Abstract

The foil activation method is usually applied to obtain the neutron spectrum at energies above 1 MeV. However, potentially useful information can be gained from detectors whose main responses lie in the low-energy region. Efficient quantitative methods are developed to extract this information using "window function" and linear programming techniques.

This method is restricted essentially to energies below 1 keV. In the 1 keV to 1 MeV regions, which is important for damage analysis, only a combination of foil detector methods and neutron transport calculations can provide satisfactory results.

Introduction

The purpose of this paper is to investigate the application of multiple foil dosimetry to the measurement of neutron fluences in the low-energy region. Foil dosimetry can determine fairly accurately the spectrum in the 1 to 18 MeV range, primarily with the use of threshold foils. Similar measurements in the low-energy region with (n, α), (n, γ), and fission detectors have been less satisfactory. Results of unfolding procedures when applied to low energies seem to depend to a large extent on the input spectrum and on the particular code. In many applications...
quotients between activity measurements are used to characterize the degree of "hardness" of the spectrum, but it is not quite clear how these quotients relate to other relevant spectral parameters.

In order to approach this problem in a precise, quantitative manner, the interpretation of spectral measurements through "window functions" will be used. This approach, combined with a linear programming technique, allows bounds to be put on selected integral quantities of the spectrum. Following this method, effective procedures for the use of foil detectors in low-energy dosimetry will be developed.

**Theory**

Only integral quantities will be considered as targets for spectral measurements. Let $\phi(E)$ be the energy-dependent neutron flux spectrum, and let $R(E)$ be any selected response function. We define the integral response, $I_R$, as

$$ I_R = \int_0^\infty \phi(E) R(E) \, dE. \quad (1) $$

It is usually not possible to measure $I_R$ directly for an arbitrary response function. This quantity can be determined, however, if $R(E)$ is a linear combination of cross sections in the detector set. Such linear combinations will be called "window functions," $W(E)$, where

$$ W(E) = \sum_{i=1}^n c_i \sigma_i(E), \quad (2) $$

for any arbitrary set of coefficients $c_i$. Obviously by replacing $R(E)$ with $W(E)$ in Eq. (1) we have

$$ I_W = \sum_{i=1}^n c_i \alpha_i. \quad (3) $$
where $a_i$ is the measured activity of the i-th detector:

$$a_i = \int_0^\infty \phi(E) \sigma_i(E) \, dE. \quad (4)$$

The term window function has been chosen to indicate that measuring the activities of a given detector set is tantamount to looking at the unknown spectrum through a set of windows that allows only certain parts of the information about the spectrum to pass.

Viewed in this context the problem of spectrum unfolding can be considered as the problem of approximating a given response function $R(E)$ by a realizable window function $W(E)$. If a window function that is consistently larger or consistently smaller than the response function is chosen, upper and lower bounds for $I_R$ can be computed. However, the primary advantage of this approach is not the ability to obtain numbers; the ability to inspect the window functions directly and to gain in this way a better insight into the inner workings of multiple foil dosimetry, regardless of whether or not a particular measurement succeeds, is much more valuable.

To keep the computations manageable and to modify unnecessarily rigid restrictions, the continuous functions are replaced by group averages. Let $F(E)$ be any energy dependent function, and let $\tilde{\phi}(E)$ be some spectrum that can pass for a reasonable estimate of the actual spectrum $\phi(E)$. Let the total energy range be subdivided into a finite number, $n$, of subintervals $(E_{i-1}^i, E_i)$, where $i = 1, 2, \ldots n$. We define

$$F_i = \int_{E_{i-1}}^{E_i} \frac{F(E) \, \phi(E) \, dE}{\int_{E_{i-1}}^{E_i} \phi(E) \, dE}, \quad \text{as the spectrum averaged group representation of } F(E). \quad (5)$$

This process is applied to the functions $R(E)$, $W(E)$, and $\sigma_i(E)$, using about 20 to 40 energy groups. This is in keeping with standard procedures in neutron transport calculations and is sufficiently accurate for our purposes.
only source of error in this procedure is the replacement of the actual unknown spectrum $\phi(E)$ with the estimate $\hat{\phi}(E)$, which usually can be disregarded. The window function $W(E)$ still can be computed in continuous form, if desired, to guard against gross misinterpretation.

All detectors with sizable low-energy response satisfy a basic $v^{-1}$ law. Thus, the window functions will be of the form

$$ W(E) = C/\sqrt{E}, $$

(6)

with some arbitrary constant $C$. Major deviations from this law appear in the form of resonances, the most prominent being that of gold, which peaks at 4.9 eV. Another resonance peak that could be used for dosimetry is that of iron and manganese (about 250 eV). Other deviations are less pronounced and are more gradual, especially for fission foils. These can be used to "truncature" the response function in order to restrict its sensitivity to energies below a given cutoff energy, $E_0$, somewhere in the range of 0.1 eV to 0.1 MeV. Response functions of the truncated $1/\sqrt{E}$ type and of the resonance type have been tried successfully. Results will be summarized in a later part of this paper.

The use of the response functions mentioned so far are restricted to energies below 1 keV. It is doubtful that with existing foils useful information can be extracted from the 1-keV to 1-MeV range. This range is, however, of the greatest interest in damage analysis because no displacement of atoms can be expected from neutron energies lower than a few keV. For this part of the spectrum, the dosimetry must rely heavily on neutron transport calculations. However, the foil detectors are not entirely useless. Much better bounds for integral responses in this region can be computed by combining activity measurements with calculated group fluxes than by using either method separately.

Frequently, quotients of detector activities are computed to determine the ratio between the high- and the low-energy content of a spectrum. A more precise interpretation of these quotients can be obtained by using
window functions. Consider two detectors with cross sections $\sigma_1(E)$ and $\sigma_2(E)$ and with corresponding activity measurements $\alpha_1$ and $\alpha_2$. Let

$$Q = \frac{\alpha_1}{\alpha_2}.$$  \hspace{1cm} (7)

If we define a window function

$$W_Q(E) = \sigma_1(E) - Q \sigma_2(E),$$  \hspace{1cm} (8)

we have

$$\int_0^\infty \phi(E) W_Q(E) \, dE = 0.$$  \hspace{1cm} (9)

The function $W_Q(E)$ separates the range of energy into two regions — a positive region and a negative region — or alternatively where

$$\frac{\sigma_1(E)}{\sigma_2(E)} > Q \text{ or } \frac{\sigma_1(E)}{\sigma_2(E)} < Q \text{ respectively.}$$

The corresponding integral response from these two regions must compensate each other so that Eq. (9) vanishes. In this way we obtain an indication of the relative magnitude of the integral flux in the two regions.

**Results**

The theoretical considerations that have been discussed in the previous section have been tested extensively using simulated activity values for a large number of foil combinations and neutron spectra. Only a few representative examples will be presented here. In the first group of experiments, the truncated $1/\sqrt{E}$ response function is investigated. The second group deals with the use of gold and iron responses.
Truncated $1/\sqrt{E}$ response

The response functions are defined as follows:

$$R(E) = \frac{1}{\sqrt{E}} , \text{ for } E \leq E_0 ,$$

$$R(E) = 0 , \text{ for } E > E_0 ,$$

with "cutoff energies" $E_0$ ranging from 0.01 eV to 18 MeV. The following 15 detectors were used: $^{115}\text{In}(n, \gamma)^{116}\text{In}, ^{197}\text{Au}(n, \gamma)^{198}\text{Au}, ^{59}\text{Co}(n, \gamma)^{60}\text{Co}, ^{55}\text{Mn}(n, \gamma)^{56}\text{Mn}, ^{23}\text{Na}(n, \gamma)^{24}\text{Na}, ^{45}\text{Sc}(n, \gamma)^{46}\text{Sc}, ^{58}\text{Fe}(n, \gamma)^{59}\text{Fe}, ^{63}\text{Cu}(n, \gamma)^{64}\text{Cu}, ^{10}\text{B}(n, \alpha)^{7}\text{Li}, ^{6}\text{Li}(n, \alpha)^{3}\text{T}, ^{235}\text{U}(n, f), ^{238}\text{U}(n, f), ^{239}\text{Pu}(n, f), ^{237}\text{Np}(n, f), ^{232}\text{Th}(n, f)$. The cross sections of these reactions were taken from the ENDF/B-IV file and were folded with several spectra from the SAND-II reference spectrum library to obtain simulated activity measurements. These values were applied to a linear programming procedure to obtain upper and lower bounds for the integral response $I_R$. These bounds, together with the correct value of $I_R$, are listed in Table 1 for different cutoff energies and for two different spectra. The first spectrum, No. 16 of the reference library, contains a high fraction of thermal neutrons, as might be found in a light water reactor a sufficient distance from the core. The second spectrum, No. 22, has its energy concentrated around 50 eV. This spectrum has been chosen to demonstrate the feasibility of the method for detecting high fluxes in the low intermediate energy region of the spectrum. Graphs of the spectra are given in Figs. 1 and 2. Graphs of the window functions related to upper and lower bounds for different cutoff energies are given in Figs. 3 and 4. These examples show that the upper window function coincides essentially with the untruncated $1/\sqrt{E}$ response function and is barely affected by the cutoff point. However, the lower window is forced to be negative or zero at energies above cutoff. It follows that the upper and lower bound for any truncated $1/\sqrt{E}$ response can be close together and close to the true value of the integral response only if this value in turn is near the value for the untruncated $1/\sqrt{E}$ integral response. This is true for values above 0.1 eV in spectrum No. 16 and for
Table 1. Integral responses and bounds for the truncated $1/\sqrt{E}$ response function

<table>
<thead>
<tr>
<th>Cutoff energy</th>
<th>Spectrum No. 16</th>
<th></th>
<th></th>
<th>Spectrum No. 22</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Upper bound</td>
<td>Lower bound</td>
<td>Correct value</td>
<td>Upper bound</td>
<td>Lower bound</td>
<td>Correct value</td>
</tr>
<tr>
<td>18 MeV</td>
<td>$6.18 \times 10^3$</td>
<td>$5.07 \times 10^3$</td>
<td>$5.60 \times 10^3$</td>
<td>$1.17 \times 10^2$</td>
<td>$9.53 \times 10^1$</td>
<td>$1.11 \times 10^2$</td>
</tr>
<tr>
<td>1 MeV</td>
<td>$6.19 \times 10^3$</td>
<td>$5.07 \times 10^3$</td>
<td>$5.60 \times 10^3$</td>
<td>$1.17 \times 10^2$</td>
<td>$9.52 \times 10^1$</td>
<td>$1.06 \times 10^2$</td>
</tr>
<tr>
<td>100 keV</td>
<td>$6.19 \times 10^3$</td>
<td>$5.07 \times 10^3$</td>
<td>$5.60 \times 10^3$</td>
<td>$1.17 \times 10^2$</td>
<td>$8.92 \times 10^1$</td>
<td>$1.06 \times 10^2$</td>
</tr>
<tr>
<td>10 keV</td>
<td>$6.19 \times 10^3$</td>
<td>$5.07 \times 10^3$</td>
<td>$5.60 \times 10^3$</td>
<td>$1.17 \times 10^2$</td>
<td>$8.21 \times 10^1$</td>
<td>$1.05 \times 10^2$</td>
</tr>
<tr>
<td>1 keV</td>
<td>$6.19 \times 10^3$</td>
<td>$5.06 \times 10^3$</td>
<td>$5.60 \times 10^3$</td>
<td>$1.17 \times 10^2$</td>
<td>$8.89 \times 10^1$</td>
<td>$1.04 \times 10^2$</td>
</tr>
<tr>
<td>100 eV</td>
<td>$6.19 \times 10^3$</td>
<td>$5.06 \times 10^3$</td>
<td>$5.60 \times 10^3$</td>
<td>$1.10 \times 10^2$</td>
<td>$7.54 \times 10^1$</td>
<td>$9.59 \times 10^1$</td>
</tr>
<tr>
<td>10 eV</td>
<td>$6.19 \times 10^3$</td>
<td>$4.99 \times 10^3$</td>
<td>$5.60 \times 10^3$</td>
<td>$3.06 \times 10^1$</td>
<td>$0.0$</td>
<td>$2.23 \times 10^{-1}$</td>
</tr>
<tr>
<td>1 eV</td>
<td>$6.19 \times 10^3$</td>
<td>$4.83 \times 10^3$</td>
<td>$5.60 \times 10^3$</td>
<td>$2.25 \times 10^1$</td>
<td>$0.0$</td>
<td>$3.37 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.1 eV</td>
<td>$6.19 \times 10^3$</td>
<td>$2.79 \times 10^2$</td>
<td>$5.33 \times 10^3$</td>
<td>$2.65 \times 10^1$</td>
<td>$0.0$</td>
<td>$2.10 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.01 eV</td>
<td>$5.42 \times 10^3$</td>
<td>$0.0$</td>
<td>$8.39 \times 10^2$</td>
<td>$2.68 \times 10^1$</td>
<td>$0.0$</td>
<td>$2.13 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

values above 100 eV for spectrum No. 22. Thus, by computing upper and lower bounds for the truncated $1/\sqrt{E}$ response for different cutoff points, the energy can be determined at which truncated and untruncated $1/\sqrt{E}$ integral responses for a given spectrum coincide for the first time. The values in Table 1 confirm this prediction. The method also has been tested for other spectra, including the pure Watt fission spectrum (No. 2) whose flux is concentrated at energies above 1 MeV.

**Resonances**

The response function is defined as

$$R(E) = 1, \text{ for } E_1 \leq E \leq E_2,$$

$$R(E) = 0, \text{ otherwise.}$$
For the resonance of gold, \( E_1 \) equals 2.5 eV, and \( E_2 \) equals 8.0 eV; for the resonance of iron, \( E_1 \) equals 150 eV, and \( E_2 \) equals 600 eV. (The iron resonance stands here for the resonances of several detectors in this region, such as manganese, whose combined contributions make up the window function.) The foil set and procedures are the same as in the other experiment. Upper and lower bounds are listed together with the correct values in Table 2 for a variety of spectra from the reference library. Spectra Nos. 11, 13, 14, and 16 differ only by their fraction of thermal neutrons, which increases with increasing number. Spectrum No. 21 is similar to No. 22, but its peak is around 5 eV instead of 50 eV. Spectrum No. 6 is a \( 1/E \) spectrum; No. 7 is a combination of No. 6 with Watt's fission spectrum.

The values in Table 2 show that reasonable bounds can be obtained only if a sizable fraction of the activity is caused by neutron fluxes at the resonance peak of the respective foil. Contributions from thermal neutrons can be suppressed to some extent through cadmium cladding. Such cladding is assumed in this experiment for the computation of the simulated activities but is not assumed for the other experiment. Even this does not

<table>
<thead>
<tr>
<th>Spectrum No.</th>
<th>Resonance peak of gold</th>
<th>Resonance peak of iron</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Upper bound</td>
<td>Lower bound</td>
</tr>
<tr>
<td>6</td>
<td>( 8.66 \times 10^{-2} )</td>
<td>( 6.80 \times 10^{-2} )</td>
</tr>
<tr>
<td>7</td>
<td>( 7.79 \times 10^{-2} )</td>
<td>( 6.12 \times 10^{-2} )</td>
</tr>
<tr>
<td>11</td>
<td>( 8.29 \times 10^{-2} )</td>
<td>( 6.25 \times 10^{-2} )</td>
</tr>
<tr>
<td>13</td>
<td>( 2.18 \times 10^{-2} )</td>
<td>( 1.72 \times 10^{-2} )</td>
</tr>
<tr>
<td>14</td>
<td>( 1.18 \times 10^{-3} )</td>
<td>( 9.28 \times 10^{-4} )</td>
</tr>
<tr>
<td>16</td>
<td>( 1.35 \times 10^{-7} )</td>
<td>0.0</td>
</tr>
<tr>
<td>21</td>
<td>( 6.21 \times 10^{-1} )</td>
<td>( 5.07 \times 10^{-1} )</td>
</tr>
<tr>
<td>22</td>
<td>( 5.83 \times 10^{-3} )</td>
<td>0.0</td>
</tr>
</tbody>
</table>
prevent the failure in spectrum No. 16. Whenever the method works, bounds within 20 to 30% of the correct values can be obtained. The graph of the upper and lower window function is given in Figs. 5 and 6.
Fig. 3.

Fig. 4.
Fig. 5.

Fig. 6.
References


