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Critical CRBR Core Pressure

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CRITICAL CRBR CORE PRESSURE

by

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ABSTRACT

The conditions are detailed under which gas pressure will cause or initiate failure in the structural containment of the fuel core. The Clinch River Breeder Reactor Plant is the prototype structure. Two general classes of problems have been studied, representing two entirely distinct configurations of containment failure. The first model determines the minimum pressure to lift a portion or the entire core from its containment. The second model estimates the critical pressure above which the fuel rods interior to the hexagonal fuel can warp, leading to blockage of the gas passages. Such blockage might cause further buildup of the gas pressure to a level causing the failure of the fuel rod containment in the hexagonal fuel container.

I. INTRODUCTION

An aspect of the safety analysis of the Clinch River Breeder Reactor Plant (CRBR)¹ fuel core is a criterion that the core or parts thereof should be contained under adverse incidents, such as a core meltdown that results in excessive production of gas. The gas pressure under design reliability should not cause a breakdown of the containment supplied by the reactor structure.

Two configurations of containment breakdown will be presented. In the first model, the gas pressure at a critical value will lift a portion or the entire fuel core against the core weight and whatever friction force exerted on the core by the former rings. In the second model, the individual fuel pins inside a hexagonal can may buckle under pressure. Such deformation would lead to blocked passage of gas and fluid, eventually to either fracturing the can or lifting the cap, and effectively breaking down the containment of the fuel rods.

For both cases, it is assumed that at core meltdown, the structures at the Top Load Plane (TLP) and at the Above-Core Load Plane (ACLP) are essentially at the operating temperature of the reactor.

II. GEOMETRY AND MATERIAL PROPERTIES

The CRBR reactor core is formed by groups of stainless steel (316ss) hex cans arranged in concentric hexagonal arrays (Fig. 1). The fuel cans are in the center of the core, surrounded by a radial blanket that consists of cans

Fig. 1. Schematic cross section of CRBR core indicating the hexagonal array arrangement.

containing the depleted UO_2 for breeding. The entire assembly is then shielded from the core barrel by shielding materials also contained in hex cans. The entire group of cans is supported vertically by the coolant inlet nozzle. The cans, being long and slender, are then constrained laterally at the TLP and the ACLP by the upper and lower core formers, respectively, which are integrally mounted to the core barrel. The hex cans are strengthened structurally at those two planes with thickened walls to withstand the anticipated pressure (load pads in Fig. 2).

In every fuel can, there are 216 fuel rods spaced by a spiral wire wrap (Fig. 2). The fuel rod is essentially a cylindrical shell containing singly stacked fuel pellets in the lower portion of the rod and a long fission gas plenum in the upper portion (Fig. 3).

The pertinent geometrical and material data are listed in Tables I and II.

III. MODEL I - BULK LIFT OF CORE BY MELTDOWN GAS PRESSURE

The hexagonal cans are placed vertically in the core barrel with some initial clearance. At the operating temperature, the clearance is taken up by thermal expansion. Moreover, the unconstrained cans could expand more than the clearance space allows in the design. Consequently, the core assembly of hex cans will be pressed against the upper and the lower core formers under the operating and the accident conditions. This pressure will provide for the friction force that will prevent the lifting of the core block as a result of buildup gas pressure.

A. Mathematical Model

It is assumed conservatively that at the transient stage of the core meltdown the above-core level will be at the sodium vapor temperature. The interior of the core has a uniform temperature of 900°C up to the blanket-shield interface. The temperature then drops steadily to 600°C at the shield-barrel interface. It is 300°C outside of the core barrel. Based on the temperature difference, the reactor core and the core assembly are modeled by a solid cylinder interior to two hollow concentric cylinders, Fig. 4.

The core barrel and core former assembly (region III) are modeled by a moderately thick-walled cylinder in plane strain. The constitutive constants in that region are μ , the modulus of rigidity and κ , the Poisson's plane

coefficient ($\kappa = 3 - 4\nu$ for plane strain). The deformation of the cylinder is axi-symmetrical and is caused by the temperature gradient and the interior pressure from the core shield (region II in Fig. 4).

Regions I and II consist of hollow hex cans. However, because of the multitude of cans, the analysis may be simplified by considering the regions to contain a homogeneous continuum. The effect of individual, hollow hex cells is taken into consideration by an adjusted value of the modulus of rigidity (μ_g), which defines the global stiffness of the region.

TABLE I
GEOMETRY

<u>Core Barrel and Core Former Assembly.</u>	
	<u>Length m, (in.)</u>
Outside radius (c)	1.91, (75.25)
Effective inside radius (b)	1.81, (71.25)
<u>Core Assembly</u>	
Effective radius of shield-barrel interface (b)	1.81, (71.25)
Effective radius of shield-blanket interface (a)	1.216, (47.88)
<u>Hex Can</u>	
Across flat (2d)	0.12052, (4.745)
Reference across-flat spacing	0.12090, (4.76)
Thickness at load pad (h)	0.00521, (0.205)
Length of load pad	0.1016, (4)
<u>Fuel Rod</u>	
Plenum length	1.2192, (48)
Wire wrap pitch	0.3023, (11.9)
Rod outside diameter	0.00584, (0.23)
Wall thickness	0.00038, (0.015)

TABLE II
MECHANICAL PROPERTIES
Thermal Expansion (ϵ_T)

Temperature

900°C	17.2×10^{-3}
600°C	10.74×10^{-3}
300°C	4.78×10^{-3}

Yield Strength at 0.2% (σ_y)

900°C	77 MPa (11.17×10^3 psi)
600°C	114.4 MPa (16.6×10^3 psi)
450°C	119.7 MPa (17.36×10^3 psi)

Ultimate Strength (σ_u)

900°C	94 MPa (13.6×10^3 psi) @38%
600°C	361.1 MPa (52.4×10^3 psi) @25%
450°C	438.8 MPa (63.6×10^3 psi) @25.2%

Young's Modulus (E_o)

900°C	1.17×10^5 MPa (17×10^6 psi)
750°C	1.332×10^5 MPa (19.3×10^6 psi)
600°C	1.491×10^5 MPa (21.6×10^6 psi)
450°C	1.637×10^5 MPa (23.7×10^6 psi)
300°C	1.767×10^5 MPa (25.6×10^6 psi)

Poisson's ratio (ν)

	<u>Nominal</u>	<u>Plane Strain</u> $\nu^* = \nu/(1-\nu)$
900°C	0.342	0.520
750°C	0.321	0.473
600°C	0.310	0.449
450°C		0.298
300°C		0.287

Coefficient of Friction (chrome carbide and 316ss ~ 600 C)

Breakaway	0.68-1.74
Moving (kinematic)	0.28-0.62

*All structural materials are 316ss. The properties at various temperatures are listed. The properties of 316ss annealed are given in Ref. 2.

Fig. 2. Hexagonal fuel can configuration and assembly.

Fig. 3. Fuel pin configuration.

Fig. 4. Mathematical idealization of the core cross section.

The interacting pressure at the shield-barrel interface (b) may then be determined by the compatibility conditions of the radial displacement at the interfaces at $r=a$ and $r=b$. We have at $r=b$,

$$u_r^{III}(P,b) - u_r^{II}(P,b) = u_r^{II}(T,b) - u_r^{III}(T,b) - C_t^{II}, \quad (1)$$

where $u_r(T)$ is the free thermal radial expansion, $u_r(P)$ is the radial displacement caused by the pressure at the interface in the isothermal state, II and III designate the region, and C_t^{II} is the clearance space taken up in region II along the radial direction. Similarly, at $r=a$

$$u_r^{II}(P,a) - u_r^I(P,a) = u_r^I(T,a) - u_r^{II}(T,a) - C_t^I . \quad (2)$$

The clearance per unit radial length is $(0.12090 - 0.12052)/0.12090 = 3.15 \times 10^{-3} \text{ m/m}$.

The total radial clearance is then

$$C_t = C_t^I + C_t^{II} = 3.15 \times 10^{-3} b . \quad (3)$$

B. Thermal Expansion

The coefficient of thermal expansion for the prescribed temperature range is not constant. However, its variation is not so large as to preclude using the standard thermoelastic equations simply by replacing the product of the coefficient of thermal expansion and the temperature rise with the thermal strain (ϵ_T) at that temperature. The unconstrained thermal expansions and the boundaries of cylinders may be obtained for axi-symmetrical cases as for solid cylinders by

$$u_r(r) = \epsilon_T r \quad (\epsilon_T = \text{const}) . \quad (4)$$

For hollow cylinders

$$u_r(r) = \frac{A}{2} \left\{ (1 + \nu^*) \left[r \log r + \frac{r_o^2 r_i^2}{r_o^2 - r_i^2} \left(\log \frac{r_o}{r_i} \right) \frac{1}{r} \right] - r + (1 - \nu^*) \left[\log r_o + \frac{r_i^2}{r_o^2 - r_i^2} \log \frac{r_o}{r_i} \right] r \right\} + Br, \quad (5)$$

where r_o and r_i are the outer and the inner radii respectively, and

$$A = \frac{(\epsilon_T^0 - \epsilon_T^i)}{\log \frac{r_o}{r_i}} ; \quad B = \frac{(\epsilon_T^i \log r_o - \epsilon_r^0 \log r_i)}{\log \frac{r_o}{r_i}} .$$

With the temperature boundary conditions shown in Fig. 4 and the corresponding thermal strains in the Table I, we obtain that

$$u_r^I(T, a) = \epsilon_T^a a = 17.2 \times 10^{-3} a, \quad (6)$$

$$u_r^{II}(T, a) = \left[\frac{b^2 \epsilon_T^b - a^2 \epsilon_T^a}{b^2 - a^2} + \frac{\epsilon_T^a - \epsilon_T^b}{2 \log \left(\frac{b}{a} \right)} \right] a = 13.546a \times 10^{-3}, \quad (7)$$

and

$$u_r^{II}(T, b) = \left[\frac{b^2 \epsilon_T^b - a^2 \epsilon_T^a}{b^2 - a^2} + \frac{\epsilon_T^a - \epsilon_T^b}{2 \log \left(\frac{b}{a} \right)} \right] b = 13.546b \times 10^{-3}, \quad (8)$$

$$u_r^{III}(T, b) = \left[\frac{c^2 \epsilon_T^c - b^2 \epsilon_T^b}{c^2 - b^2} + \frac{\epsilon_T^b - \epsilon_T^c}{2 \log \left(\frac{c}{b} \right)} \right] b = 7.705b \times 10^{-3}. \quad (9)$$

C. Radial Displacement Because of Pressure

1. Core Stiffness. We shall first justify the use of the continuum approximation for the core because of its large number of hex cells. The global stiffness of the core will be computed so that the entire core can be treated as a solid mass. The computation will be referred to in a separate report.³ Only qualitative discussion of the result is presented here.

It is adequate for the present case to consider the core to be in a planar locally uniform pressure field. By uniform, we only require that the total pressure per side on individual hex cells is a constant. It is stipulated that the sides of adjacent cells will maintain contact throughout deformation. As a consequence, because of the isotropy condition, the sides of the hex cans will remain flat. No shear exists between cells under this condition.

The mathematical model of the hollow hex can is described by a plane region bounded by two concentric hexagons. The interior boundary condition is traction free; on the exterior boundary there is a mixed boundary condition of

zero shear and a uniform normal displacement. The finite element method was used for this problem to determine an equivalent modulus of rigidity³. A plane version of finite element code TSAAS⁴ was used. When compared with a solid hex cylinder of the same cross-flat dimension, in order to produce the corresponding displacement under the same total pressure per side, we have determined the equivalent modulus of rigidity of the solid cylinder to be 0.0455 of the modulus of the hex can at 900°C. The equivalent modulus thus defines the bulk stiffness of the core, that is

$$\mu_b = 0.0455 \mu \quad (10)$$

for a thickness ratio of the can ($\frac{h}{d}$) of 0.086 and $\nu = 0.342$.

2. Radial Displacement. By assuming a uniform radial pressure, we may readily state the elastic solutions for the radial displacements as

$$2\mu u_r = -\frac{\kappa-1}{2} pr \quad (\text{for a solid cylinder}) \quad , \quad (11)$$

and

$$2\mu u_r = -\left[\frac{\kappa-1}{2} (p_o r_o^2 - p_i r_i^2) r + (p_o - p_i) \frac{r_o^2 r_i^2}{r} \right] / (r_o^2 - r_i^2) \quad (12)$$

(for a hollow cylinder) where for the regions I, II, and III,

$$\begin{aligned} \mu_b^I &\approx 0.0444 \mu_{900^\circ} & \nu &= 0.342 \\ \mu_b^{II} &\approx 0.0485 \mu_{750^\circ} & \nu &= 0.321 \\ \mu_b^{III} &= \mu_{450^\circ} & \nu &= 0.298 \end{aligned} \quad (13)$$

The pressures for individual regions are

$$p^I = p_i^{II} = p_a = p; \quad p_o^{II} = p_i^{III} = p_b = p; \quad p_o^{III} = 0 \quad .$$

Equations (11) and (12) thus yield

$$u_r^I(p,a) = -7.966 \times 10^{-5} \text{ pa} , \quad (14)$$

$$u_r^{III}(p,a) = -7.366 \times 10^{-5} \text{ pa} , \quad (15)$$

$$u_r^{II}(p,b) = -7.366 \times 10^{-5} \text{ pb} , \quad (16)$$

$$u_r^{III}(p,b) = 10.45 \times 10^{-5} \text{ pb} . \quad (17)$$

The modulus of rigidity at a specific temperature is computed from

$$\mu = \frac{E}{2(1 + \nu)} .$$

For instance,

$$\mu^{III} = \mu_{450^\circ} = \frac{E_{450^\circ}}{2(1 + \nu_{450^\circ})} = 6.296 \times 10^4 \text{ MPa} . \quad (18)$$

D. Pressure at the Interface $r = b$

The expressions in Eqs. (6) through (9) and (14) through (17) are substituted into the compatibility Eqs. (1) and (2), resulting in

$$5.841 \times 10^{-3} = 17.816 \times 10^{-5} p + c_t^{II}/b ,$$

$$3.654 \times 10^{-3} = 6.0 \times 10^{-6} p + c_t^I/a ,$$

$$3.15 \times 10^{-3} b = c_t^I + c_t^{II} .$$

The pressure at the interface is thus solved as $p = 28.25 \text{ MPa}$ ($4.1 \times 10^3 \text{ psi}$).

E. Friction

At the elevated temperature and in the hot sodium bath environment, the nichrome plate at the former contact surface could very well be removed. A lower value for the breakaway and the kinematic coefficients of friction (re: Table II) should be used. We shall use for the coefficients of friction

breakaway	0.70
kinematic	0.30 .

The corresponding friction force per unit area is then

breakaway	19.85 MPa (2880 psi)
kinematic	8.45 MPa (1225 psi) .

Both values are far less than the ultimate shear stress in the former material.

F. Vapor Pressure

There is a total of 186 contacting hex sides per load plane. Each side length is 6.98 cm (2.748 in). The contacting circumference is therefore $186 \times 6.98 = 1298$ cm. If it is assumed that the full depth of 10.16 cm (4 in.) of the load pad is in contact, the circumferential contact area per load plane is 1.32 m^2 , for a total of 2.64 m^2 of the contact surface between the core and the barrel former assembly. The total friction force available to resist the uplifting vapor pressure is then

$$\begin{aligned} P(\text{breakaway}) &= 52.5 \text{ MN } (11.7 \times 10^6 - \text{lb}) \\ P(\text{kinematic}) &= 22.4 \text{ MN } (4.92 \times 10^6 - \text{lb}) . \end{aligned}$$

If the gas pressure is provided by the inner core only, which has a cross-sectional area of 4.646 m^2 , we may compute the critical uplifting vapor pressure as

$$\begin{aligned} p(\text{breakaway}) &= 11.3 \text{ MPa } (1630 \text{ psi}) \\ p(\text{kinematic}) &= 4.82 \text{ MPa } (680 \text{ psi}) . \end{aligned}$$

The pressure to lift a single hexagonal can from the assembly may be computed readily. The total lateral friction surface of a single hex can is

$$2[6.98 \times 10.16 \times 6] = 851 \text{ cm}^2,$$

while its cross sectional area is 126.6 cm^2 . Hence, the breakaway and the kinematic lifting vapor pressures are respectively,

$$p(\text{breakaway}) = 133.5 \text{ MPa (19285 psi)}$$

$$p(\text{kinematic}) = 57 \text{ MPa (8130 psi)} .$$

Thus, we may conclude that the lifting of any individual can is most unlikely unless there is a high localized pressure in the core plane.

If we consider only the region of the inner core, of which the peripheral length is 9.63 m, the friction surface areas at TLP and ACLP combine to 1.9573 m^2 . Correspondingly, we have for the inner core

$$P(\text{breakaway}) = 38.75 \text{ MN}$$

$$P(\text{kinematic}) = 16.35 \text{ MN} .$$

The vapor pressure applies to the entire cross-sectional area of the inner core (4.646 m^2), resulting in

$$p(\text{breakaway}) = 8.3 \text{ MPa (1210 psi)}$$

$$p(\text{kinematic}) = 3.5 \text{ MPa (510 psi)}.$$

The vapor pressure indeed is the lowest yet.

IV. MODEL II - BREAKDOWN OF HEX CAN CONTAINMENT

Under high meltdown pressure, the fuel rods are pushed up against the hex can cap. If either the local deformation or the bulk warpage of rods becomes so severe as to block the free passage of gas, the pressure will build up continuously. Ultimately, either the hex can wall fractures or the cap would be forced off the can as a potential missile.

The current analysis will define the unwarped fuel rod as a safe configuration. Any gas pressure that initiates warpage is the critical pressure above which the fuel rod will continue its deformation, culminating in the total blockage of the passage.

The deformation of the fuel rod can be either overall or local buckling of the rod. The critical load will be computed for both cases and the lower value will be used for the failure pressure.

We assumed that the fuel pack section of the fuel rod is at the molten state and the structural resistance is maintained by the section of the fission gas plenum (see Fig. 3). The geometry of the plenum is described in Table I. The plenum is modeled by a long hollow cylinder, subjected to an axial load from the meltdown gas pressure.

A. Local Buckling of the Plenum

Thin-walled cylinders under axial load generally undergo local buckling by folding up the cylindrical shell. The simplest case is the axisymmetrical elastic buckling for which the differential equation is

$$w_{,xxxx} + \frac{N_{xx}}{D} w_{,xx} + \frac{12(1 - \nu^2)}{a^2 h^2} w = 0 \quad , \quad (19)$$

with the comma denoting differentiation with respect to the independent variable. The critical stress $\sigma_{xx} = N_{xx}/h$ is well known⁵ to be

$$\sigma_{cr} = \frac{hE}{a\sqrt{3(1 - \nu^2)}} = 9.88 \times 10^3 \text{ MPa} \quad .$$

This value is well above the material's ultimate strength. Equation (19) was developed for a very thin elastic shell of which the thickness-radius ratio is less than 0.05. In the present case, the thickness-radius ratio is closer to 0.14. Certain terms in the differential equation may not be omitted as being of small order. The revised differential equation is then

$$w_{,xxxx} + \left(\frac{N_{xx}}{D} + \frac{2\nu}{a^2} \right) w_{,xx} + \frac{12(1 - \nu^2)}{h^2} \frac{w}{a^2} = 0 \quad . \quad (20)$$

This small adjustment brings the critical stress value down to

$$\sigma_{cr} = 9.755 \times 10^3 \text{ MPa} \quad .$$

This value is only a little over one percent lower and still much above the maximum value of 94 MPa ($\sigma_u^{900^\circ\text{C}}$). Later we shall investigate other configurations of local buckling that can result in a lower buckling load and a consistent buckling stress within the elastic range.

For a thin cylindrical shell under a uniform axial load, the lowest buckling stress is at the second mode, for which⁵

$$\sigma_{cr} = \frac{3}{5} \frac{Eh}{a\sqrt{3(2-\nu^2)}} = 5.928 \times 10^3 \text{ MPa} .$$

With the allowance for thickness, the critical stress is still around 5.8×10^3 MPa (if $\sigma_u = 94$ MPa @ 900°C). Of course, if the local buckling is to be the mode of warpage, the values of those critical stresses must be re-examined with the use of the tangent modulus in the plastic range. The critical stress certainly will be in the plastic range; that is, higher than 77 MPa (σ_y @ 900°C). The plenum section, therefore, will first be analyzed as a column first. If the result also shows a high plastic buckling mode, the local plastic buckling of the cylindrical shell then will be determined as a meaningful buckling load.

B. Plenum Buckling as a Column

When the fuel rods are pushed up against the cap of the hex can, the shape of the cap is such that the top ends of the fuel rods will be held without displacement or rotation. The plenum section of the individual rod may be displaced laterally except at points and directions where its lateral motion is constrained by the wrapping wire. A planar deflection curve is shown in Fig. 5. The plenum section is modeled by a column, clamped at one end and pivoted at the other. There are lateral constraints at discrete locations representing the constraining effect of the wire wrap. The figure shows a buckled configuration of the column, in which the column deflects laterally against the three stops positioned at a , $2a$, $3a$ as a lower potential compared to the one buckled in the other lateral direction. There are three positions ($\bar{b}_1$, $\bar{b}_2$, $\bar{b}_3$) that locate the unknown nodes, but between the limits of adjacent pairs of the four wire stops. Their positions will be determined as the set of b_i that renders the critical load to a minimum.

$$\frac{a}{2} \leq \bar{b}_1 \leq \frac{3a}{2}, \quad \frac{3a}{2} \leq \bar{b}_2 \leq \frac{5a}{2}, \quad \frac{5a}{2} \leq \bar{b}_3 \leq \frac{7a}{2}$$

Fig. 5. Planar deflection curve for the fuel pin buckling model.

1. General Mathematical Model. Because values of $\{\bar{b}\}$ are uncertain, the solution is best suited for the energy method. A Ritz function will be assumed of the form

$$\bar{y}(\bar{x}) = \bar{x}^2(\ell - \bar{x})(\bar{x} - a)(\bar{x} - 2a)(\bar{x} - 3a)(\bar{x} - \bar{b}_1)(\bar{x} - \bar{b}_2)(\bar{x} - \bar{b}_3)(\bar{c}_1 + \bar{c}_2\bar{x} + \dots). \quad (21)$$

The potential energy corresponding to the deflection function is

$$\bar{V} = \frac{1}{2} \int_0^\ell [EI(\bar{y}'')^2] d\bar{x} - \int_0^\ell P d\lambda = \frac{1}{2} \int_0^\ell [EI(\bar{y}'')^2 - P(\bar{y}')^2] d\bar{x},$$

where $d\lambda$ is the axial contraction such that

$$d\lambda = ds - d\bar{x} = \left(d\bar{x}^2 + d\bar{y}^2 \right)^{\frac{1}{2}} - d\bar{x} = \frac{1}{2} (\bar{y}')^2 d\bar{x} .$$

By using the nondimensionalized quantities

$$x = \frac{\bar{x}}{\ell} , y = \frac{\bar{y}}{\ell} , \phi^2 = \frac{p\ell^2}{EI} , v^* = \frac{V\ell}{EI} ,$$

we have then

$$v^* = \frac{1}{2} \int_0^1 \left[(y'')^2 - \phi^2 (y')^2 \right] dx . \quad (22)$$

If we incorporate the differential equation that

$$EI\bar{y}'' + P\bar{y} = 0 \quad \text{or} \quad y'' + \phi^2 y = 0 ,$$

and introduce $V = -V^*/\phi^2$, Eq. (22) becomes

$$v = \int_0^1 \left[(y')^2 - \phi^2 y^2 \right] dx . \quad (23)$$

Expression (23) is the Timoshenko formulation, which because of the lower order derivative is in general more accurate than the Ritz formulation (22). The Ritz function (21) is also expressed in the dimensionless form and in an approximate value of a ; that is,

$$\frac{a}{\ell} = 0.25, b_i = \frac{\bar{b}_i}{\ell}, c_1 = \bar{c}_1 \ell^8, \bar{c}_2 \ell^9, \dots, \text{ and}$$

$$0.125 \leq b_1 \leq 0.375, 0.375 \leq b_2 \leq 0.625, 0.625 \leq b_3 \leq 0.875. \quad \text{Then,} \quad (24)$$

$$y(x) = x^2 (1 - x)(x - \frac{1}{4})(x - \frac{1}{2})(x - \frac{3}{4})(x - b_1)(x - b_2)(x - b_3)(c_1 + c_2x + \dots).$$

2. First Approximation. Let $c_i = 0$ for $i \neq 1$; the displacement function Eq. (24), thus becomes

$$\begin{aligned} y(x) &= c_1 \left(-x^6 + \frac{5}{2} x^5 - \frac{35}{16} x^4 + \frac{25}{32} x^3 - \frac{3}{32} x^2 \right) (x^3 - I_1 x^2 + I_2 x - I_3) \\ &= c_1 \underset{\sim}{X}^T \underset{\sim}{A} \underset{\sim}{I} \end{aligned} \quad (25)$$

for every x on the closed interval $[0, 1]$,

where

$$I_1 = b_1 + b_2 + b_3, \quad I_2 = b_1 b_2 + b_2 b_3 + b_3 b_1, \quad I_3 = b_1 b_2 b_3, \quad \text{and} \quad (26)$$

$$\underset{\sim}{I}^T = \begin{bmatrix} 1 & I_1 & I_2 & I_3 \end{bmatrix}, \quad (27)$$

$$\underset{\sim}{X}^T = \begin{bmatrix} x^9 & x^8 & x^7 & x^6 & x^5 & x^4 & x^3 & x^2 \end{bmatrix}, \quad (28)$$

$$\underset{\sim}{A} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ \frac{5}{2} & 1 & 0 & 0 \\ -\frac{35}{16} & -\frac{5}{2} & -1 & 0 \\ \frac{25}{32} & \frac{35}{16} & \frac{5}{2} & 1 \\ -\frac{3}{32} & -\frac{25}{32} & -\frac{35}{16} & -\frac{5}{2} \\ 0 & \frac{3}{32} & \frac{25}{32} & \frac{35}{16} \\ 0 & 0 & -\frac{3}{32} & -\frac{25}{32} \\ 0 & 0 & 0 & \frac{3}{32} \end{bmatrix} \quad (29)$$

We shall define the derivatives as

$$\underline{Y}(x) = \underline{X}'(x) \text{ and } \underline{Z}(x) = \underline{X}''(x) .$$

The integrals in Eq.(23) then may be expressed as

$$\int_0^1 y^2 dx = c_1^2 (\underline{I}^T \underline{A}^T \int_0^1 \underline{X} \underline{X}^T dx \underline{A} \underline{I}) ,$$

$$\int_0^1 (y')^2 dx = c_1^2 (\underline{I}^T \underline{A}^T \int_0^1 \underline{Y} \underline{Y}^T dx \underline{A} \underline{I}) .$$

For stationary values of V in Eq. (23), the gradient of $V(c_1)$ must be zero, resulting in

$$\phi^2(I_1, I_2, I_3) = \frac{\underline{I}^T \underline{A}^T \int_0^1 \underline{Y} \underline{Y}^T dx \underline{A} \underline{I}}{\underline{I}^T \underline{A}^T \int_0^1 \underline{X} \underline{X}^T dx \underline{A} \underline{I}} . \quad (30)$$

The expressions in Eq. (30) are algebraically cumbersome and ill-conditioned. The computation with high precision was carried out on MACSYMA.⁶ The solution is a rational function of (I_1, I_2, I_3) . The value of ϕ^2 thus obtained has a minimum in the coordinate space $\{b_i\}$ at a boundary point

$$\{b_i\}^T = (0.125, 0.375, 0.625),$$

for which $\phi^2 = 355$.

It is noticed that the value is only a one-term approximation. For the same approximation with the Ritz formulation, even though the minimum point (b_i) is still the same, the corresponding value of ϕ^2 is a high upper limit of 1185.

3. Second Approximation.

If we consider two terms in Eq. (24), then

$$y(x) = (c_1 \underset{\sim}{x}^T + c_2 x \underset{\sim}{x}^T) \underset{\sim}{AI} . \quad (31)$$

The integral expressions in the Timoshenko formula (4.5) are

$$\int_0^1 y^2 dx = \underset{\sim}{I}^T \underset{\sim}{A}^T \int_0^1 (c_1^2 + 2c_1 c_2 x + c_2^2 x^2) \underset{\sim}{x} \underset{\sim}{x}^T dx \underset{\sim}{AI} = D_{11} c_1^2 + 2D_{12} c_1 c_2 + D_{22} c_2^2 , \quad (32)$$

where

$$\begin{aligned} D_{11} &= \underset{\sim}{I}^T \underset{\sim}{A}^T \int_0^1 \underset{\sim}{x} \underset{\sim}{x}^T dx \underset{\sim}{AI} , \\ D_{12} &= \underset{\sim}{I}^T \underset{\sim}{A}^T \int_0^1 x \underset{\sim}{x} \underset{\sim}{x}^T dx \underset{\sim}{AI} , \\ D_{22} &= \underset{\sim}{I}^T \underset{\sim}{A}^T \int_0^1 x^2 \underset{\sim}{x} \underset{\sim}{x}^T dx \underset{\sim}{AI} , \end{aligned} \quad (33)$$

$$\begin{aligned} \int_0^1 (y')^2 dx &= \underset{\sim}{I}^T \underset{\sim}{A}^T \int_0^1 \left[c_1^2 \underset{\sim}{y} \underset{\sim}{y}^T + c_1 c_2 (\underset{\sim}{y} \underset{\sim}{y}^T + \underset{\sim}{y} \underset{\sim}{y}^T) + c_2^2 \underset{\sim}{y} \underset{\sim}{y}^T \right] dx \underset{\sim}{AI} \\ &= E_{11} c_1^2 + 2E_{12} c_1 c_2 + E_{22} c_2^2 , \end{aligned} \quad (34)$$

where

$$E_{11} = \underset{\sim}{I}^T \underset{\sim}{A}^T \int_0^1 \underset{\sim}{y} \underset{\sim}{y}^T dx \underset{\sim}{AI} ,$$

$$E_{12} = \frac{1}{2} \underset{\sim}{I} \underset{\sim}{A}^T \underset{\sim}{A} \int_0^1 (\underset{\sim}{Y} \underset{\sim}{Y}^T + \underset{\sim}{Y} \underset{\sim}{Y}^T) dx \underset{\sim}{A} \underset{\sim}{I} , \quad (35)$$

$$E_{22} = \underset{\sim}{I} \underset{\sim}{A}^T \underset{\sim}{A} \int_0^1 \underset{\sim}{Y} \underset{\sim}{Y}^T dx \underset{\sim}{A} \underset{\sim}{I} ,$$

$$\underset{\sim}{Y} = \underset{\sim}{X}'(x) ,$$

$$\underset{\sim}{Y} = \frac{d}{dx} \left[x \underset{\sim}{X}(x) \right] .$$

Expressions in Eq. (32) and (34) then are substituted in the Timoshenko formulation (Eq. (23) for $V(c_1, c_2)$. The stationary value of V in $\{c_i\}$ is obtained again by letting the gradient of V go to zero, resulting in

$$E_{11}c_1 + E_{12}c_2 - \phi^2(D_{11}c_1 + D_{12}c_2) = 0 ,$$

$$E_{12}c_1 + E_{22}c_2 - \phi^2(D_{12}c_1 + D_{22}c_2) = 0 .$$

The requirement of nontrivial solutions for (c_i) yields

$$\begin{vmatrix} E_{11} - \phi^2 D_{11} & E_{12} - \phi^2 D_{12} \\ E_{12} - \phi^2 D_{12} & E_{22} - \phi^2 D_{22} \end{vmatrix} = 0 , \quad (36a)$$

or

$$(D_{11}D_{22} - D_{12}^2) \phi^4 - (D_{11}E_{22} + D_{22}E_{11} - 2D_{12}E_{12}) \phi^2 + (E_{11}E_{22} - E_{12}^2) = 0 . \quad (36b)$$

Equation (36b) is in the familiar form of quadratic function of ϕ^2 . The result is an irrational function of (I_1, I_2, I_3) . The expression was again generated with the help of MACSYMA. The lengthy expression is omitted as too cumbersome. The minimum point in $\{b_i\}$ is again the same as in the first approximation. The corresponding value of ϕ^2 is 196.7. From the definition of ϕ^2 , it is thus concluded with adequate accuracy* that

$$P_{cr} = \frac{196.7EI}{L^2} = 3.791 \times 10^{-4} \text{ MN(85-lb)} \quad (37)$$

The corresponding critical stress is

$$\sigma_{cr} = \frac{P_{cr}}{A} = 58 \text{ MPa (8400 psi)} < \sigma_y (900^\circ\text{C}) \quad (38)$$

If we compare this value with the critical stresses obtained in Sec. IV-A for the local buckling mode, we can conclude that the plenum column will buckle as a column. The real critical stress is actually slightly less than the value in Eq. (38). The gas pressure at the critical state should be somewhere between the high limit of the buckling stress and the low limit of an entire fuel rod cross-sectional area subjected to the vapor pressure, for which

$$p = 58 \times \frac{0.584^2 - 0.508^2}{0.584^2} = 14.14 \text{ MPa (2050 psi)} \quad .$$

The low limit is most unlikely, as the assumption is that debris from meltdown would block the plenum such that the vapor pressure is taken up entirely at the lower surface of the plenum section.

*The critical load should be higher than the buckling load of a quarter length with pivoted ends ($\phi^2 = 158$) but less than that of the fixed-pivot which yields $\phi^2 = 323$.

V. CONCLUSION

The analysis in this report gives a comprehensive estimate of the vapor pressure from a meltdown incident that might result in an uplifting of the above-core structure (either the entire core assembly, a single hexagonal can from the assembly, or a part thereof). We found that during the thermal transient stage, when the above-core structure is at the sodium vapor temperature, it takes a quite large vapor pressure to effect an incident. The lifting of a single can is most unlikely, while the lifting of the inner core (fuel and blanket cans) takes the least vapor pressure to initiate (8.3 MPa or 121.0 psi) and to sustain (3.5 MPa or 5.10 psi).

Missile generation as a result of blocked flow in a hex can might be initiated, if locally in some can the pressure could reach above 14.14 MPa (2050 psi). This is the minimum vapor pressure required under a unique occurrence to start buckling of rods in the can and to result in blockage of flow. Also, it can be shown by the use of a single, pressurized, thin-cylinder model that the hex can will split along the longitudinal direction at its thin section under a pressure of 5.2 MPa (754 psi). It is not possible, therefore, to develop a buckling pressure interior to a single can without some constraint applied to the can walls.

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