Numeric Modeling Sensitivity Analysis of the Performance of Wind Turbine Arrays

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June 1982

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(a) Now at Flow Research in Seattle, Washington

SUMMARY

An evaluation of the numerical model created by Lissaman for predicting the performance of wind turbine arrays has been made. Model predictions of the wake parameters have been compared with both full-scale and wind tunnel measurements. Only limited, full-scale data were available, while wind tunnel studies showed difficulties in representing real meteorological conditions. Nevertheless, several modifications and additions have been made to the model using both theoretical and empirical techniques and the new model shows good correlation with experiment. The larger wake growth rate and shorter near wake length predicted by the new model lead to reduced interference effects on downstream turbines and hence greater array efficiencies. The array model has also been re-examined and now incorporates the ability to show the effects of real meteorological conditions such as variations in wind speed and unsteady winds. The resulting computer code has been run to show the sensitivity of array performance to meteorological, machine, and array parameters. Ambient turbulence and windwise spacing are shown to dominate, while hub height ratio is seen to be relatively unimportant. Finally, a detailed analysis of the Goodnoe Hills wind farm in Washington has been made to show how power output can be expected to vary with ambient turbulence, wind speed, and wind direction.

NOMENCLATURE

C_D	drag coefficient of blade profile
D	turbine diameter
m	U_o/U_{mo}
r	local radius
r_c	wake radius at end of near wake
r_f	wake radius at start of far wake
r_o	initial wake radius
r_w	local wake radius
r'	core radius
R	turbine radius
u	local wind speed
U_o	free stream wind speed
U_{mo}	core wind speed
X	downstream distance
X_c	distance to end of near wake
X_F	distance to start of far wake
α	transverse turbulence intensity
σ	rotor solidity
λ	tip speed ratio
Δu	local velocity deficit
ΔU_m	centerline velocity deficit
ΔU_{mo}	core velocity deficit

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1. INTRODUCTION

1.1 Background

Three fundamental questions a customer for a wind energy assessment typically asks are:

1. What specific sites are suitable for wind turbine placement or where can machines be placed?
2. What is the projected energy output of wind turbines installed at these sites or how well will the machine operate?
3. What is the projected capacity of these areas or how many machines can be installed?

A knowledge of wind turbine performance, coupled with the effects of array spacing, is necessary to answer these questions.

The successful installation of wind turbine systems depends, to a large extent, upon an effective siting methodology and technology. The siting technology must provide detailed guidelines for the optimum distribution and installation of wind turbines. The knowledge of wind turbine performance as a function of array spacing is an important consideration for siting technology. Remote sites which can only accommodate a limited number of turbines may well be uneconomical since site development costs cannot be balanced by revenues (Zalay and Arcemont, 1981).

Wind turbine operation within existing electrical networks is important for utility integration. Efforts to date have been focused on the integration of one or more independent wind turbine units with the utility grid (Justus and Mikhail 1978; Schlueter et al., 1979). However, the sensitivity of the power output of integrated wind turbine arrays to variations in wind characteristics has not been established. Therefore,

the operation of large wind turbine arrays within existing electrical networks has been modeled only approximately.

The significance of array design on the total output of a wind farm is well recognized. Currently, wind turbine arrays are in the planning or construction phase in Hawaii, California and New Hampshire. A cluster of three MOD-2 turbines is undergoing testing at Goodnoe Hills in Washington to determine the influence of wake interference on turbine operation. In order to meet the needs of planners and designers, efforts are under way to develop methods for predicting performance of wind turbine arrays.

Tools being developed to characterize wind turbine array performance include mathematical models and experimental techniques such as wind-tunnel simulation or field surveys. The first mathematical model for evaluation of array effectiveness for arbitrary arrays of turbines in a general wind field was published by Lissaman and Bate (1977). This paper has provided the basis for essentially all analytical models of wind turbine array effectiveness.

Since the publication of the original Lissaman-Bate model, additional information has been obtained on turbine wake interference from wind tunnel tests and field surveys. Recognizing the need to validate and expand the Lissaman-Bate model, AV undertook the present program under the sponsorship of the U.S. Department of Energy (DOE) through PNL.

1.2 Program Objectives

The analysis of the performance of wind turbine arrays is focused on the following objectives:

- Verification of the wind turbine wake interference model.
- Analysis of the effects of wind characteristics, wind turbine operating characteristics and array configuration.
- Applications of the model to the MOD-2 test program at Goodnoe Hills.

1.3 Approach

To carry out the sensitivity study, an evaluation was made of the original wake model and available wake measurements. The wake model was modified using, as inputs, the additional data base, and the new wake model was evaluated and incorporated into a new array performance model. The approach for evaluating the wake model included (1) survey and summary of available wake measurements, (2) simulation of measurements using the original wake model, and (3) comparison of measured and predicted wake parameters to identify systematic sources of error and to assess the accuracy of the model and the validity of the underlying assumptions.

Modifications were made to the original model to reconcile inconsistencies noted in these comparisons. Consideration was given, in particular, to modifying the turbulence growth algorithms and to including the effects of viscous rotor drag.

Additional new capabilities were sought for the upgraded model including (1) incorporation of wind turbine performance characteristics of power output versus wind speed, (2) ability to model the effects of unsteady wind velocity, and (3) ability to model the effects of nonuniform wind direction.

The updated and validated model was exercised to examine the sensitivity of the performance of wind turbine arrays to wind characteristics, wind turbine operating characteristics, and array configuration. The sensitivity analysis considered variations in array power output as functions of (1) atmospheric turbulence level, (2) mean wind speed, (3) ratio of wind turbine hub height to rotor diameter, (4) wind turbine performance curve, and (5) longitudinal and lateral array spacing.

Finally, the model was exercised to provide information on the anticipated performance of the array of three MOD-2 wind turbines at Goodnoe Hills, Washington. Predictions of power output as a function of wind speed, wind direction, and turbulence were made for individual machines as well as for the array as a whole.

The following chapters discuss the details of the above approach.

2. EVALUATION OF THE ORIGINAL WAKE MODEL

2.1 Descriptive Summary of the Original Wake Model

A full description of the theory and concepts used in the original model has been given by Lissaman (1977) and Lissaman and Bate (1977). It is useful at this stage to summarize the approach which was taken.

The air passing through a wind turbine is slowed down as kinetic energy is removed by the power extraction process. The wake of a single turbine may therefore be expected to be quite similar, fluid mechanically, to that of a slower moving circular jet or wake immersed in a faster moving, uniform outer flow. This assumed similarity acts as a basis for the model, and the work of Abramovich (1963) on co-flowing jets has provided valuable experimental results on the characteristics of turbulent axisymmetric wakes.

Figure 2-1 shows the three basic regions which can be distinguished in the wake -- an initial region (near wake), a fully-developed region (far wake), and between these a transition region. The initial velocity ratio of the jets is determined by the drag coefficient at which the turbine is operating since this is a measure of the momentum removed by the machine. The inner low-speed flow, sometimes called the potential core, is gradually eroded away in the near wake by turbulent mixing. The outer wake radius also grows by turbulent entrainment at its edge. However, the momentum deficit in the wake, which represents the drag on the turbine, must be conserved at all downstream stations. If, therefore, we know the shape of the velocity profile in the wake and the turbine drag, the actual magnitude of the velocity deficit at a given downstream station is determined by the wake radius alone. In the near wake, this profile consists of a characteristic shear layer in the mixed zone outside the core and a uniform deficit within the core. The far wake has a self-similar profile, which is assumed to be of the form given by Abramovich for co-flowing jets. The transition region allows for a smooth transformation between these shapes. Figure 2-2 compares the velocity profiles in the various regions.

The wake radius grows at a rate which is directly proportional to the total cross-flow turbulence intensity at the wake edge. Turbulence generated by the profile drag of the turbine blades was not included in the original model so that this turbulence

FIGURE 2-1. Basic wake geometry.

FIGURE 2-2. Comparison of normalized velocity profiles in the original wind turbine wake model.

level was taken to be a combination of turbulence generated by shears within the wake and the ambient turbulence in the incident flows. The shear-generated turbulence is a function of both the centerline velocity deficit and the wake radius, and becomes smaller and smaller downstream. The original model divided the far wake into two parts: in the first, both shear-generated turbulence and ambient turbulence were included in the expression for wake growth rate; in the second, further downstream, shear-generated turbulence was shown to be negligible, so that ambient turbulence controls wake growth here.

Appendix A summarizes the wake equations used in the original model.

2.2 Review of Available Wind Turbine Wake Measurements

Although wind turbine wake studies have been made since the completion of the first analytic model by Lissaman in 1977, there is not enough data available for definitive conclusions to be drawn as to the model's validity because of the many variables and the inherent scatter in measurements of turbulent processes. There is, however, enough data to suggest that the model is not seriously incorrect. A survey has been made of all known data sources and, wherever applicable, model simulations have been compared with these experimental results in an attempt to improve and consolidate the model. Section 2.3 reports on the most significant comparisons which have been made and Chapter 3 explains the modifications to the original wake model which have been found necessary.

The 2nd and 3rd International Symposia on Wind Energy Systems (1978 and 1980) provide a useful review of recent wind turbine wake measurement programs. Of particular interest is the paper by Vermeulen (1980) who has already attempted to compare and modify the original Lissaman model using semi-empirical techniques. His work has served as a starting point in the development of a new model.

Unfortunately, most of the available measurements have been made within about 10 diameters from the rotor so that information is not available at extended downstream distances, which is of practical interest for array planning. The majority of experimental data relates to wind tunnel studies using model turbines. However the wake of a model wind turbine may be different from a full scale wake because of Reynolds number effects or

because only a simple gauze screen model of the turbine was used to simulate the energy extraction process. This will transfer mean flow energy to the turbulence but not remove energy from the flow, as is the case with a wind turbine. In wind tunnel studies there are also difficulties in representing real meteorological conditions. In the equilibrium planetary boundary layer, the shear profile continually generates turbulence at a rate which is just balanced by the decay rate of the small-scale turbulent eddies. This produces an ambient turbulence intensity which varies only with height and not with downstream distance. However, in a wind tunnel test, if turbulence is created by an upstream grid, then the turbulence intensity decays continually downstream since there is no further generating mechanism. This effect is shown in Figure 2-3, which is taken from the work of Alfredson et al. (1980). Because of this, the growth rate of the turbine wake in a wind tunnel test will decrease downstream and the measured velocity deficits will be larger than expected. If a proper boundary layer is simulated in the tunnel, however, the turbulence will remain approximately constant, as in Figure 2-4, again taken from the work of Alfredson et al. (1980).

In all the experiments, only longitudinal turbulence levels were measured, whereas it is the transverse value which is important, because this is what actually causes lateral wake expansion. Since turbulence is generally not isotropic in a boundary layer, a relationship between these two turbulence components is required in order to exercise the model. Section 4.1 describes such a relationship.

In addition to these problems, the values of critical operating parameters have not been reported in some of the experiments so that complete model simulations have not always been possible.

Despite these difficulties, some important conclusions have been made from the comparisons discussed in Section 2.3. Following the modifications to the model described in Chapter 3, the comparisons have been repeated and show an improved correlation with experiment.

A list of relevant work which has been studied is given in Table 2-1.

FIGURE 2-3. Decay of grid-generated wind tunnel turbulence.

FIGURE 2-4. Persistence of boundary layer-generated wind tunnel turbulence.

TABLE 2-1. Sources of experimental data reviewed for comparison with model.

- Alfredson, P.H., and J.A. Dahlberg (1979): "A Preliminary Wind Tunnel Study of Windmill Wake Dispersion in Various Flow Conditions." FFA-TN-AV-1499, Part 7, Stockholm, Sweden.
- Alfredson, P.H., J.A. Dahlberg, and F.H. Bark (1980): "Some Properties of the Wake Behind Horizontal Axis Wind Turbines." Third International Symposium on Wind Energy Systems, Copenhagen, Denmark.
- Boschloo, G. (1977): "Wake Structure of a Darrieus Rotor." TNO Report 77-07244, Apeldoorn, The Netherlands.
- Brons, M. B. Jensen, N. Jensen, and I.N. Petersen (1979): "Darrieus Mollen." Report Fysik 3, University of Copenhagen, Denmark.
- Builtjes, P.J.H. (1978): "The Interaction of Windmill Wakes." Second International Symposium on Wind Energy Systems, Amsterdam, The Netherlands.
- Builtjes, P.J.H. and D.J. Milborrow (1980): "Modeling of Wind Turbine Arrays." Third International Symposium on Wind Energy Systems, Copenhagen, Denmark.
- Faxen, T. (1978): "Wake Interaction in an Array of Windmills." Second International Symposium on Wind Energy Systems, Amsterdam, The Netherlands.
- Hansen, A.C. (1980): "Early Results From the SWECS Rotor Wake Measurement Project." Rocky Flats Wind Systems Program Report TM-TD/81-4.
- Milborrow, D.J. (1980): "The Performance of Arrays of Wind Turbines." Journal of Industrial Aerodynamics 5.
- Vermeulen, P.E.J. (1978): "A Wind Tunnel Study of the Wake of a Horizontal Axis Wind Turbine." TNO Report 78 -09674, Apeldoorn, The Netherlands.
- Vermeulen, P.E.J. (1979): "Mixing of Simulated Wind Turbine Wakes in Turbulent Shear Flow." TNO Report 79-09974, Apeldoorn, The Netherlands.
- Vermeulen, P.E.J. (1979): "Studies of the Wake Structure of Model Wind Turbine Generators." TNO Report 79-012904, Apeldoorn, The Netherlands.
- Vermeulen, P.E.J. (1980): "An Experimental Analysis of Wind Turbine Wakes." Third International Symposium on Wind Energy Systems, Copenhagen, Denmark.
- Vermeulen, P.E.J., P. Builtjes, J. Dekker, and G.L. vanBueren (1979): "An Experimental Study of the Wake Behind a Full-Scale Vertical Axis Wind Turbine." TNO Report 79-06118, Apeldoorn, The Netherlands.

2.3 Comparison of Observed and Predicted Wake Characteristics

While the model assumes a constant centerline velocity deficit in the near wake and transition region, experimental measurements (Alfredson et al., 1980) indicate an initially large deficit close to the turbine which falls off continually downstream (see Figure 2-5). This raises some questions about the experimental measurement, since deficits this large are not consistent with realistic drag coefficients. However, the distance to the start of the far wake, X_F , is typically only a few (~ 5) turbine diameters and it is unlikely that machines will ever be packed closer than this in the wind direction because of the severe interference which would result. Normally, downwind turbines will be positioned to experience only the far wake effects of upwind machines. Thus, the near and transition wakes are of interest primarily for establishing the two initial conditions for the far wake -- the downstream distance, X_F , and the initial centerline velocity deficit. The assumed far wake velocity profile and the turbine drag then give the initial far wake radius r_F and the corresponding flow velocity at any radial station. The growth rate and the assumed self-similarity of the far wake can then be used to calculate the condition at all points further downstream.

In order to compare experimental and simulated values of X_F , it is convenient to define the experimental value as the downstream distance at which the centerline velocity deficit equals the simulated value at the end of the transition region (i.e., the downstream distance at which $\Delta U_m/U_o = (m-1)/m$). This follows the definition given by Vermeulen (1979).

Table 2-2 compares model predictions of X_F with the measured values given by Vermeulen in his summary of experimental work. All these experiments were carried out in wind tunnel tests in which only longitudinal turbulence measurements were made. Also, most of the experiments employed grid-generated turbulence so that strong turbulence decay occurred. In the case of the two simulated boundary layer tests, since the turbulence level varies with height, an estimate has to be made of the mean turbulence intensity at the wake edge.

The values for cross-flow turbulence which appear in Table 2-2 are best estimates based on the above considerations. It can be seen that there is a large discrepancy between the values for X_F predicted by the original model and the experimental data.

FIGURE 2-5. Comparison of centerline velocity deficit predictions of original model with measured values.

TABLE 2-2. Predicted and observed distances to start of far wake.

Reference	Type of Wind Turbine	Cross-Flow Turbulence (%)	m	X_F/D		
				Measured	Original Model	New Model
Alfredson, Dahlberg	Horizontal Axis	0.6	2.03	7.2	14.0	6.6
		3	2.03	5.2	10.5	5.7
		6.5	2.03	4.3	6.5	4.1
		7*	2.03	3.2	6.1	3.9
		8.5*	2.03	2.7	5.2	3.4
Vermeulen	Horizontal Axis	1	1.61	6.6	18.1	7.8
		1	1.96	5.4	14.1	5.5
		1	2.56	4.2	11.4	4.2
		3.5	1.96	4.0	9.9	4.8
Boschloo	Vertical Axis	1	1.44	7.0	21.7	9.5
Vermeulen	Vertical Axis	3.5	2.00	4.5	9.8	5.8

*experiment used a simulated boundary layer

This is attributed to the basic assumptions made in the original model, such as the neglect of viscous rotor drag. In the original model, the far wake distance is given by the equations

$$X_C = r_o \left\{ \left(\frac{dr}{dx} \right)_a^2 + \left(\frac{dr}{dx} \right)_m^2 \right\}^{-1/2} \quad (1)$$

and

$$X_F = n X_C ,$$

where n and r_o are fixed functions of m , $(dr/dx)_a$ is the growth rate due to ambient turbulence (generated by boundary layer shears), and $(dr/dx)_m$ is the growth rate due to turbulence generated by wake shears (also a fixed function of m).

As a first attempt to improve on this correlation the expression for $(dr/dx)_a$ has been modified in the light of further experimental data. Figure 2-6 compares the variation of $(dr/dx)_a$ with transverse turbulence intensity given by experiment with model predictions. The experimental data points shown are those given by Vermeulen (1980) and all refer to various wind tunnel tests. The transverse turbulence intensity must be assumed because only longitudinal turbulence values were reported and because of the problems of decaying turbulence discussed in Section 2.2. The observed mismatch led to a re-evaluation of the turbulent growth algorithms and the derivation of a new relationship which is explained in Section 3.1. The larger values of $(dr/dx)_a$, which results from the new expression, also shown in Figure 2-6, improve the model predictions of X_F , but correlation with experiment was still considered unsatisfactory.

As a second attempt to improve the prediction, an expression for the growth rate due to turbulence generated by the viscous drag of the rotor, $(dr/dx)_\lambda$, was included in the expression for X_C so that in the new model,

$$X_C = r_o \left\{ \left(\frac{dr}{dx} \right)_a^2 + \left(\frac{dr}{dx} \right)_m^2 + \left(\frac{dr}{dx} \right)_\lambda^2 \right\}^{-1/2} \quad (2)$$

FIGURE 2-6. Comparison of wake growth models due to ambient turbulence.

This method of addition of growth rates results from a simple linear addition of turbulent energy components, assuming their length scales are comparable. Section 3.1 explains the assumption that wake growth rate is directly proportional to the rms cross flow turbulent velocity. Section 3.2 explains how an expression for $(dr/dx)_\lambda$ was derived. Table 2-2 also shows the predictions of X_F by the new model which includes both of these modifications.

The new model predicts the far wake distance, X_F , to within approximately 25% of the measured value. This is considered to be within the uncertainty in the value of transverse turbulence intensity, α . In order to make the model simulations, the values for tip speed ratio and rotor solidity given by Vermeulen (1980) were used and a blade profile drag coefficient of 0.01 was assumed.

3. MODIFICATION OF THE WAKE MODEL

3.1 Turbulent Growth Algorithm

As shown earlier in Figure 2-6, a new relationship between wake growth rate and turbulence intensity has been derived which matches available experimental data. The derivation of this new turbulent growth algorithm is discussed here. Since a wind turbine wake grows by the same mechanism as a plume, by turbulent mixing at its boundary, it is reasonable to assume that their respective expressions for growth rate will be of the same form. Using this assumption, the growth rate of a plume was first found and then rewritten for a turbine wake.

The initial growth rate of a plume is given by standard statistical theory as

$$s = v't \quad , \quad (1)$$

where s is the dispersion width of the plume, v' is the transverse turbulence velocity, and t is the time of growth. While this is theoretically valid only when t is less than the Lagrangian time scale of the motion (Slade, 1968), Pasquill (1962) has shown that in the planetary boundary layer, wake growth rate of a plume is constant for approximately 5 km in conditions of neutral stability so that this model is valid over the distance of interest. Thus, if the plume is immersed in a flow of mean speed, U_∞ , so that it convects a distance, $x = U_\infty t$ in time t , then the growth rate is given by

$$\frac{ds}{dx} = \frac{v'}{U_\infty} \quad . \quad (2)$$

We now define $\alpha = v'/U_\infty$ as the transverse turbulence intensity.

In order to rewrite the above expression for a wake, a relationship between s and the wake radius, R , must be found. A plume has a Gaussian distribution, given by

$$q = q_0 e^{-(r^2/2s^2)} \quad , \quad (3)$$

where q is the concentration at radius, r , and q_0 is the centerline value. The Lissaman wake model uses the distribution given by Abramovich for the velocity profile in the wake of co-flowing jets,

$$q = q_0 \left\{ 1 - \left(\frac{r}{R} \right)^{1.5} \right\}^2 . \quad (4)$$

Unlike the Gaussian, this profile is based on a finite wake radius, R , at which the velocity deficit reaches zero. When these profiles are matched we obtain

$$s = 0.36R , \quad (5)$$

and hence for the wake, we can write

$$\frac{dR}{dx} = \frac{\alpha}{0.36} . \quad (6)$$

The matched velocity profiles are shown in Figure 3-1 and the matching technique is discussed in Appendix B. Figure 2-6 shows the correlation of this new expression with experimental data.

Figure 3-2 shows, for both the new and old models, the relationship between $(dr/dx)_e$, the effective growth rate in the first part of the far wake and $(dr/dx)_a$, the growth rate due to ambient turbulence alone. These curves may also be interpreted in terms of effective and ambient transverse turbulence intensities, α_e and α , respectively, for either model by using the appropriate expression for growth rate. If we consider the case of a turbine operating at a typical power coefficient ($m = 2$), it can be seen that for the new model the difference between α and α_e is very small ($\sim 1\%$) even at low values of ambient turbulence ($\sim 3\%$). At the higher ambient turbulence levels which may be observed at wind farm sites, the difference between α and α_e becomes even smaller.

In view of other approximations and uncertainties, the two far wake regions have been merged in the new model, each now having a growth rate of $\alpha/0.36$ (the parameter α_e being eliminated from the model). This has simplified the computer program significantly without loss of accuracy for typical values of α and m . However, this new model should only be exercised for ambient turbulence levels greater than 3%. At values of $\alpha < 3\%$, error is introduced by the neglect of shear-generated turbulence in the far wake.

FIGURE 3-1. Comparison of Gaussian and model velocity profiles in the far wake.

FIGURE 3-2. Relationship between the wake growth rates in the two regions of the far wake.

3.2 Effects of Viscous Rotor Drag

As discussed earlier in Section 2.3, comparison with experiment suggests the need for including in the new model the turbulence generated by viscous rotor drag. Appendix C shows the derivation of an expression for α_λ^* , the turbulence intensity produced by the profile drag of the turbine blades:

$$\alpha_\lambda^* = \sqrt{\frac{C_D \sigma m \lambda^3}{6(m+1)}} .$$

This expression assumes that all energy lost due to blade profile drag appears as turbulent kinetic energy. In practice, this will not be so. Some of the energy lost will appear in an organized form as the wake of the blades which will be arrayed in helicoidal sheets by the rotation of the turbine disk. The balance of the energy will appear as approximately isotropic turbulence. The magnitude of this small-scale turbulence will quickly decay downstream, not only by dissipation into heat, but also as the volume through which it is dispersed increases. To take into account all these effects, a scaling factor, k , has been introduced so that

$$\alpha_\lambda = k \alpha_\lambda^* , \quad (7)$$

where α_λ is the effective turbulence intensity produced by viscous rotor drag in the near wake. Using the same relationship between turbulence intensity and wake growth rate as before, we have

$$\left(\frac{dr}{dx}\right)_\lambda = \frac{\alpha_\lambda}{0.36} . \quad (8)$$

In order to include this term, as described by Equation (2) (Section 2.3), the value of k should also include any effects produced by a difference in scale between the turbulent components. Coincidentally, comparison with experimental data, shown in Table 3-1, suggests a value for k of 0.36, so that

$$\left(\frac{dr}{dx}\right)_\lambda = \frac{C_D \sigma m \lambda^3}{6(m+1)} . \quad (9)$$

TABLE 3-1. Comparison of growth rates caused by rotor-generated turbulence.

Reference	Type of Wind Turbine	m	Tip Speed Ratio	Solidity	C_D	$(dr/dx)_x$	
						New Model ($k = 0.36$)	Measured
Alfredson	Horizontal Axis	2.03	4.6	0.140	0.01	0.12	0.11
Vermeulen	Horizontal Axis	1.61	5.0	0.071	0.01	0.10	0.12
		1.96	6.6	0.071	0.01	0.15	0.16
		2.56	8.5	0.071	0.01	0.23	0.23
Boschloo	Vertical Axis	1.44	4.4	0.064	0.01	0.07	0.10

This component of wake growth rate is then combined, as described before, assuming algebraic addition of turbulent energy components with that due to ambient and shear-generated turbulence to give the erosion rate of the potential core, so that in the new model,

$$\frac{r_o}{x_c} = \left\{ \left(\frac{dr}{dx} \right)_a^2 + \left(\frac{dr}{dx} \right)_m^2 + \left(\frac{dr}{dx} \right)_\lambda^2 \right\}^{1/2} . \quad (10)$$

This is the same as Equation (2). The new model assumes that the turbulence generated by viscous rotor drag has decayed to a negligibly small value in the far wake because of its small scale and, hence, short lifetime.

3.3 Pressure Variations in the Wake

In practice, significant pressure variations in the wake of a wind turbine only occur close to the turbine itself. The tubular vortex system shed by the rotor induces axial velocity and pressure variations in the flow through the turbine. Continuity requires that the radius of the stream tube passing through the edge of the turbine disk varies with downstream distance. If U_o is the freestream wind speed, b is the axial interference factor at a distance x (normalized to the turbine radius) downstream of the turbine, and a is the axial interference factor at the turbine disk, it can be shown (Wilson et al., 1976) that due to the wake vortex system,

$$\frac{b}{a} = 1 + \frac{x}{\sqrt{1+x^2}} . \quad (11)$$

But continuity requires that

$$r^2 (1 - b) = (1 - a) , \quad (12)$$

where r is the radius of the stream tube (normalized to the turbine radius) of the air passing through the turbine. These equations completely define the shape of this stream tube in terms of a , which in turn is a function of the power coefficient of the turbine.

Downstream, the stream tube radius quickly becomes asymptotic to the value, r_o , given by

$$r_o^2 = \frac{1 - a}{1 - 2a} \quad . \quad (13)$$

The model assumes that this expansion occurs at the disk itself so that the wake model develops from a "virtual" turbine disk of radius, $r = r_o$, at $x = 0$. In this way, the effects of pressure variations are properly accounted for. Figure 3-3 shows this geometry to scale for a 10% turbulence intensity and a worst case value for m of 3.

To test the accuracy of this assumption, the length of potential core, X_c , predicted by the model has been compared with an exact solution. The erosion rate of the core, (r_o/X_c) , has already been defined in Section 3.2. If the same constant erosion rate relative to the stream tube is assumed to define the core, an exact solution for the core length X_c can be found. Appendix D shows the details of this procedure. For typical turbine parameters, the model predictions are within 0.3% of the "exact" solution.

3.4 Wake Radius at the End of the Potential Core Region

In the original model, Equation 5.21' given by Abramovich was used to calculate the wake radius, r_c , at the end of the potential core region. This equation is an approximate solution to Equation 5.18. However, an exact solution exists for r_c given by Abramovich as

$$r_c = r_o (0.214 + 0.144 m)^{-1/2} \quad . \quad (14)$$

In the new model, this simple expression replaces the approximation used previously. Although this change will make an insignificant difference to the value of r_c , typically less than one percent, it has been included in the interests of completeness and simplicity.

3.5 Effect of the Planetary Boundary Layer

So far, no mention has been made of the effect of the wind speed profile in the planetary boundary layer on wind turbine performance. In order to estimate the

FIGURE 3-3. Effects of pressure variations on geometry of the near wake.

magnitude of this effect, a simple calculation has been made. We shall first make the approximation that the wind speed profile across the rotor is linear so that the velocity at any point on the turbine disk is given by

$$U = U_h \left(1 + \Delta u \left(\frac{r}{R} \right) \sin \theta \right) ,$$

where U_h is the velocity at hub height, R is the turbine radius, r is the radial distance at angle θ from the horizontal, and Δu is the normalized maximum velocity perturbation from the velocity at the hub height.

The mean effective wind speed, $\bar{U}$, can then be found by calculating the incoming power to the turbine

$$R^2 \bar{U}^3 = \int_0^R \int_0^{2\pi} U_h^3 \left(1 + \Delta u \left(\frac{r}{R} \right) \sin \theta \right)^3 r dr d\theta .$$

Thus, the ratio of the mean effective speed, $\bar{U}$, to the hub height speed is given by

$$\frac{\bar{U}}{U_h} = \left(1 + \frac{3}{4} \Delta u^2 \right)^{1/3} .$$

For the MOD-2 machine operating in a boundary layer with a wind exponent of 1/7, $\Delta u \approx 0.1$, so that

$$\frac{\bar{U}}{U_h} \approx 1.0025 .$$

This difference is small enough to be neglected, and thus validates the model assumption that the turbines are exposed to a uniform wind having the speed of the boundary layer at the turbine hub height.

4. EVALUATION OF THE NEW WAKE MODEL

4.1 Comparison of Observed and Predicted Wake Characteristics

Some details of the new wake model have already been compared with experimental measurements (Figure 2-3, Tables 2-2 and 3-1) and show an improved correlation. However, the most important parameter which the wake model provides is the magnitude of the velocity deficit in the far wake region. This parameter is very sensitive to wake growth rate and hence cross-flow turbulence intensity.

Since most tests have been done in wind tunnels with grid-generated turbulence and thus decaying turbulence levels, the data from such experiments must be interpreted very carefully. Figure 4-1 shows this effect in one of the tests run by Alfredson et al. (1980) when the ambient turbulence was generated by a coarse grid placed upstream of the wind turbine model. The turbulence intensity which was measured was the longitudinal value; however, it is the cross-flow or transverse value which is required by the model.

A scaling factor of 0.6 was used to convert longitudinal values to transverse turbulence intensities. This factor is based on the graph given by Reynolds (1974) for the component turbulence intensities in the fully turbulent region nearest the wall in pipe flow. Reynolds shows the two cross-flow components to be different as would be the case in the planetary boundary layer. This would produce an elliptical wake cross-section. To find an effective transverse component we assume that it would produce a circular cross-section wake of equivalent area; i.e., of radius which is the geometric mean of the major and minor half axes of the ellipse. Since the length of these axes is proportional to their respective components of turbulence intensity, we shall define the mean transverse component as the geometric mean of the two individual cross-flow components. The work of Counihan (1973) also suggests a value of about 0.6 in a simulated boundary layer.

Using this scaling factor and local values of coarse grid turbulence shown in Figure 2-3, the expression for centerline velocity deficit used by the model was exercised using variable decaying turbulence levels to simulate the experiment. The measured value of X_F and the centerline velocity deficit there were used as the initial conditions of the far

FIGURE 4-1. Centerline velocity deficit in the far wake using actual turbulence.

wake. The results of the simulation are shown in Figure 4-1, together with the experimental values measured by Alfredson et al. (1980). Also shown are curves for two constant transverse turbulence intensities.

Figure 2-4 shown earlier described the variation of longitudinal turbulence intensity in the simulated boundary layer used in another experiment performed by Alfredson. Also shown is the relative size and height of the model turbine used in a selected test. In the case of a properly simulated boundary layer, although the turbulence intensity does not decay downstream, it is difficult to estimate an effective transverse turbulence intensity causing wake growth because the principal effect is the value at the edge of the wake. The figure shows the variation in turbulence with height.

Using the same scaling factor of 0.6 as before, and estimating a mean longitudinal turbulence intensity of about 7%, a constant transverse value of 4% was used to simulate the test using the experimental values for the initial conditions of the far wake. This simulation, together with the measured values, is also shown in Figure 4-1.

Figure 4-2 compares a model simulation with data taken downstream of a full-scale, 5.3-m diameter, vertical axis wind turbine (Vermeulen et al., 1979). Vertical axis machines which sweep out an approximately circular area, and for which the ratio of sideways force to downstream force is small, may also be expected to produce axisymmetric wakes. The measured longitudinal turbulence intensity in the experiment was 20 to 30%. Using a scaling factor of 0.6 as before, we predict a transverse turbulence intensity of 12 to 18%. The figure shows the model simulation for this range and it can be seen that the two results do indeed overlap, although more data is needed to confirm this correlation.

Accurate far wake simulations depend on both accurate predictions of the initial conditions, X_F and r_F (whose values have already been compared with experiment in Section 2.3), and on the model of the far wake region itself. The growth rate of the far wake region has been discussed earlier, and we may have confidence that the momentum deficit in the wake is conserved downstream so that the only variable left is the actual shape of the far wake velocity profile. This has been taken to be that given by Abramovich for co-flowing jets in the fully developed region. Figure 4-3 compares this

FIGURE 4-2. Comparison of full-scale measurements of centerline velocity deficit with the predictions of the new model

FIGURE 4-3. Comparison of velocity deficit profiles behind a full-scale HAWT measured at approximately 5.5D downstream

profile with measurements made by Faxen (1978) behind a full-scale (60 kW) horizontal axis machine at two different wind speeds and shows excellent correlation.

4.2 Comments on the New Comparisons

The new wake model shows good correlation with the limited wake measurements available at this time. However, it must be remembered that the model predictions are highly sensitive to the value of transverse turbulence intensity. The measurements did not specify this value adequately for a definitive comparison to be made; transverse turbulence is not a parameter that is typically recorded in meteorological surveys or experiments, but in the future, careful attention will need to be given to this value if reliable predictions of array performance are to be made.

Figure 4-4 compares the wakes predicted by both models at representative values of C_p and turbulence. Because of the increased wake growth rate and shorter far wake distance predicted by the new model, the velocity deficit experienced by downwind machines is reduced. This means that the new array model will return slightly higher energy output predictions for a given wind farm than before and, thus, it will predict closer packing of wind turbines for the same value of array efficiency.

FIGURE 4-4. Comparisons of wake models.

5. MODIFICATION OF ARRAY PERFORMANCE MODEL

5.1 Linear Wake Superposition

To examine the validity of using linear wake superposition as a model for wake interaction, the behavior of a single line array of wind turbines was analyzed as follows. The turbines are numbered from 0 (the most upwind machine) and are equally spaced a distance, S , apart. The wake of each machine is assumed to grow at the same rate, so that at distance, nS , downstream of a turbine, its wake radius is R_n . If v is the velocity (normalized to the freestream value) incident on the turbine, then the velocity perturbation which it causes at this same station is given by vk_n . Linear wake superposition assumes that k_n only varies with n , whether or not other wakes are present.

Using this assumption, we can write that the velocity perturbation caused by turbine j and seen by turbine i , is given by

$$\delta v_{i,j} = v_j k_{i-j}, \quad (\text{where } i \geq j). \quad (1)$$

Thus, the total perturbation at turbine i is

$$\delta v_i = \sum_{j=0}^i v_j k_{i-j}. \quad (2)$$

The normalized velocity incident on the turbine is then

$$v_i = 1 - \sum_{j=0}^i v_j k_{i-j}. \quad (3)$$

But v_j is also of this form so that, neglecting second-order terms,

$$v_i = 1 - \sum_{j=0}^i k_{i-j}, \quad (4)$$

or, since $k_0 = 0$,

$$v_i = 1 - \sum_{j=1}^i k_j \quad (5)$$

It can be shown that application of this expression does not appear to conserve the momentum deficit produced by upstream turbines. However, the momentum deficit produced by the drag on each turbine must be conserved since this is the only force interaction with the wind produced by the presence of the array. To remedy this, a correction factor, μ , is introduced so that all velocity perturbations caused by turbine j are scaled by μ_j . When this is done, we obtain

$$\delta v_{i,j} = v_j \mu_j k_{i-j} \quad (6)$$

Now, the momentum deficit produced by turbine j , as seen at turbine i , is (as a first approximation) given by

$$D_{i,j} = \Phi R_{i-j}^2 \delta v_{i,j} \quad (7)$$

since, if S is large so that δv is small, it can be shown that for constant momentum deficit, $\delta v R^2$ is approximately constant (Φ being the necessary scaling factor to include density and numerical constants). Thus, the total momentum deficit seen at turbine i due to all upwind machines is

$$D_i = \sum_{j=0}^i \Phi R_{i-j}^2 v_j \mu_j k_{i-j} \quad (8)$$

But $\Phi R_m^2 k_m$ equals the momentum deficit, D_0 , produced by an isolated turbine and since it is assumed that momentum deficit is conserved, D_0 can be taken outside the summation sign to give

$$D_i = D_0 \sum_{j=0}^i v_j \mu_j \quad (9)$$

This momentum deficit must equal the sum of the momentum deficits of the individual upwind machines as obtained using the approximation that turbine drag varies as the square of the incident velocity. Hence,

$$D_i = D_o \sum_{j=0}^i v_j^2 . \quad (10)$$

Equating these expressions for D_i , we obtain

$$u_j = v_j , \quad (11)$$

so that if momentum deficit is to be conserved, the incident velocity at turbine i is given correctly by

$$v_i = 1 - \sum_{j=0}^i v_j^2 k_{i-j} , \quad (12)$$

which, when second-order quantities are neglected, reduces to

$$v_i = 1 - \sum_{j=0}^i k_{i-j} , \quad (13)$$

$$= 1 - \sum_{j=0}^i k_j . \quad (14)$$

It will be noted this is precisely the same expression produced by assuming linear wake superposition. Thus, although such an assumption does not appear to conserve momentum deficit, this is only because the resulting expression is a first-order approximation.

In view of this analysis, the linear wake superposition model will continue to be used in the new program.

5.2 Effect of Wind Turbine Performance Characteristics

Changes in wind turbine performance with wind speed directly affect array performance. Figure 5-1 compares the single machine characteristic of a typical pitch-

FIGURE 5-1. Effect of real wind turbine performance characteristic on array performance.

controlled wind turbine, such as the Boeing MOD-2, with the type of characteristic, normalized by total rotor area, which could be expected of an array of such machines where the entire array is regarded as a single wind machine. The difference between these curves can be explained in terms of the interference effects of the wakes which occur in an array.

Turbines normally operate at varying C_p , due principally to control action; however, constant C_p is a good approximation between cut-in and rated wind speeds (Figure 5-1). When the wind speed is above rated and power is spilled downstream by the feathering action of the machine control system, the power coefficient is reduced. Thus, arrays operate at a C_p which is a function of wind speed so that the array characteristic has the form shown in Figure 5-1.

In the new program, the array model is run for each wind angle, assuming that all machines are operating at rated power coefficient; the resulting efficiency, A , is then extrapolated to predict the array performance at other wind speeds. This method is simpler and quicker and consistent with other approximations than the laborious procedure of programming to account for individually varying power coefficients and hence different wake shapes. The method of developing the array characteristic curve from the single value of array efficiency, A , at machine rated wind speed, V_r , is given below.

We shall first consider a general array of N pitch-controlled turbines, with all machines operating at the same fixed power coefficient and with a freestream wind speed, V . If the effective incident wind speed at the i -th turbine is V_i , and the total normalized perturbation there is ΔV_i , then

$$V_i = V (1 - \Delta V_i) \quad . \quad (15)$$

If P_r is the rated power and V_c is the cut in velocity of a single machine, then the power produced by the i -th turbine, P_i , is given by the machine characteristic as

$$P_i = \frac{P_r}{V_r - V_c} (V_i - V_c) \quad . \quad (16)$$

If we define P_v as the slope of the machine characteristic, i.e.,

$$P_v = \frac{P_r}{V_r - V_c} , \quad (17)$$

then substituting for V_i , we can write

$$P_i = P_v (V - V_c - V \Delta V_i) . \quad (18)$$

Therefore, the power produced by an average machine in the array is

$$P_a = P_v (V - V_c - V \sum_{i=0}^N \frac{\Delta V_i}{N}) . \quad (19)$$

At rated freestream wind speed, this becomes

$$P_{ar} = P_v (V_r - V_c - V_r \sum_{i=0}^N \frac{\Delta V_i}{N}) . \quad (20)$$

But for an isolated machine

$$P_r = P_v (V_r - V_c) , \quad (21)$$

and the array efficiency at rated freestream, wind speed is given by

$$A = \frac{P_{ar}}{P_r} , \quad (22)$$

so that

$$P_r (1 - A) = P_v V_r \sum_{i=0}^N \frac{\Delta V_i}{N} . \quad (23)$$

Substituting into the expression for P_a gives

$$P_a = V \left\{ P_v - \frac{P_r}{V_r} (1 - A) \right\} - P_v V_c , \quad (24)$$

$$= P_v \left[V \left\{ 1 - \left(1 - \frac{V_c}{V_r} \right) (1 - A) \right\} - V_c \right] . \quad (25)$$

We now define $B = 1 - \left(1 - \frac{V_c}{V_r} \right) (1 - A)$

so that $P_a = B P_v \left(V - \frac{V_c}{B} \right)$.

Thus, the effective array cut in wind speed, V_{ca} , is given by

$$V_{ca} = \frac{V_c}{B} , \quad (26)$$

the slope of the array characteristic, P_{av} , is

$$P_{av} = B P_v , \quad (27)$$

and the array reaches rated power ($P_a = P_r$) at a wind speed

$$V_{ra} = \frac{V_r}{B} . \quad (28)$$

The cut-out wind speed of the array is the same as for a single machine, since as soon as the most upwind turbine sees a cut-out wind speed, it shuts down and no longer interferes with those machines further downstream which, in turn, also cut out. Since A varies with wind direction, θ , a separate array characteristic should be calculated for each value of θ .

This model becomes somewhat inaccurate at low wind speeds when only part of the array is operating and at speeds just above V_r when the most upwind machines have reached P_r and are spilling power. Figure 5-2 shows the approximation for the simple case of a 3-turbine, in-line array. It can be seen that the errors are of opposite sign and, as an estimation of the overall error which is introduced, a comparison of the energy output predicted by the array characteristic with that predicted by the proposed approximation was made for this simple array. This was done by assuming that the wind always blew along the line of turbines (a worst-case estimate) and using a Rayleigh distribution to describe the variation in wind speed (Cliff, 1977). This distribution was based on a site wind speed of 14 mph at 10 m (or 18 mph at MOD-2 hub height) to represent a suitable wind farm site. A turbulence level of 10% was assumed. To simplify the calculation, the wake of each turbine was taken to be that produced at a power coefficient of 0.56 (i.e., $m = 2$) until all three machines were producing rated power. This approximation provided a worst-case estimate since if the true effects of power spillage by upwind turbines had been included near V_r , more power would have been available to downstream machines, consequently the discrepancy between the array characteristic and the approximation would have been reduced. A simple numerical integration of the product of array power output and wind speed probability distribution gave energy outputs which differed by less than 1.5% for each case. The array efficiency model described by Equations (26) through (28) is, therefore, believed to be adequate for array performance predictions.

5.3 Application to Meteorological Data

The approximation described above provides a simple method for obtaining array efficiency as a function of both wind speed and wind direction in either graphical or matrix form. The output from the new program is now a matrix of efficiencies, the rows of which correspond to wind direction bins, and the columns of which correspond to wind speed bins. Each element of this matrix can then be multiplied by the corresponding element from a joint wind speed/wind direction frequency distribution and then summed to obtain the array energy output for a given period. Since each element of the output matrix is for a particular wind speed and wind direction bin, its value is calculated from the mean wind direction of the direction bin and from the mean power in the wind speed bin. The accuracy of this technique can be controlled by varying the bin size of the data matrix.

FIGURE 5-2. Characteristics used to estimate the energy output of an array.

5.4 Effects of Nonuniform Wind Direction

While the new program predicts array efficiencies at particular wind angles, there is often meandering of the wind. For example, if meteorological data records a mean wind direction over a given period of θ^0 , there will also be a standard deviation σ_θ associated with this, the magnitude of which depends on the lateral turbulence, α , the amount of meander and their respective time scales. This effect is crudely accounted for by the use of bins in the efficiency matrix, but if the bin size is reduced sufficiently so that it is significantly less than σ_θ , then another technique can be used to allow for meander. To do this, the program takes the efficiency values predicted for a given speed and, as θ varies, averages them over the interval of meander described by σ_θ . This averaging is weighted in a Gaussian way to model more closely the way in which the wind behaves. To do this, the properties of σ_θ must be known and this value becomes an input parameter to the program.

5.5 Effects of Unsteady Wind Velocity

The effect of unsteady wind velocity can be modeled in a way similar to that described in Section 5.4 for wind meander. By defining the mean wind speed, $\bar{V}$, and standard deviation, σ_v , for a given period, and assuming a Gaussian distribution, the values in each column of the efficiency matrix are averaged. The most representative results are obtained by using a large number of small wind speed bins.

6. ARRAY PERFORMANCE SENSITIVITY ANALYSIS

6.1 Baseline Parameters

The new computer program has been run for a variety of input parameters to investigate the sensitivity of wind farm performance to meteorology, machine characteristics and array configuration. In order to do this, some representative baseline parameters had to be chosen so that variations about these values would demonstrate the sensitivity trends.

To represent site meteorology, a Rayleigh distribution of wind speed was assumed (Cliff, 1977) with a mean value of 6.7 m/s (15 mph). To generalize the wind direction, two canonical cases were assumed, one for wind only in one direction (unidirectional), the other for wind from all directions (omnidirectional); real joint frequency distributions will be somewhere between these extremes. Each sensitivity analysis was carried out over a range of transverse turbulence intensities because of the large sensitivity of array performance to this parameter.

The Boeing MOD-2 was taken as the baseline wind turbine for the study. The values of the parameters necessary to describe this machine are listed in Table 6-1 (Boeing, 1979).

Two different array configurations were studied. The first was a single line of six machines oriented in the windwise direction of a unidirectional flow, and the second a rectangular array of 36 machines arranged in six rows and six columns. The six-in-a-line array was chosen because of its fundamental nature which shows clearly the effects of variations in other parameters. The rectangular array, at a baseline longitudinal spacing of 10 rotor diameters, was used to show the sensitivity to lateral spacing and the difference between unidirectional and omnidirectional flows.

6.2 Discussion

The effects of windwise and crosswind spacing, hub height, and mean wind speed for arrays of baseline units were studied, and the array percentage loss calculated. This loss, ΔE , is defined as:

TABLE 6-1. Baseline wind turbine characteristics.

Rated Power	2500 kW
Cut-in speed (hub ht.)	6.3 m/s (14 mph)
Rated speed (hub ht.)	12.3 m/s (27.5 mph)
Cut-out speed (hub ht.)	20.1 m/s (45 mph)
Rotor radius	45.7 m (150 ft.)
Hub height	61.0 m (200 ft.)
C_p (rated)	0.38
m (based on C_p)	1.33
C_D (for blade profile)	0.015
Rotor solidity	0.03
Tip speed ratio	6.82

$$\frac{\Delta E}{100} = 1 - \frac{\text{Energy output from whole array}}{N \times \text{Energy output from an unshadowed turbine}} \quad (29)$$

where N is the number of turbines in the array.

Figure 6-1 shows the importance of adequate longitudinal (i.e., windwise) spacing and accurate measurement of the transverse turbulence intensity when planning and assessing the economics of a windfarm. There will obviously be tradeoffs at the planning stage between, for example, turbine spacing, land area and number of machines, but these will depend on the associated relative costs and availabilities.

Figure 6-2 indicates the sensitivity of array performance to lateral (i.e., crosswind) spacing. For the case of a unidirection wind, the windwise lines of the array become almost independent of each other at lateral spacings of more than 5D. However, an omnidirectional wind appears to give the highest array efficiency. This is because for many directions the wind sees an array with staggered rows at larger windwise spacings. This effect then raises the average power output of the array.

Figure 6-2 indicates that the turbulence level in an omnidirectional wind is less important than in unidirectional flow. To understand this it will be necessary to consider carefully what happens in a large rectangular array. When the wind blows parallel with the rows of turbines, as in the case of unidirectional flow, the rows operate approximately independently of one another, as shown by Figure 6-1, with a subsequent large sensitivity to turbulence intensity. This happens because only a small fraction of the total momentum deficit (or wake cross-section) of an upstream machine is seen by those downstream; the rest of the momentum deficit spreads out laterally in the crosswind direction at a rate dependent upon the transverse turbulence intensity. However, when the wind is omnidirectional, the array no longer appears to be arranged in regular rows and columns but appears irregular to the incident flow and a much larger proportion of the momentum deficit of upstream turbines reaches those downstream. This is because most of that part of the wake spreading horizontally at hub height is now seen by downstream machines; this is not the case for a single line array.

Figure 6-3 shows that the hub height ratio of the wind turbine is a relatively unimportant parameter in determining array efficiency. The graph was drawn assuming a

FIGURE 6-1 Effect of windwise spacing, single line array.

FIGURE 6-2. Effect of crosswind spacing, rectangular array.

FIGURE 6-3. Effect of hub height ratio, single line array.

mean wind speed at hub height of 15 mph; the curves are therefore independent of the boundary layer profile. Of course, in the same meteorological conditions, a large hub height is generally favorable so that the turbine is located in the highest winds.

Figure 6-4 demonstrates the significance of mean wind speed on array performance. Array efficiency becomes very small at low wind speeds since the machines only rarely reach rated power and spend a considerable time inactive when the wind is below cut-in speed.

Finally, Figure 6-5 shows the sensitivity of array output to the turbine drag coefficient, C_T , at rated wind speed, V_r . This drag coefficient is related to the initial velocity ratio, m , at V_r required by the computer code and rotor power coefficient, C_p , at V_r by the expressions

$$C_T = \frac{m^2 - 1}{m^2} \quad (30)$$

$$C_p = \frac{(m^2 - 1)(m + 1)}{2m^3}, \quad (31)$$

although these are strictly true only for an ideal actuator disk. The C_p used here is the ideal power coefficient, which is normally higher than the aerodynamic power coefficient because of viscous losses, and still higher than the system power coefficient which includes electrical and mechanical losses as well. The figure shows that while it may seem desirable to design wind turbines with high power coefficients, it must be remembered that such a design produces increased interference effects in an array; the tradeoff between the economic effects of these quantities has not been determined.

FIGURE 6-4. Effect of mean wind speed, single line array.

FIGURE 6-5. Effect of turbine drag coefficient, single line array.

7. APPLICATION TO THE GOODNOE HILLS ARRAY

7.1 Baseline Parameters

The updated computer code has been applied to the array of three MOD-2 wind turbines sited at Goodnoe Hills, Washington. To do this, the machine characteristics listed in Table 6-1 were used for the simulations; the value of m shown is based on C_p using Equation (31), although it is recognized that the MOD-2 actuator disk is not ideal. The true value of m may be slightly larger than this due to losses.

7.2 Discussion

Figures 7-1 to 7-3 show how the power outputs of the individual machines vary with wind direction (for 10° increments). Troughs in the power curves can be seen for wind directions for which the turbine is in the wake of an upstream machine. At sufficiently high wind speeds, these troughs disappear. This occurs when all machines are operating in the rated condition and the excess wind power is being spilled downstream. Figure 7-4 shows these effects of wake interference on the output of the entire array.

Figures 7-5 to 7-8 show how turbulence influences machine and array performance. The troughs in the performance curves are quite deep and narrow at low turbulence intensities since the turbine wakes then narrow with correspondingly large velocity deficits. However, as the turbulence intensity increases, the wake growth rate increases, and the troughs become shallower and extend over a slightly larger range of wind angles.

Finally, Figures 7-9 to 7-11 show, for particular wind directions corresponding to maximum interference, how machine power output varies with hub height wind speed for a range of turbulence intensities. In correspondence with earlier results, it can be seen that power output increases with turbulence intensity. Although the curves appear close together, their slopes are large and thus show a large sensitivity to turbulence intensity. It must be remembered that these curves represent the approximation discussed in Section 5.2 and lose accuracy near cut-in and rated wind speeds.

FIGURE 7-1. Output of Turbine 1 at 12% turbulence.

FIGURE 7-2. Output of Turbine 2 at 12% turbulence.

FIGURE 7-3. Output of Turbine 3 at 12% turbulence.

FIGURE 7-4. Output of entire array at 12% turbulence.

FIGURE 7-5. Output of Turbine 1 at 11 m/s hub height wind speed.

FIGURE 7-6. Output of Turbine 2 at 11 m/s hub height wind speed.

FIGURE 7-7. Output of Turbine 3 at 11 m/s hub height wind speed.

FIGURE 7-8. Output of entire array at 11 m/s hub height wind speed.

FIGURE 7-9. Output of Turbine 1 when downwind of Turbine 2.

FIGURE 7-10. Output of Turbine 1 when downwind of Turbine 3.

FIGURE 7-11. Output of Turbine 2 when downwind of Turbine 3.

8. CONCLUSIONS

Conclusions of this research can be divided into three basic areas, that of the fundamental fluid mechanics, that of the general features of array performance, and that of the specific characteristics of the Goodnoe Hills array.

Fluid Mechanics

1. The basic features of the original Lissaman/Bate model are supported by recent wind tunnel and full-scale experiments, although the wind tunnel turbulence generally does not exhibit the same characteristics as atmospheric turbulence.
2. Adjustments to the original model have been made to correlate it with applicable experimental data. The major modifications involve the near wake length and the far wake growth, but test data are still inadequate to fully define these; a thorough, full-scale wake measurement program is recommended to further validate the trends predicted by the new model. With the above adjustments included, the new wake model fits the available experimental data well.
3. Various simplifications were made to the computer code to make it process more rapidly and to widen its versatility.
4. A new approach to array performance has been developed from which array characteristics similar in format to single turbine characteristics can be derived.

Features of Array Performance

1. The new model was exercised to investigate the effects of windwise and crosswind turbine spacing, of ambient turbulence, of turbine hub height ratio, and of machine characteristics.

2. The effects of spacing and ambient turbulence dominate the array effectiveness.
3. As an illustration of the magnitude of the interference, it is noted that a 36-turbine array with square 10D spacing experiences an interference loss in 10% ambient transverse turbulence of about 5% in an omnidirectional wind.

Interference at the Goodnoe Hills Installation

1. The maximum interference loss experienced by the array for a wind speed of 11 m/s in a representative ambient transverse turbulence of 12% is 6%. For a turbulence level of 6%, this interference loss increases to 10%.
2. Maximum interference occurs for northeasterly and southwesterly winds.

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APPENDIX A

Mathematical Description of the Two Wake Models

APPENDIX A

Mathematical Description of the Two Wake Models

Near Wake

Parameter	Original Model	New Model
Initial Radius, r_o	$R \sqrt{\frac{m+1}{2}}$	No change
Final Radius, r_c	$\frac{-s + \sqrt{s^2 + 4t}}{(2t/r_o)}$ where $s = 0.416 + 0.134 m$ $t = 0.021 (1 + 0.8 m - 0.45 m^2)$	$\frac{r_o}{d}$ where $d = (0.214 + 0.144 m)^{1/2}$
Downstream Extent, X_c	$r_o \left\{ \left(\frac{dr}{dx} \right)_a^2 + \left(\frac{dr}{dx} \right)_m^2 \right\}^{-1/2}$ where $(dr/dx)_a = \alpha / 0.51$ $(dr/dx)_m = 0.27d$ $(m-1)/(m+1)$	$r_o \left\{ \left(\frac{dr}{dx} \right)_a^2 + \left(\frac{dr}{dx} \right)_m^2 + \left(\frac{dr}{dx} \right)_\lambda^2 \right\}^{-1/2}$ where $(dr/dx)_a = \alpha / 0.36$ $(dr/dx)_m = 0.27d$ $(m-1)/(m+1)$ $(dr/dx)_\lambda = \sqrt{\frac{C_D \sigma m \lambda^3}{6(m+1)}}$
Growth Rate, dr/dx	$(r_c - r_o)/X_c$	No change

Parameter	Original Model	New Model
Velocity Deficit, Δu	ΔU_{mo} (inside core) $\Delta U_{mo} = 1 - (1 - \eta^{1.5})^2$ (outside core) where $\Delta U_{mo} = U_o (m - 1)/m$ $\eta = \frac{r_w - r}{r_w - r'}$	No change

Transition Region

Parameter	Original Model	New Model
Final Radius, r_F	r_o/e where $e = (0.134 + 0.124 m)^{1/2}$	No change
Downstream Extent, X_F	$n X_c$ where $n = \frac{d(1 - e)}{e(1 - d)}$	No change
Growth Rate, dr/dx	$\frac{r_c - r_o}{X_c}$ (same as near wake)	No change
Velocity Deficit, Δu	Linear combination of profiles at extremes of transition region	No change

Far Wake

Parameter	Original Model		New Model
	Upstream Region	Downstream Region	
Final Radius r_F	$r_F + 10r_o \alpha_e/0.51$	—	—
Downstream Extent X_F	$X_F + 10r_o$	—	—
Growth Rate, dr/dx	$\alpha_e/0.51$	$\alpha/0.51$	$\alpha/0.36$
Velocity Deficit, Δu	$\Delta U_m \left\{ 1 - \left(\frac{r}{r_w} \right)^{1.5} \right\}^2$ <p style="text-align: right;">No change</p> $\text{where } \frac{\Delta U_m}{\Delta U_{mo}} = \frac{-0.258 m + \sqrt{(0.258 m)^2 + 0.536 (1-m) (r_o/r_w)^2}}{0.268 (1-m)}$		

APPENDIX B

Matching the Gaussian and Abramovich Profiles

APPENDIX B

Matching the Gaussian and Abramovich Profiles

In the far wake region of a wind turbine, we shall assume that the velocity profile is that given by Abramovich for co-flowing jets,

$$q = q_0 \left\{ 1 - \left(\frac{r}{R} \right)^{1.5} \right\}^2$$

where q is the velocity deficit at radius r , q_0 is the centerline value and R is the wake radius. The Gaussian distribution which exists in a plume is given by

$$q = q_0 \exp(-r^2/2\sigma^2)$$

where σ is the dispersion width. To find the relationship between σ and R which results in the best matching of these profiles, we shall equate first the areas under these curves. For the Abramovich expression,

$$\begin{aligned} I_A &= \int_0^R \left\{ 1 - \left(\frac{r}{R} \right)^{1.5} \right\}^2 dr \\ &= 0.45R \end{aligned}$$

For the Gaussian form

$$I_G = \int_0^\infty \exp\left(-\frac{r^2}{2\sigma^2}\right) dr$$

Substituting $t = r/\sigma\sqrt{2}$ we have

$$I_G = \int_0^\infty \sigma\sqrt{2} \exp(-t^2) dt$$

$$\begin{aligned}
&= \sigma \sqrt{\frac{\pi}{2}} \operatorname{erf}(\infty) \\
&= 1.25 \sigma .
\end{aligned}$$

Equating $I_A = I_G$ we obtain

$$\sigma = 0.36R .$$

Although this is a simple approach, the same result is also obtained when the profiles are matched by first moment of area. For the Abramovich profile,

$$\begin{aligned}
I_{AA} &= \int_0^R r \left\{ 1 - \left(\frac{r}{R} \right)^{1.5} \right\}^2 dr \\
&= 0.129R^2 .
\end{aligned}$$

For the Gaussian form,

$$I_{GG} = \int_0^\infty r \exp\left(-\frac{r^2}{2\sigma^2}\right) dr .$$

Again, substituting $t = r^2/\sigma^2$ gives

$$\begin{aligned}
I_{GG} &= \int_0^\infty \sigma^2 e^{-t} dt , \\
&= \sigma^2 .
\end{aligned}$$

Equating $I_{AA} = I_{GG}$ gives

$$\sigma = 0.36R .$$

APPENDIX C

Derivation of Expression for Turbulence Generated by Viscous Rotor Drag

APPENDIX C

Derivation of Expression for Turbulence Generated by Viscous Rotor Drag

Consider a wind turbine with B blades of radius, R , and constant chord, c , rotating at an angular velocity, Ω . If the drag coefficient, C_D , is constant along the blade and the tip speed ratio is large, then the power lost due to viscous drag is

$$\begin{aligned} P &= B \int_0^R \frac{1}{2} \rho (\Omega r)^2 C_D \Omega c r dr \\ &= \frac{1}{8} \rho C_D B c \Omega^3 R^4 . \end{aligned}$$

If all this lost power were to appear as turbulent kinetic energy of rms fluctuation, q_t , then

$$P = \frac{1}{2} \rho q_t^2 U (1-a) \pi R^2 ,$$

where U is the incident wind velocity and a is the axial interference factor at the turbine disk. Equating the expressions for P gives

$$\begin{aligned} \left(\frac{q_t}{U} \right)^2 &= \frac{1}{4} \left(\frac{C_D}{1-a} \right) \left(\frac{c R B}{\pi R^2} \right) \left(\frac{\Omega R}{U} \right)^3 \\ &= \frac{C_D \sigma \lambda^3}{4 (1-a)} \end{aligned}$$

where σ is the rotor solidity and λ is the tip speed ratio. However, the total turbulent energy, as measured by q_t , is made up of three separate velocity components, u' , v' and w' , so that

$$q_t^2 = u'^2 + v'^2 + w'^2 .$$

The only components causing wake re-energization are the cross-flow components, u' and w' . If we assume that each component has the same magnitude, then

$$v' = w' = \frac{q_t}{\sqrt{3}} .$$

The cross-flow turbulence intensity is therefore given by

$$\alpha_\lambda^* = \frac{v'}{U} = \frac{1}{3} \left(\frac{q_t}{U} \right) .$$

Thus,

$$\alpha_\lambda^* = \sqrt{\frac{C_D \sigma \lambda^3}{12 (1 - a)}} .$$

Substituting for a in terms of the model input parameter, m , this becomes

$$\alpha_\lambda^* = \sqrt{\frac{C_D \sigma m \lambda^3}{6 (m + 1)}} .$$

This expression gives the cross-flow turbulence intensity just downstream of the rotor due to viscous rotor drag if it is assumed that all energy lost due to viscous rotor drag appears as turbulence.

APPENDIX D

"Exact" Solution for the Near Wake Length

APPENDIX D

"Exact" Solution for the Near Wake Length

The equation of the stream tube passing through the edge of the rotor in the absence of turbulent mixing can be shown to be

$$r^2 = \frac{1 - a}{1 - a \left(1 + \frac{x}{\sqrt{1 + x^2}} \right)}$$

where r is the stream tube radius and x is the downstream distance (both normalized to the rotor radius). We shall write this as

$$r = f_1(x) .$$

If we assume that this inner jet is eroded away by turbulent mixing at a constant rate so that the angle between the imaginary stream tube and the "core" (the remains of the uniform inner jet) is a constant, $\tan^{-1}(s)$, then if $f_2(x)$ is the function defining the shape of this core, we can write

$$\tan^{-1}\{f_1'(x)\} - \tan^{-1}\{f_2'(x)\} = \tan^{-1}\{s\} ,$$

where the prime indicates differentiation with respect to x . Thus,

$$\begin{aligned} f_2'(x) &= \tan\left\{\tan^{-1}\{f_1'(x)\} - \tan^{-1}\{s\}\right\} \\ &= \frac{f_1'(x) - s}{1 + s f_1'(x)} . \end{aligned}$$

This cannot be integrated directly, and so a numerical method is required. The method chosen was the Runge-Kutta technique because of its simplicity and high accuracy (it is a 4th order method). For this particular problem the solution reduces to

$$f_2(x + \Delta x) = f_2(x) + \frac{\Delta x}{6} \left\{ f_2'(x) + 4 f_2'\left(x + \frac{\Delta x}{2}\right) + f_2'(x + \Delta x) \right\},$$

the initial conditions being

$$x = 0, \quad f_1(x) = f_2(x) = 1.$$

The length of the core X_c is given by the value of x when $f_2(x) = 0$. This "exact" solution for X_c has been found for two particular cases for comparison with predictions by the existing model. To do this, the erosion rate, s , was taken as that given by Abramovich, when the effects of ambient and rotor-generated turbulence are also included; i.e.,

$$s = \left\{ \left(\frac{dr}{dx} \right)_a^2 + \left(\frac{dr}{dx} \right)_m^2 + \left(\frac{dr}{dx} \right)_\lambda^2 \right\}^{1/2}.$$

Table D-1 compares the "exact" and wake model results for an ambient turbulence level of 10% for two different power coefficients. (The following values were also assumed: $C_D = 0.01$, $\lambda = 5$, $\sigma = 0.05$). The table shows that the maximum discrepancy was less than 0.3%, which is satisfactory.

TABLE D-1. Comparison of near wake length predictions.

m	X_c	
	Wake Model	"Exact" Solution
1.5	3.84	3.85
3.0	4.55	4.54

APPENDIX E

Use of New Computer Code

APPENDIX E

Use of New Computer Code

The new code has been designed to work in an interactive mode and most of the data requests are self-explanatory. A simple run, shown in Figure E-1, is described below together with the corresponding output data (Figure E-2).

Questions 1, 2

The output can be written either to a file or directly to line printer (LU6). In the former case, the file name must be specified and, in either case, a title of up to 80 characters can be read in to describe the output.

Questions 3, 4, 5

The coordinates of the wind turbines in the array can be input either from a file or manually (at the terminal). In the former case, a file name will be required; the file should contain data in the same format as for manual input. The coordinate system is orientated with the x axis increasing in the easterly direction and the y axis increasing in the northerly direction. The coordinates should be measured in rotor radii and may be negative. After the data have been read in, individual turbine positions may be altered to allow for typing errors or to investigate modified array configurations. The turbines are numbered in the order in which the data were read in.

Question 6

The effects of unsteady wind speed and wind meander can be included by enabling the "fuzzing" feature of the program. This is described in Sections 5.4 and 5.5. If fuzzing is selected, then either or both effects can be incorporated and the corresponding standard deviations must be specified.

```

:PNL
Q1  OUTPUT TO PRINTER OR FILE?
P
Q2  TITLE OF PRINTOUT?
GOODNOE HILLS EXAMPLE RUN
Q3  MANUAL OR FILE INPUT FOR TURBINE COORDS?
F
Q4  COORDINATE FILE NAME?
HILLS
      0.0000      0.0000
      -13.400      1.5400
      -19.300      -6.5000
Q5  DO YOU WISH TO CHANGE ANY TURBINE COORDINATES?
N
Q6  ENABLE FUZZING?
N
Q7  MANUAL OR FILE INPUT FOR TURBINE PARAMETERS?
F
Q8  PARAMETER FILE NAME?
M2
      2500.0
      14.000      45.000      27.500
      1.3300      1.3333      150.00
      1.50000E-02  3.00000E-02  6.8200
Q9  TRANSVERSE TURBULENCE INTENSITY (%)?
10
Q10 BOUNDARY LAYER EXPONENT?
.14
Q11 HEIGHT OF WIND SPEED BINS (FEET)?
200
Q12 START,STOP WIND DIRECTIONS AND INCREMENT (DEG)?
0,330,30
Q13 INITIAL VELOCITY AND INCREMENT?
0.5
Q14 OUTPUT OF WHICH TURBINE (0 FOR ENTIRE ARRAY)?
0
Q15 DO YOU WISH TO LOOK AT OTHER MACHINES?
N
Q16 AGAIN?
N

```

FIGURE E-1. Example of data input to computer model.

PROGRAM PNL

GOODNOE HILLS EXAMPLE RUN

THE 3 WINDMILL COORDINATES ARE

#	X	Y
1	0.00	0.00
2	-13.40	1.54
3	-19.30	-6.50

INPUT VARIABLES:

P-RATED (KW)= 2500.0
V-CUT IN= 14.00 V-RATED= 27.50 V-CUT OUT= 45.00 AT HUB HEIGHT (MPH)
V-CUT IN= 14.00 V-RATED= 27.50 V-CUT OUT= 45.00 AT BIN HEIGHT (MPH)
R= 1.33 AH= 1.333 ROTOR RADIUS (FEET)=150.0
COO= .015 SOLIDITY= .030 TSR= 6.82

ALPHA (%)= 10.0 EXPON= .140 BIN HT (FEET)= 200.0
FI= 0.0 FI2= 330.0 DG= 30.0
VO= 0.00 DV= 5.00
V-SD= 0.000 TH-SD= 0.000 NUMX= 0

OUTPUT VARIABLES:

XC= 3.571 XF= 5.191 RO= 1.079 RC= 1.695 RF= 1.974 DRDXN= .172 DRDXF= .278

```
*****
#      X      Y      A      WIND DIRECTION  0.0
2     -13.40   1.54   1.0000
1       0.00   0.00   1.0000
3     -19.30  -6.50   1.0000
ARRAY A = 1.0000
*****
#      X      Y      A      WIND DIRECTION  30.0
1       0.00   0.00   1.0000
2     -13.40   1.54   1.0000
3     -19.30  -6.50   .8479
ARRAY A = .9493
*****
#      X      Y      A      WIND DIRECTION  60.0
1       0.00   0.00   1.0000
2     -13.40   1.54   1.0000
3     -19.30  -6.50   .9789
ARRAY A = .9930
*****
#      X      Y      A      WIND DIRECTION  90.0
1       0.00   0.00   1.0000
2     -13.40   1.54   .8983
3     -19.30  -6.50   1.0000
ARRAY A = .9661
*****
#      X      Y      A      WIND DIRECTION 120.0
1       0.00   0.00   1.0000
2     -13.40   1.54   1.0000
3     -19.30  -6.50   1.0000
ARRAY A = 1.0000
*****
```

FIGURE E-2. Example of output from computer model.

```

*****
#      X      Y      A      WIND DIRECTION 150.0
1      .00      0.00      1.0000
3     -19.30     -6.50      1.0000
2     -13.40      1.54      1.0000
ARRAY A = 1.0000
*****
#      X      Y      A      WIND DIRECTION 180.0
3     -19.30     -6.50      1.0000
1      .00      0.00      1.0000
2     -13.40      1.54      1.0000
ARRAY A = 1.0000
*****
#      X      Y      A      WIND DIRECTION 210.0
3     -19.30     -6.50      1.0000
2     -13.40      1.54      .8479
1      .00      0.00      1.0000
ARRAY A = .9493
*****
#      X      Y      A      WIND DIRECTION 240.0
3     -19.30     -6.50      1.0000
2     -13.40      1.54      1.0000
1      .00      0.00      .9789
ARRAY A = .9930
*****
#      X      Y      A      WIND DIRECTION 270.0
3     -19.30     -6.50      1.0000
2     -13.40      1.54      1.0000
1      .00      0.00      .8983
ARRAY A = .9661
*****
#      X      Y      A      WIND DIRECTION 300.0
3     -19.30     -6.50      1.0000
2     -13.40      1.54      1.0000
1      .00      0.00      1.0000
ARRAY A = 1.0000
*****
#      X      Y      A      WIND DIRECTION 330.0
2     -13.40      1.54      1.0000
3     -19.30     -6.50      1.0000
1      .00      0.00      1.0000
ARRAY A = 1.0000

```

```

                      ARRAY AVERAGED POWER OUTPUT(KW) PER TURBINE
TH / V      0.0      5.0      10.0      15.0      20.0      25.0      30.0      35.0      40.0
TH / V      5.0      10.0      15.0      20.0      25.0      30.0      35.0      40.0      45.0
0.0 0.000E+00 0.000E+00 1.852E+01 6.481E+02 1.574E+03 2.384E+03 2.500E+03 2.500E+03 2.500E+03
30.0 0.000E+00 0.000E+00 7.459E+00 5.675E+02 1.470E+03 2.315E+03 2.500E+03 2.500E+03 2.500E+03
60.0 0.000E+00 0.000E+00 1.671E+01 6.369E+02 1.560E+03 2.376E+03 2.500E+03 2.500E+03 2.500E+03
90.0 0.000E+00 0.000E+00 1.060E+01 5.942E+02 1.505E+03 2.340E+03 2.500E+03 2.500E+03 2.500E+03
120.0 0.000E+00 0.000E+00 1.852E+01 6.481E+02 1.574E+03 2.384E+03 2.500E+03 2.500E+03 2.500E+03
150.0 0.000E+00 0.000E+00 1.852E+01 6.481E+02 1.574E+03 2.384E+03 2.500E+03 2.500E+03 2.500E+03
180.0 0.000E+00 0.000E+00 1.852E+01 6.481E+02 1.574E+03 2.384E+03 2.500E+03 2.500E+03 2.500E+03
210.0 0.000E+00 0.000E+00 7.458E+00 5.675E+02 1.470E+03 2.315E+03 2.500E+03 2.500E+03 2.500E+03
240.0 0.000E+00 0.000E+00 1.671E+01 6.369E+02 1.560E+03 2.376E+03 2.500E+03 2.500E+03 2.500E+03
270.0 0.000E+00 0.000E+00 1.060E+01 5.942E+02 1.505E+03 2.340E+03 2.500E+03 2.500E+03 2.500E+03
300.0 0.000E+00 0.000E+00 1.852E+01 6.481E+02 1.574E+03 2.384E+03 2.500E+03 2.500E+03 2.500E+03
330.0 0.000E+00 0.000E+00 1.852E+01 6.481E+02 1.574E+03 2.384E+03 2.500E+03 2.500E+03 2.500E+03

```

FIGURE E-2 (continued).

Questions 7, 8

The turbine characteristics can be input either from a file or directly from the keyboard. If read from a file, the parameter values are written to the user's terminal as a check on the data. The input data file must be in the same format as for manual input and its file name must be specified.

Question 9

The transverse turbulence intensity must be input as a percentage.

Questions 10, 11

The wind speed bins which appear in the output data refer to the height specified in answer to Question 11. Since the program works in terms of hub height wind speeds, the value of boundary layer exponent is only important if the bin height is not to be the same as hub height.

Questions 12, 13

If the fuzzing capability has been selected, then a full 0 to 360° range of wind angles and a full 0 to $V_{\text{cut-out}}$ range of wind speeds must be specified. Otherwise, any range of these parameters is acceptable. For accurate predictions of the effects of wind meander and unsteady wind speed, the increments in wind angle and speed should be at least as small as their respective standard deviations.

Question 14

The output data can show the performance of a particular turbine in the array or of the array as a whole.

Question 15

Finally, the calculations can be repeated with as many or as few parameter changes as required.

The output from this run is shown in Figure E-2. The input data are repeated for reference and the important wake parameters are then given (DRDXN is the near wake growth rate and DRDXF is the far wake growth rate). The values of A (Equation 22) are then given for each direction and for each turbine, as well as for the whole array. These values can be used to derive approximately the performance characteristics of the machines in the array relative to the freestream wind (e.g., Figures 7-9 through 7-11). The turbines are listed in upstream order. Finally, a table is given to determine the power output of the selected turbine at any wind speed and any wind direction. This data can then be combined with a joint wind frequency distribution to predict the total energy output for that period.

APPENDIX F

Listing of New Computer Code

```

FTN4.L
$FILES(0,2)
$EMA (LARG,0)
PROGRAM PNL
C
C A PROGRAM TO CALCULATE THE OUTPUT OF WECS ARRAYS AS A FUNCTION OF
C WIND SPEED AND DIRECTION. THIS IS AN UPDATED VERSION OF THE MODEL
C DEvised IN 1977 BY AEROVIRONMENT FOR THE SWEDISH BOARD OF ENERGY.
C
C DIMENSION ADD(44),NR(100),CO(6),VEL(100),PRAT(100)
C DIMENSION TITL(20),OUTFIL(10),NUMFUZ(2),SD(2),A(20)
C INTEGER P9,OUTFIL,NUMV,VBIN,ANS,N2,LU,FLAG,NUMFUZ,
C +NUMX,A,Q,OLD
C LOGICAL LOOP,AGAIN
C COMMON XG(100),YG(100),X(100),Y(100),/LARG/ARRAY(90,360),
C +B(360),ARRFUZ(360)
C
C READ THE TITLE OF THE OUTPUT LISTING
C WRITE(1,(' OUTPUT TO PRINTER OR FILE?'))
C READ(1,1099) ANS
1099 FORMAT(1A1)
C IF (ANS.EQ.1HF) THEN
C LU=5
C WRITE(1,(' FILE NAME?'))
C READ(1,1002) OUTFIL
1002 FORMAT(10A2)
C OPEN(UNIT=LU,FILE=OUTFIL,ACCESS='SEQUENTIAL',STATUS='NEW')
C ELSE
C LU=6
C ENDIF
C WRITE(1,(' TITLE OF PRINTOUT?'))
C READ(1,500)TITL
500 FORMAT(20A4)
C WRITE(LU,(' PROGRAM PNL'))
C WRITE(LU,501)TITL
501 FORMAT(/' '20A4/)
C
C READ WINDMILL COORDINATES
C WRITE(1,(' MANUAL OR FILE INPUT FOR TURBINE COORDS?'))
C READ(1,1099) ANS
C IF (ANS.EQ.1HM) THEN
C WRITE(1,(' NUMBER OF TURBINES IN THE ARRAY?'))
C READ(1,100)N1
100 FORMAT(I2)
C WRITE(1,(' X AND Y COORDINATES FOR EACH TURBINE?'))
C DO 120 I=1,N1
C WRITE(1,507)I
507 FORMAT(' WINDMILL NUMBER ',I2)
C READ(1,*)XG(I),YG(I)
C NR(I)=I
120 CONTINUE
C ELSE
C WRITE(1,(' COORDINATE FILE NAME?'))
C READ(1,1002) OUTFIL
C OPEN(UNIT=10,FILE=OUTFIL,ACCESS='SEQUENTIAL',STATUS='OLD')
C READ(10,*) N1

```

```

      DO 8000 I=1,N1
        READ(10,*) XG(I),YG(I)
        WRITE(1,*) XG(I),YG(I)
        NR(I)=I
8000    CONTINUE
        CLOSE(10)
      ENDIF
      AGAIN=.FALSE.
      LOOP=.FALSE.
999    LOOP=.FALSE.
520    WRITE(1,(' DO YOU WISH TO CHANGE ANY TURBINE COORDINATES?'))
      READ(1,1099)ANS
      IF (ANS.NE.1HY) GOTO 997
      WRITE(1,(' WHICH TURBINE?'))
      READ(1,100)P9
      WRITE(1,507)P9
      READ(1,*) XG(P9),YG(P9)
      GOTO 520
997    WRITE(LU,508)N1
508    FORMAT(' THE 'I2' WINDMILL COORINATES ARE'// ' #'I4X'X'9X'Y'//)
      DO 509 I=1,N1
        WRITE(LU,111)I,XG(I),YG(I)
111    FORMAT(1X,I2,6X,2F10.2)
509    CONTINUE
      C
      C
      C
      WIND MEANDER AND UNSTEADY VELOCITY
      WRITE(1,(' ENABLE FUZZING?'))
      READ(1,1099) ANS
      FLAG=0
      IF (ANS.EQ.1HY) FLAG=1
      SD(1)=0
      SD(2)=0
      IF (FLAG.GT.0) THEN
        WRITE(1,(' ENABLE V-FUZZ?'))
        READ(1,1099) ANS
        IF (ANS.EQ.1HY) THEN
          WRITE(1,(' V-SD: '))
          READ(1,*) SD(2)
          NUMFUZ(2)=1
        ELSE
          NUMFUZ(2)=0
        ENDIF
        WRITE(1,(' ENABLE TH-FUZZ?'))
        READ(1,1099) ANS
        IF (ANS.EQ.1HY) THEN
          WRITE(1,(' TH-SD: '))
          READ(1,*) SD(1)
          NUMFUZ(1)=1
        ELSE
          NUMFUZ(1)=0
        ENDIF
        NUMX=25
      ENDIF
      C
      C
      C
      PEAD TURBINE CHARACTERISTICS
      IF (AGAIN) THEN
        WRITE(1,(' DO YOU WISH TO CHANGE TURBINE CHARACTERISTICS?'))
        READ(1,1099) ANS
        IF (ANS.NE.1HY) GOTO 1000

```

```

ENDIF
WRITE(1,(' " MANUAL OR FILE INPUT FOR TURBINE PARAMETERS?"'))
READ(1,1099) ANS
IF (ANS.EQ.1HF) THEN
  WRITE(1,(' " PARAMETER FILE NAME?"'))
  READ(1,1002) OUTFIL
  OPEN(UNIT=10,FILE=OUTFIL,ACCESS='SEQUENTIAL',STATUS='OLD')
  READ(10,*) PMAX
  READ(10,*) VIN,VRAT,VOUT
  READ(10,*) CPM,AH,ROTOR
  READ(10,*) CD,SIGMA,TSR
  WRITE(1,*) PMAX
  WRITE(1,*) VIN,VOUT,VRAT
  WRITE(1,*) CPM,AH,ROTOR
  WRITE(1,*) CD,SIGMA,TSR
  GOTO 1000
ENDIF
519 WRITE(1,502)
502 FORMAT(' P-RATED PER MACHINE (kW)?')
  READ(1,*) PMAX
  WRITE(1,503)
503 FORMAT(' V-CUT IN, V-RATED, V-CUT OUT (MPH)?')
  READ(1,*) VIN,VRAT,VOUT
  WRITE(1,504)
504 FORMAT(' INITIAL VEL RATIO, HUB HT RATIO, ROTOR RADIUS (FEET)?')
  READ(1,*) CPM,AH,ROTOR
  WRITE(1,505)
505 FORMAT(' BLADE DRAG COEFF, ROTOR SOLIDITY, TIP SPEED RATIO?')
  READ(1,*) CD,SIGMA,TSR
C
C
C  READ METEOROLOGICAL DATA
1000 WRITE(1,(' " TRANSVERSE TURBULENCE INTENSITY (%)?"'))
  READ(1,*)ALPHA
  WRITE(1,(' " BOUNDARY LAYER EXPONENT?"'))
  READ(1,*)EXPON
  WRITE(1,(' " HEIGHT OF WIND SPEED BINS (FEET)?'))
  READ(1,*)BIN
C
C
C  OUTPUT TURBINE DATA
  WRITE(LU,(' " INPUT VARIABLES:"'))
  WRITE(LU,1005)PMAX
1005 FORMAT(' P-RATED (kW)=',F7.1)
  WRITE(LU,1004) VIN,VRAT,VOUT,4HHUB
  AHT=AH*ROTOR
  VIN=VIN*(BIN/AHT)**EXPON
  VOUT=VOUT*(BIN/AHT)**EXPON
  VRAT=VRAT*(BIN/AHT)**EXPON
  WRITE(LU,1004) VIN,VRAT,VOUT,4HBIN
1004 FORMAT(' V-CUT IN=',F6.2, ' V-RATED=',F6.2, ' V-CUT OUT=',F6.2,
+ ' AT ',1A4, 'HEIGHT (MPH)')
  WRITE(LU,132)CPM,AH,ROTOR
132 FORMAT(' M='F5.2' AH='F6.3' ROTOR RADIUS (FEET)='F5.1)
  WRITE(LU,133)CD,SIGMA,TSR
133 FORMAT(' CD='F5.3' SOLIDITY='F5.3' TSR='F5.2')
  AHR=AHT*12.0*0.0254
C
C
C  OUTPUT METEOROLOGICAL DATA

```

```

WRITE(LU,1001)ALPHA,EXPON,BIN
1001 FORMAT(' ALPHA (X)=',F5.1,' EXPON=',F5.3,' BIN HT (FEET)=',F6.1)
ALPHA=ALPHA/100
BINH=BIN*12.0*0.0254
C
C
C READ WIND DIRECTIONS
1200 WRITE(1,(' START,STOP WIND DIRECTIONS AND INCREMENT (DEG)?'))
READ(1,*)F1,F12,DG
IF ((F12-F1)/DG.GT.360) THEN
    WRITE(1,(' GREATER THAN 360 ANGULAR BINS'))
    GOTO 1200
ENDIF
IF (FLAG.NE.0.AND.(F1.NE.0.OR.(F12+DG).NE.360)) THEN
    WRITE(1,(' FUZZ MODE, FULL CIRCLE [0..360] ONLY!'))
    GOTO 1200
ENDIF
C
C
C READ WIND SPEEDS
1300 WRITE(1,(' INITIAL VELOCITY AND INCREMENT?'))
READ(1,*)V0,DV
IF (V0.GT.VOUT) THEN
    WRITE(1,(' INITIAL SPEED GREATER THAN CUT-OUT SPEED!'))
    GOTO 1300
ENDIF
IF (FLAG.NE.0.AND.(V0.NE.0.OR.(VOUT/DV).NE.INT(VOUT/DV))) THEN
    WRITE(1,(' FUZZ MODE, SET V0=0 AND VOUT/DV=INTEGER'))
    GOTO 1300
ENDIF
IF (INT((VOUT-V0)/DV).EQ.(VOUT-V0)/DV) THEN
    NUMV=INT((VOUT-V0)/DV)
ELSE
    NUMV=INT((VOUT-V0)/DV)+1
ENDIF
IF (NUMV.GT.90) THEN
    WRITE(1,(' GREATER THAN 90 VELOCITY BINS'))
    GOTO 1300
ENDIF
WRITE(LU,162)F1,F12,DG,V0,DV
162 FORMAT(' F1='F6.1' F12='F6.1' DG=' F6.1' V0='F6.2' DV='F6.2')
WRITE(LU,8001)SD(2),SD(1),NUMX
8001 FORMAT(' V-SD=',F6.3,' TH-SD=',F6.3,' NUMX=',I3/)
C
C
C CALCULATE HOW MANY STEPS THERE ARE
N2=(ABS(F1-F12))/DG+0.5+1
C
C
C CALCULATE ALL THE CONSTANTS
CALL CONST(ALPHA,CPM,AH,RO,RD1,RC,RF,XF,XC,DRDXN,
+DRDXF,CO,CD,SIGMA,TSR)
WRITE(LU,(' OUTPUT VARIABLES:'))
WRITE(LU,170)XC*RO,XF*RO,RO,RC*RO,RF*RO,DRDXN,DRDXF
170 FORMAT(' XC='F6.3' XF='F6.3' RO='F6.3' RC='F6.3' RF='F6.3'
+' DRDXN='F5.3' DRDXF='F5.3'/)
C
C
C START THE CALCULATION OF POWER RATIO FOR EACH WINDMILL
FI1=FI

```

```

300 PRAT(1)=1
FI=FI1-DG
WRITE(1,(' OUTPUT OF WHICH TURBINE (0 FOR ENTIRE ARRAY)?'))
READ(1,*)IWM
DO 250 IFI=1,N2
FI=(FI+DG)*2.*3.1415926/360.0
C
C EMPTY THE ARRAY FOR THE VELOCITY PROFILES
CALL ZERO(ADD,44)
C
C REARRANGE THE ARRAY WITH RESPECT TO THE INCOMING WIND
CALL COORD(FI,N1)
C
C RENUMBER THE ARRAY
CALL SORT(N1,NR,X,Y)
C
C CONTINUE THE CALCULATION OF THE POWER RATIO
DO 215 K1=2,N1
K2=1
K3=K1-K2
DO 210 K2=1,K3
DX=(X(K1)-X(K2))/R0
DY=(-Y(K2)+Y(K1))/R0
IF(DX,LT.0) WRITE(LU,(' ERROR'))
C
C CHECK IF WINDMILL NUMBER K2 IS INFLUENCING NUMBER K1, IF NOT,
PRINT THAT OUT AND JUMP TO THE NEXT WINDMILL
IF((ABS(DY)-2.*1.)/(ABS(DX)+0.00001).GT.DRDXF)GO TO 210
C
C CALCULATE THE DEFICIT FOR ALL THE 44 SQUARES AND ADD TO THE
PREVIOUS CALCULATED DEFICIT
CALL VELOC(K2,CPM,AH,RD1,RC,RF,XF,XC,DRDXN,DRDXF,CO,
+DX,DY,VEL,ADD)
210 CONTINUE
C
C WE HAVE NOW CALCULATED THE DEFICITS FROM ALL THE WINDMILLS THAT
INFLUENCED K1. NOW WE MUST CALCULATE THE POWER RATIO AND THE
AVERAGED VELOCITY OVER THE DISC.
CALL POWER(K1,VEL,PRAT,ADD)
C
C EMPTY THE REGISTERS BEFORE THE NEXT ROUND
CALL ZERO(ADD,44)
215 CONTINUE
CALL AVER(N1,P,PRAT)
C
C WE HAVE NOW CALCULATED THE POWER RATIOS FOR ALL THE WINDMILLS
IN THE ARRAY FOR A GIVEN WIND DIRECTION. WE SHALL NOW PRINT
THE RESULTS
IF (.NOT.LOOP) WRITE(LU,230)
230 FORMAT(' *****
*****')
FI=FI*360./(2*3.1415926)
IF (.NOT.LOOP) WRITE(LU,240)FI
240 FORMAT(4X,'#',8X,'X',9X,'Y',8X,'A',12X,'WIND DIRECTION',F6.1)
IF (LOOP) GOTO 260
DO 260 I=1,N1

```

```

      K=NR(I)
      WRITE(LU,280)K,XG(K),YG(K),PRAT(I)
280  FORMAT(2X,I3,2F10.2,F10.4)
260  CONTINUE
      WRITE(LU,290)P
290  FORMAT(' ARRAY A =',F7.4)
      IF (IWM.EQ.0) GOTO 4999
      OLD=0
2999  OLD=OLD+1
      IF (IWM.NE.NR(OLD)) GOTO 2999
      P=PRAT(OLD)
4999  DO 3999 ITK=1,N1
3999  NR(ITK)=ITK
      B(IFI)=(1-(1-VIN/VRAT))*(1-P)
      DO 1100 VBIN =1,NUMV
1100  ARRAY(VBIN,IFI)=EFF(V0,DV,VBIN,VIN/B(IFI),VRAT/B(IFI),VOUT,PMAX)
250  CONTINUE
      IF (FLAG.GT.0) THEN
        DO 8100 FLAG=2,1,-1
          IF (NUMFUZ(FLAG).LT.1) GOTO 8100
          DO 8200 IS=1,NUMFUZ(FLAG)
            DO 8300 I=1,NUMV
              DO 8400 J=1,N2
                IF (FLAG.EQ.1) THEN
                  FIXED=(2.*I-1.)*DV/2.+V0
                  PARA=DG*(J-1)+FI1
                  ARRFUZ(J)=FUZZ(V0,FLAG,FIXED,SD(FLAG),NUMX,DV,DG,
+                               FI1,N2,VIN,VRAT,VOUT,PARA,PMAX)
                IF (J.EQ.N2) THEN
                  DO 1003 Q=1,N2
1003  ARRAY(I,Q)=ARRFUZ(Q)
                ENDIF
              ELSE
                FIXED=(J-1)*DG+FI1
                PARA=(2.*I-1.)*DV/2.+V0
                ARRAY(I,J)=FUZZ(V0,FLAG,FIXED,SD(FLAG),NUMX,DV,DG,
+                               FI1,N2,VIN,VRAT,VOUT,PARA,PMAX)
              ENDIF
            CONTINUE
          CONTINUE
        CONTINUE
      CONTINUE
8100  CONTINUE
8300  CONTINUE
8200  CONTINUE
8100  CONTINUE
      ENDIF
      CALL OUTPT(V0,LU,AHR,BINH,NUMV,N2,DV,DG,N1,FI1,12,56,A,IWM)
      WRITE(1,(' DO YOU WISH TO LOOK AT OTHER MACHINES?'))
      READ(1,1099)ANS
      IF (ANS.EQ.1HY) THEN
        LOOP=.TRUE.
        GOTO 300
      ENDIF
      WRITE(1,(' AGAIN? '))
      READ(1,1099)ANS
      IF (ANS.EQ.1HY) THEN
        WRITE(LU,('1'))
        AH=AH*RO
        AGAIN=.TRUE.
        GOTO 999
      ENDIF
      IF (LU.EQ.5) CLOSE(5)
      END

```



```

C
C
C      CALCULATION OF CONSTANTS
SUBROUTINE CONST(ALPHA,CPM,AH,RO,RD1,RC,RF,XF,XC,DRDXN,
+DRDXF,CO,CD,SIGMA,TSR)
  DIMENSION CO(6)
  RO=SQRT((CPM+1.0)/2.0)
  CO(3)=SQRT(0.214+0.144*CPM)
  XC=1.0/SQRT((ALPHA/0.36)**2+(0.27*(1.0-CPM)*CO(3)/(1.0+CPM))**2
++CD*SIGMA*CPM*TSR**3/(6*(CPM+1)))
  CO(4)=SQRT(0.134+0.124*CPM)
  RC=1/CO(3)
  DRDXN=(RC-1.)/XC
  CO(6)=CO(3)*((1.0-CO(4))/((1.0-CO(3))*CO(4)))
  RF=1/CO(4)
  XF=CO(6)*XC
  DRDXF=ALPHA/0.36
  RD1=1./RO
  AH=AH/RO
  RETURN
END

C
C
C      RESET STORAGE ARRAY TO ZERO
SUBROUTINE ZERO(A,N)
  DIMENSION A(44)
  DO 20 I=1,N
    A(I)=0
20  CONTINUE
  RETURN
END

C
C
C      REARRANGE THE ARRAY WITH RESPECT TO THE INCOMING WIND
SUBROUTINE COORD(FI,N1)
  COMMON XC(100),YC(100),X(100),Y(100)
  DO 20 I=1,N1
    R=SQRT(XC(I)*XC(I)+YC(I)*YC(I))
    IF(0.00001.GE.XC(I).AND.XC(I).GE.-0.0001)XC(I)=0.00005
    A=ATAN(YC(I)/XC(I))
    IF(XC(I).LT.0.)A=A+3.1415926
    Y(I)=R*COS(A+FI)
    X(I)=-R*SIN(A+FI)
20  CONTINUE
  RETURN
END

C
C
C      RENUMBER THE ARRAY
SUBROUTINE SORT(K,M,B,C)
  DIMENSION M(100),B(100),C(100)
  N2=K-1
  DO 40 I=1,N2
    IND=I
    IP1=I+1
    DO 50 J=IP1,K
      IF (B(IND).GT.B(J)) IND=J
50  CONTINUE
    IF(IND.EQ.I)GO TO 40
    CHANGE PLACES
  
```

```

TEMP=B(IND)
TOMP=C(IND)
L=M(I)
M(I)=M(IND)
M(IND)=L
B(IND)=B(I)
C(IND)=C(I)
B(I)=TEMP
C(I)=TOMP
40 CONTINUE
RETURN
END

C
C
C CALCULATE THE DEFICIT
SUBROUTINE DEF(R,VINT,RD1,RC,RF,XF,XC,DRDXN,DRDXF,
+CO,DX,CPM)
DIMENSION CO(6)
IF(XF.LE.DX)GO TO 10
IF(XC.LE.DX)GO TO 30

C
C DX BELONGS TO NEAR WAKE
R1=1.-DX/XC
R2=1.+DX*DRDXN
IF(ABS(R1-R2).LE.0.00001)R2=R1+0.000001
ETA=(R2-R)/(R2-R1)
IF(ETA.GE.0.)GO TO 35

C
C ETA < 0, IMPLIES R > R2, THEREFORE WE ARE OUTSIDE THE WAKE
VINT=1
GO TO 100
35 IF(ETA.LE.1.)GO TO 40
C
C ETA > 1 MEANS R < R1, THEREFORE WE ARE IN THE CORE REGION
VINT=1./CPM
GO TO 100
40 VINT=(1.-(1.-CPM)*(1.-ETA**1.5)**2)/CPM
GO TO 100

C
C DX BELONGS TO TRANSITION REGION
30 R2=1.0+DX*DRDXN
XI=R/R2
IF(XI.GT.1) THEN
DU=0
ELSE
DU=(-0.258*CPM+SQRT((0.258*CPM)**2+0.536*(1.0-CPM)/(RF*RF)))
+ *(1.-1./CPM)/(0.268*(1.0-CPM))
DU=((DX-XC)/(XF-XC))*DU*(1-XI**1.5)**2+((XF-DX)/(XF-XC))
+ *(1.0-(1.0-(1.0-XI)**1.5)**2)
ENDIF
VINT=1.-DU
GO TO 100

C
C DX BELONGS TO FAR WAKE
10 R2=RF+(DX-XF)*DRDXF
IF(R.GT.R2) THEN
DU=0
ELSE
DU=(-0.258*CPM+SQRT((0.258*CPM)**2+0.536*(1.0-CPM)/(R2*R2)))
+ *(1.-1./CPM)/(0.268*(1.0-CPM))

```

```

      DU=DU*(1.-(R/R2)**1.5)**2
    ENDIF
    VINT=1.-DU
100  RETURN
    END

C
C
C  POWER CALCULATION
    SUBROUTINE POWER(K1,VEL,PRAT,ADD)
    DIMENSION VEL(100),PRAT(100),ADD(44)
    PRAT(K1)=0
    VEL(K1)=0
    DO 10 I=1,44
      PRAT(K1)=PRAT(K1)+((1.-ADD(I))**3)/44
      VEL(K1)=VEL(K1)+((1.-ADD(I))/44
10  CONTINUE
    RETURN
    END

C
C
C  AVERAGE
    SUBROUTINE AVER(N1,A,PRAT)
    DIMENSION PRAT(100)
    A=0
    DO 10 I=1,N1
      A=A+PRAT(I)
10  CONTINUE
    A=A/N1
    RETURN
    END

C
C
C  CALCULATE THE DEFICITS
    SUBROUTINE VELOC(K2,CPM,AH,RD1,RC,RF,XF,XC,DRDXN,
+DRDXF,CO,DX,DY,VEL,ADD)
    LOGICAL YA,YB,ZA
    DIMENSION CO(6),VEL(100),ADD(44)
    VEL(1)=1.
    Y=DY-0.945*RD1
    Z=0.135*RD1
    K5=1
10  DO 20 I=1,22
      R=SQRT(Y*Y+Z*Z)
      CALL DEF(R,VINT,RD1,RC,RF,XF,XC,DRDXN,DRDXF,CO,DX,CPM)
      K4=(1-K5)/2
      J=I+22*K4
      DU1=1.0-VINT
      R=SQRT(Y*Y+(Z+2.0*AH)**2)
      CALL DEF(R,VINT,RD1,RC,RF,XF,XC,DRDXN,DRDXF,CO,DX,CPM)
      DU2=1.-(VINT-DU1)
      ADD(J)=ADD(J)+DU2*VEL(K2)
      YA=I.LE.7.OR.(I.GE.15.AND.I.LE.19)
      YB=I.GE.8.AND.I.LE.13.OR.(I.EQ.21)
      ZA=I.EQ.8.OR.I.EQ.14.OR.I.EQ.20
      IF(YA) Y=Y+0.27*RD1
      IF(YB) Y=Y-0.27*RD1
      IF(I.EQ.20) Y=Y-0.54*RD1
      IF(ZA) Z=Z+0.27*RD1*K5
20  CONTINUE
    IF(K5.EQ.-1)GO TO 30

```

```

Y=DY-0.945*RD1
Z=-0.135*RD1
K5=-1
GO TO 10
RETURN
END
30
C
C
REAL FUNCTION EFF(V0,DV,VBIN,VIN,VRAT,VOUT,PMAX)
INTEGER VBIN
REAL DUMMY
SPEED2=DV+VBIN+V0
SPEED1=DV*(VBIN-1)+V0
DUMMY=PMAX*((VRAT-VIN)/2+(VOUT-VRAT))
IF (SPEED1.LE.VRAT) THEN
  DUMMY=DUMMY-CURVE(SPEED1,VIN,VRAT,VOUT,PMAX)*(SPEED1-VIN)/2
ENDIF
IF (SPEED1.LE.VOUT.AND.SPEED1.GT.VRAT) THEN
  DUMMY=DUMMY-PMAX*(SPEED1-VRAT)-PMAX*(VRAT-VIN)/2
ENDIF
IF (SPEED2.LE.VOUT.AND.SPEED2.GT.VRAT) THEN
  DUMMY=DUMMY-PMAX*(VOUT-SPEED2)
ENDIF
IF (SPEED2.LE.VRAT) THEN
  DUMMY=DUMMY-(PMAX+CURVE(SPEED2,VIN,VRAT,VOUT,PMAX))*(VRAT-
+ SPEED2)/2-PMAX*(VOUT-VRAT)
ENDIF
IF (SPEED1.GT.VOUT.OR.SPEED2.LE.VIN) THEN
  DUMMY=0
ENDIF
EFF=DUMMY/(SPEED2-SPEED1)
RETURN
END
C
C
REAL FUNCTION CURVE(V,VIN,VRAT,VOUT,PMAX)
IF (V.LT.VIN.OR.V.GE.VOUT) THEN
  CURVE=0
ELSE
  CURVE=PMAX*(V-VIN)/(VRAT-VIN)
ENDIF
RETURN
END
C
*ENH(LARG,0)
SUBROUTINE OUTPT(V0,LU,HUBH,BINH,NUMV,NUMTH,DV,DTH,NUM,TH0,
+ XSIZ,YSIZ,A,IWM)
+ INTEGER NUMV,NUMTH,NUM,NUMX,NUMY,THP,VP,A1,A2,B1,B2,TH,
+ LU,XSIZ,YSIZ,STR,STR2
REAL HUBH,BINH,DV,ARRAY,A
DIMENSION A(XSIZ),STR(16),STR2(15)
COMMON /LARG/ARRAY(90,360),B(360),ARRFUZ(360)
ENCODE(31,1,STR) XSIZ
ENCODE(29,4,STR2) XSIZ
4  FORMAT('3X,F7.1,1H ',I2,'(1PE9.3,1H )')
1  FORMAT('11H TH / V ',I2,'(2X,F5.1,3X)')
NUMX=(NUMV-1)/XSIZ+1
NUMY=(NUMTH-1)/YSIZ+1
DO 200 THP=1,NUMY
  DO 300 VP=1,NUMX

```

```

        J=J-NUMTH
    ENDIF
ENDIF
IF (J.GT.NUMTH.OR.J.LT.1) GOTO 50
IF (FLAG.EQ.1) THEN
    AREA=AREA+(GAUSS(SD, PARA, X1+(N-1)*DX)+GAUSS(SD, PARA, X1+DX*N
+      ))*DX/2*ARRAY(I, J)
    ELSE
    AREA=AREA+(GAUSS(SD, PARA, X1+(N-1)*DX)+GAUSS(SD, PARA, X1+DX*N
+      ))*DX/2*EFF(X1, DX, N, VIN/B(J), VRAT/B(J), VOUT, PMAX)
    ENDIF
100 CONTINUE
FUZZ=AREA
RETURN
END

```

```

REAL FUNCTION GAUSS(SD, AVE, X)
REAL SD, AVE, X
GAUSS=EXP(-.5*((X-AVE)/SD)**2)/(SD*SQRT(2*3.14159))/.9974
RETURN
END
END*

```

```

DO 150 I=1,XSIZ
A(I)=DV*(I+XSIZ*(VP-1)-1)+V0
IF (IWM.EQ.0) THEN
  WRITE(LU,2)
ELSE
  WRITE(LU,5) IWM
ENDIF
2   FORMAT('1',45X,'ARRAY AVERAGED POWER OUTPUT(kW) PER',
+   'TURBINE/')
5   FORMAT('1',49X,'POWER OUTPUT(kW) FOR TURBINE #',I3/)
  WRITE(LU,STR) A
DO 250 I=1,XSIZ
250  A(I)=DV*(I+XSIZ*(VP-1))+V0
  WRITE(LU,STR) A
  A1=1+YSIZ*(THP-1)
  A2=YSIZ*THP
  IF (NUMTH.LT.A2) A2=NUMTH
  DO 400 TH=A1,A2
    THETA=TH0+DTH*(TH-1)
    B1=1+XSIZ*(VP-1)
    B2=XSIZ*VP
    IF (B2.GT.NUMV) THEN
      B2=NUMV
      DO 800 I=NUMV+1,B2
800    ARRAY(I,TH)=0
      ENDIF
      WRITE(LU,STR2)THETA,(ARRAY(I,TH),I=B1,B2)
400    CONTINUE
300    CONTINUE
200    CONTINUE
  RETURN
END

$EMA (LARG,0)
REAL FUNCTION FUZZ(V0,FLAG,FIXED,SD,NUMX,DV,DTH,TH0,NUMTH,
+   VIN,VRAT,VOUT,PARA,PMAX)
COMMON /LARG/ARRAY(90,360),B(360),ARRFUZ(360)
INTEGER FLAG,NUMX,N,I,J,NUMTH
REAL SD,X1,X2,DX,FIXED,DV,DTH,TH0,AREA,VOUT,PARA,V0
AREA=0.
X1=PARA-3.*SD
X2=PARA+3.*SD
IF (FLAG.EQ.2) THEN
  IF (X1.LT.0) X1=0
  IF (X2.GE.VOUT) X2=VOUT
ENDIF
DX=(X2-X1)/NUMX
IF (FLAG.EQ.1) THEN
  I=INT((FIXED-V0)/DV)+1
ELSE
  J=NINT((FIXED-TH0)/DTH)+1
ENDIF
DO 100 N=1,NUMX
  IF (FLAG.EQ.1) THEN
    J=NINT(((X1+N*DX)-TH0)/DTH)+1
  ENDIF
50  IF (J.LE.0) THEN
    J=NUMTH+J
  ELSE
    IF (J.GT.NUMTH) THEN

```

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