$\rho_1(n,U,J) = \omega_1(p,h,U)\, R(n,U,J)\ . \tag{12}$$

It has been shown that the two spin cutoff factors in $R(U,J)$ and $R(n,U,J)$ are related by[7]

$$\sigma^2(U) = \sigma^2(U,\hat{n}) = g\,\langle m^2\rangle\,[(U-U_0)/a]^{1/2}\ . \tag{13}$$

Therefore, $\sigma^2(U)$ is a limiting case of $\sigma^2(U,n)$ and both are consistently defined by the same three parameters, g, U_0, and $\langle m^2\rangle$.

4. PAIRING INTERACTION PROBLEMS IN THE TWO-FERMION CASE

The two-Fermion total state-density formula, most commonly used in Hauser-Feshbach codes, is given by[2]

$$\omega_2(U) = \frac{\sqrt{\pi}}{12}\ \frac{\exp[2(aU^*)^{1/2}]}{a^{1/4}U^{*5/4}} \tag{14}$$

where the subscript "2" indicates the two-Fermion formulation in which neutrons and protons occupy separate sets of single particle states.

Equation (14) was derived under the assumptions that $g_\pi = g_\nu = g/2$, where g_π and g_ν are constants. The quantities g_π and g_ν are, respectively, single-proton and single-neutron state densities. Making the same assumptions as for Eq. (14) and introducing the pairing correction P_2 into the two-Fermion exciton state-density formula of Williams,[3] we have

$$\omega_2(p_\pi,h_\pi,p_\nu,h_\nu,U) = (\frac{g}{2})^n \frac{(U-A_2-P_2)^{n-1}}{p_\pi!h_\pi!p_\nu!h_\nu!(n-1)!}\ , \tag{15}$$

where n is the exciton number equaling $p_\pi+h_\pi+p_\nu+h_\nu$, p_π is the proton-particle number, h_π the proton-hole number, p_ν the neutron-particle number, h_ν the neutron-hole number. The quantity A_2 is the correction factor for the Pauli exclusion principle and is a simple extension of that for one kind of Fermion modified by pairing effects[5].

If $U_0 = 0$ and $P_2 = 0$, Eqs. (14) and (15) are based on the same assumptions and are consistent in principle. In fact, it can be shown numerically that if $U_0 = 0$ and $P_2 = 0$,

$$\omega_2(U) \simeq \sum_{p_\pi = h_\pi, p_\nu = h_\nu} \omega_2(p_\pi, h_\pi, p_\nu, h_\nu, U) \ , \tag{16}$$

within 10%. This small difference is due to different simplifications in deriving Eq. (14) and Eq. (15).

The spin distribution for the total state-density formula for two kinds of Fermions corresponding to Eq. (14) has the same form as that for one kind of Fermion given by Eq. (5). The product of Eqs. (14) and (5) defines the total level-density formula in the two-Fermion formulation:

$$\rho_2(U, J) = \omega_2(U)\, R(U, J) \ . \tag{17}$$

The spin distribution formula for the exciton state-density formula for two kinds of Fermions corresponding to Eq. (15) is obtained by replacing n by (n_π, n_ν) in the one-Fermion formula, Eq. (7), and is given by

$$R(n_\pi, n_\nu, U, J) = \frac{(2J+1)}{2(2\pi)^{1/2}\sigma^3(U, n_\pi, n_\nu)} \exp[\frac{-(J+1/2)^2}{2\sigma^2(U, n_\pi, n_\nu)}] \ , \tag{18}$$

where $n_\pi = p_\pi + h_\pi$, $n_\nu = p_\nu + h_\nu$, and $\sigma^2(U, n_\pi, n_\nu)$ is the spin cutoff factor to be obtained in this paper.

The product of Eqs. (15) and (18) defines the exciton level-density formula in the two-Fermion formulation:

$$\rho_2(p_\pi, h_\pi, p_\nu, h_\nu, U, J) = \omega_2(p_\pi, h_\pi, p_\nu, h_\nu, U)\, R(n_\pi, n_\nu, U, J) \ . \tag{19}$$

We now face the questions of (1) what is the value of P_2, (2) whether the two pairing correction factors U_0 and P_2 can be related in a consistent manner, (3) what is the value of $\sigma^2(U, n_\pi, n_\nu)$, and (4) whether the two spin cutoff factors $\sigma^2(U)$ and $\sigma^2(U, n_\pi, n_\nu)$ can also be related consistently. The goal for the remainder of this paper is to answer these four questions.

5. PAIRING CORRECTIONS IN THE TWO-FERMION FORMULAS

We begin with the assumption that there is no pairing interaction between the protons and the neutrons, then

$$P_2(U, n_\pi, n_\nu) = P_1(U_\pi, n_\pi) + P_1(U_\nu, n_\nu) \ , \tag{20}$$

where

$$P_1(U_\pi, n_\pi) = \frac{g_\pi}{4}[\Delta_{0\pi}^2 - \Delta_\pi^2(U_\pi, n_\pi)] \ , \tag{21}$$

$$P_1(U_\nu, n_\nu) = \frac{g_\nu}{4}[\Delta_{0\nu}^2 - \Delta_\nu^2(U_\nu, n_\nu)] \ , \tag{22}$$

and $U = U_\pi + U_\nu$, $n = n_\pi + n_\nu$, $n_\pi = p_\pi + h_\pi$, and $n_\nu = p_\nu + h_\nu$. In Eq. (20), the dependence of P_2 on U_π and U_ν is implicit since their values can be obtained from U, n_π, and n_ν, as shown in Sect. 6.

The problem now is to find Δ_π and Δ_ν. To find the solution, we must deal with the pairing theory for two kinds of Fermions.

For a nucleus described by constant single-particle state densities g_π and g_ν, and constant pairing strengths G_π and G_ν, the system of equations is given by[9]

$$\frac{2}{G_\pi} = g_\pi \int_0^S \frac{1}{E_\pi} \tanh[(E_\pi - \xi_\pi)/2T] d\epsilon_\pi \ , \tag{23}$$

$$\frac{2}{G_\nu} = g_\nu \int_0^S \frac{1}{E_\nu} \tanh[(E_\nu - \xi_\nu)/2T] d\epsilon_\nu \ , \tag{24}$$

$$n_\pi = 2g_\pi \int_0^S \frac{d\epsilon_\pi}{1 + \exp[(E_\pi - \xi_\pi)/T]} \ , \tag{25}$$

$$n_\nu = 2g_\nu \int_0^S \frac{d\epsilon_\nu}{1 + \exp[(E_\nu - \xi_\nu)/T]} \ , \tag{26}$$

$$U_\pi = -g_\pi \int_0^S \frac{\epsilon_\pi^2}{E_\pi} \tanh[(E_\pi - \xi_\pi)/2T] d\epsilon_\pi - \frac{\Delta_\pi^2}{G_\pi} + \frac{g_\pi}{2} S^2 + \frac{1}{4} g_\pi \Delta_{0\pi}^2 \ , \tag{27}$$

$$U_\nu = -g_\nu \int_0^S \frac{\epsilon_\nu^2}{E_\nu} \tanh[(E_\nu - \xi_\nu)/2T] d\epsilon_\nu - \frac{\Delta_\nu^2}{G_\nu} + \frac{g_\nu}{2} S^2 + \frac{1}{4} g_\nu \Delta_{0\nu}^2 \ , \tag{28}$$

$$U = U_\pi + U_\nu \ , \tag{29}$$

where

$$E_\pi = (\epsilon_\pi^2 + \Delta_\pi^2)^{1/2} \ , \tag{30}$$

$$E_\nu = (\epsilon_\nu^2 + \Delta_\nu^2)^{1/2} \ . \tag{31}$$

The quantities E_π and E_ν are, respectively, the quasi-proton and the quasi-neutron energies, while ϵ_π and ϵ_ν are the single-particle energies for protons and neutrons, respectively. In Eqs. (23) to (29), the known quantities are $g_\pi, g_\nu, \Delta_{0\pi}, \Delta_{0\nu}, n_\pi, n_\nu$, and U, the quantities to be solved are $\Delta_\pi, \Delta_\nu, \xi_\pi, \xi_\nu, U_\pi, U_\nu$, and T. The quantities ξ_π and ξ_ν are Lagrange multipliers constraining the excited system to fixed values of n_π and n_ν, and T is the thermodynamical temperature. The quantities G_π and G_ν are related to $\Delta_{0\pi}$ and $\Delta_{0\nu}$, respectively, by Eqs. (23) and (24) at the ground-state conditions: $T = 0$, $\xi_\pi = 0$, $\xi_\nu = 0$, $\Delta_\pi = \Delta_{0\pi}$, and $\Delta_\nu = \Delta_{0\nu}$. The integration limit S has no effect on the solutions provided S is much greater than $\Delta_{0\pi}$, $\Delta_{0\nu}$, and T.

Equations (23) to (29) can be solved for the seven unknown quantities for each needed combination of the seven input quantities by a numerical iteration technique similar to that used for the one-Fermion case.[4] However, the time-consuming nature of the present problem is many times worse. In the one-Fermion case, it is possible to factor out g and Δ_0, so the input combinations involve only U and n. But in the present case, nothing can be factored out and a numerical iteration has to be made for solutions for each needed combination of the seven input quantities. The possible combinations of the seven input quantities can be so large that the resulting computational costs could prevent this advanced theory from being used at all. For this reason, we seek approximate solutions, described in the following section, that can be easily used and still achieve our goal stated above.

6. APPROXIMATIONS FOR THE PAIRING CORRECTIONS IN THE TWO-FERMION FORMULATION

If $\Delta_{0\pi} = \Delta_{0\nu}$ (mean-gap approximation), approximate values of Δ_π and Δ_ν can be obtained from the existing results of the one-Fermion case given in Sect. 2 without having to solve Eqs. (23) to (29). In this section, we describe how these approximate results can be obtained and examine their uncertainties.

It is seen in Eqs. (23) to (28) that the same temperature T appears. In other words, the proton system and the neutron system are excited isothermally or the two points (U_π, n_π) and (U_ν, n_ν) must fall on the same T curve in an isothermal plot on the (U, n) plane. If it is assumed that $\Delta_{0\pi} = \Delta_{0\nu}$ (remember $g_\pi = g_\nu$ was assumed in the beginning), the same plot applies to both the proton system and the neutron system. If we can find an approximate solution for U_π and U_ν from given U, n_π and n_ν, satisfying the isothermal requirement, then Eqs. (23) to (28) can be separated into two sets of 3 equations, each set representing an independent one-Fermion system. Therefore, the results given above for the one-Fermion system can be used for each of these two systems.

The following simple procedures define U_π and U_ν to approximately 10% of the exact values except for energies near the threshold:

$$U_\pi = n_\pi U/n \ , \tag{32}$$

and

$$U_\nu = n_\nu U/n \ , \tag{33}$$

that give $U = U_\pi + U_\nu$.

For each U_π obtained from Eq. (32), Eqs. (23), (25), and (27) can be solved for Δ_π, ξ_π, and T from given $g_\pi, \Delta_{0\pi}$, and n_π in the same manner as in the one-Fermion model. Similarly, the neutron system can be solved from Eqs. (24), (26), and (28). Each system of equations is identical to that already solved for in Ref. 4 and needs not be repeated. The formula for $\Delta(U, n)$ given in Ref. 4 can be applied for $\Delta_\pi(U_\pi, n_\pi)$ and $\Delta_\nu(U_\nu, n_\nu)$ which, in turn, define the pairing correction $P_2(U, n_\pi, n_\nu)$ by Eqs. (20), (21), and (22).

Detailed examination of this approximation showed that the average error in P_2 due to our simplified approach in determining U_π and U_ν is about 2% except for energies near

the threshold. The reason this error is so small is partly due to the compensating nature in the errors of U_π and U_ν.

Similar to the one-Fermion case, the pairing correction U_0 for the total state density is related to $P_2(U, n_\pi, n_\nu)$ by

$$U_0 = P_2(U, \hat{n}_\pi, \hat{n}_\nu) \ . \tag{34}$$

From Eqs. (20), (21), (22), and (34), and noting that $\Delta_\pi = 0$ at $\hat{n}_\pi$ and $\Delta_\nu = 0$ at $\hat{n}_\nu$, we have

$$\Delta_{0\pi}^2 = \Delta_{0\nu}^2 = 4\,U_0/g \ , \tag{35}$$

which says that the ground-state pairing gaps in the proton system and the neutron system are both equal to that of the one-Fermion system if the same input values of g and U_0 are used. This feature is not surprising since the condensation energies $U_{0\pi} = \frac{1}{4} g_\pi \Delta_{0\pi}^2$ for the proton system and $U_{0\nu} = \frac{1}{4} g_\nu \Delta_{0\nu}^2$ for the neutron system are each one-half of $U_0 = \frac{1}{4} g \Delta_0^2$, the condensation energy of the one-Fermion system.

Therefore, the pairing correction $P_2(U, n_\pi, n_\nu)$ in the mean-gap approximation depends on only two parameters, g and U_0. The fact the pairing-corrected two-Fermion particle-hole state-density formula in the present approach can be consistently defined by the same two parameters as in the total state-density formula is the most desirable feature of the mean-gap approximation. For even-even and odd-odd nuclides, $\Delta_{0\pi}$ and $\Delta_{0\nu}$ are generally not very different, the mean-gap approximation is expected to be reasonable. The approximation could be poor for an odd-A nuclide for which $\Delta_{0\pi}$ and $\Delta_{0\nu}$ can be quite different. In this case, exact solution of the pairing equations may be necessary and has to be done with numerical iterations for large combinations of $g_\pi, g_\nu, \Delta_{0\pi}, \Delta_{0\nu}, n_\pi, n_\nu$, and U. Therefore, the simplicity offered in the mean-gap approximation remains appealing even in the odd-A case.

We have now answered the first two questions posed at the end of Sect. 4, and turn to the remaining two questions.

7. PARTICLE-HOLE SPIN CUTOFF FACTORS IN THE TWO-FERMION FORMULAS

The particle-hole spin cutoff factor appearing in Eq. (18) for two kinds of Fermions is by definition the sum of the two one-Fermion components:

$$\sigma^2(U, n_\pi, n_\nu) = \sigma^2(U_\pi, n_\pi) + \sigma^2(U_\nu, n_\nu) \ , \tag{36}$$

where

$$\sigma^2(U_\pi, n_\pi) = 2 g_\pi \langle m_\pi^2 \rangle \int_0^\infty f(\epsilon_\pi)\,[1 - f(\epsilon_\pi)]\, d\epsilon_\pi \ , \tag{37}$$

$$\sigma^2(U_\nu, n_\nu) = 2 g_\nu \langle m_\nu^2 \rangle \int_0^\infty f(\epsilon_\nu)\,[1 - f(\epsilon_\nu)]\, d\epsilon_\nu \ , \tag{38}$$

and

$$f(\epsilon_\pi) = \frac{1}{1+\exp[(E_\pi - \xi_\pi)/T]} , \tag{39}$$

$$f(\epsilon_\nu) = \frac{1}{1+\exp[(E_\nu - \xi_\nu)/T]} . \tag{40}$$

The system of seven pairing equations given by Eqs. (23) to (29) must be solved for the seven variables, which, in turn, can be used in Eqs. (36) to (40) to calculate rigorously the two-Fermion spin cutoff factors. This calculation, as discussed above, is prohibitively tedious. Reasonably good results can be obtained by using the same approximations as used for the pairing corrections, namely, by using the mean-gap approximation and the approximation for U_π and U_ν given in Eqs. (32) and (33). This way the parameterized function[8] for the spin cutoff factors for one kind of Fermion can be used to replace Eqs. (37) and (38). Thus we have answered the third question posed at the end of Sect. 4.

Combining Eqs. (13) and (36), we have

$$\sigma^2(U, \hat{n}_\pi, \hat{n}_\nu) = g_\pi \langle m_\pi^2 \rangle [(U_\pi - U_{0\pi})/a_\pi]^{1/2} + g_\nu \langle m_\nu^2 \rangle [(U_\nu - U_{0\nu})/a_\nu]^{1/2} , \tag{41}$$

and recalling $g_\pi = g_\nu = g/2$, $U_{0\pi} = U_{0\nu} = U_0/2$, and setting $U_\pi = U_\nu = U/2$, $\langle m_\pi^2 \rangle = \langle m_\nu^2 \rangle = \langle m^2 \rangle$, we have

$$\sigma^2(U, \hat{n}_\pi, \hat{n}_\nu) = \sigma^2(U) . \tag{42}$$

Therefore, $\sigma^2(U)$ is again a limiting case of $\sigma^2(U, n_\pi, n_\nu)$ as in the one-Fermion case, however, this occurs at the additional conditions: $U_\pi = U_\nu$ and $\langle m_\pi^2 \rangle = \langle m_\nu^2 \rangle = \langle m^2 \rangle$. This is the answer for the fourth question posed at the end of Sect. 4.

Both the exciton level densities and the spin cutoff factors calculated for the one-Fermion and the two-Fermion formulations are further clarified by numerical examples in the next section.

8. ILLUSTRATIONS

Several interesting points, difficult to visualize from the formulas, can be made using numerical examples.

^{41}Ca, ^{94}Nb, and ^{241}Pu with realistic parameters were chosen for computational tests. All results show that the consistency relation, Eq. (16), relating the total and exciton state-density formulas, holds within 10% for $U \geq 3.15U_0$ using the constant-pairing-energy correction U_0 for the left side and the generalized pairing correction factor $P_2(U, n_\pi, n_\nu)$ for the right side. When ω_2 on both sides of Eq. (16) is replaced by ρ_2, the approximation still holds. For $U < 3.15U_0$, the agreement in not as good because the constant-pairing-energy correction U_0 is invalid at low energies as shown previously.[4]

The exciton level density for $n = 2$ is the most dominant component for calculating the precompound reaction cross sections while the total level density accounts for compound reaction. The level densities and the spin cutoff factors for these two components

are illustrated for ^{41}Ca using the parameters g=3.937 MeV^{-1} and U_0=1.83 MeV. The parameter $\langle m_\pi^2 \rangle$ and $\langle m_\nu^2 \rangle$ were set equal to $\langle m^2 \rangle$ taken from Ref. 10 to be $0.24A^{2/3}$ where A is the mass of the nuclide. The conclusions drawn below for ^{41}Ca are also valid for ^{94}Nb and ^{241}Pu.

Figure 1 compares $\rho_1(n = 2, U)$ in the one-Fermion formula with $\rho_2(p_\pi = 1, h_\pi = 1, p_\nu = 0, h_\nu = 0, U) + \rho_2(p_\pi = 0, h_\pi = 0, p_\nu = 1, h_\nu = 1, U)$ for ^{41}Ca. These were obtained from Eqs. (12) and (19), respectively, by summing over J. For simplicity, these two components are labeled in Fig. 1 as $\rho_1(n = 2)$ and $\rho_2(n = 2)$. Similarly, $\rho_1(\text{all } n)$ and $\rho_2(\text{all } n)$, obtained by the additional summation over n, are shown. The "all n" component can be regarded through Eq. (16) as the total level density. The one-Fermion densities are larger, as expected.

Figure 2 shows the ratios $\rho_1(n = 2)/\rho_2(n = 2)$ and $\rho_1(\text{all } n)/\rho_2(\text{all } n)$. The former is nearly constant in U while the latter increases with increasing U. This is an interesting result for the following reason. In Hauser-Feshbach codes, it is a common practice to use the two-Fermion level-density formula given in Eq. (17) for the compound part, while the one-Fermion exciton level-density formula of Eq. (12) for the precompound effects. This is simply because the two-Fermion exciton level-density formula of Eq. (17) is complicated and the (U, n)-dependent pairing correction and spin cutoff factor (subject of this paper) in the formula have not been implemented. Therefore, a popular approach[11] in Hauser-Feshbach codes with precompound effects is to normalize $\rho_2(\text{all } n)$ to $\rho_1(\text{all } n)$ at each energy U. This normalization would make the U-dependence in $\rho_2(n = 2)$ incorrect. Consequently, the major achievement of the present work is to make all level-density formulas consistent. No normalization is needed and the consistent formulation of compound and precompound reactions with angular momentum conservation[12] can be fully realized.

Figure 3 shows the spin cutoff factors for $n = 2$ and "all n". For simplicity, the component $\sigma^2(U, n = 2)$ is labeled $\sigma_1^2(n = 2)$ and the average of $\sigma^2(U, n_\pi = 2, n_\nu = 0)$ and $\sigma^2(U, n_\pi = 0, n_\nu = 2)$ is labeled $\sigma_2^2(n = 2)$, and similarly for the "all n" components. The latter were obtained by averaging Eq. (7) over all n and averaging Eq. (36) over all allowed combinations of n_π and n_ν using the respective state densities as weighting functions. These factors in the one-Fermion and the two-Fermion formulations are quite close, differing by less than 2% for $n = 2$ and less than 10% for "all n". The reason for this agreement, for n=2, can be seen from Fig. 3 of Ref. 4 in which is shown that for a fixed small n the spin cutoff factor is nearly independent of energy and its value is proportional to g. For the "all n" components, the similarity in the the spin cutoff factors of the two approaches can be understood from the following reasons: (1) Fig. 3 of Ref. 4 shows that the largest spin cutoff factor among all n for a fixed U is near $\hat{n}$, (2) the state densities (the weighting functions) are also the largest near $\hat{n}$, and (3) the spin cutoff factors at $\hat{n}$ in the one-Fermion formulation and at $(\hat{n}_\pi, \hat{n}_\nu)$ in the two-Fermion formulation, given by Eqs. (13) and (42) respectively, are both equal to $\sigma^2(U)$.

9. CONCLUSIONS

It has been shown that the total state-density formula for two kinds of Fermions, Eq. (14), with pairing correction factor U_0, and the particle-hole state-density formula, Eq.

(15), with pairing correction factor $P_2(U, n_\pi, n_\nu)$, can be calculated approximately but consistently by the same two parameters, namely, the single-particle state density g and the constant-pairing-energy correction U_0. Although the pairing correction P_2 depends on the excitation energy U, the proton exciton number n_π, and the neutron exciton number n_ν, it has been shown that a good approximation to P_2 can be derived from g and U_0 as well. Simultaneously, the spin cutoff factors in the two-Fermion formulation have been derived, defining the corresponding particle-hole level-density formula. It has been shown that consistency between the one-Fermion and the two-Fermion level-density formulas cannot be obtained by normalizations at each U because all results are also n-dependent. In particular, normalization at each U of the one-Fermion "all n" component to the corresponding two-Fermion component would make the U-dependence in the $n = 2$ component in the two-Fermion level density incorrect. On the other hand, numerical results have shown that the spin cutoff factors in the one-Fermion and the two-Fermion formulations agree within 10% and probably need not be distinguished.

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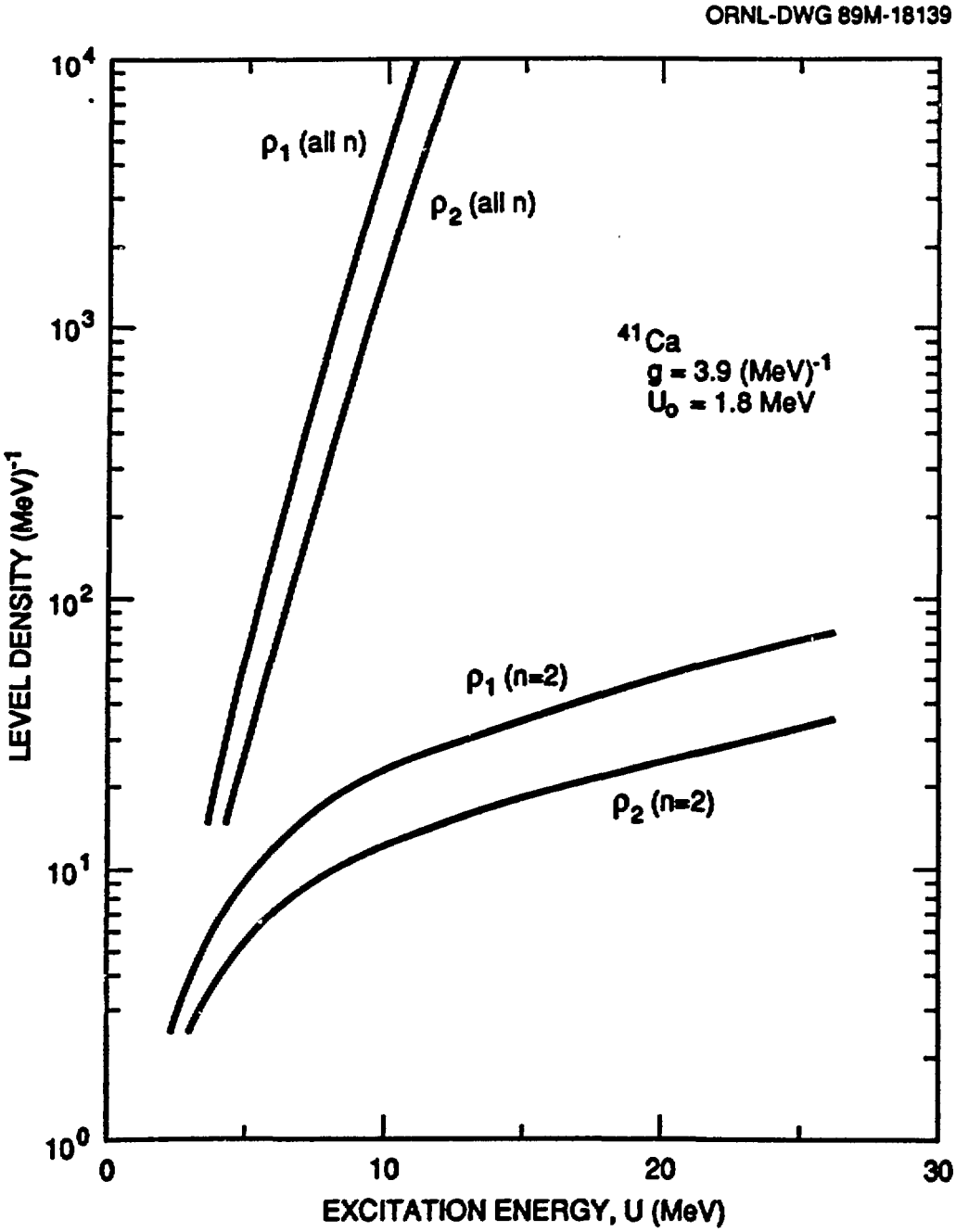

Fig. 1. Comparison of the one-Fermion and two-Fermion level densities calculated for ^{41}Ca with a consistent pairing interaction theory. Subcripts 1 and 2 denote the one-Fermion and the two-Fermion cases, respectively. Two exciton components, n=2 and "all n", are shown. The "all n" component is obtained by summing the exciton level densities over all of the allowed exciton numbers.

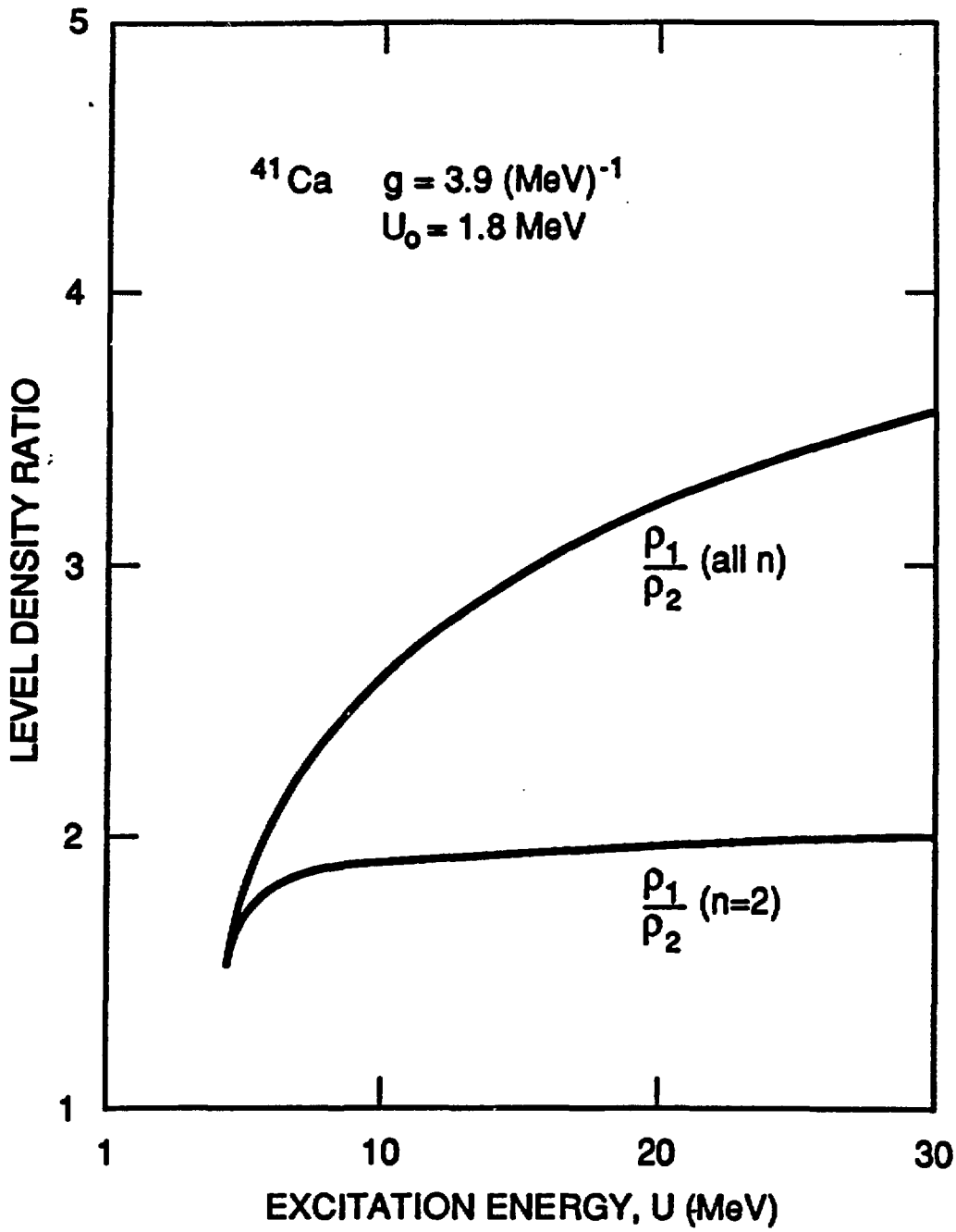

Fig. 2. Ratios of the one-Fermion to two-Fermion level densities calculated for ^{41}Ca as shown in Fig. 1. The ratio for the n=2 component, important for the precompound part of the cross-section calculation, is nearly flat in excitation energy. The ratio for the "all n" component, important for the compound part, increases with increasing excitation energy. This means that the compound to precompound ratios calculated by the one-Fermion and two-Fermion reaction theories will be different in absolute magnitudes and in their energy dependence.

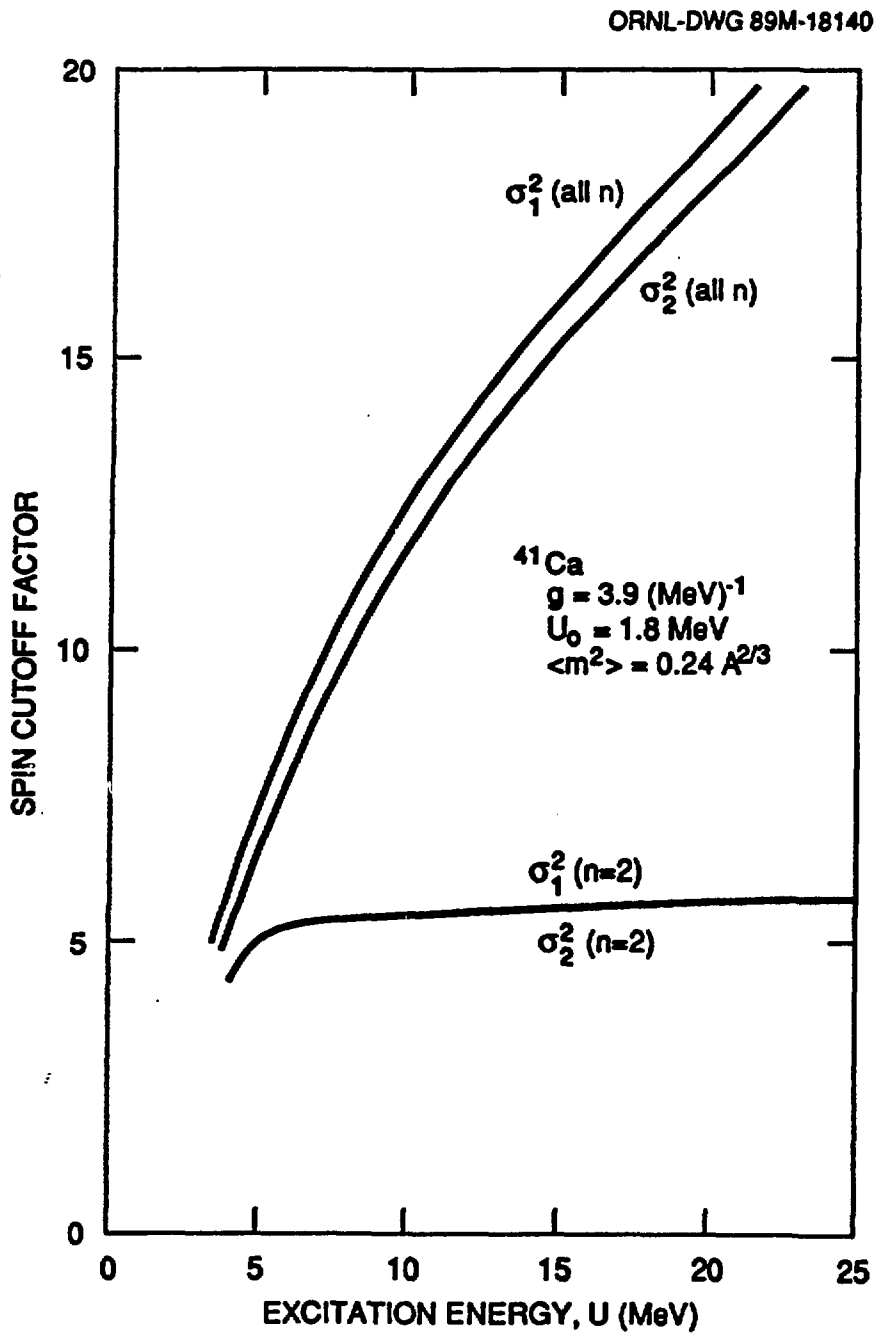

Fig. 3. Comparison of the one-Fermion and two-Fermion spin cutoff factors calculated for ^{41}Ca with a consistent pairing interaction theory. The n=2 components in the two different approaches are nearly identical and are nearly independent of excitation energy. The "all n" component increases with increasing excitation energy. The differences between the one-Fermion and the two-Fermion "all n"components remain small.