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LASER PUMPING OF A RELATIVISTIC ION BEAM*

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A TECHNIQUE TO PRODUCE COHERENT X-RAY RADIATION[†]
VIA LASER PUMPING OF A RELATIVISTIC ION BEAM

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ABSTRACT

The level population of a beam of relativistic positive ions with $Z \geq 2$ having a single bound electron may be inverted by the application of a " π pulse" of laser radiation tuned to the Doppler shifted $1s-2p$ transition. When the laser beam and ion beam move in opposite directions the required laser frequency is reduced by a factor 2γ . Subsequently applied short wavelength resonant radiation moving in the same direction as the ion beam (with an inverted population) will be amplified via stimulated emission; the wavelength in the lab frame now being shorter than the original laser wavelength by a factor $(2\gamma)^2$.

In this letter we propose a scheme which could result in the production of coherent electromagnetic radiation in the x-ray region. At present the most popular approach to generating short wavelength radiation involving particle beams (electrons) is the free electron laser (FEL)¹. The efficiency of the FEL is basically governed by the Thomson scattering cross section. Qualitatively, if one were to use a beam of particles having a resonant scattering cross section for some frequency, a greatly enhanced gain might be expected. A natural

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candidate is a hydrogenic positive ion, having $Z \geq 2$, with a single bound electron. The fact that the ion is charged allows the beam to be accelerated to (and stored at) relativistic energies ($\gamma \gg 1$) where one can exploit the properties of relativistic kinematics which dictate that the "back scattered" radiation will have its wavelength shortened by a factor $(2\gamma)^2$.

The idea is as follows. Suppose the ion beam encounters a laser beam traveling in the opposite direction. In the rest frame of the ions the frequency of the light is enhanced by a factor 2γ (extreme relativistic case, $\gamma \gg 1$). Assume that the rest frame frequency matches, say, the $1s-2p$ transition frequency, and that the interaction path length in the lab frame corresponds to a time in the rest frame necessary to invert the level population (a so-called π pulse). If we subsequently apply a different wave, traveling parallel to the ion beam, we may achieve stimulated emission. The wavelength of this signal would be reduced by a factor 2γ from that of the resonant radiation in the rest frame, and by a factor $(2\gamma)^2$ from the original laser wavelength. If the ions were at rest, there is no reduction in the wavelength since the frequencies of the pump and the stimulated emission would be identical; for a relativistic beam with $\gamma=50$ the frequency of the stimulated radiation is higher by a factor 10^4 ! Feasibility, of course, rests on whether the necessary pump power is achievable and whether the gain is sufficient to overcome the losses associated with typical x-ray mirrors; both of these are governed by the spontaneous emission rate. We now proceed with a detailed discussion and evaluation of our model.

We require expressions for the time evolution of the $1s$ and $2p$

level occupations (in the rest frame) as a function of an applied oscillatory electric field of strength E and frequency ω for some initial occupation of the levels. Since we will be concerned with frequencies in the vicinity of the $1s$ - $2p$ transition, we truncate our basis set to these two levels. We introduce a two-by-two density matrix of the form:

$$N = \begin{bmatrix} a_{\uparrow} & a_{\uparrow}^* & a_{\uparrow} & a_{\downarrow}^* \\ a_{\downarrow} & a_{\downarrow}^* & a_{\downarrow} & a_{\uparrow}^* \end{bmatrix},$$

where the symbols $a_{\uparrow}$ and $a_{\downarrow}$ denote the amplitudes of $|2p\rangle$ and $|1s\rangle$ states respectively. The time dependence of N (in the interaction picture) is governed by the following equation:

$$\dot{N} = i\hbar [U, N] - \frac{1}{\tau} (N - N_{eq}), \quad (1)$$

where U denotes the perturbing Hamiltonian given by:

$$U = |U| \begin{bmatrix} 0 & e^{i\epsilon t} \\ e^{-i\epsilon t} & 0 \end{bmatrix}.$$

Here $|U| = \frac{1}{2} eE |\langle 1s | z | 2p \rangle|$, $\langle 1s | z | 2p \rangle = 4\sqrt{2} (2/3)^{5/2} \frac{a_0}{Z}$, $\epsilon = \omega_{1s \rightarrow 2p} - \omega$, and $\hbar \omega_{1s \rightarrow 2p} = E_{2p} - E_{1s} = (3/4) R Z^2$, where R and a_0 are the Rydberg and Bohr radius respectively. The last term in Eq. (1) accounts for spontaneous emission², where τ is the lifetime and $N_{eq} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ is the equilibrium density matrix (at zero temperature). By defining quantities P_0 and $\vec{P}$ through

$$N = P_0 \mathbf{1} + \vec{P} \cdot \vec{\sigma}, \quad (2)$$

we can rewrite Eq. (1) as

$$\dot{P}_0 = 0 \quad (3a)$$

and

$$\dot{\vec{P}} = [\vec{P} \times \vec{\Omega}] - \frac{1}{\tau} (\vec{P} - \vec{P}_{eq}) \quad (3b)$$

where $\vec{\Omega} = \hat{x} \Omega$ and $\Omega = 2|U|/\hbar$. The particular choice of the geometry (the precession axis along the x-axis) fixes the components of $\vec{P}$ and P_0 as follows:

$$\begin{aligned} P_0 &= \frac{1}{2} (N_{\uparrow\uparrow} + N_{\downarrow\downarrow}) , \\ P_1 &= \text{Re}(N_{\uparrow\downarrow} e^{-i\epsilon t}) , \\ P_2 &= \text{Im}(N_{\downarrow\uparrow} e^{-i\epsilon t}) , \end{aligned} \quad (4)$$

and

$$P_3 = \frac{1}{2} (N_{\uparrow\uparrow} - N_{\downarrow\downarrow}) .$$

Eq. (3a) guarantees probability conservation while Eq. (3b) is analogous to the Bloch equation and describes the precession of an electric polarization vector induced by the perturbing electric field.³ Since we wrote the equation of motion for the density matrix in the interaction picture, our Bloch equation, Eqs. (3) and (4), is formulated in the rotating frame. Solving Eq. (3b) with the appropriate initial conditions one can obtain the required time evolution of the excited state population,

$$N_{\uparrow\uparrow}(t) = P_3(t) + 1/2 .$$

In the chosen geometry, Eq. (3b) becomes

$$\begin{cases} \dot{P}_3 = -\Omega P_2 - \frac{1}{\tau} (P_3 + 1/2) \\ \dot{P}_2 = \Omega P_3 - \frac{1}{\tau} P_2 \end{cases} \quad (5a)$$

$$\begin{cases} \dot{P}_2 = \Omega P_3 - \frac{1}{\tau} P_2 \end{cases} \quad (5b)$$

and the general on resonance ($\epsilon=0$) solution can be written as

$$P_3 = -\frac{1}{2} \frac{1}{1+(\Omega\tau)^2} + e^{-t/\tau} (C_1 e^{i\Omega t} + C_2 e^{-i\Omega t}) \quad (6a)$$

and

$$P_2 = -\frac{1}{\Omega} \left[\dot{P}_3 + \frac{1}{\tau} (P_3 + \frac{1}{2}) \right] \quad (6b)$$

To describe the pumping stage of our model we begin with the initial population entirely in the ground state; i.e., at $t = 0$, $P_3 = -1/2$ and $P_2 = 0$. For this initial condition Eq. (6a) leads to:

$$N_{\uparrow\uparrow}(t) = \frac{1}{2} \frac{(\Omega\tau)^2}{1+(\Omega\tau)^2} \left\{ 1 - e^{-t/\tau} \left(\frac{\sin \Omega t}{\Omega\tau} + \cos \Omega t \right) \right\} \quad (7)$$

Therefore population inversion, $N_{\uparrow\uparrow} = 1 - \mathcal{O}(\Omega\tau)^{-1}$, requires $\Omega\tau \gg 1$ (strong field limit) and occurs after a time $T_{\text{rest}}^\pi = \pi/\Omega$; (this corresponds to the π -pulse in NMR). Evaluating τ from the Einstein relation⁴,

$$\tau = \frac{3hc^3}{4e^2 \omega^3} |\langle 1s|z|2p \rangle|^{-2} = 1.59 \times 10^{-9} Z^{-4} \text{ sec}, \quad (8)$$

and using the explicit expression for Ω ,

$$\Omega = 4\sqrt{2} (2/3)^5 \frac{eEa_0}{\hbar Z}, \quad (9)$$

one can rewrite the strong field condition ($\Omega\tau \gg 1$) as an inequality on the required electric field:

$$E_{\text{rest}} \gg 0.349 \cdot Z^5 \text{ st. V cm}^{-1}, \quad (10)$$

In the lab-frame the E-field is reduced by a factor 2γ and the pulse width $T_{\text{rest}}^\pi = \pi/\Omega$ translates to an interaction length

$$L = \gamma c T_{\text{rest}}^\pi = 2.61 \times 10^1 \frac{Z}{E_{\text{lab}}} \text{ cm} \quad (11)$$

and a power flux, $\langle S \rangle = \frac{c}{8\pi} E_{\text{lab}}^2$, of

$$\langle S \rangle \gg 3.64 \frac{Z^{10}}{\gamma^2} \text{ W/cm}^2, \quad (12)$$

which is readily achievable in steady state for low Z .⁵ Finally the Doppler-shifted resonance condition: $2\pi c/\lambda = \omega_{1s \rightarrow 2p}/(2\gamma)$ relates γ and λ as

$$\gamma = 4.12 \times 10^4 Z^2 \lambda. \quad (13)$$

We now examine the second stage of our model, where the inverted population ion beam is subjected to an incoming electromagnetic wave, which is to be amplified by stimulated emission. The field strength will be assumed small, ie. $\Omega' \tau \ll 1$, where we use Ω' for the precession frequency associated with the amplified wave. Our initial condition now assumes complete inversion (achieved by the previously described pumping); at $t=0$ $N_{\uparrow\uparrow} = 1$ or $P_3 = 1/2$. To obtain the gain⁶ we note that the rate of production of coherent photons, $\dot{N}_p$, is governed by the first term on the right of Eq. (5a) (taken with the opposite sign); ie.,

$$\dot{N}_p = \Omega' P_2; \quad (14)$$

(the second term in Eq. (5a) involves spontaneous emission, which is radiated isotropically and incoherently). The required solution, obtained from Eq. (6), with an inverted initial population is

$$P_2(t) = e^{-t/\tau} \sin \Omega' t + \frac{1}{2} \frac{\Omega' \tau}{1 + (\Omega' \tau)^2} \left[e^{-t/\tau} (\cos \Omega' t - \Omega' \tau \sin \Omega' t) - 1 \right]. \quad (15)$$

The total number of coherent photons emitted during some time t (to be discussed shortly) results in an increment to the energy density of the initial wave, $\Delta \mathcal{E}$, given by

$$\Delta \mathcal{E} = \hbar n \omega_{1s \rightarrow 2p} \int_0^t \dot{N}_p(t') dt', \quad (16)$$

where n is the ion density in the rest frame. After carrying out the above integration using Eq. (15) in the weak field limit ($\Omega' \tau \ll 1$) one obtains:

$$\Delta \mathcal{E} = \frac{1}{4} n (\Omega' \tau)^2 A \hbar \omega_{1s \rightarrow 2p} \quad (17)$$

where $A = x \frac{3-2x}{2x+1}$ and $x = t/\tau$.

The relationship between Ω' and the energy density $\mathcal{E}$,

$$(\Omega')^2 = 8\pi e^2 |\langle 1s | z | 2p \rangle|^2 \mathcal{E} / \hbar^2, \quad (18)$$

allows us to rewrite Eq. (17) as

$$\Delta \mathcal{E} = 2\pi A n e^2 \omega_{1s \rightarrow 2p} \tau^2 \mathcal{E} / \hbar. \quad (19)$$

The total gain, G , is defined as

$$G = (\mathcal{E} + \Delta \mathcal{E}) / \mathcal{E}, \quad (20)$$

and lasing requires $GR^2 > 1$; here R is the mirror reflectivity of the second (lasing) cavity. We must optimize the time t so as to produce the largest number of coherent photons; this follows on setting Eq. (15) equal to zero and leads to the criteria $t/\tau = 1.256$. Inserting

this time into Eq. (17) yields the optimized gain as:

$$G = 1 + 2\pi A n e^2 \omega_{ls+2p} \tau^2 / \hbar, \quad (21)$$

with $A = 0.174$.

Assuming mirrors with an intensity reflection efficiency of 50% leads to a required ion density (in the rest frame) of

$$n_{\text{rest}} = 6.84 \times 10^9 \text{ Z}^3 \text{ cm}^{-3}. \quad (22)$$

In the lab frame this requires an ion beam current density of

$$j_{\text{lab}} = (Z-1)e n_{\text{rest}} c \gamma. \quad (23)$$

For the case of Li^{++} ions the threshold current density would be

$$j_{\text{lab}} = 4.31 \times 10^5 \text{ A cm}^{-2}. \quad (24)$$

Note that the above discussion has neglected the inhomogeneous broadening resulting from the fractional momentum spread in the ion beam⁷, which we write as $\Delta\gamma/\gamma$. Thus, we are implicitly assuming $\gamma/\Delta\gamma \gg \omega T_{\text{lab}}^\pi$ and $\gamma/\Delta\gamma \gg \omega\tau$, for the pump and stimulated emission steps respectively. Presently obtainable particle beams are in the opposite limit. For the pump phase we may, in principle, overcome the momentum spread by increasing the pump laser field strength thus shortening the pulse width T_{lab}^π . However in the second or lasing stage, induced radiation would be obtained only for a fraction of order $\frac{\gamma}{\Delta\gamma\omega\tau}$ of the ions, which will severely degrade the gain.

Whether particle beams having the requisite current density, momentum spread, and energy can be achieved should be a challenge to accelerator physicists.

REFERENCES AND FOOTNOTES

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2. R. Karplus and J. Schwinger, Phys. Rev. 73, 1020 (1948).
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5. Were one to operate in the fundamental Gaussian mode of an optimized confocal resonator, the condition on the total power flux, P , can be written as

$$P \gg 10^{-2} \times Z^4 \text{ [W]},$$

and may be enhanced over the drive power by a factor involving the Q of the resonator.

6. One could also proceed by an alternate treatment involving an inhomogenous wave equation where the source term involves the polarization vector,³ the latter being governed by Eq. (5). Such an approach involving the amplitude of the E-field would yield the full dispersive behavior of the process, i.e., the phase velocity shift as well as the gain. The approach given here was chosen for simplicity.
7. Intrabeam collisional relaxation has not been examined.