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The Gyrocon: A Deflection-Modulated, High-Power Microwave Amplifier

Paul J. Tallerico

MASTER

los alamos
scientific laboratory
of the University of California
LOS ALAMOS, NEW MEXICO 87545

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THE GYROCON: A DEFLECTION-MODULATED, HIGH-POWER MICROWAVE AMPLIFIER

by

Paul J. Tallerico

ABSTRACT

A large-signal, relativistic theory of the electron-field interaction in a new class of microwave amplifiers is presented and applied to the analysis of a high-power, 450-MHz amplifier for accelerator applications. The analysis indicates that electronic efficiencies in excess of 90% are obtainable and that overall efficiencies of 90% are possible. The amplifier is unique in several respects; the electron velocity is perpendicular to the circuit energy flow, the device uses a fast-wave circuit, and the electron beam is deflection modulated.

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I. INTRODUCTION

A novel electron device, called a gyrocon or bermutron* and capable of functioning as a highly efficient, very high power microwave amplifier, oscillator, or frequency multiplier, is analyzed in this report. The gyrocon, invented in 1946,¹ has only recently been shown² capable of producing very high output power and excellent dc-to-rf conversion efficiency. Such an efficient energy converter makes practical several types of large, high-power rf systems. It can reduce drastically the construction and operating costs of very high power accelerators, especially those that call for high-energy, high-beam currents, or a high duty factor. The feasibility of high-current, cw accelerators for producing fissile material may depend on amplifier properties.

The gyrocon operates by deflection, rather than velocity modulation, of an electron beam. Because the deflection is produced by the magnetic fields of a microwave cavity, operation at very high beam voltage is possible. The basic device, shown in Fig. 1, comprises an electron gun, a deflection cavity, an output resonator, a collector, and an output coupler. The electron gun produces a cylindrical, solid electron beam. The deflection cavity must support two modes with magnetic fields perpendicular to the beam direction. When the two component fields are 90° out of phase in time and space, the resultant field rotates, exactly like the field in a two-phase electric motor. The rotating deflection field causes the beam to travel in an expanding helical path, although the envelope of the trajectories is conical. The beam enters a slot in the output resonator, a resonant ring of rectangular waveguide. The phase velocity of the waveguide field is greater than

*Russian workers use the name "gyrocon." Gyrocon implies *gyrating constantly*, the feature of the device that they consider essential. "Bermutron," an acronym for beam-rotating multiplier, has been used in American literature. We use gyrocon here because the device is more efficient as an amplifier than as a multiplier.

the velocity of light, c , but the deflection velocity of the beam, also a phase velocity, can be synchronized with the field. The collector is the back of the waveguide. A suitable output coupler extracts the microwave power from the output resonator. With the proper choice of output resonator geometry and rf voltage, the electron beam can be almost halted in transit across the waveguide. Thus, the dc-to-rf conversion efficiency can be very high. The waveguide field is the dominant TE_{01} mode, and the electrons are stopped by the electric field. The electrons need not be bunched, because each part of the electron beam experiences the same fields. The deflection cavity may operate at a subharmonic n of the output frequency if the output resonator is n wavelengths long.

The basic device was patented in 1946 by J. W. McRae¹ to generate high frequencies by frequency multiplication. Kaufman and Oltman,² in 1965, built a frequency multiplier that generated 0.4 W of power at 34 GHz with a 65-W drive power at 8.5 GHz. Their device had a linear output cavity, a length of shorted rectangular guide. Their paper contains several references to unpublished work on similar devices in the 1950s. In 1958, P. B. Wilson³ at Stanford investigated a very large family of deflection modulation devices, including the gyrocon, theoretically, but no devices were built.

Recently, G. I. Budker⁴ at the Institute for Nuclear Research in Novosibirsk has shown visitors a 1.3-MV, 40-A gyrocon that is alleged to produce 40 MW of rf power at 430 MHz. Budker's improvement in the basic device is a bending magnet, shown in Fig. 2, that bends the beam into the output waveguide. The bending magnet greatly reduces the deflection power needed to modulate the beam, because the deflection power is proportional to the square of the tangent of the deflection angle.

This report develops the first-order theory for the dc-to-rf conversion efficiency $\bar{\eta}$ of the gyrocon. The theory also is useful for designing gyrocons. Most of the equations have been programmed into GYRO, an analysis code used to investigate the effects of beam voltage, current, and frequency on gyrocon performance.

In the gyrocon, it is very easy to obtain an overall $\bar{\eta}$ of 80 to 90%, whereas in the klystron, for example, the calculated $\bar{\eta}$ is usually 50 to 70%. Thus, accelerator applications that are impractical using conventional rf sources may be practical with the gyrocon. The power per rf source may be increased substantially because

Fig. 1.
The basic gyrocon.

Fig. 2.
The improved gyrocon with a bending magnet
to reduce deflection power.

of the large output circuit and excellent efficiency of the gyrocon. Although the power gain of a gyrocon is much smaller than that of a klystron or a traveling-wave tube, the possibility of high output power and high efficiency justifies further analysis and experimentation.

For analytical purposes, it is convenient to write $\bar{\eta}$ as

$$\bar{\eta} = \bar{\eta}_e \bar{\eta}_c \bar{\eta}_d \bar{\eta}_f, \quad (1)$$

where η_e = the electronic conversion efficiency

η_c = the output circuit efficiency

η_d = the deflection circuit efficiency

η_f = the focusing and bending system efficiency.

Each term in Eq. (1) can be calculated independently, and, then, the overall efficiency calculated.

First, the output interaction and circuit are analyzed to establish the electronic and circuit efficiencies. Next, the deflection circuit is analyzed. The bending system is discussed only in general terms because the author has not yet analyzed this part of the gyrocon. The trajectory from the electron gun to the output circuit is analyzed, and the report concludes with several specific design examples that illustrate the performance of gyrocon amplifiers as functions of current, voltage, and frequency.

II. OUTPUT INTERACTION

A. First-Order Analysis

The basic principle of the gyrocon is the injection of an unbunched electron beam into a resonant ring of TE_{01} -mode waveguide. If the conditions are proper, the beam will be halted as it travels across the waveguide, and the dc-to-rf conversion efficiency will be very high. A highly idealized case with the following assumptions clarifies the problem.

1. The electric field E_y in the output circuit parallel to the beam direction provides the only significant interaction between the electrons and the rf field. (See Fig. 3.)
2. The electron beam enters the waveguide at $x = a/2$, $y = b$ with velocity u_0 in the minus y -direction. The beam is so small that it is considered infinitely thin.
3. The effects of space charge are negligible.
4. The dc beam velocity u_0 is so small that relativistic effects are neglected.
5. The waveguide field results from a straight, uniform, rectangular waveguide; that is, output-waveguide curvature effects are neglected.
6. All higher order modes in the waveguide are neglected.

These assumptions define the ideal gyrocon. An analysis of a more general case in which the first four restrictions are removed is presented later, but the ideal case has simple equations that can be integrated easily. Without loss of generality, the z -axis origin is placed in the output section at a traveling-wave minimum, at $t = 0$. The beam is injected at $z = \xi/\beta$, and $t = 0$, as shown in Fig. 4. The waveguide dimensions, electric field, and power losses are calculated next for the ideal case.

The electric field in the waveguide is

$$E_y = -E_0 \cos(\omega t - \beta z), \quad (2)$$

where ω is the radian frequency, β is the propagation constant of the wave, and E_0 is the field amplitude. The force equation is

Fig. 3.
The output interaction region.

Fig. 4.
The electric field in the output waveguide at $t = 0$.

$$\frac{dv_y}{dt} = \eta E_0 \cos(\omega t - \phi), \quad (3)$$

where $\eta = |e|/m_0$ and $\phi = \beta z$. The electrons in the ideal case have no motion in the z - or x -directions. Equation (3) may be integrated to determine the velocity;

$$v_y(t) = -u_0 + \frac{\eta E_0}{\omega} [\sin(\omega t - \phi) + \sin \phi]. \quad (4)$$

The velocity may be integrated to find the position;

$$y(t) = b - \left(u_0 - \frac{\eta E_0}{\omega} \sin \phi \right) t - \frac{\eta E_0}{\omega^2} [\cos(\omega t - \phi) - \cos \phi]. \quad (5)$$

From Eq. (4), the velocity is zero when

$$E_0 = \frac{\omega u_0}{\eta [\sin(\omega t - \phi) + \sin \phi]}. \quad (6)$$

The lowest loss solution will have the minimum E_0 , so from Eq. (6) the optimum transit time τ and entry phase ϕ are

$$\omega \tau = (2m + 1)\pi, \quad m = 0, 1, 2, \dots, \quad (7a)$$

and

$$\phi = \pi/2. \quad (7b)$$

The condition $y(\tau) = 0$ yields

$$b = \left(u_0 - \frac{\eta E_0}{\omega} \sin \phi \right) \tau + \frac{\eta E_0}{\omega^2} [\cos(\omega\tau - \phi) - \cos \phi] \quad (8)$$

Putting the optimum values from Eq. (7) into Eqs. (4) and (8) produces

$$E_0 = \frac{\omega u_0}{2\eta} \quad (9)$$

and

$$b = \pi u_0 / (2\omega) \quad (10)$$

where the $m = 0$ trajectory is chosen as the minimum loss case. The rf voltage in the output waveguide is

$$V_{rf} = E_0 b = \frac{\pi}{2} V_0 \quad (11)$$

where V_0 is the dc beam voltage that corresponds to u_0 ; that is, $V_0 = u_0^2/(2\eta)$. All the beam's kinetic energy is converted to electric energy in the waveguide; thus, $\eta_0 = 1.0$. In Sec. II.C, large-signal equations for determining η_0 more accurately are derived.

B. Output-Circuit Efficiency

The output circuit is a resonant ring of waveguide. For simplicity, consider a rectangular waveguide bent in a circle, shown in Fig. 5 in cross section. Because the fields in the waveguide are TE_{01} , the mode can be described⁹ as

$$E_y = E_0 \sin \frac{\pi x}{a} \cos(\omega t - \beta z) \quad (12)$$

$$H_x = \frac{E_0}{Z_{TE}} \sin \frac{\pi x}{a} \cos(\omega t - \beta z) \quad (13)$$

and

$$H_z = \frac{E_0}{\eta_0} \frac{\lambda}{2a} \cos \frac{\pi x}{a} \sin(\omega t - \beta z) \quad (14)$$

where $\lambda = c/f$, c is the velocity of light in vacuum, $\eta_0 = (\mu_0/\epsilon_0)^{1/2}$,

$$Z_{TE} = \eta_0 \left[1 - \left(\frac{\lambda}{2a} \right)^2 \right]^{-1/2}, \text{ and } \beta = \frac{2\pi}{\lambda_g}, \text{ where } \lambda_g = \lambda \left[1 - \left(\frac{\lambda}{2a} \right)^2 \right]^{-1/2}.$$

The circuit efficiency is defined as

$$\eta_c = 1 - \frac{P_L}{P_T}, \quad (15)$$

where P_T is the power transmitted in the waveguide and P_L is the power dissipated in the waveguide walls. The transmitted power is

$$\begin{aligned} P_T &= \frac{1}{2} \int_0^b \int_0^a E_y H_x dx dy \\ &= \left[1 - \left(\frac{\lambda}{2a} \right)^2 \right]^{1/2} E_0^2 ab / (4\eta_0). \end{aligned} \quad (16)$$

The resonant ring is designed so that

$$2\pi r_g = n\lambda_g, \quad n = 1, 2, \dots \quad (17)$$

A bit of algebra shows the power lost in the waveguide walls to be

$$P_L = \frac{n\pi \left(1 + \frac{b\lambda^2}{2a^3} \right) a E_0^2}{\eta_0 \left(2\sigma\mu_0 \omega \right)^{1/2} \left[1 - \left(\frac{\lambda}{2a} \right)^2 \right]^{1/2}}, \quad (18)$$

where σ is the conductivity of the waveguide material. The overall circuit efficiency can be obtained by evaluating Eqs. (16) and (18) and placing the results in Eq. (15). The parameter n should be set to 1 for greatest efficiency, unless r_g from Eq. (17) must be increased. At high frequencies, the circuit efficiency decreases because the losses are proportional to $\omega^{1/2}$.

C. Output-Section Electron Dynamics

The relativistic force equation is

$$\frac{d}{dt}(\gamma \vec{v}) = -e(\vec{E} + \mu_0 \vec{v} \times \vec{H}) \quad (19)$$

where $\gamma = \left(1 - \frac{v_x^2 + v_y^2 + v_z^2}{c^2} \right)^{-\frac{1}{2}}$. (20)

After expansion of the force equation with the relation

$$\frac{d\gamma}{dt} = -\frac{n}{c^2} v_y E_y \quad (21)$$

and the fields of Eqs. (12)-(14), the component equations are

$$\frac{dv_y}{dt} = -\frac{n}{\gamma} \left\{ E_y \left[1 - \left(\frac{v_y}{c} \right)^2 \right] + \mu_0 (v_z H_x - v_x H_z) \right\} , \quad (22)$$

$$\frac{dv_x}{dt} = -\frac{nv}{\gamma} \left(\mu_0 H_z - \frac{v_x E_y}{c^2} \right) , \quad (23)$$

and

$$\frac{dv_z}{dt} = \frac{nv}{\gamma} \left(\frac{v_z E_y}{c^2} + \mu_0 H_x \right) . \quad (24)$$

A second approximation of the electronic efficiency (see Sec. II.A) is obtained by integrating Eqs. (21)-(24) through the output waveguide, for a given set of fields. The beam is divided into one central core and N_R rings; each ring is subdivided into N_T segments, as shown in Fig. 6. The first-order effect of space charge is calculated by allowing the potential, hence the y-directed beam velocity, to vary with the beam radius. Each beamlet in Fig. 6 contains an equal amount of current. If the beam has current I_0 and an initial radius r_0 , the potential at any radius $r < r_0$ is

$$V(r) = V_0 + \frac{I_0 r^2}{4\pi u_0 r_0^2 \epsilon_0} , \quad (25)$$

where V_0 is the potential on the axis and I_0 is positive. The numerical integration procedure is checked by calculating γ , the energy factor for each electron, by using Eqs. (20) and (21). The results would be identical if the integration were perfect. The difference between the two calculated energies usually is $< 2\%$.

D. Other Output Waveguide Considerations

The presence of the slot shown in Fig. 5 increases the waveguide wavelength⁶ by

Fig. 5.
The circular output waveguide.

Fig. 6.
Division of the beam into N_R rings and N_T subdivisions. Here $N_R = 3$ and $N_T = 4$.

$$\frac{\Delta \lambda_g}{\lambda_g} \approx \frac{\pi s^2}{16ab} \left(\frac{\lambda_g}{2a} \right)^2 \quad \text{if } t' \approx 0 \quad (26a)$$

or

$$\frac{\Delta \lambda_g}{\lambda_g} \approx \frac{s^2}{2\pi ab} \left(\frac{\lambda_g}{2a} \right)^2 \quad \text{if } t' \gg s \quad (26b)$$

In each case, it is assumed that $s/b \ll 1$. Thus, in a typical case, r_g is a few percentage points larger than Eq. (17) indicates. The slot radiates no energy to first order.

A major omission in the equations presented in Sec. C is the effect of space charge on the beam within the output waveguide. As the beam is almost stopped in the output region, space-charge forces are large as the beamlets land on the collector. These low-energy electrons generate secondary electrons that can extract energy from the output fields.

The output coupler is not considered here. Russian reports² indicate that two couplers are needed; one probably extracts the power and the other probably attenuates the wave that rotates in the wrong direction. Wilson³ has shown how to build a simple two-aperture coupler to extract the energy from a gyrocon. The apertures may be on either surface of the output circuit, but it seems prudent to locate them in the narrow wall, farthest from the electrons. An output coupler with many apertures will work better than a coupler with only two apertures because the output circuit voltage is more constant with angle in the former.

III. THE DEFLECTION CAVITY

Microwave cavities have been used to deflect electron beams in several accelerators. Panofsky and Wenzel⁴ analyzed the problem in general terms and showed that TM modes will work, whereas TE modes

produce no net transverse deflection because the electric and magnetic forces cancel each other at any beam velocity. Haimson⁶ has studied the problem using several modes in rectangular and cylindrical cavities. His analysis is fairly complete, even to tracing rays through the deflection fields to find the aberration.

In this section the deflection produced by a TM_{102} field in a rectangular deflection cavity is analyzed. Then the power required to support these fields is calculated. Some of Haimson's aberration results are discussed and applied to this problem.

A. Deflection Angle and Fields

The TM_{102} mode seems best for a gyrocon deflection cavity. Either rectangular or cylindrical cavities can be used, but only the rectangular one is considered here. Figure 7 shows the cavity and its coordinate system. The beam is injected parallel to the y-axis at $z = a_1/2$, $x = b_1/2$, and we assume that the beam entrance and exit apertures do not affect the cavity fields.

The electric field, in the y-direction, is

$$E_y = E_0 \sin \frac{2\pi z}{a_1} \sin \frac{\pi x}{b_1} \quad (27a)$$

where E_0 is an arbitrary constant. The magnetic field components are

$$H_z = \frac{j\omega E_0 \epsilon_0 \pi}{b_1 k_c^2} \sin \frac{2\pi z}{a_1} \cos \frac{\pi x}{b_1} \quad (27b)$$

and

$$H_x = - \frac{j\omega E_0 \epsilon_0 2\pi}{a_1 k_c^2} \cos \frac{2\pi z}{a_1} \sin \frac{\pi x}{b_1} \quad (27c)$$

where

$$k_c^2 = \left(\frac{2\pi}{a_1} \right)^2 + \left(\frac{\pi}{b_1} \right)^2 \quad (28)$$

and the resonant wavelength is

$$\lambda = 2\pi/k_c \quad (29)$$

The very attractive feature of this mode is that, for a thin electron beam, $E_y = H_z = 0$ at $z = a_1/2$,

Fig. 7.
The TM_{102} deflection cavity.

$x = b_i/2$. Thus, there is no velocity modulation to first order for a small-diameter beam. Therefore, the transverse force equation is

$$\frac{d(\gamma_0 v_z)}{dt} = - \frac{2\pi\eta\omega_0}{a_1 k_c^2} \sin(\omega t + \phi) \quad , \quad (30)$$

where u_0 is the axial velocity and γ_0 is the mass factor. With a small deflection angle, the beam passes through no electric field, so γ_0 is constant. In Eq. (30), ϕ is an arbitrary phase of the magnetic field relative to the entrance time of the particles being deflected. Equation (30) may be integrated to yield

$$v_z(y = d) = \frac{2\pi\eta u_0 E_0}{\gamma_0 a_1 \omega^2} [\cos(D + \phi) - \cos \phi] \quad , \quad (31)$$

where $D = \omega d/u_0$, the transit angle. Maximum deflection occurs when $\phi = (\pi - D)/2 + n\pi$, and the deflection velocity then is

$$v_z = \frac{4\pi\eta E_0 u_0}{\gamma_0 a_1 \omega^2} \sin D/2 \quad . \quad (32)$$

The deflection angle is ψ , where

$$\tan \psi = \frac{v_z}{u_0} = \frac{4\pi\eta E_0}{\gamma_0 a_1 \omega^2} \sin D/2 \quad . \quad (33)$$

B. Power Loss in Cavity Walls

The surface currents on the cavity walls are given by

$$\vec{J} = \hat{n} \times \vec{H} \quad , \quad (34)$$

where $\hat{n}$ is the unit normal to the cavity surface and the power dissipated in the walls is

$$P_G = \frac{R_s}{2} \iint_S |\vec{J}|^2 dS \quad , \quad (35)$$

where S is the surface of the cavity and R_s is the skin-effect resistance,

$$R_s = [\omega\mu_0/(2\sigma)]^{1/2} \quad . \quad (36)$$

After considerable algebra, Eqs. (34)-(36) yield

$$P_o = \frac{E_o^2 \epsilon_o^2 \omega^2 R_s a_1^4}{\pi^2 \left[4 + \left(\frac{a_1}{b_1} \right)^2 \right]^2} \left(\frac{a_1}{4b_1} + \frac{b_1}{a_1} + \frac{2b_1 d}{a_1^2} + \frac{da_1}{2b_1^2} \right) \quad (37)$$

To achieve a rotating deflection, another mode, 90° out of time and space phase, must be added. Therefore, a_1 must be equal to b_1 , and P_o must be doubled. Then Eqs. (37) and (29) become

$$P_o = \frac{R_s \left(E_o a_1^2 \omega \epsilon_o \right)^2}{10\pi^2} \left[1 + \frac{2d}{a_1} \right] \quad (38)$$

and

$$\lambda = \frac{2a_1}{\sqrt{5}} = 0.89443 a_1 \quad (39)$$

C. Optimum Length of Deflection Cavity

Equation (39) determines the dimension a_1 of the cavity, and it is convenient to combine Eqs. (33), (36), and (38) to obtain

$$P_o = C_1 \gamma_o^2 f^{1/2} \frac{\tan^2 \psi}{\sin^2 \left(\frac{\pi f d}{u_o} \right)} \left(1 + \frac{4df}{c\sqrt{5}} \right) \quad (40)$$

where

$$C_1 = \frac{125\pi^2 c^5 \epsilon_o^{1.5}}{160\sigma^{1/2} n^2} = 2.0862 \text{ for copper.}$$

The deflection power varies with the square root of frequency and with the square of the deflection-angle tangent. For any given deflection angle, beam voltage, deflection length, and frequency, we can use Eq. (40) to calculate the deflection power. However, there is an optimum cavity length, found by differentiating Eq. (40) and setting the result equal to zero. The author has not been able to solve the resulting transcendental equation exactly, but

$$d_{\text{opt}} \approx \frac{u_0}{4f} \left[1 - \alpha + \sqrt{(1 + \alpha)^2 - \frac{8}{\pi^2}} \right] , \quad (41)$$

where

$$\alpha = \frac{c\sqrt{5}}{2u_0} \quad (42)$$

is an approximate solution. The quantities γ_0 and u_0 are found from

$$\gamma_0 = 1 + \frac{nV_0}{c^2} \quad (43)$$

and

$$u_0 = c \left(1 - 1/\gamma_0^2 \right)^{-1/2} \quad (44)$$

where V_0 is the beam voltage on the axis.

D. Aberrations and Neglected Quantities in the Deflection Cavity

Haimson⁸ has integrated the force equations through a TM_{102} deflection cavity and a 0.2-m drift space. He neglected space-charge and aperture effects and found few aberrations for a beam size of 0.1λ , whereas for 0.2λ , the beam was highly elliptical. The required force equations are Eqs. (21)-(24), but Eqs. (12) to (14) must be used for the fields. Thus, CYRO, which integrates Eqs. (21)-(24), can be modified to integrate the deflection-cavity equation. This work has not yet been done. The effects of space charge in the drift space beyond the deflection cavity are calculated approximately below.

IV. THE BENDING SYSTEM

Budker's improvement to the gyrocon, the bending magnet shown in Fig. 2, is a major achievement because it reduces the required deflection angle by a factor of about 5. This reduces the required deflection power by the square of the factor. In many cases, the gyrocon power gain is a factor of only 10 to 15, and an additional 10 to 25 makes the device much more attractive.

The author has not analyzed the bending system, but he makes the following points. There are at least four methods, illustrated in Fig. 8, that can be used for a bending system. The first-order system uses a current sheet; it has the advantage that the magnetic field can be perpendicular to the electron velocity. There is a serious problem where the current sheet crosses the beam path, but the deflection is proportional to the current. The second-order bending system has a simple solenoid within the cone of the beam trajectories, and it is easier to build, but the deflection is proportional to the square of the current, so many more

Fig. 8.
Four possible beam-bending systems.

ampere turns are required. Miller⁸ has pointed out that there must be an optimum current-sheet shape for deflection or aberration characteristics, but that shape will be a challenge to find. The magnetic cusp system has two solenoids whose fields oppose each other; it works like a magnetic mirror and deflects the beam outward. It also is a second-order system; it requires even more ampere turns because the fields partially cancel each other. For a high-power, high-voltage gyrocon, a magnetic bending system is best, but for lower power models, it may be possible to use an electrostatic bending system. In each case, the detailed optics must be studied carefully. The focusing properties of each bending system are important in selection of the bending system.

V. SPACE-CHARGE EFFECTS

The major space-charge effect outside the output cavity is to force the beam to grow as it travels. The waveguide slot must be kept as small as possible, but the beam must not touch the slot edges. Thus, the beam size must be estimated.

The radial force equation is¹⁰

$$\gamma \frac{dv_r}{dt} = -\eta E_r \left[1 - \left(\frac{v_r}{c} \right)^2 \right] + \eta v_z \left(B_\theta + \frac{v_r}{c^2} E_z \right) + \frac{r \omega_c^2}{4\gamma} \left[1 - K \left(\frac{r_0}{r} \right)^4 \right], \quad (45)$$

where K is the cathode flux parameter

$$K = \left(\frac{r_c}{r_0} \right)^4 \left(\frac{B_c}{B_0} \right)^2. \quad (46)$$

The subscripts c and o refer to quantities evaluated at the cathode and the magnetic field entrance region, respectively, and $\omega_c = \eta B_0$. We assume that the flow is laminar and that both the radial and azimuthal velocities are small compared to the velocity of light. The axial velocity is constant at u , and the force equation can be transformed to

$$\frac{d^2 r}{dz^2} = \frac{\eta I_0}{2\pi \epsilon_0 u_0^3 \gamma_0^3 r} - \frac{\omega_c^2}{4\gamma_0^2 u_0^2} \left(r - K \frac{r_0^4}{r^3} \right), \quad (47)$$

where we have used the static fields

$$E_r = \frac{I_0}{2\pi r \epsilon_0 u_0}, \quad (48a)$$

$$B_\theta = \frac{\mu_0 I_0}{2\pi r}, \quad (48b)$$

and

$$E_z = 0. \quad (48c)$$

We assume that Eq. (47) holds true even though the central trajectory is not straight, and the beam may not even be circular because of aberration from the deflection cavity. Equation (47) has been integrated for several configurations, as shown in Fig. 9. There may be a magnetic lens at the beginning of the trajectory computation, and, beyond the lens, the magnetic field is zero. Figure 10 shows the trajectories for a LAMPF klystron gun operating at 86 kV and 11.6 A with $B_z = 0, 100$, and 200 G. Trajectories for higher voltages and no magnetic lenses are shown in Fig. 11. The beam spread is much less at 200 kV, the highest voltage considered.

Fig. 9.

The model used for beam-spread calculations.

Fig. 10.

Beam spread of the LAMPF klystron gun with a 10-cm-long magnetic lens.

VI. GYROCON DESIGN EXAMPLES

Most of the preceding equations have been assembled into the FORTRAN computer code GYRO. Using GYRO, one can get most first-order design parameters for any gyrocon amplifier quickly. The bending system equations are not included, so only the simple device shown in Fig. 1 can be analyzed. The primary design consideration is that, at any dc beam voltage V_0 , the rf voltage in the output waveguide must be approximately $\pi V_0/2$ to stop the electrons and give high conversion efficiency. With such a waveguide voltage, power losses in the output waveguide will be tens of kilowatts; therefore, for high efficiency, beam power must be hundreds of kilowatts. Deflection power is independent of beam power because beam loading has been neglected; output power must be large for high deflection efficiency and power gain.

The first design, shown in Fig. 12(a), has a 30° deflection angle and operates at 450 MHz with no bending system, and the distance from the center of the deflection cavity (the beam minimum) to the entrance of the output waveguide is 40 cm. The second design, shown in Fig. 12(b), has a 10° deflection angle and operates at 450 MHz, and the distance from the center of the deflection cavity to the entrance of the output waveguide is 110 cm. In each design, the waveguide radius is 19.15 cm, or a few percentage points greater, to account for the slot effects. No focus lens was used, and the minimum beam radius provided by the electron gun was assumed to be 1.5 cm, about that of the beam from the LAMPF klystron at 86 kV.

First, a matrix of calculations on the beam size at the output waveguide was made for these initial conditions as a function of beam voltage and current. The results are shown in Table I. The step size used to integrate Eq. (47) was 0.5 cm, and the cathode flux parameter was set to zero. The slot width in the output waveguide must be as small as possible, but it should be about 40% larger than the beam diameter. With a waveguide width of 40 cm, a beam radius of <4 cm gives a slot width of <11 cm. If the slot width is nearly equal to the waveguide height, Eq. (26) will not be valid for estimating the guide wavelength increase due to the slot. A more important problem could be radiation from the slot. Both problems should be attacked by an experimental program, and both are eliminated by operating the device at a dc voltage of several hundred kilovolts or higher. At 20 kV, the beam is too large at any power level, and at 50 kV, the amplifier can operate only at 100 KW input. Thus, dc beam-spread calculations alone show that high output power gyrocons require high dc beam voltages.

Fig. 11.

LAMPF klystron gun trajectories as a function of beam voltage with no magnetic lens.

Fig. 12.

Scale drawings of two gyrocons for 450-MHz: (a) 30° deflection; (b) 10° deflection.

Next, the deflection system parameters were calculated. The square deflection cavity is 74.5 cm on a side and resonates at 450 MHz. The optimum cavity length and the power required to deflect the beam by 10 and 30° are listed in Table II. Estimates of the deflection power are optimistic, because beam-loading effects have been neglected. The 30° deflection system becomes impractical at voltages above 500 kV, but the 10° system is practical up to 2 MV.

TABLE I
BEAM RADIUS AT THE OUTPUT WAVEGUIDE VERSUS BEAM V_0

V_0 (kV)	$P_0=100$ (kW)	$r(40 \text{ cm})$			$r(10 \text{ cm})$			
		300 (kW)	1000 (kW)	3000 (kW)	100 (kW)	300 (kW)	1000 (kW)	3000 (kW)
20	9.90	20.00	42.90	83.60	35.60	66.60	154.00	273.00
50	2.71	4.66	9.71	19.60	8.09	16.40	35.1	69.20
100	1.72	2.13	3.40	6.26	2.97	5.28	11.1	22.50
200	1.53	1.60	1.84	2.46	1.76	2.23	3.68	6.90
500	1.50	1.51	1.53	1.58	1.52	1.56	1.69	2.04
1000	1.50	1.50	1.50	1.51	1.50	1.51	1.52	1.56
2000	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.51

The most time-consuming part of the analysis was integration of Eqs. (21)-(24) with the central core and four beamlets spaced equally around it. The rf fields used in these equations are from Eqs. (12)-(14) except that the time and the z-variations are now relative to an entrance phase angle, ξ .

For each combination of waveguide field E_0 and waveguide height b , the entrance phase angle was swept in 5° steps to find the highest energy conversion efficiency. The time step was 2° of the 450-MHz rf fields, and the beam radius at the output-waveguide entrance was from Table I. Calculations were made for the cases in which this radius was smaller than 20 cm, half the output waveguide width. Enough runs were made to optimize E_0 to $\pm 5\%$ and b to ± 1 cm. The values of b under 10 cm were optimized to $\pm 1/2$ cm. A very nice feature is that the electronic efficiency varies very slowly with b and E_0 . Changes in b of 2 to 4 cm and in E_0 of 10% change the optimum value of the entrance phase significantly, but change the electronic efficiency by only 1 or 2%. The optimum values found to date are listed in Table III for the 30° system and in Table IV for the 10° system. At the optimum entrance phase, the electrons are not stopped in the injection direction because they gain energy rapidly in the z-direction as y decreases. The optimum condition gives equal amounts of energy in the y- and z-directions. The z-directed velocity comes from the magnetic force on the electrons in the waveguide; this force was neglected in the one-dimensional results. The dc beam power for the three sets of results are 100, 300, and 1000 kW. Higher power devices could be built, but the 1-MW beam input limit demonstrates the potential of the gyrocon.

The electronic efficiency of the 30° system (Table III) reaches a maximum of about 95% at 100 kV at each beam power, and it is independent of the beam current. If space-charge effects can be included more accurately, the beam current will have a stronger effect on the electronic efficiency. The overall efficiency, the total rf output power divided by the beam input plus the deflection power, also is shown in Table III. Usually, overall efficiencies increase as dc beam power increases, because waveguide losses depend on beam voltage, not on beam power. At 2 MV, at least 2 MW of power is lost in the waveguide, so a gyrocon must be very high powered to be efficient at 2 MV. Budker's design,² with a 100-MW beam power could be very efficient, as only 2% of the beam power is lost in the waveguide. The rf power gain (Table III) also increases with beam power, because the deflection power depends on beam voltage, not on beam power. The rf power gain for the 10° device (Table IV) is about 10 dB higher in each case, but the electronic efficiency is lower because the beam is larger when it reaches the output waveguide.

Calculations were made for a six-times frequency multiplier with a 30° deflection system. A 200-KV beam voltage and a 5-A current were assumed. Figure 12(a) applies, except that the waveguide broad dimension is now 6.67 cm. The beam radius is still 1.84 cm, because the diameter of the output guide has not

TABLE II
OPTIMUM DEFLECTION CAVITY LENGTH, FIELD
AND POWER VERSUS BEAM VOLTAGE

V_0 (kV)	d (cm)	E_0 (MV/m)	P_0 , 30° (kW)	P_0 , 10° (kW)
20	8.5	1.62	17.52	1.64
50	13.0	1.71	21.44	2.01
100	17.0	1.87	27.58	2.58
200	21.0	2.19	40.50	3.78
500	27.0	3.11	89.00	8.32
1000	27.0	4.67	207.10	19.32
2000	30.0	7.76	579.80	54.08

TABLE III
30° DEFLECTION-ANGLE AMPLIFIER SUMMARY

V_0 (kV)	I_0 (A)	r_0 (cm)	b (cm)	E_0 (MV/m)	ξ (deg)	Electronic Efficiency	P_L (kW)	P_{out} (kW)	Gain (dB)	Overall Efficiency
<u>100-kW dc Beam Power</u>										
20	5.00	9.90	3.0	0.80	40	0.9242	6.6	85.8	17.2	0.7303
50	2.00	2.71	8.0	1.00	95	0.9574	12.0	83.8	16.9	0.6895
100	1.00	1.72	10.0	1.50	90	0.9587	28.4	67.5	3.9	0.5288
200	0.50	1.53	10.0	2.40	70	0.9328	72.7	20.6	-0.3	0.1463
500	0.20	1.50	14.0	4.20	75	0.8755	245.6	-	-	-
1000	0.10	1.50	14.0	7.00	55	0.8134	682.3	-	-	-
2000	0.05	1.50	10.0	16.00	35	0.7435	3231.9	-	-	-
<u>300-kW dc Beam Power</u>										
20	15.00	20.00	3.0	0.90	30	0.6736	8.4	193.7	20.8	0.6100
50	6.00	4.66	5.0	1.15	50	0.9455	14.6	269.1	22.0	0.8370
100	3.00	2.13	9.0	1.50	80	0.9516	27.7	257.8	9.7	0.7890
200	1.50	1.73	12.0	2.20	80	0.9299	64.3	214.7	7.2	0.6306
500	0.60	1.50	16.0	4.00	85	0.8689	233.2	27.5	-5.1	0.0706
1000	0.30	1.50	20.0	6.40	95	0.8069	650.2	-	-	-
2000	0.15	1.50	15.0	12.00	70	0.7496	2098.7	-	-	-
<u>1000-kW dc Beam Power</u>										
50	20.00	9.70	5.0	1.20	40	0.9154	11.0	899.6	27.2	0.8807
100	10.00	3.40	7.0	1.70	60	0.9509	33.7	917.2	15.2	0.8926
200	5.00	1.84	12.0	2.20	80	0.9333	64.3	869.1	13.3	0.8353
500	2.00	1.53	16.0	4.00	85	0.8765	233.2	643.3	8.6	0.5907
1000	1.00	1.50	18.0	6.50	85	0.7733	643.2	170.1	-0.9	0.1409
2000	0.50	1.50	16.0	12.00	70	0.7496	2098.7	-	-	-

changed. The best *electronic* efficiency (86.5%) occurred with a 1.2-cm waveguide height and an 18.0-MV/m field. The circuit losses exceed the rf power at this high field, so the device cannot operate. A 2.4-cm-high waveguide gave an optimum overall efficiency of 29.1%, with a 6-MV/m field in the output guide. The overall efficiency is this low because the waveguide fields are this high; thus, the waveguide losses are high. A smaller initial beam diameter and operation as a straight amplifier would raise the overall efficiency, but whether this type of amplifier can compete with a well-designed klystron at operating frequencies above 3 GHz is doubtful. A reentrant output circuit, such as a ridge-loaded guide, would probably improve the efficiency at these higher frequencies. When analyzed previously³ for low-efficiency operation at very high frequencies, the device showed little promise.

VII. SUMMARY AND CONCLUSIONS

A class of deflection-modulated, high-power amplifiers has been analyzed. The large-signal calculations indicate that overall efficiencies approaching 90% and 10- to 30-dB power gains are achievable in the 450-MHz region. Design parameters such as the electron gun and waveguide width were not varied to achieve the

TABLE IV
10° DEFLECTION-ANGLE AMPLIFIER SUMMARY

V_0 (kV)	I_0 (A)	r_0 (cm)	b (cm)	E_0 (MV/m)	ξ (deg)	Electronic Efficiency	P_L (kW)	P_{out} (kW)	Gain (dB)	Overall Efficiency
<u>100-kW dc Beam Power</u>										
50	2.00	8.09	5.0	1.20	45	0.9402	15.8	78.2	26.9	0.7664
100	1.00	2.97	7.0	1.65	55	0.9556	31.7	63.3	13.9	0.6223
200	0.50	1.76	11.0	2.30	75	0.9349	68.5	25.0	8.2	0.2408
500	0.20	1.52	16.0	4.00	85	0.8771	233.2	-	-	-
1000	0.10	1.50	14.0	7.60	55	0.8134	682.3	-	-	-
2000	0.05	1.50	10.0	16.00	35	0.7435	3221.9	-	-	-
<u>300-kW dc Beam Power</u>										
50	6.00	16.40	4.0	1.50	25	0.7958	24.0	214.7	31.3	0.7110
100	3.00	5.28	6.0	1.85	45	0.9459	38.8	245.0	19.8	0.8097
200	1.50	2.23	9.0	2.50	55	0.9348	76.9	203.6	17.3	0.6702
500	0.60	1.56	16.0	4.00	85	0.8764	233.2	8.9	10.3	0.0286
1000	0.30	1.51	16.0	6.60	70	0.8146	634.8	-	-	-
2000	0.15	1.50	16.0	12.00	70	0.7496	2098.7	-	-	-
<u>1000-kW dc Beam Power</u>										
100	10.00	11.10	7.0	1.75	45	0.8899	35.7	854.2	25.2	0.8520
200	5.00	3.68	8.0	2.70	45	0.9307	87.3	843.4	23.5	0.8402
500	2.00	1.69	14.0	4.10	70	0.8769	234.1	642.8	18.9	0.6375
1000	1.00	1.52	15.0	6.80	65	0.8148	658.9	155.9	9.1	0.1529
2000	0.50	1.50	16.0	12.00	70	0.7496	2098.7	-	-	-

above results; this makes them remarkable. The devices seem to operate best at frequencies below a few gigahertz.

Several problems have been neglected; most important are analysis of the possible bending systems, analysis of beam effects on the circuit loading, and space charge in the output region. The first problem is straightforward; the author is programming the equations for a magnetic bending system. The second problem should not be difficult; a generalization of Rowe's methods,¹¹ in which the circuit equations are derived to relate the circuit wave amplitude to an induced current integral, can be used. This method is used often in traveling-wave tube analysis. The space-charge problem is the most difficult because the electron beam is almost halted in the output region. The conventional Green's-function methods in Refs. 10 and 11 lose validity in these conditions.

The author has worked extensively on the optimization of klystrons and traveling-wave amplifier efficiency. Very rarely, if all parameters are optimized carefully, one can calculate an electronic efficiency of over 70% for a klystron. For the gyrocon, it is easy to find combinations of parameters that predict >80% overall efficiency. Because the output circuit is a resonant ring, the gyrocon bandwidth will be narrow, but adequate for accelerators. Thus, the author believes that the gyrocon will find many applications where very efficient, high-power, low-frequency microwave amplifiers are required.

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SYMBOLS AND (UNITS)

a	broad dimension of output waveguide (m)
a_1	broad dimension of deflection cavity (m)
b	narrow dimension of output waveguide (m)
b_1	narrow dimension of deflection cavity (m)
B	magnetic flux density (T)
B_c	axial magnetic flux density at cathode (T)
B_o	axial magnetic flux density at lens (T)
c	velocity of light in vacuum (m/s)
C_1	numerical constant in Eq. (40) ($Ws^{1/2}$)
d	length of deflection cavity (m)
D	transit angle of deflection cavity (rad)
e	electronic charge (C)
E, E_y	electric field (V/m)
E_o	rf electric field (V/m)
f	frequency (Hz)
H	rf magnetic flux (Wb)
I_o	dc beam current (A)
J	rf current density (A/m ²)
k_c	wave number in output waveguide (m ⁻¹)
K	cathode flux parameter
m_o	electronic mass (kg)
n	harmonic frequency multiplier
$\hat{n}$	unit normal vector
N_R	number of rings in electron beam model
N_T	number of angular subdivisions per ring in beam model
P_L	rf power dissipated in output waveguide (W)
P_o	rf power dissipated in deflection cavity (W)
P_{out}	rf output power (W)
P_T	rf power circulating in output waveguide (W)
r	radial coordinate (m)
r_c	cathode radius (m)
r_s	beam radius at slot in output waveguide (m)
r_o	beam radius at entrance to magnetic lens (m)
R_s	skin effect resistance (Ω)
s	width of slot in output waveguide (m)
S	area (m ²)
t	time (s)
t'	thickness of slot in output waveguide (m)
u_o	dc beam velocity (m/s)
v_x, v_y, v	beam velocity components (m/s)
$V(r)$	dc beam voltage (V)
V_o	dc beam voltage on axis (V)
V_{rf}	rf voltage in output waveguide (V)
x, y, z	cartesian coordinates (m)
Z_{TE}	waveguide impedance (Ω)
α	numerical factor in Eq. (42)
β	waveguide phase constant (m ⁻¹)
γ	ratio of electron mass to rest mass
γ_o	ratio of electron mass to rest mass evaluated at velocity u_o
$\Delta\lambda_g$	increase in guide wavelength due to output-waveguide slot

ϵ_0	permittivity of free space (F/m)
e/m	electronic charge-to-mass ratio (C/kg)
η	overall dc-to-rf energy conversion efficiency
η_c	output circuit efficiency
η_d	deflection circuit efficiency
η_e	electronic conversion efficiency
η_f	focus and bending system efficiency
η_0	characteristic impedance of free space (Ω)
λ	free-space wavelength (m)
λ_g	guide wavelength in output waveguide (m)
μ_0	permeability of free space (H/m)
ξ	normalized entrance position of beam into output waveguide
σ	conductivity of a conductor ($m^{-1} \Omega^{-1}$)
τ	transit time of electron (s)
ϕ	phase angle (rad)
ψ	deflection angle (rad)
ω	radian frequency (rad/s)