Distribution Categories:
Particle Accelerators and
High-Voltage Machines (UC-28)
Mathematics and Computers (UC-32)

ANL-80-27

ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60439

SLOW NEUTRON LEAKAGE SPECTRA
FROM SPALLATION NEUTRON SOURCES

by

S. G. Das, J. M. Carpenter,
and R. E. Prael

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

February 1980

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

TABLE OF CONTENTS

	Page
I. Introduction	2
II. Calculational Details	5
A. Radial Distribution	6
B. Axial Distribution	10
C. Angular Distribution	10
D. Energy Distribution	15
E. Neutron Histories and Scoring	17
F. Statistics	21
G. Normalization of Results	22
H. Relationship of Results to Neutron Spectra Below E_{min}	24
III. Results	25
A. Calculations on the ZING Mockup	25
B. Studies of the Effect of Source Size (Proton Energy) in One Moderator Geometry	31
C. Calculations for a Vertical Beam Three-Moderator Geometry	37
D. Calculations of the ZING Prototype, Two Moderator Geometry	42
E. The Calculations of the IPNS Three Moderator Geometry	42
F. Calculation of an Electron-Driven U^{235} Fission Multiplier	42
G. Time-Dependent Neutron Beam Current Calculations	59
IV. Conclusion	63
References	64

LIST OF FIGURES

	Page
1. Simple geometry for studying the slow neutron leakage spectrum	3
2. Radial and axial distribution of the production of evaporation neutrons in a 7.5-cm-radius, 30-cm-long ^{238}U target with 800 MeV protons incident on axis (Ref. 3)	7
3. The assumed radial distribution of neutron production density in the target cylinder of radius R. The proton beam has a finite radius $D/\sqrt{2}$. The density of neutrons produced within a distance b from a narrow proton ray is about ten times that produced outside this region	8
4. The axial distribution of the evaporation neutrons in ^{238}U cylinder of height Z, produced by a proton beam of energy 800 MeV. The histogram is from Ref. 2	11
5. The rejection function $R(\mu)$ used for forward biasing of the source neutrons as a function of μ for values of $\rho = 3$ and 10	14
6. The assumed energy distribution of the neutrons in a source containing 80% ^{238}U and 20% sodium, as they are first born (curve A) and when they first emerge from the source cylinder (curve B)	16
7. The method of studying the leakage spectrum of slow neutrons. The probability for a neutron to emerge in the direction Ω_0 through the face A of the moderator is calculated at energy collision points inside the hatched region and summed to give the leakage spectrum	18
8. Mockup of the ZING source, moderator and reflector with a beam port. The assembly was surrounded by 6" Benelex . . .	26
9. The neutron beam current $EI(E)$ as a function of energy E for the ZING Mockup geometry (Ref. 3). The source is a ^{238}U fission spectrum and the size of the moderator is 4 x 4 x 2 in ³ polyethylene	27
10. The neutron beam current $EI(E)$ for the ZING mockup geometry with no beryllium reflector	29
11. The geometrical arrangement (simple wing geometry) used in studying the variation of emerging neutron beam current as a function of proton energy, the moderator size and distance of moderator from source	32

LIST OF FIGURES (cont.)

	Page
12. The neutron beam current $EI(E)$ as a function of energy E for the simple wing geometric arrangement of Fig. 11. The energy of incident protons is 500 MeV and the size of the polyethylene moderator is $10 \times 10 \times 5 \text{ cm}^3$, placed at a distance of 6 cm away from the source	34
13. The neutron beam current per source neutron $EI(E, \Omega_0)$, ev and per incident proton $EI_p(E, \Omega_0)$, ev , as a function of the energy of the incident protons, plotted vs. the square of the corresponding height H_s of the source cylinder. Curve A represents results for moderator of size $10 \times 20 \times 5 \text{ cm}^3$ placed at a distance of 6 cm from the moderator, curve B corresponds to moderator size $10 \times 10 \times 5 \text{ cm}^3$ at a distance of 6 cm and curve C corresponds to moderator size $10 \times 10 \times 5 \text{ cm}^3$ at a distance of 8 cm from the source	38
14. The source, reflector and radial-beam moderator arrangement for neutron yield measurements	39
15. The neutron beam current $EI(E)$ for incident proton energy of 650 MeV in the radial-beam arrangement of Fig. 14	40
16. The vertical proton beam arrangement with three 5 cm thick polyethylene moderators used for the ZING neutron beam current calculations	41
17. The neutron beam currents $EI(E)$ for incident proton energy of 800 MeV and for the vertical proton beam moderator arrangement considered in the ZING studies	43
18. The ZING-Prototype arrangement (horizontal section)	45
19. The ZING-Prototype arrangement (vertical section)	46
20. The neutron beam currents $EI(E)$ for the two polyethylene moderators of the ZING-Prototype arrangement with protons of energy 200 MeV on a lead target	47
21. The IPNS neutron scattering source, moderator, reflector arrangement	48
22. The neutron beam currents $EI(E)$ for the three polyethylene moderators of the IPNS arrangement with an incident proton energy of 800 MeV	49
23. The arrangement of primary electron driven photo-neutron source (cross hatched), U^{235} fission booster surrounded by boron carbide (shaded) water moderator and the defined source area (shaded)	52
24. Multiplication constant k_{eff} vs. geometric buckling B_g^2 for the	54

LIST OF FIGURES (cont.)

	Page
25. The neutron beams current $EI(E)$ for the fission booster arrangement	58
26. Beam intensity of 1 eV as a function of time (expressed as a function of vt) for the ZING Mockup. The symbol 'o' corresponds to setup with no boron liner, 'x' to one with boron liner, whereas solid line is for an infinite medium of polyethylene	61

LIST OF TABLES

	Page
1. Slow Neutron Leakage Currents for Several Moderator Materials in the ZING-Mockup Geometry	30
2. The extrapolated value of $EI(E, \Omega_0)$ at $E = 1$ eV, neutrons/ steradian-source neutron leaking from $H_M \times 10 \times 5.1 \text{ cm}^3 \text{ CH}_2$ moderators normal to the surface as a function of proton energy and source-moderator geometry	35
3. The extrapolated value of $EI_p(E, \Omega_0)$ at $E = 1$ eV, neutrons/ ster-proton for $H_M \times 10 \times 5.1 \text{ cm}^3 \text{ CH}_2$ moderators as a function of proton energy and source moderator geometry, and the neutron yield as a function of proton energy	36
4. Slow Neutron Beam Currents	44
5. Slow Neutron Beam Currents IPNS Geometry	50
6. Pulse Shape Parameters for ZING Mockup Calculations	62

SLOW NEUTRON LEAKAGE SPECTRA FROM SPALLATION NEUTRON SOURCES

Shashikala G. Das, J. M. Carpenter and R. E. Prael

ABSTRACT

We describe an efficient technique for Monte Carlo simulation of neutron beam spectra from target-moderator-reflector assemblies typical of pulsed spallation neutron sources. The technique involves the "scoring" of the transport-theoretical probability that a neutron emerge from the moderator surface in the direction of interest, at each collision. We have also introduced an angle-biasing probability which further enhances efficiency in simple problems.

We have introduced these modifications into the VIM low energy neutron transport code, representing the spatial and energy distributions of the source neutrons approximately as those of evaporation neutrons generated through the spallation process by protons of various energies. We have studied the intensity of slow neutrons leaking from various reflected moderators for various neutron source arrangements. These include computations relating to early measurements on a mockup-assembly, a brief survey of moderator materials and sizes, and a survey of the effects of varying source and moderator configurations with a practical, liquid metal cooled Uranium source.

We compare "wing" and "slab" i.e., "tangential" and "radial" moderator arrangements, and Be vs. CH_2 reflectors. We also present results for several complicated geometries which more closely represent realistic arrangements for a practical source, and for a subcritical fission multiplier such as might be driven by an electron linac.

We have developed an adaptation of the code to enable time dependent calculations, and investigated the effects of the reflector, decoupling and void liner materials on the pulse shape.

I. INTRODUCTION

The importance of the spallation process as an intense pulsed neutron source for obtaining both fast and slow neutrons is well known. For the production of slow neutron pulses, the fast source neutrons must be slowed down in a hydrogenous moderator placed close to the source. To estimate the intensity of slow neutron beams from such moderators, we have computed the energy distribution of the neutron beam intensity leaking from a moderator per source neutron $I(E, \underline{\Omega})$, which is defined as

$$I(E, \underline{\Omega}) = \int_A v n(E, \underline{\Omega}, r) da, \quad (1)$$

$v n(E, \underline{\Omega}, r)$ being the neutron angular current density distribution per source neutron. $I(E, \underline{\Omega})$ is the total number of neutrons emitted from the moderator surface, per unit solid angle, per unit energy, per source neutron. This quantity is referred to as the "beam current", though this designation does not emphasize its full meaning; it is a density with respect to solid angle and energy, relative to the number of source neutrons. When other related quantities are introduced, the required modifiers will be added. Fig. 1 illustrates the quantity computed, and, in simplified form, the geometry considered. A is the area of the surface of the moderator from which the neutrons emerge. The neutron beam current $I(E, \underline{\Omega})$ depends strongly on the dimensions of the target and the source (and thus on the energy of the protons), the moderator size, temperature and material (that is, the proton density), and on reflector and voids (that is, the surroundings of the target and moderator). The energy dependence of the beam intensity is smooth and monotonic for low energies, but it is necessary to have information about the effect of these various factors on $I(E, \underline{\Omega})$ to design and understand efficient moderators for an intense pulsed

Fig. 1. Simple geometry for studying the slow neutron leakage spectrum. ANL Neg. No. 125-6870.

neutron source. The beam current is the fundamental quantity needed for computing data rates in experiments which use neutron beams. We consider in particular various cylindrical neutron sources, about which are placed polyethylene moderators. These are surrounded by reflector material of beryllium and viewed by one or more beam holes.

We have used a Monte Carlo method of simulating the source-moderator arrangement using an efficient computer code for obtaining $I(E, \underline{\Omega})$. Our code is based on the continuous-energy Monte Carlo code VIM¹, with some additional techniques in scoring which have made our program considerably faster than programs based on "direct" or analog simulation. The modifications of the VIM program involve the following two changes:

(1) In conventional analog Monte Carlo calculations, one follows the path of a neutron through the medium, calculating its energy and angle of scattering at every collision point until the neutron is absorbed or leaks from the system, or perhaps emerges from the surface of the moderator in the direction of interest. It is found that the success rate for such a process is very small, less than only about one neutron per 1000 histories. In this "direct" method of simulation, the calculations of $I(E, \underline{\Omega})$ become quite expensive. To overcome this problem, we determine at each collision point in the path traversed by a neutron, the probability that the neutron will leak through the surface of the moderator in the direction of interest, $\underline{\Omega}_0$. In order to find this probability one has to find: (a) whether or not the ray from the point of collision along the direction $\underline{\Omega}_0$ passes through the surface of the moderator, (b) the optical path traversed by the neutron from the collision point to the surface of the moderator, (c) the energy of the neutron if it had been scattered along the direction of interest, and (d) the probability

of scattering into this direction and energy. This requires the determination of the isotope from which the neutron is scattered and the process of scattering (isotropic elastic, anisotropic elastic, n-2n scattering, discrete level excitation, excitation into the continuum, etc.). Although these steps involve some extra calculations at each collision point, the efficiency of the program is increased by a factor of ≈ 10 , compared to a "direct" simulation.

(2) Instead of considering an isotropic angular distribution of source neutrons, we have introduced for some problems a fictitious angular probability distribution function $P(\underline{\Omega})$ such that the probability of neutrons starting in a favorable initial direction $\underline{\Omega}^*$, say, in the direction toward the moderator, is larger (≈ 2 times) than in the opposite direction. Then, in order to be consistent with the real situation, the initial weight $W(\underline{\Omega})$ of the neutron is adjusted to compensate. This biasing process has increased the efficiency of the program by about a factor of 2 for the simple geometries in which it is appropriate.

The basic computer code, VIM,¹ is provided with a special package called the "combinatorial geometry package" which allows one to express a body of complicated shape as a combination of simple standard geometrical objects like a cylinder, rectangular parallelepiped, cone, etc., and thus without changing the computer program one can handle relatively trivially even quite complicated geometrical setups.

II. CALCULATIONAL DETAILS

The source of neutrons was considered to be cylindrically shaped, containing 80% by volume U^{238} , and 20% sodium representing a coolant, cladding, and other structure. The spatial distribution of the neutrons was chosen to represent approximately that of neutrons generated through the spallation process by a wide beam of protons of $D/\sqrt{2}$ radius (D being the radius of the source cylinder) derived as follows.

A. Radial Distribution

According to Fullwood et al.,² the distribution of the evaporation neutrons produced by a narrow ray of protons of energy 800 MeV in a 7.5 cm radius and 30 cm long uranium target is of the form shown in Fig. 2. Thus the density of neutrons produced within a distance of radius $b \approx 0.75$ cm from the proton ray is about 10 times larger than that produced outside this region. We have used this information to calculate the radial distribution of source neutrons appropriate for our case as follows. If $f(r)$ represents the radial density distribution function for a narrow proton ray and $f_s(r)$ that for the wide proton beam of radius $D/\sqrt{2}$, then we can write

$$f_s(r) = \frac{\int_0^D S(\underline{r}') f(|\underline{r}-\underline{r}'|) d^2 \underline{r}'}{\int_0^D S(\underline{r}') d^2 \underline{r}'} \quad , \quad (2)$$

where

$$\begin{aligned} S(\underline{r}) &= 1 \text{ for } |\underline{r}| < D/\sqrt{2} \\ &= 0 \text{ for } |\underline{r}| > D/\sqrt{2} \\ f(r) &= a \text{ for } r < b \\ &= a/10 \text{ for } r > b \end{aligned}$$

The function $f_s(r)$ can be calculated for various values of r as follows.

If $r > D/\sqrt{2} + b$, as at point P in Fig. 3, then a small circle of radius b with the point P as center does not intersect the circle of radius $D/\sqrt{2}$, bounding the region where $S(\underline{r}') = 1$. Thus where $S(\underline{r}') = 1$, $f(|\underline{r}-\underline{r}'|) = a/10$. Where $f(|\underline{r}-\underline{r}'|) = a$, $S(\underline{r}') = 0$. Hence for $r > D/\sqrt{2} + b$

Fig. 2. Radial and axial distribution of the production of evaporation neutrons in a 7.5-cm-radius, 30-cm-long ^{238}U target with 800 MeV protons incident on axis (Ref. 3)

Fig. 3. The assumed radial distribution of neutron production density in the target cylinder of radius R . The proton beam has a finite radius $D/\sqrt{2}$. The density of neutrons produced within a distance b from a narrow proton ray is about ten times that produced outside this region. ANL Neg. No. 125-6873.

$$f_s(r) = \frac{a}{10} \frac{\int_0^{D/\sqrt{2}} S(\underline{r}') d^2 \underline{r}'}{\int_0^{D/\sqrt{2}} S(\underline{r}') d^2 \underline{r}'} = \frac{a}{10} \quad (3)$$

If on the other hand $r < D/\sqrt{2} - b$, as at point P' in Fig. 3, the small circle of radius b where $f(|\underline{r}-\underline{r}'|) = a$ is in a region where $S(\underline{r}') = 1$. Then for $r < D/\sqrt{2} - b$

$$\int_0^D S(\underline{r}') f(|\underline{r}-\underline{r}'|) d^2 \underline{r}' = \pi b^2 + \frac{a}{10} (\pi \frac{D^2}{2} - \pi b^2) \quad (4)$$

Hence,

$$f_s(r) = \frac{a}{10} + \frac{18}{10} a \frac{b^2}{R^2} \quad \text{for } r < \frac{D}{\sqrt{2}} - b \quad (5)$$

In our calculations of the beam current we have considered the radius of the source cylinder to be $D = 5$ cm, and thus with $b = 0.75$ cm, we have

$$\frac{f_s(r)|_{r < D/\sqrt{2} - b}}{f_s(r)|_{r > D/\sqrt{2} + b}} = \frac{\frac{a}{10} + \frac{18}{10} a (\frac{0.75}{5})^2}{\frac{a}{10}} = 1.405 \quad (6)$$

In the region $(D/\sqrt{2} - b) < r < (D/\sqrt{2} + b)$ the distribution function drops smoothly from 0.14 a to 0.1 a . Thus for a wide proton beam the source neutrons are sufficiently nearly uniformly distributed that we have assumed a uniform distribution.

B. Axial Distribution

The distribution of the source neutrons along the axis of the source cylinder was taken to be of the form

$$f(z) = A \sinh \frac{z_0 - z}{L}, \quad (7)$$

where z_0 is the range in cm of protons in target material at an energy E , empirically calculated by

$$z_0 = \frac{640 \times E^{1.41}}{\rho_{\text{source}}}, \quad .5 < E < 1. \text{ GeV} \quad (8)$$

where E is the energy of the incident protons in GeV, ρ_{source} is the average density of target material in gm/cm^3 , and z is the distance in cm measured along the axis from the surface of the cylindrical source on which the protons impinge. The constant L in cm is taken to be $L = 12.3 \rho_{\text{U238}}/\rho_{\text{source}}$ based on the results of Fullwood et al.,² and A is a normalization constant. L is approximately equal to the mean free path of high energy nucleons in target material. The function $f(z)$ is plotted in Fig. 4 for proton energy $E_0 = 800$ MeV, the corresponding value of z_0 being 29.38 cm. The histograms represent the distribution obtained by Fullwood et al. which can be approximated satisfactorily at this energy by the function $f(z)$, with only a small discrepancy near $z = 0$ associated with a buildup factor.

C. Angular Distribution

The evaporation neutrons from a spallation target are nearly isotropically distributed in the laboratory system, and are treated as isotropic in these computations. In our calculations for the simple geometries, however, we have made the following modifications to bias the angular distribution of the source neutrons for the purpose of improving the statistical accuracy of the results.

Fig. 4. The axial distribution of the evaporation neutrons in U^{238} cylinder of height Z , produced by a proton beam of energy 800 MeV. The histogram is from Ref. 2. ANL Neg. No. 125-6874 Rev. 1.

Let $\underline{\Omega}$ be the direction of emission of a source neutron, and $\underline{\Omega}^*$ be the direction in which the distribution is to be biased, with

$$\mu \equiv \underline{\Omega} \cdot \underline{\Omega}^* \quad . \quad (9)$$

In simple geometries with only one moderator, $\underline{\Omega}^*$ has been chosen as the direction from the center of the source to the center of the moderator.

For an isotropic source distribution

$$P(\underline{\Omega}) W(\underline{\Omega}) = \frac{1}{4\pi} \quad , \quad (10)$$

where $W(\underline{\Omega})$ is the initial weight given to the neutron, and $P(\underline{\Omega})$ the probability of emission per unit solid angle around direction $\underline{\Omega}$, which is

$$P(\underline{\Omega}) = \frac{1}{2\pi} p(\mu) \quad (11)$$

for the isotropic case. To accomplish the desired biasing, we have introduced an angular biasing distribution function

$$p(\mu) = \frac{\rho}{\rho + 1} R(\mu) \quad , \quad (12)$$

where

$$R(\mu) = \frac{4 + (\rho - 1)(\mu + 1)^2(2 - \mu)}{4\rho} \quad (13)$$

is called the "rejection function." In this distribution, the parameter ρ ($\rho \geq 1$) determines the extent of biasing; $\rho = 1$ corresponds to no biasing, $\rho \rightarrow \infty$ corresponds to complete forward biasing. As the maximum value of $R(\mu) = 1$, this can be used to select random numbers to give the desired, biased, angular distribution.

The function $R(\mu)$ is plotted in Fig. 5 for values of $\rho = 3$ and 10.

The extent of biasing is shown by considering the ratio of forward to backward emission

$$\begin{aligned} p_{>} &= \int_0^1 p(\mu) d\mu = \frac{13\rho + 3}{16(\rho + 1)} \\ p_{<} &= \int_{-1}^0 p(\mu) d\mu = \frac{3\rho + 13}{16(\rho + 1)} \\ \frac{p_{>}}{p_{<}} &= \frac{13\rho + 3}{3\rho + 13} \end{aligned} \quad (14)$$

For $\rho = 3$, $p_{>}/p_{<} = 1.91$; about twice as many neutrons emerge in the forward direction as in the backward direction. For $\rho = 10$, $p_{>}/p_{<} = 3.09$.

The corresponding weights of the neutrons emitted in the direction $\underline{\Omega}$ are adjusted so that $W(\underline{\Omega})P(\underline{\Omega}) = 1/4\pi$;

$$W(\underline{\Omega}) = \left(\frac{1}{4\pi}\right)/P(\underline{\Omega}) = \frac{1}{2p(\mu)} = \frac{(\rho + 1)}{2\rho R(\mu)} \quad (15)$$

One must not bias the distribution too strongly against backward directions, otherwise the small value of $p(\mu)$ for those directions with their correspondingly large weights may cause the small number of histories for backward emitted neutrons to make adversely large contributions to the statistics of the calculation, which is opposite to the effect desired. One must seek an optimum biasing; we have used $\rho = 3$. Biasing with this distribution, we have increased the efficiency of calculations by a factor of about 2.

$$R(\mu) = \frac{4 + (\rho - 1)(\mu + 1)^2(2 - \mu)}{4\rho}$$

Fig. 5. The rejection function $R(\mu)$ used for forward biasing of the source neutrons as a function of μ for values of $\rho = 3$ and 10

D. Energy Distribution

The energy distribution of source neutrons has been described as an evaporation spectrum, essentially identical to that of fission neutrons. Not accounted for in our calculations is the highest energy component of the source energy spectrum, which are cascade neutrons. Fullwood's² calculation indicate that approximately 90% of neutrons from a U^{238} spallation target are distributed in an evaporation spectrum.

The initial energies of the source neutrons were chosen from the ENDF/B-III fission spectrum of isotope U^{238} for which the energy distribution is given by

$$f(E) = \frac{2}{(kT)^{3/2} \sqrt{\pi}} \sqrt{E} \exp(-E/kT) \quad (16)$$

with $kT = 1.407$ MeV. The neutrons which emerge from the source may have experienced collisions in the source which modify the energy distribution of the emerging neutrons from their initial distribution.

The energy distribution of the neutrons as born is shown on curve A in Fig. 6. We have also plotted on curve B the energy distribution of the neutrons as they first emerge from a source containing 80% U^{238} and 20% sodium, 35.26 cm long and 10 cm diameter (as for our 800 MeV case--see below). Those neutrons which emerge from the source, return after scattering outside the source and then emerge from the source again are not considered in Fig. 6. A comparison with the calculations of Fullwood et al. shows that our spectrum is quite similar to theirs, except for some differences which may be attributed to the 20% sodium present in source, and to the contribution of cascade neutrons which we have ignored. As expected, the energy spectrum of emerging neutrons is degraded in energy due to collisions in the source material.

Fig. 6. The assumed energy distribution of the neutrons in a source containing 80% U^{238} and 20% sodium, as they are first born (curve A) and when they first emerge from the source cylinder (curve B)

E. Neutron Histories and Scoring

After choosing the position coordinates, directions of motion and energies of all the source neutrons by the process described above, each of these neutrons is followed to its subsequent collision points, which are determined by considering the mean free path at its present energy, and the medium in which it is located. The isotope with which the neutron collides, the interaction process, the direction of motion and the energy of the neutron after scattering are calculated. Thereafter the probability that the neutron will leak through the moderator surface along the direction $\underline{\Omega}_0$ is calculated, and then the neutron is followed to the next collision point repeating all the calculations described above. This process is repeated until either the neutron leaks out of the system or is absorbed, or its energy falls below a cut off energy.

The leakage spectrum is computed by means of an exact transport calculation carried out at each collision point, rather than tallying those neutrons whose paths actually carry them out of the moderator surface. This latter method, the so-called "direct" or analog simulation method is, as noted above, relatively inefficient compared to the present "scoring" method.

Fig. 7 illustrates the method of calculation. The probability that a neutron which collides at $\underline{r}$ and whose energy and direction before collision were E and $\underline{\Omega}$, emerges from an area A on the moderator surface in direction $\underline{\Omega}_0$ and with energy E' is

$$P_S(\underline{r}, \underline{\Omega}_0, E') = P_M(\underline{r}, \underline{\Omega}_0) \left[\exp - \int_0^{\ell(\underline{r}, \underline{\Omega}_0)} \Sigma_T(x, E') dx \right] \times \sum_{i,n} \frac{\Sigma_{i,n}(\underline{r}, E)}{\Sigma_T(E)} F_i^n(E, E', \underline{\Omega}, \underline{\Omega}_0) \quad (17)$$

$\Sigma_T(x, E')$ is the total cross section for neutrons with energy E' at point x along the path from $\underline{r}$ in direction $\underline{\Omega}_0$, and the quantity $P_M(\underline{r}, \underline{\Omega}_0)$ is

Fig. 7. The method of studying the leakage spectrum of slow neutrons. The probability for a neutron to emerge in the direction Ω_0 , through the face A of the moderator is calculated at energy collision points inside the hatched region and summed to give the leakage spectrum. ANL Neg. No. 125-6876.

either 1 or 0, according to whether or not the line along the direction $\underline{\Omega}_0$ through the point $\underline{r}$ passes through the moderator exit surface A. In Fig. 7 the shaded volume represents the region where $P_M = 1$; elsewhere, $P_M = 0$. Only when $P_M = 1$ does the rest of the leakage calculation need to be carried out. Some efficiency might be gained if the rest of the calculation were carried out only when the collision point is close to the moderator exit surface; the judgement could be made in a simple way without computing the factor $\exp -\int_0^L \Sigma_T(E) dx$, but we have not done this. $F_i^n(E, E', \underline{\Omega}, \underline{\Omega}_0)$ is the probability per unit scattered energy E' and per unit solid angle that a neutron of energy E traveling in direction $\underline{\Omega}$, which collides with a nucleus of type n through a scattering process of type i , travels in direction $\underline{\Omega}_0$ with energy E' after the collision. $\Sigma_{i,n}(\underline{r}, E)$ is the macroscopic scattering cross section for neutrons of energy E at point $\underline{r}$, for collision with a nucleus of type n and process of type i . The ratio $\Sigma_{i,n}(\underline{r}, E)/\Sigma_T(\underline{r}, E)$ is therefore the probability that if a collision takes place, it is with a nucleus of type n and process of type i . The summation runs over all types of nuclei at $\underline{r}$, and all processes of scattering. Rather than performing the indicated sums i, n , and computing $\Sigma_{i,n}/\Sigma_T \cdot F_i^n$ at each collision, the choice of i, n is taken at random in the simulation for the collision at $\underline{r}$. For processes in which E' is independent of direction after scattering, E' is taken to be the same for direction $\underline{\Omega}_0$ as for the next leg of the simulated path. For processes in which E' is dependent on direction after scattering, E' is computed for the direction $\underline{\Omega}_0$, and then F_i^n is computed. The cross sections and other physics data for this calculation are taken from ENDF/B-III.

In the calculation, the N^{th} neutron is assigned a weight w_{KN} at the K^{th} collision, which may vary along the neutron path from collision to collision, depending on the types of interactions that have occurred. The computed quantity

(the total score per history) $J_n(\underline{\Omega}_0)$, which is given by

$$J_n(\underline{\Omega}_0) = \frac{1}{N_H} \sum_N \sum_K w_{KN} \int_{\substack{E' \text{ in } \Delta E_n \\ \text{around } E_n}} P_s(\underline{r}_{KN}, \underline{\Omega}_0, E') dE' \quad (18)$$

is stored in the n^{th} energy bin corresponding to energy E_n of the outcoming neutrons. Here, $\underline{r}_{KN}$ is the position of the K^{th} collision point in the N^{th} neutron history, and N_H is the total number of histories. The width of the n^{th} bin is chosen to correspond to a constant interval of lethargy

$$u = \ln(E_{\text{Ref}}/E), \text{ with a width equal to } 1/3 \text{ decade in energy,} \\ \Delta u = \frac{1}{3} \ln(10) = 0.768 \quad (19)$$

E_{Ref} is an arbitrarily chosen reference energy, taken here to be 10 MeV, the highest energy allowed in the calculations. Due to limitations imposed by the cross section library, the highest and lowest neutron energies which can be treated are respectively 10 MeV and 10 eV (in the latest calculations, $E_{\text{min}} = 1$ eV). The energy corresponding to the center of the n^{th} bin is

$$E_n = E_{\text{Ref}} \times 10^{-(n+0.5)/3} = 6.813 \times 10^6 \times 10^{-n/3} \text{ eV} \quad (20)$$

The computed quantity $J_n(\underline{\Omega}_0)$ is thus related to $I(E, \underline{\Omega}_0)$ defined in Eq. (1) by

$$\begin{aligned}
J_n(\underline{\Omega}_0) &= \int_{\substack{E' \text{ in } \Delta E_n \\ \text{around } E_n}} I(E', \underline{\Omega}_0) dE' \\
&= \int_{\substack{u \text{ in } \Delta u \\ \text{around } u_n}} \left(\frac{dE'}{du} \right) I(E', \underline{\Omega}_0) du = E_n I(E_n, \underline{\Omega}_0) \Delta u,
\end{aligned}$$

so that,

$$E_n I(E_n, \underline{\Omega}_0) = \frac{1}{\Delta u} J_n(\underline{\Omega}_0). \quad (22)$$

F. Statistics

The quantity $J_n(\underline{\Omega}_0)$ is calculated from Eq. (18) by using the Monte Carlo sampling technique in our calculations; thus we have to compute large numbers of histories to reduce the statistical error in $J_n(\underline{\Omega}_0)$. The statistical errors are estimated as follows: The calculations are pursued in batches. Suppose there are ν number of neutrons in every batch and, say, N number of batches have been run. Let y_{N-1} and y_N be the accumulated scores for a particular bin after the $(N-1)^{\text{th}}$ and N^{th} batch, respectively, has been run. Then (here we suppress the bin index) $J(\underline{\Omega}_0)$, which is the mean score per history after N batches are run, is given by

$$J(\underline{\Omega}_0) = \frac{y_N}{\nu N} \pm \Delta J(\underline{\Omega}_0) \quad (23)$$

where $\Delta J(\underline{\Omega}_0)$ is the statistical error in $J(\underline{\Omega}_0)$. ΔJ is calculated as follows: The mean contribution per neutron from the N^{th} batch to the accumulated score in a given bin is given by

$$x_N = \frac{y_N - y_{N-1}}{u} \quad (24)$$

Then if we define the variance V_N of $J(\underline{\Omega}_0)$ after the N^{th} batch has been run as

$$V_N = \frac{1}{N(N-1)} \sum_{i=1}^N \left(x_i - \frac{1}{N} \sum_{j=1}^N x_j \right)^2, \quad (25)$$

one can prove the following recurrence relation

$$V_N = \frac{N-2}{N} V_{N-1} + \frac{1}{u^2} \left(\frac{y_N}{N} - \frac{y_{N-1}}{N-1} \right)^2, \quad (26)$$

where $y_0 = 0$ and $V_1 = 0$.

The statistical error $\Delta J(\underline{\Omega}_0)$ of the mean score is $\sqrt{V_N}$ if N batches have been already run. The number of batches necessary to be run and the number of neutrons per batch to be considered depends on the particular problem of interest, but usually about 2000 neutrons per batch, with 6 or 7 batches at least, should be run for an estimate of $J(\underline{\Omega}_0)$ within about 10% accuracy in each 1/3-decade lethargy bin.

G. Normalization of Results

The calculated spectra are presented as the function $EI(E)$ derived from computed quantities as in Eq. (22). All spectra have a peak at about 1 MeV representing the neutrons which have collided only once or a few times before emerging. Below 100 keV the function $EI(E)$ is nearly constant, or perhaps varying as E^α ; α may be as great as ~ 0.15 in high-leakage moderators. In the tables below, 1 (1 eV) is quoted, which is extrapolated from the range 10 eV - 10 keV over which $EI(E)$ shows smooth behavior. In the latest calculations, the cross section library has been extended down to $E_{\min} = 1$ eV.

In our calculation of $I(E, \Omega)$, neutron histories are initiated in an evaporation spectrum distributed in space and angle as described above in Section II. In U_{238} , these primary source neutrons cause some fissions which produce many generations of secondary neutrons. In U^{238} each generation is smaller than the preceding generation by a factor of about 0.1. These secondary neutrons are also distributed in an evaporation spectrum, but in a somewhat different spatial distribution from the primary source neutrons. We ignore the secondary neutrons in our calculations for U^{238} based on the reasoning that the assumed primary source represents all the source neutrons. We have performed a few calculations which include the secondary fission neutrons, and have found that to within the statistical accuracy of our calculations (typically 5% in each of energy bin) the history of fission neutrons so generated are equivalent to the primary source evaporation neutrons. The total number of primary plus secondary neutrons in the source is approximately 1.1 times the number of primary neutrons. The total leakage is also approximately 1.1 times the leakage calculated for primaries alone. Therefore our results, which are based on the total leakage from the moderator surface normalized to the number of primary source evaporation neutrons, should be changed by an amount less than the statistical accuracy of any of our calculations, if they were to be normalized to the total number of neutrons produced in the source. We therefore consider our results to represent the leakage per total of evaporation neutrons generated in the source by energetic protons. This number must be obtained by separate calculations such as those of Fullwood et al.²

H. Relationship of Results to Neutron Spectra Below $E_{\min}$

The present calculations extend downward to the energy $E_{\min} = 10$ eV, or 1 eV. Nevertheless the results are applicable to estimate the spectra at lower energies, if the shape of the spectrum below the lowest energy of the calculation is known for a moderator of the same configuration as the one calculated. Materials having insignificant cross sections in the energy range of the calculations may be incorporated into the moderator and affect the spectrum at energies lower than $E_{\min}$. In particular cadmium, which has been used to decouple moderators from reflectors and also to act as heterogeneous poison within moderators, is a material of this kind.

We have in mind the use of spectra measured in the range up to $E = E_{\min}$ although without the normalization appropriate to the present case. These spectra can be attached smoothly to the calculated spectra at energies of about $E_{\min}$, to provide estimates of the thermal neutron beam current. If the joining point is 1 eV, and if the measured beam current at energies below 1 eV normalized to its value at 1 eV is

$$[I(E)/EI(E)]_{1 \text{ eV}}^{\text{measurement}}, \quad (27)$$

then the beam current at low energy is given by

$$I(E) = [I(E)/EI(E)]_{1 \text{ eV}}^{\text{measurement}} [EI(E)]_{1 \text{ eV}}^{\text{calculation}}, \quad (28)$$

so that this result has the same normalization as the calculation.

III. RESULTS

A. Calculations on the ZING Mockup

As a check on the accuracy of the calculations, we have computed the neutron beam intensity for the ZING Mockup for which measurements have been performed.³ Similar calculations have also been performed by Russell.⁴ The source was taken to be a cylinder of 2.54 cm diameter x 2.54 cm high to represent roughly the small Cf²⁵² and Am²⁴¹ - Be sources used in the measurements, and the moderator 4 x 4 x 2 in³ CH₂. The energy spectrum was represented approximately as being the same as the fission spectrum of U²³⁸ (Eq. 16). Fig. 8 shows the ZING Mockup, geometry the results for which are given in Fig. 9. The function EI(E) is nearly constant in the range below 10 KeV, and the average value of EI(E) for the interval 10 eV - 10 KeV is

$$EI(E) = 9.76 \times 10^{-4} \text{ n/ster/n}_f \pm 2\% \quad . \quad (29)$$

This can confidently be extrapolated to $E = 1 \text{ eV}$, so that $I(E)|_{1 \text{ eV}} = 9.76 \times 10^{-4} \text{ n/ster-eV/n}_f \pm 2\%$. The ZING mockup results³ gave the thermal neutron beam intensity, averaged for three measurements, of

$$I_{Th} = 4.15 \times 10^{-3} \text{ n}_{Th}/\text{ster/n}_f \pm 4\% \quad . \quad (30)$$

Since the shape of the spectrum in the thermal energy range is known from measurements on a similar moderator in the ZING prototype,^{5,6} the relationship between I_{Th} and $I(E)$ at higher energies is known. From measurements at ZING-P on a polyethylene moderator of dimensions 5.08 x 10 x 10 cm³, it was found that $I_{Th}/I(1 \text{ eV}) = 5.59 \text{ n}_{Th}/(n/\text{eV})$. Thus we find for the ZING mockup,

Fig. 8. Mockup of the ZING source, moderator and reflector with a beam port. The assembly was surrounded by 6 in. Benelex

Fig. 9. The neutron beam current $EI(E)$ as a function of energy E for the ZING mockup geometry (Ref. 3). The source is a U^{238} fission spectrum and the size of the moderator is $4 \times 4 \times 2$ in.³ polyethylene. ANL Neg. No. 125-77-82.

$$\begin{aligned}
 I(1 \text{ eV}) &= (I_{Th}) / (I_{Th} / I(1 \text{ eV})) \\
 &\quad \text{spectrum measurement} \\
 &= \frac{4.15 \times 10^{-3} \text{ n}_{Th}/\text{ster-n}_f}{5.59 \text{ n}_{Th}/(\text{n/eV})} = 7.42 \times 10^{-4} \text{ n/ster-eV/n}_f \pm 4\% ,
 \end{aligned}
 \tag{31}$$

which is in satisfactory agreement with the calculated result, the calculation being higher by 30%.

In Fig. 10, we have plotted $EI(E, \Omega_0)$ for the ZING mockup geometry without the Be reflector, i.e., a bare moderator; a least-squares fit to the function in the interval $10 \text{ eV} < E < 10 \text{ keV}$ gives

$$EI(E) = 8.59 \times 10^{-5} (E_{\text{eV}})^{0.13} . \tag{32}$$

The shape of the spectrum is consistent with Russell's result,⁴ which gave $\alpha = 0.117$. Comparison of the results with and without the Be reflector indicates that the reflector provides a gain of $9.76 \times 10^{-4} / 8.59 \times 10^{-5} = 11.4$ over the intensity of the bare moderator. The availability of an order of magnitude increase in neutron flux by the use of a beryllium reflector on a spallation neutron source is an important result. This computational demonstration corroborates the conclusions derived from the experimental results of the ZING Mockup.³

A brief set of Be-reflected calculations was performed, in the ZING-Mockup geometry to survey the differences of beam current spectra for different moderator materials, thicknesses and decoupling. Polyethylene represents a typical material for 300 K neutrons. Liquid hydrogen is an excellent candidate

Fig. 10. The neutron beam current $EI(E)$ for the ZING mockup geometry with no beryllium reflector

for cryogenic moderators, while TiH_2 is a candidate giving shortest pulses of epithermal neutrons. All moderators were of 10 x 10 cm in lateral dimension. Results are summarized in Table I, as determined from visual fits to the calculated spectra. Except for the calculation for $2''\text{CH}_2$, the statistical accuracy in each 1/3-decade lethargy bin was approximately 10%. The accuracy of $EI(E)|_{1 \text{ eV}}$ is expected typically to be $\pm 3\%$, and of the exponent α , about $\pm .01$.

These calculations cost only approximately \$25 each (about 10 min.) on the IBM 360-195, demonstrating the efficiency of the program.

Table I

Slow Neutron Leakage Currents for Several Moderator Materials in the ZING-Mockup Geometry

Moderator*	$EI(E) _{1 \text{ eV}}$	α^{**}
2.54 cm $(\text{CH}_2)_n$	9.20×10^{-4}	.026
2.54 cm $(\text{CH}_2)_n$, B-decoupled	8.16	.040
5.48 cm $(\text{CH}_2)_n$	9.76	.000
5.48 cm $(\text{CH}_2)_n$, B-decoupled	7.67	.020
2.54 cm Liq. H_2	11.52	.038
2.54 cm Liq. H_2 , B-decoupled	6.06	.119
5.08 cm Liq. H_2	9.94	.034
5.08 cm Liq. H_2 , B-decoupled	7.15	.076
2.54 cm TiH_2	9.47	.015
2.54 cm TiH_2 , B-decoupled	8.91	.004
5.08 cm TiH_2	8.47	.000
5.08 cm TiH_2 , B-decoupled	7.49	.019

*All 10 x 10 cm in lateral dimensions. CH_2 density .94 gm/cm³. Liquid H_2 density .070 gm/cm³. TiH_2 with .094 H/b-cm, Boron decoupling $.18 \times 10^{24} \text{ B}^{10}/\text{cm}^2$.

** α is the exponent in $EI(E)\alpha E^\alpha$

B. Studies of the Effect of Source Size
(Proton Energy) in One Moderator Geometry

A simple target-reflector-moderator arrangement used for studying the variation of $J_n(\underline{\Omega}_0)$ as a function of proton energy, moderator size and distance of the moderator from the source is shown in Fig. 11. The radius of the source cylinder containing 80% U^{238} and 20% Na was kept 5 cm throughout the calculations. The moderator was placed adjacent to a void surrounding the source whose radius D was 5 cm, and two distances (6 and 8 cm) from source centerline to moderator were studied. The height of the source H_S was 1.2 times the range of protons z_0 at the corresponding energy. The source is surrounded by a Be cylinder of radius 50 cm and height 100 cm, such that the centers of the source and the outer Be cylinder coincide. The center of the moderator was kept at a distance $\bar{z}$ from the top of the source cylinder, where $\bar{z}$ is the mean axial position at which neutrons are emitted from the source, and is given by

$$\bar{z} = \frac{\int_0^{z_0} z \sinh\left(\frac{z_0-z}{L}\right) dz}{\int_0^{z_0} \sinh\left(\frac{z_0-z}{L}\right) dz} = \frac{L \sinh\left(\frac{z_0}{L}\right) - z_0}{\cosh\left(\frac{z_0}{L}\right) - 1} \quad (33)$$

The calculation of L is discussed earlier in Section II.B. Although we have not investigated the best relative positioning of moderator and source, we assume that the leakage from the moderator is maximum for this position. The values of z_0 , H_S and $\bar{z}$ for various proton energies under consideration are shown in Table II. The neutrons are scored along the direction $\underline{\Omega}_0$ normal to the $10 \times 10 \text{ cm}^2$ surface of the moderator through a rectangular void in the Be cylinder.

The variation of $EI(E, \underline{\Omega}_0)$ with the proton energy in the range from 500 MeV to 950 MeV was calculated for a fixed distance of $d = 6 \text{ cm}$ between source

Fig. 11. The geometrical arrangement (simple wing geometry) used in studying the variation of emerging neutron beam current as a function of proton energy, the moderator size and distance of moderator from source. ANL Neg. No. 125-6871 Rev. 2.

and moderator, and for a single polyethylene moderator of size $10 \times 10 \times 5.08 \text{ cm}^3$ for the geometry of Fig. 11. A typical result, that for $E_p = 500 \text{ MeV}$ and $d = 6 \text{ cm}$, is shown in Fig. 12. The value of $EI(E, \underline{\Omega}_0)$ extrapolated to $E = 1 \text{ eV}$ from the range 10 eV to 10 keV (where $EI(E, \underline{\Omega}_0)$ is nearly constant) for various cases is tabulated in Table II, along with the target and moderator parameters for each case. It can be seen from these results that the value of $EI(E, \underline{\Omega}_0)|_{1 \text{ eV}}$ (beam current per source neutron) decreases as the energy of the protons causing the spallation process is increased. However, the number of neutrons produced in the spallation process per proton increases as the energy of the protons is increased.

Thus we introduce another quantity which is more closely related to the performance of an accelerator neutron source, which is the slow neutron beam current per proton. That is,

$$I_p(E, \underline{\Omega}_0) = I(E, \underline{\Omega}_0) \times Y(E_p) \quad , \quad (34)$$

where $Y(E_p)$ is the neutron yield per proton, that is, the number of neutrons produced in the source per proton on target, and E_p is the proton energy. Table III gives the slow neutron beam current per proton, $I_p(E, \underline{\Omega}_0)$ at 1 eV , for various proton energies. While the beam current $I(E, \underline{\Omega}_0)$ on a per source neutron basis decreases as proton energy increases due to increasing target length, the beam current I_p increases as proton energy increases since the yield increases. Table III also gives this yield as a function of proton energy for thick targets of U^{238} . We have chosen to use the measured yields of Milton and Fraser^{7,8} in Table III. Fullwood et al.² note a discrepancy between these measurements and their calculations which are 25% lower. The evaporation component has been estimated as 90% of the total yield, as judged from the calculations of Fullwood. For the present purposes we take all neutrons as equivalent to evaporation neutrons.

Fig. 12. The neutron beam current $EI(E)$ as a function of energy E for the simple wing geometric arrangement of Fig. 11. The energy of incident protons is 500 MeV and the size of the polyethylene moderator is $10 \times 10 \times 5 \text{ cm}^3$, placed at a distance of 6 cm away from the source.

Table II. The extrapolated value of $EI(E, \underline{\Omega}_0)$ at $E = 1$ eV, neutrons/steradian-source neutron leaking from $H_M \times 10 \times 5.1 \text{ cm}^3 \text{ CH}_2$ moderators normal to the surface as a function of proton energy and source-moderator geometry.

Moderator Height H_M (cm)	Distance of Moderator from Source d (cm)	Energy of Incident Protons (MeV)			
		500	650	800	950
10	6	0.714×10^{-3}	0.688×10^{-3}	0.581×10^{-3}	0.568×10^{-3}
10	8	0.578×10^{-3}	0.477×10^{-3}	0.486×10^{-3}	0.426×10^{-3}
20	6	1.106×10^{-3}	1.005×10^{-3}	1.003×10^{-3}	0.904×10^{-3}
Range of protons z_0 in target ($U^{238} + Na$) (cm)		15.15	21.93	29.38	37.44
Length of the source cylinder H_S (cm)		18.18	26.32	35.26	44.94
Distance of the center of moderator from the top of source cylinder $\bar{z}$ (cm).		4.89	6.84	8.72	10.4

Table III. The extrapolated value of $EI_p(E, \Omega_0)$ at $E = 1$ eV, neutrons/ster-proton for $H_M \times 10 \times 5.1 \text{ cm}^3$ CH_2 moderators as a function of proton energy and source moderator geometry, and the neutron yield as a function of proton energy.⁸

Moderator Height H_M (cm)	Distance of Moderator from Source d (cm)	Energy of Incident Protons (MeV)				l , cm
		500	650	800	950	
10	6	1.43×10^{-2}	1.82×10^{-2}	1.92×10^{-2}	2.27×10^{-2}	83.
10	8	1.16×10^{-2}	1.27×10^{-2}	1.60×10^{-2}	1.70×10^{-2}	81.
20	6	2.21×10^{-2}	2.66×10^{-2}	3.31×10^{-2}	3.62×10^{-2}	97.
$\chi(E_p)$, neutrons/proton		20	26.5	33.0	40.0	

In Fig. 13 we have plotted both $EI(E, \underline{\Omega}_0) |_{1 \text{ eV}}$ and $EI_p(E, \underline{\Omega}_0) |_{1 \text{ eV}}$ as a function of the square of the length H_s of the source cylinder. The curves indicate that to increase the slow neutron beam current, it is probably inexpedient to increase the energy of the protons very much beyond about 1. GeV. It is found that one can reasonably represent $EI(E, \underline{\Omega}_0, H_s) |_{1 \text{ eV}}$ by a Gaussian function, $EI(E, \underline{\Omega}_0, H_s) \propto \exp[-(H_s/\lambda)^2]$, where λ is a fitting constant depending on the source-moderator geometry, values of which are given to Table III.

We have also calculated the beam current from a moderator placed in a radial neutron beam configuration shown in Fig. 14. This increases the current of neutrons at high energy considerably as is shown for incident proton of energy 650 MeV in Fig. 15, but there is little difference in the low energy beam current. Since these neutrons contribute only to the background, the radial arrangement seems to be disadvantageous.

Further, we have calculated the beam current in this same geometry, replacing the Be reflector with CH_2 , so that the arrangement is just a block of polyethylene with a re-entrant hole. The low energy beam current was found to be only approximately 45% as large as in the case with a Be reflector. The effect is probably due in part to the absence of $(n, 2n)$ reactions in CH_2 , which contribute to the neutron current in the case with Be reflector.

C. Calculations for a Vertical Beam Three-Moderator Geometry

A vertical proton beam arrangement was considered in the ZING studies in which three 5 cm thick polyethylene moderators were arranged around a spallation neutron source (containing 80% U^{238} + 20% Na acting as coolant) as shown in Fig. 16. The configuration was devised to provide twelve horizontal slow neutron beams. The moderator A is intended to provide short pulses at the sacrifice of intensity and is decoupled from the Be reflector by a 1" thick layer of natural Boron carbide containing $6.5 \times 10^{22} \text{ B}^{10}/\text{cm}^2$. This layer decouples the moderator from the reflector at about 100 eV.

Fig. 13. The neutron beam current per source neutron $EI(E, \Omega)_1 \text{ eV}$ and per incident proton $EI_p(E, \Omega)_1 \text{ eV}$, as a function of the energy of the incident protons, plotted vs the square of the corresponding height H_s of the source cylinder. Curve A represents results for moderator of size $10 \times 20 \times 5 \text{ cm}^3$ placed at a distance of 6 cm from the moderator, curve B corresponds to moderator size $10 \times 10 \times 5 \text{ cm}^3$ at a distance of 6 cm and curve C corresponds to moderator size $10 \times 10 \times 5 \text{ cm}^3$ at a distance of 8 cm from the source. ANL Neg. No. 125-77-68.

Fig. 14. The source, reflector and radial-beam moderator arrangement for neutron yield measurements

Fig. 15. The neutron beam current $EI(E)$ for incident proton energy of 650 MeV in the radial-beam arrangement of Fig. 14

Fig. 16. The vertical proton beam arrangement with three 5-cm-thick polyethylene moderators used for the ZING neutron beam current calculations. ANL Neg. No. 125-76-19 Rev. 1.

Fig. 17 shows the computed spectra for an incident proton energy of 800 MeV, and the values of $EI(E)|_1$ eV are tabulated in Table IV. The added complications of this configuration reduce the intensity to about 2/3 of the value for a similar one moderator case. The neutron source corresponded to that of an 800 MeV proton beam.

D. Calculations of the ZING Prototype, Two Moderator Geometry

In the ZING Prototype studies⁶ the two polyethylene moderators were placed at a distance of 2" from the center of a lead block, acting as a neutron source. Horizontal and vertical views of the setup are shown in Figs. 18 and 19. One of the moderators (A) was surrounded by a 2.54 cm layer of B_4C and epoxy, having a decoupling energy of about 100 eV. The initial energy distribution of the neutrons was considered to be the same as that of U^{238} fission spectrum, and the spatial distribution was the same as that described in Section II. The results obtained for this geometry using protons of energy 200 MeV are presented in Fig. 20 and in Table IV.

E. The Calculations of the IPNS Three Moderator Geometry

In this geometrical setup (shown in Fig. 21), the spallation target contains 80% U^{238} and 20% sodium and is oriented horizontally; one of the three polyethylene moderators being below the source and the other two above the source. The results obtained for this geometry are presented in Fig. 22 and in Table V.

F. Calculation of an Electron-Driven U^{235} Fission Multiplier

We have calculated the beam current of neutrons leaking from a water moderator with a subcritical multiplying assembly representing a "fission booster" driven by a pulsed electron bremsstrahlung photoneutron source. Such an arrangement has been suggested⁹ for installation at the Oak Ridge Electron

Fig. 17. The neutron beam currents $EI(E)$ for incident proton energy of 800 MeV and for the vertical proton beam moderator arrangement considered in the ZING studies. ANL Neg. No. 125-77-80.

Table IV
Slow Neutron Beam Currents

ZING Geometry

($E_p = 800$ MeV, target 80% U^{238} + 20% Na)

Moderator	$EI(E) _1$ eV n/ster- n_f
A1	3.0×10^{-4}
A2	2.8×10^{-4}
B	3.4×10^{-4}
C	4.4×10^{-4}

ZING Prototype Geometry

($E_p = 200$ MeV, lead target)

Moderator	$EI(E) _1$ eV n/ster- n_f
A	3.1×10^{-4}
B	6.0×10^{-4}

Fig. 18. The ZING-Prototype arrangement (horizontal section)

Fig. 19. The ZING-Prototype arrangement (vertical section). ANL Neg. No. 125-6758 Rev. 1.

Fig. 20. The neutron beam currents $EI(E)$ for the two polyethylene moderators of the ZING-Prototype arrangement with protons of energy 200 MeV on a lead target. ANL Neg. No. 125-77-81.

Fig. 21. The IPNS neutron scattering source, moderator, reflector arrangement

Fig. 22. The neutron beam currents $EI(E)$ for the three polyethylene moderators of the IPNS arrangement with an incident proton energy of 800 MeV. ANL Neg. No. 125-77-82.

Table V
Slow Neutron Beam Currents
IPNS Geometry

($E_p = 800$ MeV, target 80% U^{238} + 20% Na)

Moderator	$EI(E) _1$ eV
A1	2.8×10^{-4}
A2	2.5×10^{-4}
B	2.5×10^{-4}
C	4.5×10^{-4}

Linear Accelerator (ORELA). The geometry is shown in Fig. 23. The reactor consists of an inner cylinder of radius R and height H (such that $H = 1.85 R$) containing a mixture of 70% U^{235} and 30% sodium, surrounded by a 2 cm thick cylinder of B_4C of density 1.6 gm/cm^3 . This ratio ($H/R = 1.85$) was chosen because the volume is minimum for a given buckling B_g^2 of the cylinder. The beam current from a window on the moderator was studied which was chosen according to the Oak Ridge specifications and placed so as to provide maximum output according to the measurements of Menzel.¹⁰ Since the number of secondary neutrons produced due to fission per source neutron for such a system is very large (≈ 1.0) compared to the usual spallation source case where it is around 0.1, many generations follow from each source neutron. Thus the relationship of the intensity of neutrons leaking from the moderator to the fission neutrons could not be evaluated for this case in the approximate way described above for U^{238} systems.

The procedure used in this calculation is as follows: The k_{eff} for the system was determined first by considering a uniform distribution of source neutrons inside the small primary source cylinder which were allowed to leak through the moderator, encountering the usual process of slowing down. After this, instead of choosing new source neutrons as was done in the previous calculations, the source for each succeeding generation was taken to be the neutrons generated by fissions caused by the preceding generation and their histories followed through the system by the usual procedure. For this calculation the VIM code was used in the manner for which it was developed; namely, as a reactor criticality code. The ratio of the number of fissions in succeeding generations was found to be decreasing from generation to generation. This procedure of following the neutrons in successive generations was repeated until the k_{eff} of the system converged and was determined to 2% accuracy.

Fig. 23. The arrangement of primary electron driven photo-neutron source (cross hatched), U^{235} fission booster surrounded by boron carbide (shaded) water moderator and the defined source area (shaded). ANL Neg. No. 125-75-103 Rev. 2.

The value of k_{eff} was determined for three different values of H . Then the plot of $1/k_{eff}$ vs. B_g^2 from these three values of H (Fig. 24) was used to find buckling of the system for which k_{eff} is 0.97; the multiplier then has a gain of about $1/(1 - k_{eff}) = 33$, which is a reasonable goal for such a multiplier. The height H and radius R of the reactor with this value of k_{eff} was then determined from the relations

$$B_g^2 = \left(\frac{\pi}{H}\right)^2 + \left(\frac{2.405}{R}\right)^2; \quad R = H/1.85 \quad (35)$$

The k_{eff} of the system with these values of H and R was then determined by following the procedure described above, up to accuracy of 0.15%. This accurate determination of k_{eff} is essential as will be explained in the following section. We had to follow about 200 generations of neutrons each containing 1000 neutrons; this was done without calculating the leakage.

After thus accurately determining k_{eff} of the system, next we concentrated on the beam current from the moderator, using 8000 starting neutrons per generation, for better statistical accuracy. About 20 generations were necessary to get the desired accuracy in the intensity of the neutrons leaking from the moderator. The total number of neutrons leaking from the moderator in the n^{th} energy interval, per fission in the booster, $J_{nF}(\underline{\Omega}_0)$, the total number of fission neutrons produced per source neutron, N_T , were evaluated in these calculations as follows.

A fairly large number of batches, M , were run. Each batch represented one generation of neutrons, but each batch consisted of the same number N_B of starting neutrons. We denote by F_i the number of fissions induced by neutrons in the i^{th} batch, per starting neutron, and v_i the number of neutrons produced per fission induced by the i^{th} batch. Then $k_i = v_i F_i$ is the number of neutrons produced in the i^{th} batch, per starting neutron in the i^{th} batch. The value of $k_{eff} = \lim_{i \rightarrow \infty} k_i$ was computed using

Fig. 24. Multiplication constant k_{eff} vs geometric buckling B_g^2 for the fission booster arrangement. ANL Neg. No. 125-77-69.

$$k_{\text{eff}} = \left(\frac{1}{M-m_1} \right) \sum_{i=M_1+1}^M k_i, \quad (36)$$

which takes advantage of the statistical accuracy in M batches, but ignores the first M_1 batches, since these k_i 's are expected to differ from k_{eff} .

The value of F_T was computed from

$$\begin{aligned} F_T &= F_0 + (v_0 F_0) F_1 + (v_0 F_0)(v_1 F_1) F_2 + \dots \\ &= F_0 + \sum_{i=1}^{\infty} \left(\prod_{j=0}^{i-1} k_j \right) F_i \end{aligned} \quad (37)$$

Since after a large number of batches (say M) have been computed, F_i approaches its limit

$$\lim_{i \rightarrow \infty} F_i = \bar{F}, \quad (38)$$

we represent

$$F_T = F_0 + \sum_{i=1}^M \left(\prod_{j=0}^{i-1} k_j \right) F_i + \left(\prod_{j=0}^M k_j \right) \bar{F} \left(\sum_{\ell=0}^{\infty} k_{\text{eff}}^{\ell} \right), \quad (39)$$

so that the last terms can be summed as a geometric series, thus

$$F_T = F_0 + \sum_{i=1}^M \left(\prod_{j=0}^{i-1} k_j \right) F_i + \left(\prod_{j=0}^M k_j \right) \bar{F} \left(\frac{1}{1 - k_{\text{eff}}} \right) \quad (40)$$

In practical terms,

$$\bar{F} = \frac{1}{M - M_1} \sum_{i=M_1+1}^M F_i \quad (41)$$

Similarly,

$$\begin{aligned} N_T &= 1 + \nu_0 F_0 + (\nu_0 F_0)(\nu_1 F_1) + \dots \\ &= 1 + \sum_{i=1}^{\infty} \left(\prod_{j=0}^{i-1} k_j \right) \\ &= 1 + \sum_{i=1}^M \left(\prod_{j=0}^{i-1} k_j \right) + \left(\prod_{j=0}^M k_j \right) \left(\frac{1}{1 - k_{\text{eff}}} \right) \quad (42) \end{aligned}$$

Similarly, we compute $R_{nT}(\underline{\Omega}_0)$ the total number of neutrons leaking from the moderator surface into unit solid angle about direction $\underline{\Omega}_0$, in the n^{th} energy bin, per starting neutron

$$\begin{aligned} R_{nT}(\underline{\Omega}_0) &= R_{n0}(\underline{\Omega}_0) + R_{n1}(\underline{\Omega}_0)(\nu_0 F_0) + R_{n2}(\underline{\Omega}_0)(\nu_0 F_0)(\nu_1 F_1) + \dots \\ &= R_{n0}(\underline{\Omega}_0) + \sum_{i=1}^{\infty} \left(\prod_{j=0}^{i-1} k_j \right) R_{ni}(\underline{\Omega}_0) \\ &= R_{n0}(\underline{\Omega}_0) + \sum_{i=1}^M \left(\prod_{j=0}^{i-1} k_j \right) R_{ni}(\underline{\Omega}_0) \\ &\quad + \left(\prod_{j=0}^M k_j \right) \bar{R}_n^-(\underline{\Omega}_0) \left(\frac{1}{1 - k_{\text{eff}}} \right) \quad (43) \end{aligned}$$

We take the limiting value of $R_{ni}(\underline{\Omega}_0)$ to be

$$\bar{R}_n^-(\underline{\Omega}_0) = \left(\frac{1}{M - M_1} \right) \sum_{i=M_1+1}^M R_{ni}(\underline{\Omega}_0) \quad (44)$$

Here, $R_{ni}(\underline{\Omega}_0)$ is the number of neutrons leaking from the moderator surface A, into unit solid angle about $\underline{\Omega}_0$, in the n^{th} energy bin, per neutron starting in the i^{th} batch.

As can be seen from the equations, unless M is quite large ($M \gg 100$) most of the contributions to the sums comes from their last terms, involving the quantity $1/(1 - k_{\text{eff}})$, which is of the order of 30 since k_{eff} was chosen to have value 0.97. Thus it is essential to determine the value of k_{eff} very accurately.

Finally $J_{nT}(\underline{\omega}_0)$ was computed from

$$J_{nT}(\underline{\omega}_0) = \frac{1}{F_T} R_{nT}(\underline{\omega}_0) \quad (45)$$

which is the number of neutrons leaking from the moderator surface A , into unit solid angle about $\underline{\omega}_0$ in the n^{th} energy bin, per fission in the booster.

Also, we identify N_T as the gain of the booster, which has sometimes been approximately represented as

$$N_T \approx \frac{1}{1 - k_{\text{eff}}} \quad (46)$$

Of course, this approximation is not exactly valid, since source neutrons are not distributed in the most nearly critical mode of the reactor, source neutrons having higher importance than critical-mode fission neutrons on the average.

The function $EI_F(E, \underline{\omega}_0) = J_{nT}(\underline{\omega}_0)/\Delta u$ is plotted in Fig. 25. For this case we find using Eqs. (40) and (42)

$$F_T = 18.3 \text{ fiss./source neutron}$$

and

$$N_T = 48.0 \text{ neutrons/source neutron} \quad .$$

The calculated value $N_T = 48$ exceeds the approximation $1/(1 - k_{\text{eff}}) = 33$ for the above mentioned reason. For this arrangement, $EI_F(E)|_{1 \text{ eV}} = 4. \times 10^{-5} \text{ n/ster-fission.}$

Fig. 25. The neutron beams current $EI(E)$ for the fission booster arrangement

G. Time-Dependent Neutron Beam Current Calculations

To investigate the time dependence of the neutron beam current, the VIM code was further modified to keep account of the total time t taken by neutrons to travel from the point at which they were produced to the point at which they emerged from the moderator surface. All neutrons were assumed to be born at time $t = 0$. The scores $J_{mn}(\underline{\Omega}_0)$ corresponding to the emerging beam current were then binned according to energy E_n , as above into 1/3-decade lethargy bins, and according to the variable $(vt)_m$. Here, v is the actual neutron speed at which neutrons emerge (as opposed to the nominal value for each energy bin.) This method is appropriate since, to a first approximation, the time dependent beam current when expressed as a function of vt is expected to be independent of the energy when cross sections are constant. Recall that for neutrons slowing down in an infinite medium of free protons, the flux per unit speed at time t ,

$$\phi(v,t) = (\Sigma_S vt)^2 e^{-\Sigma_S vt} \quad (47)$$

is a homogeneous function of the variable $(\Sigma_S vt)$. The scattering cross section Σ_S is very nearly independent of neutron speed. The beam current per unit time $I(E, \underline{\Omega}_0, t)$ was then computed for the n^{th} energy bin according to

$$EI(E, \underline{\Omega}_0, t) = \frac{J_{mn}(\underline{\Omega}_0)}{\Delta u \Delta(vt)_m} \quad (48)$$

where $\Delta u = 1/3 \ln 10$, and the values $\Delta(vt)_m$ were chosen to provide an efficient representation of the function calculated.

In order to determine the extent of pulse broadening by the reflector, and the effect of neutron streaming across the beam hole void, the time dependence of the beam current was computed for two cases in the ZING Mockup geometry (Fig. 8); viz.,

- a) Cadmium decoupling only (ignored in calculations above 1 eV).
- b) Boron liner (5×10^{21} boron-10/cm²) inside the beam hole void only (moderator decoupled by cadmium.)

The results for 1 eV neutrons are shown in Fig. 26. Russell et al.¹¹ have also computed the moments of the time distribution for the case a), though not the detailed shape. The moments determined in both calculations at 1 eV are shown in Table VI. Allowing for the differences in assumed CH₂ density, the moments determined by both calculations for case a) are consistent, and for both cases significantly greater than the values for the bare moderator. This is due to the long tail on the distribution, which is significantly suppressed by the boron liner, and is therefore partly due to streaming across the beam hole void. Also plotted in the figure is the bare moderator pulse shape evaluated from the function $I(E,t) \propto 1/v \ x^2 \ e^{-x}$, where $x = \Sigma_s vt$ using the scattering cross section for free protons at the density of polyethylene.

Comparison of the bare moderator pulse shape with that for case b) indicates the very important result that the reflector broadens the pulse little if any near the peak, but adds a tail which increases both the mean emission time $\bar{t}$ and the variance σ_t in the pulse width. In addition, it was found that, within statistical precision, the shape of $I(E,t)$ is independent of E for both cases, for all energies up to about 100 eV, above which energy the boron liner is no longer opaque.

Fig. 26. Beam intensity of 1 eV as a function of time (expressed as a function of vt) for ZING mockup. The symbol 'o' corresponds to setup with no boron liner, 'x' to one with boron liner, whereas solid line is for an infinite medium of polyethylene.

Table VI

Pulse Shape Parameters for ZING Mockup Calculations

<u>Case</u>	<u>$EI(E) \int_{n_f}^{\infty} \frac{1}{n} dv$</u>	<u>$\bar{v}t$</u> <u>cm</u>	<u>$v\sigma_t$</u> <u>cm</u>	<u>$v\tau$</u> <u>cm</u>		
a. (No boron)	7.9×10^{-4}	3.9 4.0	4.6 4.8	2.2 *	(Das) (Russell)	$(\rho = .94 \text{ gm/cm}^3)$ $(\rho = .90 \text{ gm/cm}^3)$
b. (Boron void liner)	5.1×10^{-4}	3.0	3.5	1.9	(Das)	
Infinite moderator		1.82	1.03	2.00		

*Not calculated.

IV. CONCLUSION

We have introduced a scoring method into the VIM Monte Carlo neutron transport code which enables efficient calculation of the neutron leakage from moderators for neutrons of energy above 1 eV. The modified code has been used to compute neutron beam currents for a number of unreflected and beryllium reflected source-moderator geometries appropriate to use in pulsed neutron sources. Results are in good agreement with measurements and with earlier calculations of similar cases done by others. The slow neutron beam current has been computed as a function of proton energy for primary fast-neutron sources representing those generated by 500-900 MeV protons, in an idealized source-moderator-reflector arrangement. The slow neutron beam currents for three practical arrangements of sources with several reflected moderators and for a subcritical fission multiplier neutron source with one moderator have been computed. Using a further modification of the VIM code, we have computed the time dependence of the pulse from a Be-reflected moderator, and determined the effect of a boron beam hole void liner on the time dependent pulse.

This report has been issued only after a considerable lapse of time following the actual calculations. Since that time, we have developed methods with which to compute the neutron source due to the high-energy nucleon cascade, which calculation is used as input to VIM low-energy neutron transport as in this report. Conclusions from those calculations and similar to those drawn here in most respects.

Acknowledgements

We thank Prof. Motoharu Kimura and Dr. David Mildner for critical reading of this manuscript, and Ms. Midge Thompson for her enduring patience in preparing it.

REFERENCES

1. K. E. Prael and H. Henryson II, Proc. Conf. Nuclear Cross Sections and Technology, March 1975, Washington, D.C.
2. R. G. Fullwood, J. D. Cramer, R. A. Harman, R. P. Forrest, Jr., and R. G. Schrandt, Neutron Production by Medium-Energy Protons on Heavy Metal Targets, Los Alamos Scientific Laboratory Report LA-4789 (1972).
3. J. M. Carpenter and G. J. Marmer, Argonne National Laboratory Report ANL/SSS-72-1; J. M. Carpenter, High Intensity Pulsed Thermal Neutron Source, U.S. Pat. No. 3,778,627 (1973).
4. G. J. Russell, private communication (December, 1973).
5. J. M. Carpenter, M. H. Mueller, R. A. Beyerlein, T. G. Worlton, J. D. Jorgensen, T. O. Brun, K. Sköld, C. A. Pelizzari, S. W. Peterson, N. Watanabe, M. Kimura, and J. E. Gunning, "Neutron Diffraction Measurements on Powder Samples Using the ZING-P Pulsed Neutron Source at Argonne," Proc. Conf. New Methods and Techniques in Neutron Diffraction, August 1975, Petten, Netherlands, RCN Report #234, 192.
6. J. M. Carpenter, Nucl. Instr. & Meth. 145, 91 (1977).
7. J. C. D. Milton and J. S. Fraser, "A Monte Carlo Calculation of Neutron Production in Heavy Element Target by Protons in the Range 0.3 - 1 BeV", Atomic Energy of Canada, Ltd. Report AECL-2259 (1965).
8. W. A. Gibson, E. E. Gross, Z. Zucker, J. S. Fraser, R. E. Green, J. W. Hilborn, and J. C. D. Milton, "Low-Energy Neutron Production by High-Energy Proton Bombardment of Thick Targets", Oak Ridge National Laboratory Report ORNL-3940, 110-111 (1966).
9. R. W. Peele, T. A. Lewis, J. T. Mihalczko, H. A. Mook, and R. M. Moon, "Use of ORELA to Product Neutrons for Scattering Studies on Condensed Matter," Oak Ridge National Laboratory Report ORNL-TM-4987 (1975).
10. J. H. Menzel, "Measurements of Time-Dependent Neutron Spectra in H₂O", Ph.D. Thesis, Renssalaer Polytechnic Institute (1968).
11. G. J. Russell, P. A. Seeger, R. G. Fluharty, "Parametric Studies of Target/Moderator Configurations for the Weapons Neutron Research Facility, Los Alamos Scientific Laboratory Report LA-6020 (1977).