SINGLE PASS COLLIDER MEMO

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TITLE: AN ATTEMPT TO COMPARE TWO ARC ORBIT CORRECTION SCHEMES ANALYTICALLY

Consider a transport line that consists of periodic cells. Let the beam position monitors and the orbit correctors be located with the same period as the cells and let the BPM's and the corrector distributions interlace each other as shown below:

Fig. 1

The arrangement does not always provide a stable orbit correction. The criterion for stability has been derived by Joe Murray and is reproduced as Section 1 (using different notations) for completeness. In Sections 2,3,4, we calculate the rms orbit, the effect of BPM errors and the rms corrector strength in such correction schemes, yielding analytic formulae for these quantities. In Sections 5 and 6, we apply these formulae to the SLC arcs.

1. The Stability Criterion

Consider the n -th cell in the transport line. Let errors in this cell be such that they produce an orbit displacement e_n and angle e'_n at the end of the cell where the n -th BPM is located. The orbit at the entrance to the n -th cell has been corrected so that it has a zero displacement but has an angle of x'_{n-1} .

At this point, we leave the type of orbit correctors open, except that its strength in the n -th cell will be specified by θ_n . The orbit at end of the n -th cell is then given by

$$\begin{pmatrix} x_n \\ x'_n \end{pmatrix} = T \begin{pmatrix} 0 \\ x'_{n-1} \end{pmatrix} + \theta_n \begin{pmatrix} n \\ n' \end{pmatrix} + \begin{pmatrix} e_n \\ e'_n \end{pmatrix}$$

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where T is the 2×2 transfer matrix for the cell, η and η' are the orbit responses to the corrector at the end of the cell.

If the corrector is represented as a short dipole kick, then θ_n is equal to the kick angle and η, η' are the 12- and 22- elements of the transfer matrix from the corrector to the monitor.

Eq. (1) can be rewritten as 2 equations

$$x_n = t_{12} x'_{n-1} + \eta \theta_n + e_n \quad (1a)$$

$$x'_n = t_{22} x'_{n-1} + \eta' \theta_n + e'_n \quad (1b)$$

The corrector strength is determined by the condition that $x_n = 0$. This gives, using (1a),

$$\theta_n = - \frac{t_{12} x'_{n-1} + e_n}{\eta} \quad (2)$$

Substituting into eq. (1b) then gives the angle at the exit of the n -th cell

$$x'_n = A x'_{n-1} + B_n \quad (3)$$

where

$$A = t_{22} - \frac{\eta'}{\eta} t_{12} \quad (4a)$$

$$B_n = e'_n - \frac{\eta'}{\eta} e_n \quad (4b)$$

Eq. (3) shows that the angular divergence of the orbit at the BPM's after orbit correction contains two terms. The first term comes from the propagation of the residual angular divergence upstream of the cell under consideration. The second term is the noise contribution from errors in the cell. Furthermore, the first term is "damped" by the factor A per cell (when $|A| < 1$). As a result, the rms orbit contains a damping on the one hand and a noise on the other hand, and at equilibrium it will acquire an equilibrium value. The condition $|A| < 1$ is Murray's stability criterion.

It may be instructive to apply the results to the case when the corrector is a δ -function kick. We have

$$\theta_n = \text{kick angle}$$

$$\eta = \sqrt{\beta_c \beta_m} \sin \psi \quad (5a)$$

$$\eta' = \sqrt{\frac{\beta_c}{\beta_m}} (\cos \psi - \alpha_m \sin \psi) \quad (5b)$$

where the subscripts c and m refer to the corrector and the monitor locations, respectively; ψ is the phase advance from corrector to monitor. In addition, we have

$$t_{12} = \beta_m \sin \psi_{\text{cell}} \quad (6a)$$

$$t_{22} = \cos \psi_{\text{cell}} - \alpha_m \sin \psi_{\text{cell}} \quad (6b)$$

where ψ_{cell} is the phase advance per cell. Substituting eqs. (5) and (6) into (4a), the coefficient A is found to be

$$A = - \frac{\sin(\psi_{\text{cell}} - \psi)}{\sin \psi} \quad (7)$$

The coefficient A plays the role of a "damping constant". Note that expression (7) is independent of the β -functions. A strong damping (which is preferred) requires A close to 0. This is achieved if $\psi_{\text{cell}} - \psi$ (the phase advance from monitor to corrector) is close to 0 or a multiple of π and if ψ (the phase advance from corrector to the next monitor) is away from a multiple of π . For instance, one optimum arrangement is to have 90° cells and to have the correctors immediately downstream of the monitors, which is not a surprising result.

2. RMS Orbit After Correction

We parametrize the quality of orbit correction by the rms orbit slopes at the BPM's. As mentioned before, this rms value is determined by a balance between a damping effect and a noise diffusion effect. From eq. (3), we have

$$x_n' = B_n + A B_{n-1} + A^2 B_{n-2} + \dots \quad (8)$$

Squaring the quantity and taking the expectation value assuming no correlations among errors in different cells, we obtain

$$\begin{aligned} \langle x'^2 \rangle &= \langle B^2 \rangle (1 + A^2 + A^4 + \dots) \\ &= \begin{cases} \frac{\langle B^2 \rangle}{1-A^2} & \text{if } |A| < 1 \\ \text{diverges} & \text{if } |A| \geq 1 \end{cases} \quad (9) \end{aligned}$$

The orbit correction scheme breaks down if $|A| \geq 1$. What happens then is that the orbit slope at the BPM's continue to grow as sketched below:

Fig. 2

Eq. (9) gives the value of $\langle x'^2 \rangle$ at the BPM's once A and $\langle B^2 \rangle$ are known. It is clear that to keep the rms orbit small, we need to make A not too close to 1 and to keep $\langle B^2 \rangle$ small.

To obtain an expression for $\langle B^2 \rangle$, we need to know the possible sources of errors. Let the k -th error have strength θ_k and its orbit at the end of the cell be $e = \theta_k \eta_k$ and $e' = \theta_k \eta_k'$. Then we have

$$\langle B^2 \rangle = \sum_k \langle \theta_k^2 \rangle (\eta_k' - \frac{\eta'}{\eta} \eta_k)^2 \quad (10)$$

where the summation is over all possible sources of error in one cell. Note that if the error source is the same as that used as the corrector, it does not contribute to orbit error after correction since $\eta = \eta_k$ and $\eta' = \eta_k'$.

3. BPM Errors

So far we have assumed that the orbit displacements at the BPM's are perfectly corrected. This will not be the case if the BPM readings have an rms error Δ . To include this effect, a more complicated calculation similar to that of the previous section is carried out. The result is that instead of Eq. (9) we have

$$\langle x'^2 \rangle = \frac{\langle B^2 \rangle}{1-A^2} + \langle \Delta^2 \rangle \left[\left(\frac{n}{\eta} \right)^2 + \frac{D^2}{1-A^2} \right] \quad (11)$$

where

$$D = t_{21} - \frac{n}{\eta} (t_{11} - A) \quad (12)$$

Compared with Eq. (9), the rms orbit has acquired another term proportional to $\langle \Delta^2 \rangle$. To obtain a feeling on the coefficient D, consider the case of a δ -function corrector. We have

$$t_{21} = -\frac{1}{\beta_m} (1 + \alpha_m^2) \sin \psi_{cell} \quad (13a)$$

$$t_{11} = \cos \psi_{cell} + \alpha_m \sin \psi_{cell} \quad (13b)$$

which gives

$$D = -\frac{1}{\beta_m} \frac{\sin \psi_{cell}}{\sin^2 \psi} \quad (14)$$

To minimize the effect of BPM errors, we need to have $D^2/(1-A^2)$ as small as possible. This requirement is not the same as the previous requirement that $|A|$ should be minimized. Depending on the magnitude of the BPM errors, therefore, some compromise has to be made. If we let the corrector be immediately downstream of the monitor and let $\psi_{cell} = 90^\circ$ as the example we mentioned before, we have $A = 0$ and eq. (11) reads

$$\langle x'^2 \rangle = \langle B^2 \rangle + \langle \Delta^2 \rangle \frac{1 + \alpha_m^2}{\beta_m^2} \quad (15)$$

4. RMS Corrector Strength

The corrector strength in the absence of BPM errors is given by (2). If the BPM errors are included, the rms corrector strength is found to be

$$\langle \theta_{\text{corr}}^2 \rangle = \frac{1}{\eta^2} \langle \Delta^2 \rangle + \left(\frac{t_{12}}{\eta} \right)^2 \langle x'^2 \rangle + \frac{1}{\eta^2} \langle e^2 \rangle \quad (16)$$

where $\langle x'^2 \rangle$ is given by eq. (11) and

$$\langle e^2 \rangle = \sum_k \langle \theta_k^2 \rangle \eta_k^2$$

5. Application to SLC Arc - Scheme 1

A cell in the SLC arc consists of 2 magnets. In this section, we consider horizontal(*) orbit correction with the following arrangement (which is one of the possible schemes presently considered by the Beam Dynamics Task Force):

Fig. 3 Orbit Correction Scheme 1

The corrector is assumed to be moving the F magnets horizontally. The corrector strength θ is chosen to be the amount of movement (in μm). The relevant parameters found by the TRANSPORT and TURTLE programs are

$$\begin{aligned} t_{11} &= -2.812 \\ t_{12} &= 3.821 \\ t_{21} &= -1.877 \\ t_{22} &= 2.194 \\ \eta &= -5.514 \mu\text{m}/1 \mu\text{m movement} \\ \eta' &= -3.449 \mu\text{rad}/1 \mu\text{m movement} \end{aligned}$$

The coefficient A is then found from eq. (4a) to be

$$A = -0.196$$

The orbit errors are assumed to come from magnet misalignments. The misalignment

(*) The results also apply to the vertical orbit corrections in which case the D magnets are used as correctors.

of the F magnet (which is the corrector) does not contribute to orbit error after correction. The misalignment of the D magnet gives, using eq. (10),

$$\langle B^2 \rangle = \langle \theta^2 \rangle \left(n_D' - \frac{n'}{n} n_D \right)^2 \quad (17)$$

Again using 2nd order TURTLE, we find

$$n_D = 1.702$$

$$n_D' = 1.573$$

If the rms magnet misalignment is $\langle \theta^2 \rangle^{1/2} = 100 \mu\text{m}$, we have from eq. (17)

$$\langle B^2 \rangle^{1/2} = 50.84 \mu\text{rad}$$

Substituting the values of A and $\langle B^2 \rangle$ into eq. (9) gives

$$\langle x'^2 \rangle^{1/2} = 51.85 \mu\text{rad} \quad (18)$$

The orbit angle calculated above should be rather accurate. To translate it into a rough estimate of the orbit, note that the orbit has been corrected to zero at the monitor so that the maximum orbit displacement tends to occur at the center of the D-magnets. Transferring $\langle x'^2 \rangle^{1/2}$ to the orbit in the D-magnets gives an orbit of $103.07 \mu\text{m}$. A similar transfer gives an orbit of $59.6 \mu\text{m}$ and $79.3 \mu\text{m}$ at the center of the F-magnet and the gap between the two magnets, respectively. If we take the rms orbit to be the rms of the orbit at the BPM, the F and the D magnets, and the gap, we obtain

$$\langle x^2 \rangle^{1/2} = 71.5 \mu\text{m}$$

To take into account of the BPM errors, we use eq. (11). Substituting the values into eq. (12) yields

$$D = -0.241$$

If we assume $\langle \Delta^2 \rangle^{1/2} = 100 \mu\text{m}$, then

$$\langle x'^2 \rangle^{1/2} = 84.68 \mu\text{rad} \quad (19)$$

The rms orbit error due to this $\langle x'^2 \rangle^{1/2}$ is about 116.8 μm . On top of this, there is a contribution from the BPM misalignments. Adding the contributions together gives an rms orbit error of

$$\langle x^2 \rangle^{1/2} = 153.7 \mu\text{m}.$$

Comparing (18) and (19), it is clear that the BPM errors contribute significantly to the arc orbit correction and must be taken into consideration.

To find the rms corrector strength, one can use eq. (16). The result is

$$\langle \theta_{\text{corr}}^2 \rangle^{1/2} = 121.34 \mu\text{m}$$

for the needed rms magnet movements. This number becomes 110.65 μm if there are no BPM errors. The movement doubles if magnets are moved with one end fixed.

As a comparison, one might ask what if the D magnets are used as the horizontal correctors. Such a correction scheme does not work at all. This is because $A = -1.337$ and the corrected orbit diverges as shown in Fig. 2.

6. Application to SLC Arc - Scheme 2

Another orbit correction that has been considered in the past is the following

Fig. 4 Orbit Correction Scheme 2

The difference from Scheme 1 is that now each cell contains 2 correctors and 2 monitors instead of one corrector and one monitor.

A calculation of the orbit angles at the two BPM's can be performed similarly to what was done before. The rms values of these quantities are found to be

$$\langle x_1'^2 \rangle = \frac{1 + t_{11}^2}{t_{12}^2} \langle \Delta^2 \rangle + \frac{1}{t_{12}^2} \langle e^2 \rangle \quad (20)$$

$$\langle x_2'^2 \rangle = \frac{1 + t_{22}^2}{t_{12}^2} \langle \Delta^2 \rangle + \langle (e' - \frac{t_{22}}{t_{12}} e)^2 \rangle$$

where T is the 2 x 2 transformation matrix between the two BPM's, e and e' are the orbit and angle at the second BPM due to errors of the two magnets between the two BPM's.

Note that unlike the correction scheme studied in the previous section, the orbit error does not propagate from cell to cell. The rms orbit is determined solely by the noise in the cell under consideration and there is no "damping effect" as the previous scheme.

Using the numbers from the previous section, we have

$$\langle e^2 \rangle = \langle \theta^2 \rangle (\eta_F^2 + \eta_D^2) = 33.30 \langle \theta^2 \rangle \quad (21)$$

$$\begin{aligned} \langle (e' - \frac{t_{22}}{t_{12}} e)^2 \rangle &= \langle \theta^2 \rangle [(\eta_F' - \frac{t_{22}}{t_{12}} \eta_F)^2 + (\eta_D' - \frac{t_{22}}{t_{12}} \eta_D)^2] \\ &= 0.435 \langle \theta^2 \rangle \end{aligned}$$

Substituting (21) into (20) and assuming $\langle \Delta^2 \rangle^{1/2} = \langle \theta^2 \rangle^{1/2} = 100 \mu\text{m}$, we find

$$\begin{aligned} \langle x_1'^2 \rangle^{1/2} &= 170.0 \mu\text{rad} \\ \langle x_2'^2 \rangle^{1/2} &= 91.28 \mu\text{rad} \end{aligned} \quad (22)$$

In case there is no BPM error, i.e. $\langle \Delta^2 \rangle = 0$, we find

$$\begin{aligned} \langle x_1'^2 \rangle^{1/2} &= 151.02 \mu\text{rad} \\ \langle x_2'^2 \rangle^{1/2} &= 66.0 \mu\text{rad} \end{aligned} \quad (23)$$

Comparing (22) and (23) with (18) and (19), it can be noted that the results are substantially worse than those of Scheme 1. In Scheme 1, according to eq. (17), the errors introduced by the corrector F-magnet itself is effectively suppressed. This is not the case in the present correction Scheme 2. Furthermore, the needed corrector strength will be higher in the present scheme since the D-magnets are less effective as horizontal orbit correctors.

However, our calculations do not allow one to draw the conclusion that Scheme 1 is more advantageous than Scheme 2 because we have not studied all possible sources of errors. For example, one error source that Scheme 1 is more sensitive to and we have not studied is the setting errors in magnet movers (since magnets moved in scheme 1 are the more effective ones)*.

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* The effect of magnet mover setting errors on orbit depends on how many magnets are moved at one correction step. The case when each step consists of 20 magnets (one achromat) has been studied in CN-252.

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