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IN PLASMA SIMULATION:
THE DARWIN OR MAGNETOINDUCTIVE APPROXIMATION

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Elimination of Electromagnetic Radiation in Plasma Simulation:
The Darwin or Magnetoinductive Approximation

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ABSTRACT. For many astrophysical and most magnetic fusion applications, the purely electromagnetic modes generated by real as well as simulation "plasma" fluctuations are a source of high frequency radiation that is often irrelevant to the physics of interest. Unfortunately, a numerical CFL stability limit prevents either making c infinite or Δt large while using the usual explicit Maxwell's equations for the fields. A modification of Maxwell's equations, which provides implicitly the field components, circumvents this problem. The solution is to neglect retardation effects so that the electromagnetic propagation speed is effectively infinite. The purely electromagnetic modes in this limit evolve "instantly" to a time-asymptotic configuration about the macroscopic plasma configuration at each new time level. The Darwin or magnetoinductive approximation effectively provides infinite propagation speeds for purely electromagnetic modes by converting Maxwell's equations from hyperbolic to elliptic in character. In practice, this is accomplished by neglecting the solenoidal part of the displacement current. The elimination of the CFL time step constraint more than offsets the substantially more complicated field solution that is required. The details of a numerical implementation of this model will be presented. Numerical examples will be given and extensions of the Darwin field solution to other plasma models also will be considered.

1. Introduction

The tremendous variety of collective behavior in plasma models with both electric and magnetic fields usually necessitates some approximations to narrow the range of phenomena treated by the model. This concentration of effort is desirable; there is no reason to follow low frequency behavior if it does not change appreciably during the time scales of interest. Similarly, high frequencies contained in full models can be

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so much faster that often, for good approximation, the phenomenon could have been represented by its "average" value. Numerical stability requirements of fully explicit models frequently require small enough time steps to follow the rapid oscillations about the mean even though this behavior is of no particular interest to the physics under study.

A case in point, and the subject of this paper, is the purely electromagnetic behavior of plasmas. For many astrophysical and most magnetic fusion applications the speed of light is effectively infinite; the kinetic behavior of the plasma and certainly the macroscopic movement of the plasma can be orders of magnitude slower than the transit time of light. The purely electromagnetic modes in these cases evolve "instantly" to a time-asymptotic configuration about the macroscopic plasma configuration at each new time level.

It is desirable to solve the field equations in the limit of instantaneous propagation and thus avoid retardation effects and the resulting electromagnetic modes. Unfortunately, as demonstrated in the next section, if the electric and magnetic fields are governed by the full set of Maxwell's equations, the desired limit cannot be obtained by setting the velocity of light c to infinity. If c is set to ∞ in Maxwell's equations, nothing more than electrostatics is retained. The intent of this lecture is to show how to eliminate purely electromagnetic modes by neglecting retardation while properly maintaining magnetic and inductive effects.

There is an additional advantage of neglecting the pure electromagnetic modes, as Nielson and Lewis[1] point out. In simulations representing the plasma as macroscopic particles, anomalously large fluctuations result from the relatively small number of simulation compared to real particles representing the plasma. These fluctuations may drive much more than a realistic amount of energy into purely electromagnetic modes, which can obscure and distort interesting physics.

In following sections, a discussion of a plasma model which treats the field quantities in the Darwin[2,1] or magnetoinductive[3] approximation is presented. Entailing much more than a simple neglect of the displacement current, this approximation allows the fields to be obtained implicitly, effectively defeating the rather restrictive Courant, Friedrichs and Lewy (CFL) condition[4] which requires $c \Delta t \leq \Delta x$. In effect this approximation is accomplished by neglecting only the solenoidal part of the displacement current—which leads to an elliptic rather than a hyperbolic set of field equations. The cost of this improvement, while nominal compared to the price paid for CFL-limited small Δt runs, is a substantially more complicated field solution.

In the next section a description of a numerical implementation of this model is presented. The task is shown to be a straightforward solution of the Darwin-modified model. The biggest numerical difference is the iteration needed for the inductive electric field solution, and the new conceptual difference is that more boundary conditions need to be specified. Complex geometries, such as those with internal conductors, require consideration of the interrelationship of these conditions. Extensions of the Darwin field solution to other plasma models is also considered.

2. The Darwin Approximation

The essence of the Darwin approximation can be distilled easily from the complete Maxwell's equations

$$\nabla \times \mathbf{B} = 4\pi/c \mathbf{j} + 1/c \partial \mathbf{E} / \partial t \quad (1a)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (1b)$$

$$\nabla \times \mathbf{E} = -1/c \partial \mathbf{B} / \partial t \quad (1c)$$

$$\nabla \cdot \mathbf{E} = 4\pi \rho \quad (1d)$$

in which $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields and ρ and $\mathbf{j}$ are the plasma source terms which are obtained by integrating over the ensemble of moving point charges. Although instantaneous electromagnetic (EM) propagation is an important part of the Darwin model, setting the propagation speed c to infinity in Maxwell's equations eqs. 1 leaves only electrostatic fields (ES), governed by eq. 1d. From eqs. 1a-1b the $\mathbf{B}$ field can only exist as a constant in both time and space and the inductive part of $\mathbf{E}$, governed by eq. 1c, also has only the trivial solution. Thus the most straightforward technique for enforcing instantaneous propagation ($c \rightarrow \infty$) removes all nontrivial magnetic and inductive phenomena from the field equations.

This assumption is far too restrictive for most present day topics of research; even time-independent inhomogeneous magnetic configurations such as magnetic confinement or field shear cannot be self-consistently represented in this limit. A model containing more physics than the ES model but less than the full EM model is the object of this report. A commonly used approximation is to neglect all of the displacement current (the $\partial \mathbf{E} / \partial t$ term in eq. 1a). This assumption results in a low frequency model that certainly does not support purely EM modes since the coupling between the inductive $\mathbf{E}$ and $\mathbf{B}$ no longer exists. As will be shown, however, this model is inconsistent with charge continuity;

serious errors can result when applied to the model to nonneutral plasmas.

The Darwin model[1-6] provides an avenue into the desired physics regime while avoiding the continuity problem. To introduce the Darwin model, it is convenient to consider Maxwell's equations written in terms of the scalar and vector potentials ϕ and $\underline{A}$. These equations are, in the Coulomb gauge $\nabla \cdot \underline{A} = 0$,

$$\nabla^2 \phi = -4\pi \rho \quad (2a)$$

$$\nabla^2 \underline{A} - 1/c^2 \partial^2 \underline{A} / \partial t^2 = -4\pi/c \underline{j} + 1/c \nabla \dot{\phi} \quad (2b)$$

The $\dot{}$ signifies a time derivative and

$$\underline{B} = \nabla \times \underline{A} \quad (2c)$$

and

$$\underline{E} = -\nabla \phi - 1/c \dot{\underline{A}} \quad (2d)$$

Formally the solutions can be obtained by integration and expressed by (Jackson 1962, p185)

$$\phi(\underline{r}, t) = \int d\underline{r}' 4\pi \rho(\underline{r}', t) / |\underline{r} - \underline{r}'| \quad (3a)$$

$$\underline{A}(\underline{r}, t) = 1/c \int d\underline{r}' [4\pi \underline{j} - \nabla \dot{\phi}]_{\text{ret}} / |\underline{r} - \underline{r}'| \quad (3b)$$

where "ret" means that the quantity is to be evaluated at position $\underline{r}'$ and time

$$t' = t - |\underline{r} - \underline{r}'|/c \quad (4)$$

Eq. 3a expresses the concept that given the instantaneous charge density, the instantaneous self-consistent ES potential ϕ can be obtained by integration over all space. The "propagation speed" is infinite. This is not the case in eq. 3b for the vector potential $\underline{A}$ which, at a given position $\underline{r}$ and time t , is determined by those currents at position $\underline{r}'$ and retarded time t' such that their existence has just been communicated to position $\underline{r}$ at time t . The time t' at which the current at $\underline{r}'$ must be evaluated is retarded from t by the time it takes the signal to cross the distance $|\underline{r} - \underline{r}'|$.

The essence of the Darwin model is to neglect this retardation. Eq. 3b is replaced by

$$\underline{A}(\underline{r}, t) = 1/c \int d\underline{r}' [\underline{4\pi \underline{j}} - \underline{\nabla \phi}]/|\underline{r}-\underline{r}'| \quad (5)$$

so that, like 3a, the instantaneous current at position $\underline{r}'$ and time t contributes to $\underline{A}(\underline{r}, t)$. The effective propagation speed is infinite but c is finite. (Equivalently, only the c in eq. 4 is assumed infinite.) By analogy with eqs. 2a and 3a, eq. 5 is the solution of

$$\nabla^2 \underline{A} = -4\pi/c \underline{j} + 1/c \underline{\nabla \phi} \quad (6)$$

The approximation can be expressed in a different form by noting that all vector fields can be expressed as the sum of irrotational and solenoidal parts, denoted here by subscripts l and t respectively. The irrotational part can be obtained from the gradient of a scalar; the solenoidal part, from, the curl of a vector. This means that irrotational vectors have no curl; solenoidal vectors have no divergence. Eq. 2d reveals that the $\underline{\nabla \phi}$ term in eq. 6 is the irrotational part of the displacement current. Since $\underline{A}$ is itself purely solenoidal (remember $\underline{\nabla \cdot A} = 0$ in Coulomb gauge), $\nabla^2 \underline{A} = -\underline{\nabla \times \nabla \times A} + \underline{\nabla (\underline{\nabla \cdot A})}$ is also purely solenoidal, implying that the irrotational part of the right-hand-side of eq. 6 must vanish. This condition is also easily derivable from the continuity equation and Poisson's equation eq. 1d. Models which neglect all the displacement current force the plasma current to be purely solenoidal which is inconsistent with the continuity requirement in non-neutral plasmas. Thus, the Darwin limit of eqs. 2 can be written

$$\nabla^2 \phi = -4\pi \rho \quad (7a)$$

$$\nabla^2 \underline{A} = -4\pi/c \underline{j}_t \quad (7b)$$

The irrotational part of $\underline{E}$ is obtained from

$$\mathbf{j}_t = -\nabla\phi$$

(7c)

and the solenoidal part $\mathbf{E}_t$ can be obtained from eq. 2d and the time derivative of eq. 7b

$$\nabla^2 \mathbf{E}_t = 4\pi/c \, \mathbf{j}_t \quad (7d)$$

The only difference between eqs. 2 and 7 is that the solenoidal part of the displacement current has been dropped on the right-hand-side of eq. 7b. Another identical statement is that the Darwin limit entails neglecting just the solenoidal part of the displacement current in Ampere's law eq. 1a. If all the displacement were neglected, the right-hand-side of 7b would retain a relatively large irrotational contribution which for consistency must go to zero. Careful numerical implementation is required to guarantee consistency in the Darwin model and is essential to avoid driving spurious energy into the magnetic field as the low frequency approximation breaks down and inductive effects become important.

In summary, the Darwin limit neglects only the solenoidal part of the displacement current in Ampere's law. This limit provides a model which is identical to the full Maxwell's equations through terms of order $(v/c)^2$. The first missing term is the v/c correction to the solenoidal part of $\mathbf{j}_{ret}$ in eq. 3b. The expression for $\mathbf{A}$ in eq. 5 then differs from the correct expansion 3b by terms of order $(v/c)^2$. Finally, before any fields determined by $\mathbf{A}$ interact with the particles, they are multiplied again by v/c . Terms neglected by the Darwin approximation are therefore of order $(v/c)^3$ or higher.

3. Numerical Implementation

The most important point to remember in implementing a Darwin field model is that the modified equations are no longer hyperbolic. The concept of instantaneous propagation, reflected in the elliptic eqs. 7, is evident: given the charge and current densities ρ and $\mathbf{j}$, the self-consistent fields $\mathbf{E}$ and $\mathbf{B}$ can be found if the appropriate boundary conditions are specified. As Nielson and Lewis point out, no finite differencing in time is required or allowed.

Given an adequate Poisson solver, the issues to be dealt with in this section are largely reduced to a question of the proper boundary conditions. As is now shown, even the required decomposition of vectors is reduced to a question of proper boundary conditions. We find, for example, $\mathbf{j}_t$ from $\mathbf{j}$ for use in eq. 7b by solving

$$\nabla^2 \psi = - \nabla \cdot \mathbf{J}$$

so that

$$\mathbf{J}_t = \mathbf{J} - \mathbf{J}_l = \mathbf{J} + \nabla \psi. \quad (8b)$$

The boundary conditions needed for eq. 8a take on physical significance if we realize that specifying the normal derivative of ψ on the simulation boundary is the same as specifying the normal component of the irrotational current on the boundary. Gauss's law eq. 1d gives a further constraint here because the integral of $\mathbf{J}_l \cdot \hat{\mathbf{r}}$ ($\hat{\mathbf{r}}$ is the normal vector to the surface) around the boundary must equal the rate at which the net charge within the simulation region is changing.

With $\mathbf{J}_l$ now calculated, $\mathbf{A}$ can now be computed from eq. 7b with boundary conditions which are physically motivated by specifying magnetic fields or fluxes on the simulation boundary. Notice from eq. 2c that a tangential component of $\mathbf{B}$ implies a normal derivative of $\mathbf{A}$. After $\mathbf{A}$ is computed in the simulation region, eq. 2c provides the $\mathbf{B}$ field everywhere else.

The electrostatic or irrotational part of $\mathbf{E}$ is obtained through eqs. 7a and 7c. This calculation is reminiscent of eq. 8a because the equations have similar form and the normal component of the gradient of the scalar potential provides the physical significant boundary condition. For consistency, $\mathbf{E}_l \cdot \hat{\mathbf{r}}$ integrated around the simulation region must equal the net charge contained in the simulation region.

The equation for the solenoidal part of $\mathbf{E}$, eq. 7d, follows the same pattern for boundary conditions. Frequently it is most convenient to specify the functional value $\mathbf{E}_t$ on the boundary rather than a condition on normal derivatives. Again a consistency relation is given by Stoke's law relating the integral of $\mathbf{E}_t$ at the boundary to the rate at which the net magnetic flux through the simulation surface changes. Assuming we know $\mathbf{J}_l$, the solution of eq. 7d is relatively easy.

Unfortunately, $\mathbf{J}_l$ is not as easy to obtain as the other source terms. The temptation to express $\mathbf{J}_l$ as a finite difference in time has been the source of much difficulty: temporal finite differencing in the field equations is inconsistent with "instantaneous propagation". Nielson[1] was among the first to realize that if more information about each particles kinetic tensor is determined, the divergence of this tensor may be accumulated directly for each species much the same way a species density of current is obtained. Following Nielson and Lewis[1], we see that an expression of $\mathbf{J}$ for each species is

$$\partial \mathcal{J}_s / \partial t = \mathcal{D}_s + q_s / n_s (\rho_s \mathbf{V} + \mathcal{J}_s / c) \quad (9)$$

where $\mathcal{D}_s$ is the vector that results from the divergence of the kinetic energy tensor

$$\mathcal{D}_s = - \nabla \cdot \int q_s \mathbf{v}_p \mathbf{v}_p f_s d\mathbf{v} \quad (10)$$

Another expression for $\mathcal{J}_s$ can be obtained by differentiating

$$\mathcal{J}_s = q_s \sum_p \mathbf{v}_p S(\mathbf{x}_g - \mathbf{x}_p) \quad (11)$$

where S is the particle shape factor, to obtain

$$\begin{aligned} \dot{\mathcal{J}}_s(\mathbf{x}_g) = & - q_s \sum_p \mathbf{v}_p \mathbf{v}_p \cdot \nabla S(\mathbf{x}_g - \mathbf{x}_p) \\ & + q_s^2 / m_s \sum_p (E(\mathbf{x}_p) + \mathbf{v}_p / c \times B(\mathbf{x}_p)) S(\mathbf{x}_g - \mathbf{x}_p) \end{aligned} \quad (12)$$

We have used the Lorentz force and Newton's laws to eliminate $\dot{\mathbf{v}}_p$. A comparison of eqs. 9 and 12 indicates how an expression for $\mathcal{D}_s$ may be obtained.

By accumulating a $\mathcal{D}_s$ for each species in the particle pusher, we can obtain an expression for $\mathcal{J}$ by summing eq. 9 over species. We now must find the solenoidal part using the same procedure outlined for other steps in this calculation. An exception in this case compared with eq. 8 is that now we must specify the normal component of the time derivative of $\mathcal{J}_l$ rather than $\mathcal{J}_l$ itself.

A final complication is that the expression for $\mathcal{J}_l$ contains the unknown E_l . We cannot algebraically solve for E_l and thus find E_l directly because the right-hand-side must be reduced to just its solenoidal component. As a consequence iteration is required in which the latest value of E_l is used in the expression for $\mathcal{J}_l$ in eq. 9 prior to its decomposition. Other techniques can be employed to speed convergence but in practice the iteration has converged quickly.

4. Examples

One use to which Darwin codes are particularly well suited is the investigation of the stability of the sheath regions of magnetically confined plasmas. The interface or sheath between the region of high density and relatively low $|B|$ and the region with low ρ and high $|B|$ typically must carry a diamagnetic current whose magnitude is inversely proportional to the thickness of the sheath. A wide variety of kinetic

at the other end of the such a region--the source of free energy being the localized drift kinetic energy. A typical approach is to initialize the Darwin code with an inhomogeneous density with a corresponding magnetic field such that pressure balance exists. The electron distribution is now constructed such that the electrons carry the required self-consistent current. Details of this method can be found in references 1 and 5 and references therein.

The Darwin code is now used to advance the equilibrium in time. Figure 1 indicates the evolution of ϕ and B_z , shown by contour plots in Fig. 1a and 1b, respectively. The ripples in the two quantities are localized about the region of strong electron drift and grow to large amplitude. The instability has been identified as the lower hybrid drift (LHD) instability.[5,6]. Run parameters are $\omega_{pe} \Delta t = 0.2$, $N_x = 40$, $N_y = 128$, $0 < x < 8 \text{ c}/\omega_{pe}$, $0 < y < 25 \text{ c}/\omega_{pe}$, giving $c \Delta t / \Delta x = 1.0$. The principal advantage to the Darwin model here is the reduced EM fluctuation levels. Full EM codes need many times more particles/cell to reduce noise levels so that the small collective behavior is not destroyed by large simulation "particle" fluctuations.

A second example is a recent application by J.K. Boyd[7] to the generation of relativistic electron beams. This application requires the Darwin model to be implemented in r-z geometry and as shown in Fig. 2 requires the application of internal boundary conditions to mock up the shapes of the anodes. Considerable care is required to properly set the boundary conditions required for the vector decompositions on these internal conductors. The electrons are emitted from the cathode in such a way that the surface irrotational electric field is held to zero; that is, enough electrons are "born" in cells adjacent to the cathode so that the E_t at the boundary is zero.

Ultimately the relativistic electron velocities in the axial direction approach C . Gains to be realized by the Darwin model are two-fold. The mesh cell is much elongated in the axial direction to accommodate the much greater axial dimension of the problem. The time step requirement for particle motion accuracy is that $c \Delta t < \Delta z$; the CFL condition for a full EM model is that $c \Delta t < \min(\Delta r, \Delta z)$ --enabling the Darwin model to run with Δt 2 to 5 times larger. The other advantage is again that the Darwin model eliminates "excessive bremsstrahlung" which allows significantly fewer simulation particles to be used. Figure 3 shows the electron charge density as it evolves to fill the initially-evacuated axisymmetric gun. Notice in Fig. 3b-3e the formation and subsequent evolution of a virtual cathode. This run is made with $\Delta t = 2.0 \times 10^{-11}$ seconds, $N_r = 33$, $N_z = 121$ and $\Delta r = \Delta z = 0.5 \text{ cm}$. The applied voltage is 2.4 MeV which is turned on with a rise time of 20 ns.

5. Extensions to Fluid Electrons

The early simulations of the LHD instability with the Darwin model were started with exceedingly narrow sheath equilibria so that the local electron drift velocity was quite large and so, consequently, was the LHD growth rate. The overriding physics issue in these studies is the determination of the saturation level and mechanism. As the initial sheath width increases to a more physically reasonable size, the growth rate decreases as well. As the growth rate decreases simulations require more particles/cell so that the inherent simulation noise level does not disrupt the early collective behavior of the instability. Increased particles/cell requirements are directly proportional to increased computational requirements.

A solution to this dilemma is to eliminate electron fluctuations by treating the electrons as a fluid. Further, as the thickness of the sheath increases, the assumption of quasi-neutrality becomes quite appropriate. As a consequence of this assumption, $\rho_e \sim -\rho_i$ and the irrotational electron current can be determined from $\nabla \cdot \mathbf{j} \sim 0$ since $\mathbf{j}_i$ is known from the PIC ion process. The solenoidal electron current $\mathbf{j}_{e1}$ can be advanced in time using the solenoidal part of the electron momentum eq. 9. The Darwin field equations can be solved just as before with one exception: $\mathbf{E}_1$ can no longer be obtained from Poisson's eq. 1d in the quasi-neutral limit. Fortunately a quasi-neutral Poisson replacement can be obtained by summing eq. 9 over all components. Taking the divergence of this sum and using $\nabla \cdot \mathbf{j} \sim 0$, we have

$$\nabla \cdot \mu \nabla \phi = - \nabla \cdot (D_i + D_e - \mu \mathbf{E}_1 + \zeta \times \mathbf{B}) \quad (13)$$

where

$$\begin{aligned} \mu &= \sum_s q_s \rho_s (1/m_i + 1/m_e) \\ \zeta &= \sum_s q_s / c (\mathbf{j}_i / m_i + \mathbf{j}_e / m_e) \end{aligned}$$

Eq. 13 replaces eq. 7a in the quasi-neutral limit. A final requirement is that since ϕ is now a function of $\mathbf{E}_1$ and since $\mathbf{E}_1$ is still a function of $\mathbf{E}_1$ in this model, an overall iteration is required over eqs. 7d and 13. Details and examples can be found in references 8 and 9.

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7. FIGURE CAPTIONS

1. Results of a 2-D Darwin simulation at times 50, 100, and $165 \omega_{pe}^{-1}$ showing (a) contours of constant electrostatic potential ϕ and (b) contours of constant magnetic field B_z , both displayed in the x-y computational rectangle of dimensions $8 C/\omega_{pe}$ wide and $25 C/\omega_{pe}$ high (from reference 1).
2. Axisymmetric r-z simulation region for Darwin simulation of an electron gun. (a) shows the cathode-anode placement and (b) gives the initial electric field direction. (courtesy of J.K. Boyd)
3. Time evolution of the electron charge density for the electron gun configuration shown in fig. 2. (courtesy of J.K. Boyd)

Fig. 1

Fig. 2