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COMPUTER PROGRAMS TO DETERMINE CREDIBLE
REGIONS FOR WEIBULL DISTRIBUTION

MASTER

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by

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ABSTRACT

Computer software for analyzing life test data with a Weibull distribution is given. The programs calculate and plot the 95, 90, 50 and 5% highest probability density contours and the maximum likelihood estimators on the same frame for all sets of data. They also calculate the maximum likelihood estimator of mean life and conservative 95, 90, 50 and 5% credible intervals on mean life. The programs are used to analyze data on the lifetimes of Kevlar/Epoxy spherical vessels and the results are given.

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Avinash Agrawal

1. INTRODUCTION

This report describes computer software useful in analyzing life test data. In particular, programs for obtaining contour plots corresponding to Bayesian credible regions for the scale and shape parameter of a Weibull distribution are given. The program calculates and plots the 95, 90, 50 and 5% highest probability density contours and the maximum likelihood estimators on the same frame for all sets of data. It also calculates the 95, 90, 50 and 5% credible intervals on mean life as well as the maximum likelihood estimator of mean life.

This report was motivated by the need to implement an ongoing study of lifetimes of Kevlar/Epoxy spherical pressure vessels that are subjected to a constant sustained pressure until the vessel fails. The mean rupture stress of the vessels was determined to be 5000 psi and 39 vessels were put on test under a pressure equal to 86% of the mean rupture stress (4300 psi), 24 vessels were put on test at 80% of the mean rupture stress (4000 psi). All of these vessels failed before the earlier analysis [1] was done. We shall therefore not analyse these two sets of data here. Twenty-four vessels were put on test at 74% stress (3700 psi) and 21 at 68% stress (3400 psi). At the time of this analysis 21 of those at 74% stress level and 9 of those at 68% stress level had failed. This represents an increase of 3 at 74% stress and 4 at 68% stress since the time of the last analysis [1]. Therefore, we expect an improvement in our estimates and a shortening of our credible intervals. Tables 2 and 3 give the ordered times of failure of the failed units and the time that the unfailed units have been on test for 74% and 68% stressed pressure vessels respectively.

In the previous study on these vessels [1], the data was analysed statistically. A Bayesian approach was used. The pressure vessels were assumed to have a life distribution represented by a Weibull life distribution. i.e.

$$p(\text{life} > x) = \exp(-(\lambda x)^\alpha)$$

$$\text{where } x \geq 0, \alpha > 0, \lambda > 0.$$

The mean life is given by $\frac{\Gamma(1 + 1/\alpha)}{\lambda}$.

The likelihood of α and λ , given the data D is

$$L(\alpha, \lambda | D) \propto \alpha^k \lambda^{k\alpha} \left[\prod_{i=1}^k x_{(i)} \right]^{\alpha-1} \exp \left[-\lambda^\alpha \left\{ \sum_{i=1}^k x_{(i)}^\alpha + (n-k)t^\alpha \right\} \right].$$

The test termination time is t . The $x_{(i)}$'s are the ordered times of failure, n is the total number put on test and k is the number of failed units.

If we assume a prior $\pi_o(\alpha, \lambda)$ on α and λ then the posterior probability density of α and λ is given by

$$\pi(\alpha, \lambda | D) = \frac{L(\alpha, \lambda | D) \pi_o(\alpha, \lambda)}{\int_{\alpha} \int_{\lambda} L(\alpha, \lambda | D) \pi_o(\alpha, \lambda) d\alpha d\lambda}$$

The purpose of our study was to develop computer software to analyse the data for each stress level and terminated test time (i.e. at stress levels 74% and 68% and at the end of selected calendar periods). We

1. Obtain the 95, 90, 50 and 5% highest probability density contours on the parameters, α and λ ;
2. Obtain the maximum likelihood estimators of α and λ ;
3. Observe how the contours evolve and how the correlation between α and λ changes as more data is obtained;
4. Obtain conservative 95, 90, 50, and 5% credible intervals on the mean life and the maximum likelihood estimator of mean life.

2. PROGRAM DESCRIPTION

The program developed is given in appendix 1. It

1. Calculates and plots the 95, 90, 50 and 5% highest probability density contours and the maximum likelihood estimator on the same frame for all sets of data;
2. Calculates the covariance between α and λ for each set of data;
3. Calculates conservative 95, 90, 50 and 5% credible intervals on the mean life. The 95% intervals should be close. The intervals computed are wider than the exact intervals.

If we do not wish to obtain all of the above, we can eliminate those lines from the program. Also, this program can be modified by adding a few lines to obtain other results. For example, we can plot the marginal density on α and λ or obtain credible intervals on α and λ . The main computation is the calculation of the likelihood. Once this has been done, we can do all the above and more by simple modifications of the program.

The program description is divided into the following subheadings:

Program Outline
Detailed Description
Input Description
Output Description
Bugs.

Program outline

1. The program reads in the range over which α varies α_{falo} to α_{fahi} , the range over which λ varies λ_{blo} to λ_{bli} , the number of points (num) into which each of the axes should be divided and ngraph- the number of data sets to be analysed.
2. It divides each axis into num points and calculates the likelihood at all num squared pairs of points. This gives us a three dimensional surface over the α, λ plane.
3. The probability density at each pair of points is obtained by normalizing the likelihood with respect to the specified prior. The highest point on the normalized surface gives the maximum likelihood estimator, for the parameter pair.

4. Knowing the probability density of the pair (α, λ) , the marginal density of α and λ can be determined. The expected values of α , λ , α^2 , λ^2 , and $\alpha\lambda$ are calculated. These in turn can be used to determine the correlation between α and λ .
5. A plane with height z and parallel to the α, λ plane is specified. The projections of this plane on the α, λ plane determined by the posterior density produces a region in the α, λ plane. The volume under the posterior density determined by this region is the "credible" probability attached to the region. By a sequential search, the height for the required plane is bracketed between two heights, such that, the volume over points on or in the contour for one height is more than required and for the second one is less. E.g. if we are trying to find the 95% contour, then we find z_1 and z_2 such that the volume over points corresponding to height z_1 or more is greater than or equal to .95 and over those corresponding to height z_2 or greater is less than or equal to .95. Then, by a simple binary search, the height corresponding to a contour which contains anywhere between .955 and .945 of the volume is determined. The heights for the 90, 50, and 5% contours are similarly obtained and all the contours are drawn using library programs.
6. For each contour, the coordinates of the points lying on that contour are obtained, the mean life at each of those points calculated and the maximum and minimum of those points determined. This gives credible intervals on the mean life.

Detailed description

alfa(41),alamb(41) are two arrays containing points on the α and the λ axes. The dimensions are 41 so that at most 40 points can fit on each axis.

contht(4) is an array containing the heights of the 4 contours to be drawn for each set of data.

perc(4) is an array giving the values of the credible contours i.e. .95, .90, .50, and .05 (input).

prob(41,41) is an array whose (i,j)th element prob(i,j) represents the probability density at the point alfa(i) and alamb(j).

num gives the number of points on each axis α and λ that are being considered. This is an input.

alfalo,alfahi,almblo,almbli are the extreme values of α and λ being considered. The intervals between alfalo and alfahi and almblo and almbli are each divided into num equal intervals where num is less than or equal to 40. These values are inputs.

ngraph gives the number of sets of data to be plotted on the same frame e.g. we might want to plot the data as it was on Jan 4, 1979 and also as it was on Nov 1, 1979. Here ngraph is 2. This way we can compare and see how the contours evolve over time.

x(50) This is an input and represents the times of failure of the pressure vessels.

1. The values of *alfalo*, *alfahi*, *almblo*, *almbli*, *num* and *ngraph* are read.
2. Some plotting routines set up parameters for plotting the contours. Refer to the program in appendix 1. The plotting routines have been taken from the Surface Display Library, computer Center, University of California, Berkeley [2].
3. The whole program with the exception of a few plotting routines on the bottom is executed *ngraph* number of times. Each time it analyses one set of data.
4. *N* - the number of vessels put on test and time - the time since they were put on test are read in. Time is divided by 24000 so, now, time is in units of 1000 days instead of hours as it was originally specified. Depending on the particular data that is to be analysed, this statement may be deleted if not required. If the data values seem large so that there is a danger of calculated values getting very small and the rounding off errors becoming significant then this statement should be there, otherwise not.
5. The times of failure of the vessels $x_{(i)}$ are read in, one at a time. The end of data condition is denoted by reading in a value of 0.0 for the data. They do not have to be sorted. If the time was divided by 24000 then the values of $x_{(i)}$ should also be divided by 24000 to take care of the modified scale as described above.

6. The arrays *alfa* and *alamb* are initialized by

$$alfa(1) = alfalo$$

$$alamb(1) = almblo$$

$$alfa(i + 1) = alfa(i) + diffal \text{ for all } i$$

$$alamb(i + 1) = alamb(i) + diffth \text{ for all } i$$

where

$$diffal = \frac{(alfahi - alfalo)}{(num - 1)}$$

$$diffth = \frac{(almbli - almblo)}{(num - 1)}.$$

7. The product of likelihood and prior density at each pair of points *alfa*(*i*) and *alamb*(*j*) is calculated by the function *alikhd* (described later).
8. The product of likelihood and prior density is integrated from *alfalo* to *alfahi* and *almblo* to *almbli* using the library routine *DBCQDU*. (It uses the bicubic spline method for integration and is available in the IMSL library, Computer Center, University of California, Berkeley [3].)
9. All the values obtained in 7 are normalized by dividing by the integral obtained in 8.
10. The following values are determined

$$E(\alpha) = \sum_j \sum_i diffal diffth Prob(i,j) alfa(i)$$

$$E(\lambda) = \sum_j \sum_i diffal diffth Prob(i,j) alamb(j)$$

$$E(\alpha^2) = \sum_j \sum_i diffal diffth Prob(i,j) alfa^2(i)$$

$$E(\lambda^2) = \sum_j \sum_i diffal diffth Prob(i,j) alamb^2(j)$$

$$E(\alpha, \lambda) = \sum_j \sum_i diffal diffth Prob(i,j) alfa(i) alamb(j)$$

$$Var(\alpha) = E(\alpha^2) - [E(\alpha)]^2$$

$$Var(\lambda) = E(\lambda^2) - [E(\lambda)]^2$$

$$\text{correlation } (\alpha, \lambda) = \frac{E(\alpha, \lambda) - E(\alpha)E(\lambda)}{\sqrt{\text{Var}(\alpha) \text{Var}(\lambda)}} .$$

Note: If due to some error, the $\text{var}(\alpha)$ or $\text{var}(\lambda)$ terms are negative, an error message is printed and the program proceeds by taking the absolute value.

11. The maximum of $\text{prob}(i,j)$ for all i and j is obtained and stored in ahtmax .
12. Nine equally spaced points are chosen between 0 and ahtmax . For each of these 11 heights (the nine points chosen along with 0 and ahtmax) the volume that lies under points within a contour at that height is determined. If the volume at any height as determined above is different from $\text{perc}(i)$ by less than .005, then $\text{contht}(i)$ = that height. This is done for all four $\text{perc}(i)$. If some $\text{contht}(i)$'s cannot be determined in this way then two heights are found such that

$$\text{contnt}(h) \leq \text{perc}(i) \leq \text{contnt}(h')$$

where $\text{contnt}(z)$ is defined as the volume under the points which are at a height of z or more. i.e. points which lie within a contour at height z . A binary search is conducted between h and h' to find a height z such that

$$|\text{contnt}(z) - \text{perc}(i)| \leq .005 .$$

This gives all 4 contour heights for the given set of data.

13. Library routines are used to plot contours at heights $\text{contht}(1)$ to $\text{contht}(4)$ as determined above.
14. For each contour height, subroutine 'points' estimates the points on the contour, calculates the mean life at those points and outputs them as amaxli and aminli . The amaxli and aminli on the contour at $\text{contht}(i)$ give the $\text{perc}(i)$ credible intervals on the mean life.

Function $\text{contnt}(z)$

Given the contour height, this subprogram estimates the content inside that contour i.e. the proportion of the volume which lies under the points which lie on or in the contour obtained as a projection on the α, λ plane of a plane parallel to and a height above the α, λ plane. For doing this, the probability density of points which do not lie within the contour is set to zero (points at which the density is greater than or equal to z lie within the contour) and the resulting function integrated over the entire range. This gives the content within the contour at height z .

Function $\text{alikhhd}(\lambda, \alpha)$

This function calculates the product of the prior density and likelihood of the pair (λ, α) . In our program, we have used a Weibull distribution to represent the life distribution and a flat prior on α and λ . Therefore

$$\text{alikhhd} = \alpha^k \lambda^{k\alpha} \left[\prod_{i=1}^k x(i) \right]^{\alpha-1} \exp \left[-\lambda^\alpha \left\{ \sum_{i=1}^k x(i)^\alpha + (n-k) \text{time}^\alpha \right\} \right] .$$

If we change the life distribution and/or the prior, only this function has to be changed in the program. The rest of the program remains the same.

Subroutine accurt(pp,cc,dd,z,pex)

Once the height of the pp contour has been bracketed between cc and dd, this subroutine conducts a simple binary search for at most 20 iterations and gets out with a height z , at which, the content given by pex is less than half a percent away from pp. We have assumed that 20 iterations are sufficient for this purpose. If at the end of 20 iterations, the program still doesn't know the value of z , then the mean of the two limits at that stage is taken as the approximating value for z .

Subroutine points(z,amaxli,aminli)

This subroutine finds out the maximum and minimum life on the contour at height z and outputs them as amaxli and aminli respectively. Each point is examined. If the density at that point is equal to z then this point lies on the contour and the mean life at that point is calculated. amaxli and aminli are then updated. If $\text{Prob}(i,j) \neq z$ then the program sees if

$$\begin{aligned} \text{prob}(i,j) \leq z \leq \text{prob}(i+1,j) \text{ or} \\ \text{prob}(i,j) \geq z \geq \text{prob}(i+1,j) \text{ is true .} \end{aligned}$$

If any of these is true, then a linear interpolation is performed between the points and the approximate co-ordinates of the point are obtained and the values of amaxli and aminli are updated. It also checks if

$$\begin{aligned} \text{prob}(i,j) \leq z \leq \text{prob}(i,j+1) \text{ or} \\ \text{prob}(i,j) \geq z \geq \text{prob}(i,j+1) \text{ is true .} \end{aligned}$$

If any of these two is true then again a linear interpolation is performed. This time between the new set of points between which it lies. Once the co-ordinates of the point are obtained, amaxli and aminli are updated. Caution: to start with, we had initialized amaxli=0. and aminli=100000. So in the output, if aminli=100000, then we need to initialize aminli with a higher value and make the run again.

Input description

The input consists of 3 read statements within loops as shown below.

```
.  
.   
.   
  read 3,alfalo,alfahi,almblo,almbli,num,ngraph  
3  format (1x,2f10.3,2f10.8,2i5)  
  do 100 ia=1,ngraph  
  .  
  .  
  .  
    read 64,n,time  
64 format(i5,5x,f10.3)  
  .  
  .  
  .  
    read 2, x(ncount)  
  .  
  .  
  .  
100 continue  
.  
.  
.
```

A sample input

This is the input for the 74 percent data. 24 vessels were initially put on test. We wish to calculate the quantities stated above as of January 4, 1979 (10008.0 hours) and as of November 1, 1979 (17184.0 hours). The values of α vary between .1 and 2, the values of λ between .5 and 6. We want to do our calculations on a grid 40 by 40 for two sets of data as shown in table 2(a) and 2(b). Our input data follows. (Time is in hours.)

column numbers

12345678901234567890123456789012345678901234567890123

.1 2.0 .5 6.0 40 2

24 10008.0

225.2

503.6

1087.7

1134.3

1824.3

1920.1

2383.0

2442.5

2974.6

3708.9

4908.0

5556.0

6271.1

7332.0

7918.7

7996.0

9240.3

9973.0

0.0

24 17184.0

225.2

503.6

1087.7

1134.3

1824.3

1920.1

2383.0

2442.5

2974.6

3708.9

4908.0

5556.0

6271.1

7332.0

7918.7

7996.0

9240.3

9973.0

11487.3

11727.1

14032.0

0.0

Output description

The output for the above set of data is shown in table 1 and figure 1. In table 1 the first line echo prints alfalo, alfahi, almblo almbli, num and ngraph. The second line tells you which set of data, the results following it are for. If in the output there is a statement saying "the variance of alpha is negative" or "the variance of lambda is negative" all the results following after this are wrong. The program should be run again, this time with an increased value of "num". The next statement gives the correlation between α and λ . The four statements after that give the heights at which the 95%, 90%, 50% and 5% highest probability density contours are drawn respectively along with the exact proportion that the contour contains. The small table with contour maxlife and minlife gives the 95%, 90%, 50% and 5% credible intervals on the mean life.

In figure 1, the x axis represents α and the y axis represents λ . The range of α is alfalo to alfahi and the range of λ is almblo to almbli in units of per 1000 days. The subroutine which plots contours can plot them in only 2 ways light and dark. They have therefore, been plotted alternately i.e. those due to the first set of data are dark, due to the second set light etc. The 4 contours for each set of data are the 95, 90, 50 and 5% highest probability density contours. The points in the middle represent the maximum likelihood estimators. The point at the smallest height represents the maximum likelihood estimators due to the first set of data etc.

Bugs

1. The input values of the variables alfalo, alfahi, almblo, almbli etc. should be in the correct order and format otherwise cryptic error messages from the library routines will be obtained.
2. The statements which scale the input by 24000 must be given careful thought before being removed or put in. In the equation for likelihood, there is a term

$$\exp \left[-\lambda^{\alpha} \left\{ \sum_{i=1}^k x_{(i)}^{\alpha} + (n - k) \text{time}^{\alpha} \right\} \right].$$

If the x values are high, this term becomes zero. Consequently, the probability density becomes identically zero.

3. In the function alikhd, the value of alikhd is being generated in two statements. If it is suspected that some terms are going to be small and the rounding off error will be significant, then the terms may be redistributed among the two statements.
4. If, in the output, there is a statement saying "the variance of alpha is negative" or "the variance of lambda is negative" then all the results following it are wrong. One solution is to increase the value of num and try again.

3. RESULTS

For the 74% data

For January 4, 1979,

Correlation(α , λ) = -.0715

The credible intervals on mean life

95%	188.40 days	to	731.83 days
90%	199.78 days	to	627.23 days
50%	241.37 days	to	425.07 days
5%	289.73 days	to	329.30 days.

The maximum likelihood estimator

for λ = 3.2 per 1000 days

for α = 1.07

for mean life = 304.40 days.

Standard deviation of life = 284.67 days.

For November 1, 1979

Correlation(α , λ) = -.236

The credible intervals on mean life

95%	200.39 days	to	678.86 days
90%	212.93 days	to	600.29 days
50%	257.80 days	to	438.38 days
5%	308.46 days	to	348.86 days

The maximum likelihood estimator

for λ = 3.05 per 1000 days

for α = 1.01

for mean life = 326.51 days

Standard deviation of life = 323.28 days.

For the 68% data

For January 4, 1979,

Correlation(α , λ) = .6732

The credible intervals on mean life

95%	591.76 days	to	48360.39 days
90%	622.91 days	to	16669.38 days
50%	761.35 days	to	2921.21 days
5%	1028.39 days	to	1381.01 days.

The maximum likelihood estimator

for λ = .775 per 1000 days

for α = 1.48

for mean life = 1166.78 days.

Standard deviation of life = 802.08 days.

For November 1, 1979,

Correlation(α , λ) = .4421

The credible intervals on mean life

95%	701.10 days	to	4349.57 days
90%	736.95 days	to	3147.88 days
50%	862.18 days	to	1660.65 days
5%	1048.21 days	to	1178.48 days.

The maximum likelihood estimator

for λ = .825 per 1000 days

for α = 1.6

for mean life = 1086.76 days.

Standard deviation of life = 695.42 days.

Note: The program to calculate the maximum likelihood estimator and the standard deviation of mean life, given the maximum likelihood estimators of α and λ is given in appendix 2.

Table 1

This is a sample output for the 74% data.

```
.100 2.000 .5000000006.00000000 40 2
these are the results for data set number 1
the correlation of alfa and lambda is -.71511974e-01
the height of .95336629e+00 contour is .51920693e-01
the height of .90251165e+00 contour is .10384139e+00
the height of .49988990e+00 contour is .52569702e+00
the height of .48792297e-01 contour is .99298326e+00
contour      maxlife      minlife
.950      .73183704e+00 .18840825e+00
.900      .62723262e+00 .19978271e+00
.500      .42506914e+00 .24137087e+00
.050      .32929862e+00 .28972508e+00
these are the results for data set number 2
the correlation of alfa and lambda is -.23616251e+00
the height of .95172301e+00 contour is .65480835e-01
the height of .90156525e+00 contour is .13096167e+00
the height of .49960180e+00 contour is .65480835e+00
the height of .52665815e-01 contour is .12523210e+01
contour      maxlife      minlife
.950      .67885536e+00 .20039197e+00
.900      .60028642e+00 .21292811e+00
.500      .43838020e+00 .25779388e+00
.050      .34886301e+00 .30845975e+00
```

Table 2(a)

Number of vessels put on test at 74% stress = 24

Ordered failure times of vessels up to 10008.0 hours (Jan 4,1979).

1	225.2
2	503.6
3	1087.7
4	1134.3
5	1824.3
6	1920.1
7	2383.0
8	2442.5
9	2974.6
10	3708.9
11	4908.0
12	5556.0
13	6271.1
14	7332.0
15	7918.7
16	7996.0
17	9973.3

Table 2(b)

Ordered failure times of vessels at 74% stress level which failed between 10008.0 and 17184.0 hours (Jan 4,1979 to Nov 1,1979).

1	11487.3
2	11727.1
3	14032.0

Table 3(a)

Number of vessels put on test at 68% stress level = 21.

Ordered times of failure (in hours) of vessels at 68% stress level which failed before 13488 hours (Jan 4, 1979).

1	4000
2	5376
3	7320
4	8616
5	9120

Table 3(b)

Ordered times of failure of vessels at 68% stress level that failed between 13488 hours and 20568 hours (Jan 4 1979 to Nov 1,1979).

1	14400
2	16104
3	20231
4	20233

FIGURE 1

HIGHEST PROBABILITY DENSITY CONTOURS FOR α AND λ FOR KEVLAR/EPOXY PRESSURE VESSEL LIFE TEST DATA. THE PRESSURE VESSELS WERE TESTED AT 74% STRESS LEVEL.

FIGURE 2

HIGHEST PROBABILITY DENSITY CONTOURS FOR α AND λ FOR KEVLAR/EPOXY PRESSURE VESSEL LIFE TEST DATA. THE PRESSURE VESSELS WERE TESTED AT 68% STRESS LEVEL.

Appendix 1

```
      program lllcont(input,output,tape99,tape98)
c---this program plots the highest probability density
c--- contours for 95%,90%,50% and 5% confidence contours
c--- on alfa and alamb which are the parameters of the
c--- assumed Weibull distribution.
c--- program inputs---time of each failure x(1) through x(n)
c---     where n  is the total number of failures
c---     time-time when the observations stopped for this
c---     set of data
c---     alfalo- lower limit on alfa
c---     alfahi- higher limit on alfa
c---     almblo- lower limit on alamb
c---     almbli- higher limit on alamb
c---     num- num by num is the grid we are calculating
c---           out points over
c---     ngraph- number of sets of data we wish to have
c---           plotted on the same graph
      dimension alfa(41),alamb(41),contht(4),perc(4)
      dimension zz(12)
      dimension p(12)
      common /bb/ x(50),m,n,time
      common/ /prob(41,41),num,alfa,alamb,almbli,func(41,41),
      *wk(2400),iwk(2000)
      read 3,alfalo,alfahi,almblo,almbli,num,ngraph
      3  format (1x,2f10.3,2f10.8,2i5)
      print 3,alfalo,alfahi,almblo,almbli,num,ngraph
c--- this subroutine initiates plotting
      call bgnplt
c--- this subroutine suppresses the printing of the
c--- execution log
      call setrpl(1,98)
c--- this subroutine fixes the proportion of the divisions
c--- on the x, y and z axes. Here, since we do not have a
c--- z axis, this subroutine fixes the size of the plot
c--- window to 5 inches by 7 inches
      call setprp(2,5.0,7.0,1.0)
c--- this subroutine sets the x and y axes.Each of them must
c--- have num points and the x axis should start from alfalo
c--- and end at alfahi the y axis should start at almblo and
c--- end at almbli.
      call setxyg(num,alfalo,alfahi,num,almblo,almbli)
c--- this subroutine chooses the page size
      call setpag(9.0,9.0,0.0,0.0)
c--- this subroutine chooses the window size on the page.
      call setwin(7.0,7.0,1.0,1.0)
      ithbld=1
      do 100 ia=1,ngraph
      iab=ia
      print 110,iab
110  format(1x,*these are the results for data set * i3)
      read 64,n,time
```

```
64 format(i5,5x,f10.3)
   ah=alfahi
   al=alfalo
   time=time/24000.0
c---for reading the values of x, we shall punch one value on
c--- one card and denote the eof by reading in a value 000
c--- for x. ncount gives the number of values of x read.
   ncount=0
   1  ncount=ncount+1
      read 2,x(ncount)
   2  format (f10.3)
   4  format(1x,f10.3)
      x(ncount)=x(ncount)/24000.0
      if (x(ncount).ne.0) go to 1
      ncount=ncount-1
      m=ncount
      t=almblo
   98  format (1x,i5)
c--- calculation of the values of alfa and alamb vectors and
c--- the probability matrix
      alfa(1)=alfalo
      alamb(1)=almblo
      diffal=(alfahi-alfalo)/float(num-1)
      diffth=(almbli-almblo)/float(num-1)
      do 11 i=1,num
         alfa(i+1)=alfa(i)+diffal
         alamb(i+1)=alamb(i)+diffth
   11  continue
      nump=num+1
c--- function alikhd gives the likelihood of the value of
c--- alpha which is represented by tempi and the value of
c--- alamb represented by tempii
      do 12 i=1,nump
         do 12 j=1,nump
            tempi=alfa(i)
            tempii=alamb(j)
            prob(i,j)=alikhd(tempii,tempi)
   12  continue
c---integration of the volume under the integral. using
c--- bicubic spline method over a grid of points num by num.
      aa=alfa(num)
      tt=alamb(num)
      ab=alfa(nump)
      tb=alamb(nump)
      call dbcqdu(prob,41,alfa,num,alamb,num,alfalo,aa,t,tt,q1,wk,ier)
      l=nump
c---normalizing the value of the probability and getting the
c--- value of the maximum- ahtmax. this will aid us later in
c--- getting the contour heights.
c--- here we also calculate elamb-expected value of lambda
c---          ealfa-expected value of alpha
c---          elmsq-expected value of lambda square
c---          ealfsq-expected value of alpha square
c---          elmfa-expected value of alpha*lambda
```

c---these values are used to estimate the variance of alpha
c--- varalf and the variance of lambda varlmb which together
c--- with the above values give the correlation of alfa and
c--- lambda for that particular set of data.

```
    ahtmax=prob(1,1)/q1
    elamb=0.
    ealfa=0.
    elmfa=0.
    elmbsq=0.
    ealfsq=0.
    do 66 i=1,num
        do 66 j=1,num
            prob(i,j)=prob(i,j)/q1
            ahtmax=amax1(ahtmax,prob(i,j))
            elamb=elamb+diffal*diffth*prob(i,j)*alamb(j)
            ealfa=ealfa+diffal*diffth*prob(i,j)*alfa(i)
            elmfa=elmfa+diffal*diffth*prob(i,j)*alfa(i)*alamb(j)
            elmbsq=elmbsq+diffal*diffth*prob(i,j)*alamb(j)**2.
            ealfsq=ealfsq+diffal*diffth*prob(i,j)*alfa(i)**2.
66      continue
65      format(15x,2(e15.8,10x),e15.8)
        varalf=ealfsq-ealfa**2.
        if (varalf.lt.0.) print 9
9        format(1x,*variance of alfa is negative*)
        varlmb=elmbsq-elamb**2.
        if (varlmb.lt.0.) print 10
10       format(1x,*variance of lambda is negative*)
        cov=(elmfa-elamb*ealfa)/(sqrt(abs(varalf*varlmb)))
        print 68,cov
68       format(1x,*the correlation of alfa and lambda is*,e15.8)
c--- contht(i) gives the height at which perc(i) percent
c--- contour is drawn.
    do 67 i=1,4
        contht(i)=0
67      continue
        perc(1)=.95
        perc(2)=.9
        perc(3)=.5
        perc(4)=.05
c--- to find the contour heights, we first do a sequential
c--- search in subintervals of the given interval 0 to ahtmax.
c--- We accept any value less than .5 % away from perc(i)
c--- and assign that value to contht(i) if we don't find any
c--- such value then we bracket perc(i) between two heights
c--- and call upon subroutine accurt to do a binary search
c--- between them for at most 20 iterations.
        zlo=0.0
        zhi=ahtmax
        zdiff=(zhi-zlo)/10.0
        zz(1)=zlo
        z=zlo
c--- subroutine contnt(z) gives the volume that lies inside
c--- the contour at height z
        p(1)=contnt(z)
```



```
zz(2)=zz(1)+zdiff
do 44 iij=2,11
z=zz(iij)
p(iij)=contnt(zz(iij))
c print 55,z,p(iij)
55 format(1x,e15.8,10x,e15.8)
a=p(iij-1)
b=p(iij)
cc=zz(iij-1)
dd=zz(iij)
do 13 j=1,4
if (contht(j).ne.0) go to 13
pp=perc(j)
if (abs(b-pp).gt..005) go to 45
contht(j)=dd
pex=b
go to 47
45 if(a.lt.pp.or.b.gt.pp) go to 13
call accurt(pp,cc,dd,contht(j),pex)
47 print 46,pex,contht(j)
46 format(1x,*the height of*,e15.8,* contour is*,e15.8)
13 continue
zz(iij+1)=zz(iij)+zdiff
44 continue
c--- this is the contour for 95 % hpd
call setcon(contht(1),100.0,ithbld,0,1,0)
call conmap(prob,41,41,iwk,2000)
c--- this is the contour for 90 % hpd
call setcon(contht(2),100.0,ithbld,0,1,0)
call conmap(prob,41,41,iwk,2000)
c--- this is the contour for 50 % hpd
call setcon(contht(3),100.0,ithbld,0,1,0)
call conmap(prob,41,41,iwk,2000)
c--- this is the contour for 5 % hpd
call setcon(contht(4),100.0,ithbld,0,1,0)
c---this subroutine plots the peak of the three dimensional
c--- surface that point corresponds to the maximum likelihood
c--- estimator.
call sethl(1,1.0,4.0,1,2,1,1)
call conmap(prob,41,41,iwk,2000)
c--- subroutine points gives the points with the maximum and
c--- the minimum mean life on all the contours.
print 103
103 format(2x,*contour      maxlife      minlife*)
do 101 ikl=1,4
call points(contht(ikl),amaxli,aminli)
print 102,perc(ikl),amaxli,aminli
102 format(3x,f4.3,7x,e15.8,2x,e15.8)
101 continue
c--- the next step makes the plotting of dark and light
c--- contours alternate.
ithbld=iabs(ithbld-1)
100 continue
call mapscl(1,0.0,0.0,0.1,2,0.1)
```

```
call mapscl(2,0.0,0.0,0.05,2,0.1)
call maplab(1,0.15,0.0,0.2,1,2)
call maplab(2,0.15,0.0,0.5,1,2)
call endlt
stop
end

function contnt(z)
c---*****
c---this function sees the height over all the points in our
c--- grid and sets the height of those points which are
c--- below z ( and therefore do not lie within our contour)
c--- to zero and again integrates over the whole grid. The
c--- whole grid. The output is the content inside the
c--- contour.
common/ /prob(41,41),num,alfa,alamb,almbli,func(41,41),
*wk(2400),iwk(2000)
dimension alfa(41),alamb(41)
dimension contnt(4)
do 66 i=1,num
do 66 j=1,num
func(i,j)=prob(i,j)
if (func(i,j).lt.z) func(i,j)=0.
66 continue
aa=alfa(num)
t=alamb(1)
tt=alamb(num)
af=alfa(1)
k=num
call dbcqdu(func,41,alfa,k,alamb,k,af,aa,t,tt,contnt,wk,ier)
return
end
function alikhd(alamb,alfa)
c---*****
c---this function calculates the likelihood of various values
c--- of alpha and lambda. in this program it uses a weibull
c--- distribution to represent the life distribution and a
c--- alpha and lambda. the whole program can be made to fit
c--- any prior an any model by just changing this function.
common /bb/ x(50),m,n,time
k=m
sumalf=0.
do 14 i=1,k
sumalf=sumalf+(x(i)**alfa)
14 continue
sumalf=sumalf+(n-k)*time**alfa
prod=1.0
do 2 i=1,k
prod=prod*x(i)
2 continue
alikhd=alfa**k*alamb**(k*alfa)*prod**(alfa-1.)
alikhd=alikhd*exp(-alamb**alfa*sumalf)
return
end
subroutine accurt(pp,cc,dd,z,pex)
```

```
c--- *****
c--- the inputs to this subroutine are pp-the proportion of
c--- volume that should be inside this confidence contour,
c--- dd and cc- the limits within which we should do our
c--- binary search. The outputs of this subroutine are z-the
c--- height of the contour closest to our required contour
c--- pex- the exact proportion of volume contained within
c--- that contour.
      clim=cc
      dlim=dd
      n=0
1      n=n+1
      z=(clim+dlim)/2.
      pex=contnt(z)
      if(abs(pex-pp).le..005) go to 2
      if (pex.lt.pp) dlim=z
      if (pex.gt.pp) clim=z
      if (n.lt.20) go to 1
      z=(clim+dlim)/2.
      pex=contnt(z)
2      continue
      return
      end
      subroutine points(z,amaxli,aminli)
c *****
c--- this subroutine gives the maximum and minimum mean life
c--- on all the contour at height z. it first calculates all
c--- the points that lie at a particular height z. then it
c--- calculates the mean life at those points and gets the
c--- minimum (aminli) and the maximum (amaxli) of them.
      common/ /prob(41,41),num,alfa,alamb,almbli,func(41,41),
      *wk(2400),iwk(2000)
      dimension cont(2,200),alfa(41),alamb(41)
      m=0
      amaxli=0.
      aminli=1000000.0
      numl=num-1
      do 7 i=1,numl
        do 7 j=1,numl
          if (prob(i,j).ne.z) go to 4
          cont(1,m)=alfa(i)
          cont(2,m)=alamb(j)
          amlife=(gamma(1.+1./cont(1,m)))/cont(2,m)
          amaxli=amax1(amaxli,amlife)
          aminli=amin1(aminli,amlife)
          m=m+1
4      pr=prob(i,j)
      ps=prob(i+1,j)
      pt=prob(i,j+1)
      af=alfa(i)
      tj=alamb(j)
      afi=alfa(i+1)
      tji=alamb(j+1)
      if((ps.lt.z.or.z.lt.pr).and.(pr.lt.z.or.z.lt.ps)) go to 5
```

```
cont(1,m)=af+(afi-af)/(ps-pr)*(z-pr)
cont(2,m)=alamb(j)
amlife=(gamma(1.+1./cont(1,m)))/cont(2,m)
amaxli=amax1(amaxli,amlife)
aminli=amin1(aminli,amlife)
m=m+1
5  if((pt.lt.z.or.z.lt.pr).and.(pr.lt.z.or.z.lt.pt)) go to 6
    cont(1,m)=af
    cont(2,m)=tj+(tji-tj)/(pt-pr)*(z-pr)
    amlife=(gamma(1.+1./cont(1,m)))/cont(2,m)
    amaxli=amax1(amaxli,amlife)
    aminli=amin1(aminli,amlife)
    m=m+1
6  mcount=m
7  continue
return
end
```

Appendix 2

This is the program to find the maximum likelihood estimator of mean life and standard deviation of life, given the maximum likelihood estimators of the shape and scale parameters.

```
      program mle(input,output)
      read 1,almbda,alpha
1     format(2(f5.3,4x))
2     amean=(gamma(1.+1./alpha))/almbda
      stddev=sqrt(gamma(1.+2./alpha)-(gamma(1.+1./alpha))**2.)
      stddev=stddev/almbda
      print 3,alpha,almbda,amean,stddev
3     format(5x,4(e15.8,5x))
      read 1,almbda,alpha
      if (alpha.ne.0.and.almbda.ne.0) go to 2
      stop
      end
```

The data given are the maximum likelihood estimators of λ and α . The end of data condition is denoted by reading in a value of 0 for both λ and α .

References

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