May 1977

MASTER

KAON-NUCLEUS PHYSICS AT LOW
AND MEDIUM ENERGIES *

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Series of Lectures given at Brookhaven National Laboratory
by
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Lectures 1 and 2

K^+ Interactions

- a) K^+ -nucleon (phenomenological aspects, phase shifts, Ξ^* resonances, isospin properties)
- b) K^+ -nucleus (optical potential, weak absorption, qualitative features of elastic, total cross sections, sensitivity to neutron density, comparison with α , p , π , e^- as probe of the nucleus)
- c) $(K^+, K^{*'})$ inelastic scattering (selection rules)
- d) Reactions induced by K^+ (nucleon and cluster knockout, quasielastic, knock-out stripping)

Overview:

- 1) Qualitative features of $K^\pm$ -n interaction
- 2) Attractive features of $K^\pm$ as a nuclear structure probe
- 3) Physics of weakly and strongly absorbed probes
- 4) Optical Model for $K^\pm$ -nucleus
- 5) Typical $K^\pm$ -nucleus cross sections
 - a) sensitivities to $\rho_n(r)$, $\rho_p(r)$
 - b) sensitivity to changes in $f_{K^\pm N}$
- 6) Comparison with emulsion data
- 7) Nuclear Structure Aspects:
Inelastic scattering
 - a) selectivity
 - b) Comparison of $K^\pm$, $\pi^\pm$, e^- , p , α as probes of nuclei

Attractive Features of K^+ for Nuclear Studies

1) small elementary cross section

$$\sigma_{K^+p} \approx 12-13 \text{ mb} \quad \text{for } p_{\text{lab}} \leq 800 \frac{\text{MeV}}{c}$$

2) long mean free path $\lambda \approx \frac{1}{\rho \sigma_{\text{av}}}$

$$\text{for } \rho \approx 0.16 \text{ fm}^{-3}, \quad \sigma_{\text{av}} = \frac{1}{2}(\sigma_p + \sigma_n) \approx 9 \text{ mb}$$

$$\text{we get } \lambda \approx 7 \text{ fm}$$

$\rightarrow K^+$ is sensitive to whole nucleus

3) particular spin-isospin dependence of elementary amplitude leads to selectivity in inelastic transitions.

(complements info from $e^-, \pi^\pm, p, \alpha, \text{HI}$)

4) a "controllable" probe

a) little ambiguity in reaction mech.

b) single scattering dominates

c) small wave function distortion

(PWIA good starting point for σ_{meas})

Properties of Elementary $K^\pm n$ Amplitudes

1) K^+n and K^-n interactions very different

K^-n : a) attractive potential

b) many low energy resonances
(ex. $\Lambda(1405)$, $\Sigma^*(1520)$)

c) a multichannel problem, even
at threshold ($\Lambda\pi$, $\Sigma\pi$ open)

K^+n : a) repulsive potential

b) no low energy resonances
(except possible exotic Ξ^* at ~ 16)

c) a single channel problem
below π production threshold
(except for $K^+n \rightarrow K^0 p$)

Qualitative Features of K^+n Amplitudes (Low Energy)

1) $l=0$ dominates up to 500 MeV/c

2) $\sigma_{I=1} \gg \sigma_{I=0}$ for $l=0$ part

$\therefore \sigma_{K^+p} \approx 2 \sigma_{K^+n}$ at low energy

3) $\sigma_{I=0} \gg \sigma_{I=1}$ for $l=1$

$\therefore \sigma_{K^+n}^{l=1} \gg \sigma_{K^+p}^{l=1}$

4) Scattering lengths

$$\frac{1}{k} \cot \delta_{0,1} = \frac{1}{A_{0,1}} + \frac{r_0^{0,1}}{2} k^2$$

$$A_0 \approx 0.04 \text{ fm}$$

$$A_1 \approx -0.3 \text{ fm} \quad r_0^1 \approx 0.5 \text{ fm}$$

like scattering from hard sphere
of radius 0.5 fm.

TABLE I. Possible main inelastic channels.

Inelastic channel $K^+p \rightarrow$	Production threshold ^a	
	Kaon laboratory momentum (GeV/c)	Total c.m. energy (GeV)
K^+p	525	1573
K^+p	545	1587
K^+p	525	1571
$K^+p(1230)$	570	1583
$K^+p(1300)N$	1075	1812
K^+p	1123	1840
K^+p	1220	1896
$K^+p(1400)$	1417	1984
$K^+p(1490)$	1422	1987
K^+p	1460	2003
$K^+p(1650)N$	1495	2019
$K^+p(1525)$	1500	2021

FIG. 1. Qualitative behavior of the total and partial K^+p cross sections.

Lea, Martin & Oades, Phys. Rev. 165, 1770 (1968)

Fig. 2. Total elastic and inelastic cross sections for K^+N scattering in the $J=0$ isospin state. Solid curves are eyeball interpolations of the data. The new DNL-72 data points were presented at the 1972 Batavia Conference [30] and are not included in the analysis.

Nucl. Phys. B71, 138 (1974)
BGRT Collaboration

Review of formalism: SPIN 0 + SPIN $\frac{1}{2}$

$$f(q, \theta) = \frac{1}{q} \sum_{l=0}^{\infty} [(l+1) F_{l+}(q) + l F_{l-}(q)] P_l(\cos \theta)$$

$$g(q, \theta) = \frac{i}{q} \sum_{l=1}^{\infty} [F_{l+}(q) - F_{l-}(q)] P_l^1(\cos \theta)$$

↑ spin flip ampl.

$$f_{K^+p \rightarrow K^+p} = f_1$$

$$f_{K^+n \rightarrow K^+n} = \frac{1}{2} (f_0 + f_1)$$

$$f_{K^+n \rightarrow K^0p} = \frac{1}{2} (f_1 - f_0)$$

↑ isospin index

$$\frac{d\sigma}{d\Omega} = |f|^2 + |g|^2$$

$$P \, d\sigma/d\Omega = 2 \operatorname{Re}(f^* g)$$

$$\sigma_{\text{TOT}} = \frac{4\pi}{q} \operatorname{Im} f(q, \theta=0^\circ)$$

$$f_{I,J} = \frac{\eta_{I,J} e^{2i\delta_{I,J}} - 1}{2i}$$

$$\sigma_{\text{TOT}}^I = \pi/q^2 \sum_{l=0}^{\infty} [(l+1)(1 - \eta_{I, l+\frac{1}{2}}^2) + l(1 - \eta_{I, l-\frac{1}{2}}^2)]$$

Parametrization of Amplitudes:

1) Giacomelli et al (EGRT)

$$\bar{\sigma}_{l\pm} = q^{2l+1} (a_{l\pm} + b_{l\pm} q^2 + c_{l\pm} q^4)$$

$$\eta_{l\pm} = 1/(1 + \gamma_{l\pm}^2)$$

$$\gamma_{l\pm} = \begin{cases} 0 & s \leq s_2 \\ (\frac{s-s_2}{s})^{3/2} (d_{l\pm} + e_{l\pm} (\frac{s-s_2}{s})) & s > s_2 \end{cases}$$

2) B. Martin, Nucl. Phys. B94, 412 (1975)

$$C_l(q) F_{l\pm}^{-1} = \sum_{n=0}^{N_l} a_n^{(l\pm)} (q/q_{\max})^n - i [1 + \theta(q - q_{\min}) R_{l\pm}^2] C_l(q)$$

$$C_l(q) = q^{2l+1} / (1 + (q/q_{\min})^{2l+1})$$

$$R_{l\pm}(q) = \sum_{m=1}^{M_l} b_m^{(l\pm)} (q - q_{\min} / q_{\max} - q_{\min})^m$$

Fig. 2. Fit to some typical differential cross sections from solution 2.

Fig. 3. Fit to $K^* p$ elastic and $K^* n$ charge-exchange polarization from solution 2, and predictions for unmeasured polarization distributions from the same solution.

K^+N $I=0$ BGR Collaboration

Fig. 12. Total, total elastic and partial $I=0$ cross sections predicted by the energy-dependent phase-shift analysis solutions A, C, BGR and D BGR.

The phases δ and the absorption coefficients η versus energy for the solutions A, BGR, C BGR and D BGR.

Theory for $I=0$ Exotic Resonances (Ξ^{*0})

- 1) Aaron, Amado, Silber, Phys. Rev. Lett. 26, 407 (1971).
- 2) Aaron, Rich, Hagen, Srivastava, Phys. Rev. D7, 1401 (1973)

Mechanism: rapidly opening inelastic channel
can drive elastic channel resonant

ex. onset of $\bar{s}$ production in πN system
drives D_{13} (1520) resonance

Here we have a "potential"

- Points:
- 1) Isospin factor 1 for $I=0$, $1/3$ for $I=1$
 $\Rightarrow$ favors Ξ^{*0}
 - 2) must couple to $K^{*} N$ S-waves
 $\Rightarrow S_{1/2}$ or $D_{3/2}$ KN states

Physics of weak and strong absorption:

Concept of geometricity

$$\gamma = A\sigma_T / \sigma_G$$

useful in classifying probes as weakly or strongly absorbed.

Illustration in Glauber theory:

$$\sigma_R = 2\pi \int_0^\infty (1 - e^{-\chi(b)}) b db$$

$$\chi(b) = \sigma_T \int_{-\infty}^{+\infty} dz \rho(r)$$

If we take $\rho(r) = \rho_0 e^{-\alpha r^2}$, then

$$\sigma_R / \sigma_G = \gamma + \ln \gamma + Ei(-\gamma) \quad \gamma = .577$$

$$\sigma_G = 2\pi/3 \langle r^2 \rangle \sim A^{2/3}$$

Two Limits

a) Weak absorption $\gamma \ll 1$: $\sigma_R = A\sigma_T + \mathcal{O}(A^2)$

b) Strong absorption $\gamma \gg 1$: $\sigma_R / \sigma_G \approx \ln \gamma > 1$
or $\sigma_R \sim A^{2/3} \ln(A^{1/3})$

Plot. 1.1.1.1. /

Optical Model Treatment for K^+ -nucleus

Up to $p_{lab} \approx 600 \text{ MeV}/c$, need only $L=0, 1$.

$$-2EV(r) = \Pi_{L=0} + \Pi_{L=1}$$

$$\Pi_{L=0} = 4\pi f_{K^+p \rightarrow K^+p}^{L=0} \rho_p(r) + 4\pi f_{K^+n \rightarrow K^+n}^{L=0} \rho_n(r)$$

$$\Pi_{L=0} = 4\pi f_{AV} f(r) - 2\pi f_{K^+n \rightarrow K^+p}^{L=0} (\rho_n(r) - \rho_p(r))$$

where $f_{AV} = \frac{1}{2} (f_{K^+p}^{L=0} + f_{K^+n}^{L=0})$

$$\Pi_{L=1} = -b_1 \nabla^2 f(r) \nabla^2 + \text{ch. exch.}$$

$$b_1 = \frac{4\pi}{\rho k^2} (k f_{AV}^{L=1})_{c.m.}$$

For the present, we use

$$-2EV(r) = b_0 f(r) - b_1 \nabla^2 f(r) \nabla^2$$

Estimate of corrections to unitarity pt. 11

1) finite range of $\pi^+\pi^-$ interaction

$$\rho(r) \rightarrow \int g_{K^+\pi^0}(r') \rho(r-r') d^3r'$$

2) Correlation terms (Pauli)

$$\pi_{L=0} \approx 4\pi f_{AV}^{L=0} (1 - f_{AV}^{L=0} \langle \frac{1}{r} \rangle)$$

$$\langle \frac{1}{r} \rangle \approx 3\rho_F / 2\pi \approx 0.65 \text{ fm}^{-1}$$

$$\frac{\Delta\pi_{L=0}}{\pi_{L=0}} \approx -f_{AV}^{L=0} \langle \frac{1}{r} \rangle \approx -0.15 \pm 0.05i$$

at $p \approx 800 \text{ MeV}/c$

$$\pi_{L=1} \approx b_1 g^2 (1 - b_1 \rho / 3)$$

$$\frac{\Delta\pi_{L=1}}{\pi_{L=1}} \approx -b_1 \rho / 3 \approx 0.004 \pm 0.002i$$

at $p \approx 600 \text{ MeV}/c$

using B. Martin amplitudes.

Nuclear Reaction and Structure Studies with K^+

optical model potential used to generate distorted waves for reaction calculations
(DWBA $\approx$ FWDA for K^+)

Reactions:

1) Inelastic Scattering (K^+, K^{*})
(E. Walker)

2) Knockout (K^+, K^+p)
should be simpler than (p, p')
as probe of deeply bound
hole states.

3) Exotica

$$(K^-, K^+) \quad \Delta S = 2$$

The matrix element of $T^{\dagger}$: (C. Wilkin)

for given $\vec{p}$ and $\vec{p}'$ and target and fixed $\vec{p}$ final state, we have

$$d\sigma_{\text{NR}}(e^+ \rightarrow \mu^+) \sim |T|^2$$

$$|T|^2 \sim \beta_T \left| \sum_{j_1 j_2} \langle j_1 j_2 | \vec{J}_j \cdot \vec{Y}_j | i_1 i_2 \rangle \right|^2$$

$\nearrow$ conf. mixing $\nearrow$ also for longitudinal form factor for $\vec{e}p$

1) Since $\sigma_{T=1} \gg \sigma_{T=0} \rightarrow \beta_T = \begin{cases} 9/8 & T=0 \text{ ph.} \\ 1/8 & T=\pm 1 \text{ ph.} \end{cases}$
 $\Rightarrow \Delta T = 0$ transitions

2) Since $\Delta = 0$ dominates $\rightarrow$ no spin-flip in $\vec{p}_T$
 $\Rightarrow \Delta S = 0$

3) no spin dependence of $\vec{p}_T$ $\rightarrow$ natural parity

4) because of $\vec{J}_j(\vec{p})$ in m.e., high spin states dominate the $(K^+, K^{*'})$ cross section for large momentum transfer q

Handwritten notes at the top of the page, possibly describing the system or parameters.

$$Q = 1.7 \frac{A \Delta T}{C}$$

$$Q = 2.1 \frac{A \Delta T}{C}$$

$$Q = 4.1 \frac{A \Delta T}{C}$$

Comparison of K^+ with other Pions

$$K^+ : \Delta S = 0 \quad \Delta T = 0 \quad (\text{for } p \leq 250 \text{ MeV/c})$$

$$\pi : \Delta S = 0 \quad \Delta T = 0$$

$$p : \begin{cases} \Delta S = 0 \quad \Delta T = 1 & \text{for small } q \\ \Delta S = 1 \quad \Delta T = 1 & \text{for large } q \end{cases}$$

$$\pi : \Delta S = 1 \quad \Delta T = 0$$

$$\epsilon^- : \Delta S = 1 \quad \Delta T = 1 \quad \text{for large } q$$

(transverse form factor dominates,
and isovector magn. moment
plays major role)

Lectures 3 and 4

K^- Interactions

- a) K^- -nucleon amplitudes (Υ^* resonances)
(phase shift analysis, coupled channels)
- b) K^- -nucleus interactions ("neutron halo",
non-mesonic absorption, elastic and inelastic
scattering)
- c) Theory of kaonic atoms - construction
of optical potential for K^- -nucleus
- d) Qualitative Theory of Strangeness
Analog States
- e) Formation of Λ hypernuclei in (K^-, π^-)
reactions (review of experiments + shell
model calculations of hypernuclear spectroscopy)
- f) Formation of double hypernuclei in
 (K^-, K^+) reaction (second order process,
qualitative discussion)

Outline: Lecture 5

1) Properties of elementary K^-N interaction

2) Older data on K^- Absorption

- a) Emulsions $\rightarrow$ peripheral abs.
- b) "neutron halo"
- c) non-mesonic absorption

3) Theory of Kaonic Atoms

Coupled Channels for Low Energy $K^{\pm}p$ Scattering

<u>Channel</u>	<u>Threshold</u>	<u>σ_{TOT}</u>
$K^{\bar{p}} \rightarrow K^{\bar{p}}$	0 MeV	48 mb
$K^{\bar{p}} \rightarrow \bar{K}^0 n$	5.5	8
$K^{\bar{p}} \rightarrow \Sigma^{\bar{p}} \pi^-, \Sigma^{\bar{p}} \pi^+, \Sigma^0 \pi^0$	-100	13.6, 10, 5.2
$K^{\bar{p}} \rightarrow \Lambda \pi^0$	-180	5.2
$K^{\bar{p}} \rightarrow \Lambda \pi \pi$	-40	.4
$K^{\bar{p}} \rightarrow \Sigma \pi \pi$	+40	.05
<hr/>		
$K^+p \rightarrow K^+p$	0	13

at $p_{lab} \approx 300$
MeV/c

Figure 15

K^- -nucleon total cross sections for $I=0$ and $I=1$ below 3.3 GeV/c.

Υ^* Resonances below $1 \frac{\text{GeV}}{c}$

<u>Resonance</u>	<u>J^P</u>	<u>Width</u>
$\Upsilon_0^* (1405)$	$\frac{1}{2}^-$	40 MeV
$\Upsilon_0^* (1520)$	$\frac{3}{2}^-$	16
$\Upsilon_0^* (1670)$	$\frac{1}{2}^-$	15-38
$\Upsilon_0^* (1690)$	$\frac{3}{2}^-$	27-85
$\Upsilon_1^* (1385)$	$\frac{3}{2}^+$	35
$\Upsilon_1^* (1670)$	$\frac{3}{2}^-$	35-65
$\Upsilon_1^* (1765)$	$\frac{5}{2}^-$	120

K-matrix Analysis (Dalitz)

$$\underline{\tilde{K}} = \underline{I} - i \underline{\tilde{K}} \underline{F} \underline{I}$$

$\underline{F}$ = phase space matrix

$\underline{\tilde{K}}$ is real and symmetric

For $I=0$

$$K_0 = \begin{pmatrix} \alpha & \beta \\ \beta & \gamma \end{pmatrix} \frac{\overline{KN}}{\pi \Sigma}$$

$$\text{Re} \cot \delta_0 = \left(\alpha_0 - \frac{k_z \beta_0^2}{k_z \gamma_0 + i} \right)^{-1}$$

For $I=1$, 3 channels $\overline{KN}, \Sigma\pi, \Lambda\pi$

KN DATA

B. R. MARTIN AND M. SAKITT

FIG. 1. Cross section for $K^- + p \rightarrow K^- + p$. The solid curve is the theoretical solution. Experimental points are from Sakitt et al. (Ref. 2) and Kim (Ref. 5).

FIG. 2. Cross section for $K^- + p \rightarrow \Sigma^- + \pi^+$. The solid curve is the theoretical solution. Experimental points are from Sakitt et al. (Ref. 2) and Kim (Ref. 5).

FIG. 3. Cross section for $K^- + p \rightarrow \Sigma^+ + \pi^-$. The solid curve is the theoretical solution. Experimental points are from Sakitt et al. (Ref. 2) and Kim (Ref. 5).

FIG. 4. Cross section for $K^- + p \rightarrow K^0 + n$. The solid curve is the theoretical solution. Experimental points are from Kim (Ref. 5), Abrams et al. (Ref. 9), and Kittel et al. (Ref. 10).

KN SCATTERING LENGTHS

B. R. MARTIN AND M. SAKITT

Values of the s -wave $J=0$ KN scattering length (in F) as calculated by various authors, and the result of this paper.

$A_0 = a_0 + ib_0$	Method	Ref.
$(-1.67 \pm 0.04) + i(0.72 \pm 0.04)$	Constant scattering length analysis for $k_L \leq 300$ MeV/c	3
$(-1.63 \pm 0.07) + i(0.51 \pm 0.05)$	Constant scattering length analysis for $k_L \leq 300$ MeV/c	4
$(-1.65 \pm 0.04) + i(0.73 \pm 0.02)$	Effective range K -matrix analysis for $k_L \leq 550$ MeV/c	5
$(-1.66 \pm 0.02) + i(0.69 \pm 0.02)$	Zero-range K -matrix analysis for $k_L \leq 500$ MeV/c	Present calculation

Values of the s -wave $J=1$ KN scattering length (in F) as calculated by various authors, and the result of this paper.

$A_1 = a_1 + ib_1$	Method	Ref.
$(0.00 \pm 0.06) + i(0.99 \pm 0.03)$	Constant scattering length analysis for $k_L \leq 300$ MeV/c	3
$(-0.19 \pm 0.08) + i(0.44 \pm 0.04)$	Constant scattering length analysis for $k_L \leq 300$ MeV/c	4
$(-0.13 \pm 0.02) + i(0.51 \pm 0.03)$	Effective range K -matrix analysis for $k_L \leq 550$ MeV/c	5
$(-0.07 \pm 0.03) + i(0.54 \pm 0.02)$	Zero-range K -matrix analysis for $k_L \leq 500$ MeV/c	Present calculation

MARTIN & SAKITT, Phys. Rev. 183, 1345 (1969)

$$f_0^N(W) = \frac{\gamma_0^N(W) \exp[i2\delta_0^N(W)] - 1}{2i\gamma_0^N(W)}$$

FIG. 1. RN scattering amplitude for the isotopic-spin $I=1$ channel plotted as a function of the total center-of-mass energy W . The dotted curve is the imaginary part and the solid curve is the real part of the scattering amplitude.

FIG. 3. RN scattering amplitude for the isotopic-spin $I=0$ channel plotted as a function of the total center-of-mass energy W . The dotted curve is the imaginary part and the solid curve is the real part of the scattering amplitude.

REAL
PHASE SHIFT →

FIG. 2. Phase shift δ_0^N and inelasticity γ_0^N for the RN system for both isotopic-spin $I=1$ and 0 channels plotted as a function of the laboratory momentum of the beam k_L .

INELASTICITY →

Emulsion Data

a) $\theta_{\Sigma\pi}$

b) $P_{\Sigma} + P_{\pi}$

c) Energy Spectrum of Σ or π

d) R_{pn}, R_{+-}

$$R_{pn} = \frac{N(\Sigma^{-}\pi^{+}) + N(\Sigma^{+}\pi^{-})}{N(\Sigma^{-}\pi^{0})}$$

$$R_{+-} = \frac{N(\Sigma^{+}\pi^{-})}{N(\Sigma^{-}\pi^{+})}$$

$$\frac{R_{pn}(\text{light})}{R_{pn}(\text{heavy})} \approx 5 \pm 1$$

K⁻ Absorption

$$\text{abs. rate } P \approx \overline{R_{nl}} \int R_{nl}^2 \rho \, dr$$

$$\text{or } P \sim \int r^{2l+2} \rho(r) \, dr$$

$$\Gamma_{\text{abs}} \sim P \sim E^{4 + \frac{8l}{3}}$$

Peripheral Absorption

a) $\Sigma \pi$ correlation

b) no $\Sigma^0 \pi^+$, $\Sigma^+ \pi^0$, $\Lambda^0 \pi^+$, $\Sigma^- \pi^-$

ANNALS OF PHYSICS 96, 43 (1976)

ALBERG, HENLEY AND WILETS

FIG. 1. In ^{12}C , absorption as a function of radius is plotted for perturbation theory, $c^2 \phi^2$, and for the "exact" calculation, $\phi^2 \text{Im } U^K$. In the former case, ϕ is the point Coulomb kaonic wavefunction and c is a constant; in the latter case, ϕ is the exact kaon wavefunction and U^K is the optical potential (use right-hand scale). The nuclear density ρ is also shown (left-hand scale).

LOCALIZATION OF KAON ABSORPTION IN SURFACE REGION

Neutron and proton
densities in the periphery

$$R_{pn} = \frac{\int |R_{n2}|^2 \rho_p(r) dr}{\int |R_{n2}|^2 \rho_n(r) dr}$$

a) Same radial shapes for ρ_n and ρ_p

$$\Rightarrow \frac{R_{pn}(\text{light})}{R_{pn}(\text{heavy})} \approx 1.2$$

b) $\rho_{n,p}(r) \sim e^{-\gamma_{n,p}(r-R)}$

$$\gamma_n = (2MB_n)^{1/2}$$

$$\gamma_p = (2M(B_p + E_c))^{1/2}$$

Bethe
+ Siemen

Same radius for n and p
different surface thickness

$$c) S_{n,p}(r) \sim \sum_{l \in n,p} |q_l(r)|^2$$

shell model (Brook, Aslam-Rook)

gives $\frac{R_{pn}(\text{light})}{R_{pn}(\text{heavy})} \approx 2.0 - 2.6$

Neutron halo? Not so fast!

Must correct data for

- a) $\Sigma \rightarrow \Lambda$ conversion
- b) Σ charge exchange
- c) π absorption
- d) Effect of (pn) correlations
- e) Coulomb (inhibit Σ^-)

This gives $\frac{R_{pn}(\text{light})}{R_{pn}(\text{heavy})} \approx 3.2$

for primary process.

Non-Mesonic K^- Absorption

1% in $K^- + d$, 20-30% in heavy nuclei

mom. transfer of 500 MeV

→ sensitive to high momentum components of $\phi_D(q)$

Graphs:

K^- - ATOMS

III

Klein - Gordon Equation

$$[\nabla^2 + (E - V_c)^2 + \mu^2] \psi_K(r)$$

$$= 2\mu V_{opt}(r) \psi_K(r)$$

$$\text{or } \int V_{opt}(r, r') \psi_K(r')$$

Models for $V_{opt}(r)$:

$$a) V_{opt}(r) = -\frac{2\pi}{\mu} \left(1 + \frac{m_K}{m_N}\right) \bar{A} \rho(r)$$

$$1) \bar{A} = \bar{A}(\text{free}) \rightarrow \text{Wrong!}$$

$$2) \bar{A} = \bar{A}(\text{eff})$$

averaged over energy

$$3) \text{phenomenological } \bar{A}$$

$$b) \text{Non-local (Alberg)}$$

$$c) \text{non-linear in } \rho \text{ (Wycech)}$$

DATA ON K^- -ATOMS:

Measure shift ΔE , widths Γ , Γ_{th} , Γ_{down}

Kaonic Atom Strong Interaction Shifts ϵ and Widths Γ Obtained by the CERN Group [14]

Atom	Transition	$\Gamma(\text{eV})$	$\epsilon(\text{eV})$
${}^{10}\text{B}$	$3d-2p$	810 ± 100	-258 ± 35
${}^{11}\text{B}$	$3d-2p$	700 ± 80	-167 ± 35
${}^{12}\text{C}$	$3d-2p$	1730 ± 150	-590 ± 60
	$4f-3d$	0.53 ± 0.19	—
${}^{31}\text{P}$	$4f-3d$	1440 ± 120	-330 ± 60
	$5g-4f$	1.94 ± 0.33	—
${}^{32}\text{S}$	$4f-3d$	2330 ± 200	-550 ± 60
	$5g-4f$	3.25 ± 0.41	—
${}^{35}\text{Cl}$	$4f-3d$	3500 ± 1000	-770 ± 400
	$5g-4f$	5.69 ± 1.50	—

Kaonic Atom X-ray Data From the ZGS ACS Collaboration [15]

Atom	Transition	$\Gamma(\text{eV})$	$\epsilon(\text{eV})$
Al	$4f-3d$	490 ± 160	-130 ± 50
Si	$4f-3d$	810 ± 120	-240 ± 50
Ni	$5g-4f$	590 ± 210	-180 ± 70
	$6h-5g$	6.0 ± 2.3	—
Cu	$5g-4f$	1650 ± 720	-240 ± 220
	$6h-5g$	7.1 ± 3.8	—

Effective $\bar{K}N$ Amplitude
in the medium:

112

(Bardeen-Tongue and Burhop)

$$\bar{A} = \int dE A(E) P(E)$$

↑
prob. of $\bar{K}N$ system
having c.m. en. E

$$E_i = m_K - E_K + m_i - E_i - \frac{P_{cm}^2}{2E_i}$$

The averaging procedure
produces change of
sign of real part
of V_{opt}

K $\bar{N}$ AMPLITUDES NEAR THRESHOLD

FIG. 2. Real and imaginary parts of the scattering amplitudes for the K -nucleon interaction versus center-of-mass energy, taken from Bardoun and Torigoe [2]. "Kim" and "Martin and Sakitt" refer to different sets of parameters for the two-body interaction.

BARDEEN + TORIGOE, Phys. Lett. 38B, 135 (1972)

$$\frac{1}{J} \sim \int P(E) f_{RN}(E) dE$$

Results of the Pseudopotential Calculations of Bardeen and Torigoe [29]

Atom	Transition	Kim [33]		M.S. [32]		Experimental	
		$\epsilon_2(\text{keV})$	$\Gamma_2(\text{keV})$	$\epsilon_2(\text{keV})$	$\Gamma_2(\text{keV})$	$\epsilon_2(\text{keV})$	$\Gamma_2(\text{keV})$
¹⁰ B	3d-2p	-0.30 (-0.23)	0.65 (0.24)	-0.25 (-0.22)	0.72 (0.25)	-0.203 ± 0.035	0.81 ± 0.10
¹¹ B	3d-2p	-0.30 (-0.23)	0.64 (0.24)	-0.25 (-0.23)	0.70 (0.26)	-0.167 ± 0.035	0.70 ± 0.08
¹² C	3d-2p	-0.50 (-0.49)	1.44 (0.55)	-0.57 (-0.58)	1.58 (0.58)	-0.59 ± 0.03	1.73 ± 0.15
³¹ P	4f-3d	-0.52 (-0.49)	1.65 (0.59)	-0.36 (-0.48)	1.68 (0.60)	-0.33 ± 0.08	1.44 ± 0.12
³² S	4f-3d	-0.83 (-0.82)	2.73 (0.94)	-0.61 (-0.80)	2.78 (0.98)	-0.55 ± 0.66	2.33 ± 0.06
³⁶ Cl	4f-3d	-1.44 (-1.29)	4.06 (1.45)	-1.05 (-1.27)	4.23 (1.51)	-0.77 ± 0.40	3.8 ± 1.0

*The column headings "Kim" and "M.S." refer to the set of two-body parameters with which the calculations were made. The numbers shown in parentheses are the results of calculations made with the free values of the scattering amplitudes. A harmonic oscillator form of the nuclear density distribution was used for the three light atoms, and two-parameter Fermi densities were used for the heavier atoms. The parameters ϵ_2 and Γ_2 refer to the energy shift and width of the lower level in the transition. The experimental results are those of the CERN group [14].

Non-local potential:

13

(Alberg, Henley + Wilets)

Ann. Phys. 96, 43 (1976)

1) start with separable $\bar{K}$ -potential $V(r, r')$

Adjust parameters to reproduce K -matrix

$$2) V_{opt}(r, r') = \sum_{N, I} \int d\tilde{r}_N d\tilde{r}'_N$$

$$Q_N^*(r_N) T_I(\tilde{r}_N r; \tilde{r}'_N r') Q_N(r'_N)$$

Q_N is opt. model w.f.

3) Advantages: a) off-shell effects handled properly

b) γ^* included (but no p^2 terms
à la Lorentz-Lorentz)

DETAILS OF APPROACH OF ALBERS, HENLEY, WILTS

1) First fit $\bar{K}N$ scattering lengths and effective ranges with separable potentials

ex. $I=0$ ($\psi_1 = \bar{K}N$, $\psi_2 = \Sigma\pi$)

$$(-\hbar^2 \nabla^2 / 2\mu_1 + (m_p + m_\pi) + V_{11} - E_1) \psi_1 = -V_{12} \psi_2$$

$$(-\hbar^2 \nabla^2 / 2\mu_2 + (m_\pi + m_\pi) + V_{22} - E_2) \psi_2 = -V_{12} \psi_1$$

$$V_{ij}(r, r') = \lambda_{ij} \frac{e^{-\beta r}}{r} \frac{e^{-\beta r'}}{r'} = \lambda_{ij} v(r) v(r')$$

4 parameters λ_{11} λ_{12} λ_{22} β

$$K = (T^{-1} + i k)^{-1}, \quad M = K^{-1}$$

$$\frac{1}{k} = \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}$$

$$T_{ij}(k, k') = v(k) A_{ij} v(k')$$

Derivation of Optical Potential:

$$(H_0 + V)\psi = E\psi$$

$$V = V_{NN} + V_{KN} \quad ; \quad H_0 = \sum_i h^N(i) + h_0^K$$

$$h^N(r_i) = -\hbar^2 \nabla_i^2 / 2m + V_c(r_i) + U^N(r_i)$$

$$h^N \phi_n = \epsilon_n^N \phi_n$$

$$h^K = -\hbar^2 \nabla_K^2 / 2m + V^C(r_K)$$

$$h^K \phi^K = \epsilon_0^K \phi^K$$

Now introduce $P = |\phi^N\rangle\langle\phi^N|$

$$\phi^N = (A!)^{-1/2} \det(\phi_N(r_i))$$

kaon wavefunction ψ^K

$$\langle\phi^N | P | \psi \rangle = \psi^K(r_K)$$

$$\left[-\frac{\hbar^2 \nabla^2}{2m_K} + V_C \right] \psi^K(r) + \int dr' U^K(r, r') \psi^K(r') = E \psi^K$$

$$U^K = \langle \phi^N | U | \phi^N \rangle = \bar{K} \text{ OPTICAL POTENTIAL}$$

$$U = V [1 + (E - H_0 - QV)^{-1} QV] \quad ; \quad Q = 1 - P$$

$$V \equiv V_{NN} + V_{\bar{K}N} \approx V_{\bar{K}N} \quad (\text{neglect NN residual inter.})$$

APPROXIMATION FOR U^K :

$$U^K \approx \sum_n \langle n k' | \tau(E^K) | n k \rangle$$

$$\tau(E) = v + v \frac{1}{e} \tau(E)$$

$$\frac{1}{e} = \frac{Q}{E - H^N - H_0^K - U^K}$$

GRAPHS FOR K^- -NUCLEUS OPTICAL POTENTIAL

We neglect residual nucleon-nucleon interactions. The contributions to U_K can be represented diagrammatically: A straight line represents the kaon; curved lines are used for nucleon particle or hole states. The two-body K -nucleon interaction operates at each vertex. Diagrams begin and end with a kaon line. Diagrams through fourth order are shown in Fig. 4.

FIG. 4. Perturbation diagrams for the K -nucleus optical potential. The heavy line is the kaon, light lines are nucleon particle or hole lines. The vertices are K -nucleon interactions.

$$U^K |\phi^K\rangle = \sum_n \langle \phi_n | v | \psi_n \rangle$$

ψ_n is KN wave function in the medium

$$\left[-\hbar^2 \nabla_n^2 / 2m_n - \hbar^2 \nabla_n'^2 / 2m_n + U^N(r_n) + U^K(r_n, r_n') \right. \\ \left. + v - E^K \right] \psi_n(r_n, r_n') = 0$$

this is matrix equation for $\psi_{KN}, \psi_{\pi\pi}, \psi_{\pi K}$

$$\int dr_n' U^K(r_n, r_n') \phi^K(r_n') = \sum_n \int dr_n dr' \phi_n^*(r, R) \\ v(r, R; r', R') \psi_n(r', R')$$

Rewrite in terms of τ :

$$\tau = v + v G \tau \quad ; \quad G = \frac{Q}{E_n^N + E_0^K + \frac{\nabla_N^2}{2m_N} + \frac{\nabla_K^2}{2m_K} - U_K - U_N}$$

$$\tau = \sum_{ij} v_i A^{ij} v_j$$

Use zero range approximation

$$v_j(r) = \frac{e^{-\beta_j r}}{r} \approx \frac{4\pi}{\beta_j^2} \delta(r)$$

$$\begin{aligned} U^K(r_k, r'_k) &= \frac{2}{\pi} \sum_n \int d\mathbf{K} \phi_n^*(r_k) e^{i\mathbf{K} \cdot (r_k - r'_k)} \\ &\approx \sum_{ij} \beta_i^{-2} \beta_j^{-2} A^{ij}(r_k, K^2) \phi_n(r'_k) \end{aligned}$$

Three local approximations:

1) Local equivalent potential

$$U_{EL}(r) = \frac{1}{\phi^K(r)} \int dr' U^K(r, r') \phi^K(r')$$

2) $U_L(r) = \int dr' U^K(r, r')$

3) LOCAL DENSITY APPROXIMATION

evaluate A^{ij} at average $K = K_0$

$$U_{LD}(r) = 16\pi^2 \sum_n |\phi_n(r)|^2 \sum_{ij} A^{ij}(r, K_0^2) / \beta_i^2 \beta_j^2$$

Outline :

1) Theory of Kaonic Atoms (cont.)

2) "Strangeness Analog. States"

Formation of Λ -hypernuclear states
in (K^-, π^-) reactions

3) (K^-, K^+) reactions

Double hypernuclei

$$U^k |\phi^k\rangle = \sum_n \langle \phi_n | v | \psi_n \rangle$$

ψ_n is EN wave function in the medium

$$\left[-\frac{\hbar^2 \nabla_n^2}{2m_n} - \frac{\hbar^2 \nabla_k^2}{2m_k} + U^N(r_n) + U^k(r_k, r_k') \right. \\ \left. + v - E^k \right] \psi_n(r_n, r_k) =$$

this is matrix equation for $\psi_{KN}, \psi_{\pi\pi}, \psi_{\pi\pi}$

$$\int dr_k' U^k(r_k, r_k') \phi^k(r_k) = \sum_n \int dr_n dr' \phi_n^*(r, R) \\ v(r, R; r', R') \psi_n(r', R')$$

Rewrite in terms of τ :

$$\tau = v + v G \tau \quad ; \quad G = \frac{Q}{E_n^N + E_0^k + \frac{\nabla_n^2}{2m_n} + \frac{\nabla_k^2}{2m_k} - U_k - U_n}$$

$$\tau = \sum_{ij} v_i A^{ij} v_j$$

Use zero range approximation

$$v_j(r) = \frac{e^{-\beta_j r}}{r} \approx \frac{4\pi}{\beta_j^2} \delta(r)$$

$$U^K(r_k, r_k') = \frac{2}{\pi} \sum_n \int dK \phi_n^n(r_k) e^{iK \cdot (r_k - r_k')} \\ \times \sum_{ij} \beta_i^{-2} \beta_j^{-2} A^{ij}(r_k, K^2) \phi_n(r_k')$$

Three local approximations:

1) Local equivalent potential

$$U_{el}(r) = \frac{1}{\phi^K(r)} \int dr' U^K(r, r') \phi^K(r')$$

$$2) U_L(r) = \int dr' U^K(r, r')$$

3) LOCAL DENSITY APPROXIMATION

evaluate A^{ij} at average $K = K_0$

$$U_{LD}(r) = 16\pi^2 \sum_n |\phi_n(r)|^2 \sum_{ij} A^{ij}(r, K_0^2) / \beta_i^2 \beta_j^2$$

TABLE VI
Energy Level Shifts and Widths in ^{12}C :
 $\Delta E = E^K - E_t^K = \epsilon - i\Gamma/2$

Approximation of the optical potential	ΔE (keV)
Equivalent local potential	$-0.657 - i.608$
Local potential	$-0.672 - i.569$
Local density approximation	$-0.609 - i.625$
Experiment [14]	$-0.59(\pm .08) - i.67(\pm .07)$

The wavefunction ψ^K that is found for the kaon in ^{12}C using this approximation for the optical potential is shown in Fig. 7. The imaginary part of ψ^K is dominant in the nuclear region. Because of this absorptive part, the optical potential has a repulsive effect on the kaon wavefunction, which is pushed out of the nuclear region.

FIG. 7. A comparison of the point Coulomb kaon wavefunction ψ^K with the "exact" wavefunction ψ^C . The phase of ψ^K is chosen to be real as $r \rightarrow \infty$.

K^- WAVE FUNCTION IN ^{12}C

SENSITIVITY TO FORM OF DENSITY

A comparison of calculations using different forms for the nuclear density in the local density approximation.

	ΔE	
	Harmonic oscillator nuclear density	Two-parameter Fermi nuclear density
^{11}P	-0.384 - 1.479	-0.547 - 1.709
^{12}S	-0.620 - 1.748	-0.924 - 11.20
^{11}Cl	-1.04 - 11.13	-1.51 - 11.74

TABLE XIII

A comparison of calculations for ^{12}C in the local density approximation using harmonic oscillator and shell model wavefunctions based on a Woods-Saxon potential to construct the nuclear density*

Nuclear density	$\Delta E(\text{keV})$
Harmonic oscillator	-0.619 - 1.635
Shell model	-0.915 - 1.871

* In each case the rms. radius of the nucleus was 2.45 fm.

Calculation for ^{25}S in the local density approximation, using a two-parameter Fermi distribution for the nuclear density*

d -shell binding energy (MeV)	ΔE (keV)
20	-0.868 - 11.30
16	-0.897 - 11.25
12	-0.924 - 11.20
8	-0.934 - 11.14
4	-0.952 - 11.06

* The d -shell binding energy, at which the matrix elements A_{11}^d and A^d are evaluated, is varied from the experimental value of 12 MeV.

Effects of the Pauli exclusion principle on energy level shifts and widths in several light elements, using Kim's 2-body parameters and the local density approximation; $\Delta E = \epsilon - \epsilon^*$

Nucleus	Without Pauli exclusion principle	With Pauli exclusion principle
^{10}B	-0.203 - 1.229	-0.213 - 1.238
^{11}B	-0.237 - 1.257	-0.236 - 1.236
^{12}C	-0.609 - 1.625	-0.630 - 1.645

$$U^s(r) = [-(50 \div 120) \rho(r)/\rho(o)] \text{ MeV}$$

$$U^d(r) = \{[-35 \div i(-80 \div 52\rho(r)/\rho(o))]\rho(r)/\rho(o)\} \text{ MeV}$$

Calculation for ^{25}S in the local density approximation, using a two-parameter Fermi distribution for the nuclear density*

π	ΔE (keV)
0	-0.913 - 11.18
1	-0.919 - 11.19
2	-0.924 - 11.20
3	-0.927 - 11.21
4	-0.930 - 11.22

* The S and π optical potentials used in the calculation are multiplied by a factor π .

BINDING $\rightarrow$
ENERGY

PAULI $\rightarrow$

$\pi, \Sigma \rightarrow$
POTENTIAL

$$\rho(r) = \rho(0) \left[1 + \exp \left(4 \ln 3 \frac{r-c}{t} \right) \right]^{-1}$$

changes in c or t by 0.1 fm change $\Delta E, \text{ MeV}$
by 2.5%

Calculation for ^{32}S in the Local Density Approximation*

$t = 2.60 \text{ fm}$		$c = 3.33 \text{ fm}$	
$c(\text{fm})$	$\Delta E(\text{keV})$	$t(\text{fm})$	$\Delta E(\text{keV})$
3.13	-0.523 - 11.11	2.4	-0.806 - 11.03
3.23	-0.572 - 11.15	2.5	-0.663 - 11.11
3.33	-0.924 - 11.20	2.6	-0.924 - 11.20
3.43	-0.978 - 11.25	2.7	-0.988 - 11.29
3.53	-1.04 - 11.30	2.8	-1.06 - 11.39

* A two-parameter Fermi distribution is used for the nuclear density. The half-density radius $c = 3.33 \text{ fm}$ together with $t = 2.60 \text{ fm}$ will produce an rms radius of 3.244 fm, in agreement with the muonic atom studies of Backenstoss [43].

Calculation for ^{32}S in the local density approximation, using a two-parameter Fermi distribution for the nuclear density*

V_0 variation		c variation		t variation	
$V_0(\text{MeV})$	$\Delta E(\text{keV})$	$c(\text{fm})$	$\Delta E(\text{keV})$	$t(\text{fm})$	$\Delta E(\text{keV})$
-62	-0.915 - 11.20	3.29	-0.931 - 11.21	1.98	-0.924 - 11.21
-57	-0.919 - 11.20	3.39	-0.927 - 11.20	2.20	-0.924 - 11.21
-52	-0.924 - 11.20	3.49	-0.924 - 11.20	2.42	-0.924 - 11.20
-47	-0.928 - 11.20	3.59	-0.919 - 11.20	2.64	-0.924 - 11.19
-42	-0.931 - 11.21	3.69	-0.915 - 11.19	2.86	-0.924 - 11.19

* The nuclear optical potential V is given by the expression

$$V = V_0 \left[1 + \exp \left(4 \ln 3 \frac{r-c}{t} \right) \right]^{-1}$$

$V_0 = -52 \text{ MeV}$, $c = 3.49 \text{ fm}$, $t = 2.42 \text{ fm}$ except as noted.

SENSITIVITY TO CHANGES IN
RADIUS AND DIFFUSENESS

ENERGY SHIFTS AND WIDTHS FOR K⁻-MESIC ATOMS

ALBERG, HENLEY AND WILETS

A comparison of our theory with experiment and with other calculations*

$\Delta E(\pm \Gamma/2)$					
Atom	Transition	Experiment	Theory (this work) local density approximation	Theory (Wycech)	Theory (pseudo- potential of Bardien and Torignee)
¹¹ B	3d-2p	-0.208(±.04)-i.405(±.05)	-0.233-i.266		-0.30-i.32
¹¹ B	3d-2p	-0.167(±.04)-i.350(±.04)	-0.247-i.277		-0.30-i.32
¹² C	3d-2p	-0.590(±.08)-i.865(±.08)	-0.619-i.635		-0.80-i.72
	4f-3d	-i.49(±.09)	-i.508		
³¹ P	4f-3d	-0.330(±.05)-i.72(±.05)	-0.547-i.70	-0.36-i.00	-0.52-i.82
	5g-4f	-i.97(±.16)	-i.157		
³² S	4f-3d	-0.550(±.06)-i.116(±.63)	-0.924-i.120	-0.61-i.153	-0.88-i.136
	5g-4f	-i.162(±.20)	-i.331		
³⁵ Cl	4f-3d	-0.770(±.40)-i.19(±.5)	-1.51-i.174	-1.0-i.234	-0.72-i.203
	5g-4f	-i.254(±.75)	-i.581		
Al	4f-3d	-0.130(±.05)-i.245(±.08)	-0.142-i.215		
Si	4f-3d	-0.240(±.05)-i.405(±.06)	-0.257-i.464		
Ni	5g-4f	-0.150(±.07)-i.295(±.10)	-0.195-i.405		
	6h-5g	-i3.0(±1.1)			
Cu	5g-4f	-0.240(±.220)-i.525(±.36)	-0.435-i.790		
	6h-5g	-i3.6(±1.9)			

* The lower level shifts and widths are given in keV; the upper level widths are given in eV. Calculations for ¹¹B, ¹¹D, and ¹²C were made using an harmonic oscillator form of the nuclear density, and calculations for ³¹P, ³²S, ³⁵Cl, were made using a Woods-Saxon form of the nuclear density. The parameters used in the descriptions of the nuclear density were the same in our calculation and in that of Bardien and Torignee.

Concept of "strangeness Analog States"

- Early refs:
- 1) H. J. Lipkin, Phys. Rev. Lett. 14, 18 (1965)
 - 2) Feshbach + Kerman in "Preludes in Theoret. Physics" (ed. deShalit et al), North-Holland 1966, p. 260
 - 3) L. Kisslinger, Phys. Rev. 157, 1358 (1967)
 - 4) A. Kerman & H. Lipkin Annals of Phys. 66, 738 (1971)

General Points:

- 1) Use $K^- n \rightarrow \pi^- \Lambda$ or $K^- p \rightarrow \pi^0 \Lambda$ reactions to change nucleon in nucleus into a Λ (form a hypernucleus)
- 2) Analogy to usual analog state excitation
$$V = V_0 + V_1 \underline{\tau}_1 \cdot \underline{\tau} \rightarrow \text{couples } (p, p), (p, n)$$
$$\hookrightarrow V = \hat{V}_0 + \hat{V}_1 \underline{u}_1 \cdot \underline{u} \rightarrow \text{couples } (K^-, K^-), (K^-, \pi^-)$$
- 3) Possibility of exciting a coherent state
$$|S\rangle = \frac{1}{\sqrt{N}} \sum_{\alpha} c_{\alpha}^{\dagger} b_{\alpha} |0\rangle$$

Qualitative Description of SAR :

Define $a_a^\dagger$ = creation operator for proton (p)
 $b_a^\dagger$ = " " " neutron (n)
 $c_a^\dagger$ = " " " lambda (Λ)

U and V-spin generators:

$$U_- = \sum_a c_a^\dagger b_a \quad V_- = \sum_a a_a^\dagger c_a$$

$$U_+ = \sum_a b_a^\dagger c_a \quad \vdots$$

$$U_0 = \frac{1}{2} \sum_a (b_a^\dagger b_a - c_a^\dagger c_a)$$

$|\pi\rangle$ = parent state (ground state of π jet)

$$\left. \begin{aligned} |S_p\rangle &= \frac{V_+}{\sqrt{U_-}} |\pi\rangle \\ |S_n\rangle &= \frac{U_-}{\sqrt{V_-}} |\pi\rangle \end{aligned} \right\} \text{strangeness analog states}$$

$|S_p\rangle$ Analogous to $|\Lambda\rangle = \frac{1}{\sqrt{U_-}} T_+ |\pi\rangle$ (usual analog)

Sakata triplet Model : n, p, Λ

(Octet version of $SU(3)$ less good because of $\Lambda \Sigma$ mass difference)

$SU(3)$ algebra of Sakata Model generated by operators $T_{\pm}, T_0, U_{\pm}, U_0, V_{\pm}, V_0$
($T_0 + U_0 + V_0 = 0$)

$|\pi\rangle = |\Sigma, N\rangle = \text{ground state of target}$

$$T_0 |\pi\rangle = (\Sigma - N)/2 |\pi\rangle$$

$$U_0 |\pi\rangle = N/2 |\pi\rangle$$

$$V_0 |\pi\rangle = -\Sigma/2 |\pi\rangle$$

Eigenvalues for single particle

	n	p	Λ
t_0	$-1/2$	$1/2$	0
u_0	$1/2$	0	$-1/2$
v_0	0	$-1/2$	$1/2$

← hence "U-spin flip"
for (K^-, π^-)

$$U = \sum_i u^i$$

$$V = \sum_i v^i$$

$$T = \sum_i t^i$$

Breaking of Sakata $SU(3)$ Symmetry:

$$W^{ij} = W_{nn}^{ij} + \frac{2}{3} (u_o^i + u_o^j - v_o^i - v_o^j - 1) \Delta W^{ij}$$

$$\Delta W^{ij} = W_{nn}^{ij} - W_{n\Lambda}^{ij} \quad u_o^i + u_o^j - v_o^i - v_o^j - 1 = \begin{cases} 0 & nn \\ -3/2 & n\Lambda \end{cases}$$

↑
symmetry breaking piece

$$W \equiv \frac{1}{2} \sum_{i \neq j} W^{ij}$$

Estimate energy of V-spin analog state ($T = T_\pi + 1/2$)

$$|S_p\rangle = V_+ |\pi\rangle / \sqrt{2}$$

$$\begin{aligned} \Delta E_V &= \langle S_p | H | S_p \rangle - \langle \pi | H | \pi \rangle \approx \langle S_p | W | S_p \rangle - \langle \pi | W | \pi \rangle \\ &= \frac{1}{2} \langle \pi | V_- W V_+ | \pi \rangle - \langle \pi | W | \pi \rangle \\ &= \frac{1}{2} \langle \pi | V_- [W, V_+] | \pi \rangle \\ &= \frac{1}{2} \langle \pi | \sum_{i \neq j} (v_o^i + v_o^j) \Delta W^{ij} | \pi \rangle \\ &= - \langle \Delta V_{p\Lambda} \rangle \approx +25 \text{ MeV} \quad [\text{difference betw.} \\ &\quad n, \Lambda \text{ Hartree pot.}] \end{aligned}$$

Now consider U-spin analog ; not eigenstate of T for $N \geq 2$

→ split into $T_+ = T - 1/2$, $T_- = T + 1/2$ components

$$|S_n\rangle = \frac{u_- |\pi\rangle}{\sqrt{A}} \rightarrow \begin{aligned} |S_+\rangle &= T_+ |S_p\rangle / \sqrt{2T+1} \\ |S_-\rangle &= \frac{(u_- + T_+ V_+ / (2T+1)) |\pi\rangle}{((N+1)(2T)/(2T+1))^{1/2}} \end{aligned}$$

$|S_+\rangle$ is isobaric analog of $|S_p\rangle$ ($T_+ = T + 1/2$)

$$\therefore E_{S_+} - E_{S_p} \approx E_{\text{Coulomb}}$$

$|S_-\rangle$ has $T_- = T - 1/2$

$$\therefore E_{S_+} - E_{S_-} \approx E_{\text{symmetry}} \quad (\text{from } \tau_1, \tau_2 \text{ nn force})$$

Qualitative Picture :

Enhancement factors :

$$f_p = \langle S_p | V_+ | \pi \rangle = \sqrt{\frac{2}{2T+1}}$$

$$f_s = \langle S_s | u_- | \pi \rangle = \sqrt{\frac{2}{2T+1}} \quad \leftarrow \text{less enhancement.}$$

$$f_c = \langle S_c | u_- | \pi \rangle = \sqrt{(N+1) \left(\frac{2T}{2T+1} \right)}$$

(K^-, π^-) CROSS SECTIONS ON LIGHT NUCLEI

W. Brückner et al, Phys. Letters (1975)
(CERN)

Quasifree Contribution to (K^-, π^-) Process:

Dalitz & CAL

nucleon momentum $\vec{k}$ in Fermi gas ($|\vec{k}| \leq k_F$)

momentum-energy conservation:

$$\vec{p}_K + \vec{k} = \vec{p}_\pi + \vec{p}_\Lambda \quad ; \quad \epsilon_K + \epsilon_N = \epsilon_\pi + \epsilon_\Lambda$$

$$\epsilon_N = m_N + \frac{k^2}{2m_N} - U_N$$

$$\epsilon_\Lambda = m_\Lambda + \frac{p_\Lambda^2}{2m_\Lambda} - U_\Lambda$$

momentum-energy transfer:

$$\vec{q} = \vec{p}_K - \vec{p}_\pi$$

$$\omega = \epsilon_K - \epsilon_\pi = \epsilon_\Lambda - \epsilon_N$$

$$\omega = m_\Lambda - m_N + U_N - U_\Lambda + \frac{p_\Lambda^2}{2m_\Lambda} - \frac{k^2}{2m_N}$$

$$\omega = m_\Lambda - m_N + U_N - U_\Lambda - \frac{(m_\Lambda - m_N)}{m_\Lambda} \frac{(k_\perp^2 + k_z^2)}{2m_N} + \frac{k_z q_z}{m_\Lambda} + \frac{q^2}{2m_\Lambda}$$

$$\langle k_\perp^2 \rangle = (k_F^2 - k_z^2)/2 \quad ; \quad (m_\Lambda - m_N)/m_\Lambda \approx 1/6$$

Quasifree Contribution to $(K^+ \pi^-)$ (cont.)

In Fermi gas, k_z distribution is

$$\phi(k_z) = 3(1 - k_z^2/k_F^2)/4k_F \quad -k_F \leq k_z \leq +k_F$$

So quasielastic distribution is parabola centered at $W(k_z=0) \equiv \bar{W}$

$$\bar{W} = m_\Lambda - m_N + (U_N - U_\Lambda) - \frac{(m_\Lambda - m_N)k_F^2}{4m_\Lambda m_N} + q^2/2m_\Lambda$$

and ranging from $\bar{W} - k_F q/m_\Lambda$ to $\bar{W} + k_F q/m_\Lambda$

Fit observed wide bump for $Be_\Lambda^9, C_\Lambda^{12}, O_\Lambda^{16},$
 $S_\Lambda^{32}, Ca_\Lambda^{40}$

$\therefore$ so a value $U_N - U_\Lambda \approx 31 \text{ MeV}$

using $U_\Lambda \approx 27 \text{ MeV}$ from study of ground state b.e. $\rightarrow U_N \approx 58 \text{ MeV}$ (reasonable)

Test : look for $\theta \neq 0^\circ$, position of QF peak should move upwards ($q^2/2m_\Lambda$ term)

DALITE + GAL

Fig. 1. Excitation spectra for the (K^-, π^-) strangeness exchange reaction [3] are compared with those calculated for quasi-free (QF) interactions $\text{K}^-\text{N} \rightarrow \pi^-\Lambda$.

DWIA FOR PRODUCTION OF HYPERNUCLEI:

$$\frac{d\sigma}{d\Omega_L}(\theta=0^\circ) = \sum_f \left| \int \chi_{f\pi}^{(-)}(r) \langle f | \sum_j u_+(j) \delta(r-r_j) | i \rangle \chi_{fK}^{(+)}(r) \right| \rho_n(r) \\ \times \delta(\omega - E_f + E_i) \frac{d\sigma(0^\circ)}{d\Omega_L}(\bar{K}\bar{n} \rightarrow \pi^-\lambda)$$

$$\frac{d\sigma}{d\Omega_L}(\theta=0^\circ) = N_{eff} \frac{d\sigma(0^\circ)}{d\Omega_L}(\bar{K}\bar{n} \rightarrow \pi^-\lambda)$$

$$N_{eff} = \int \rho_n(r) |\chi_{f\pi}^{(-)}(r)|^2 |\chi_{fK}^{(+)}(r)|^2 dr = \text{effective neutrons number}$$

$$\text{Now } \chi_{f\pi}^{(-)}(r) \chi_{fK}^{(+)}(r) = \sum_{L=0}^{\infty} (4\pi(2L+1))^{1/2} i^L \hat{j}_L(r) Y_L^0(\hat{r})$$

$$N_{eff}^{(j_\lambda j_n^{-1})_J} = (2J+1)(2j_\lambda+1)(2j_n+1) \left(\frac{j_\lambda j_n J}{\frac{1}{2} - \frac{1}{2} 0} \right)^2 |\langle n_\lambda l_\lambda | \hat{j}_J(r) | n_n l_n \rangle|^2$$

$$\text{for } 0^+ \text{ states } N_{eff}^{(j_\lambda j_n^{-1})_{0^+}} = (2j_n+1) |\langle n_\lambda l_\lambda | \hat{j}_0(r) | n_n l_n \rangle|^2$$

If no distortion $\hat{j}_0(r) = j_0(qr) \rightarrow 1$ for $q \rightarrow 0$

So $N_{eff} \rightarrow 2j_n+1$ for no distortion and $q \rightarrow 0$

↑ no. of neutrons in filled j_n shell.

In eikonal approximation, we have

$$\chi_{p\pi}^{(-)}(r) \chi_{pK}^{(+)}(r) = e^{i\mathbf{q} \cdot \mathbf{r}} \exp \left[-\frac{\bar{\sigma}_{KN}}{2} (1 - i\alpha_K) \int_{-\infty}^{\infty} f(b, z) dz \right. \\ \left. - \frac{\bar{\sigma}_{\pi N}}{2} (1 + i\alpha_{\pi}) \int_{-\infty}^{\infty} f(b, z) dz \right]$$

for $\alpha_{\pi} \approx \alpha_K \approx 0$ and $\bar{\sigma}_{KN} \approx \bar{\sigma}_{\pi N} = \bar{\sigma}$, we find

$$N_{eff} = \frac{N}{A} \int T(b) e^{-\bar{\sigma} T(b)} d^2 b$$

$$T(b) = \int_{-\infty}^{+\infty} f(r) dz \quad \int T(b) d^2 b = A$$

FACTOR $e^{-\bar{\sigma} T(b)}$ cuts out contribution

from small $b \rightarrow$ surface localization of π, K

Shell Model Calculation for Λ -hypernuclear states

St
{ Hüfner, Lee, Weidenmüller, Nucl. Phys. A290 (1974) 170
A. Bouyssy, Preprint MPI H-1976-V18 (Heidelberg)

- 1) Use single particle potentials $U_{\Lambda A}(r)$, $U_{\Lambda A}(r)$
to generate a basis of shell model orbitals
 $u_{\Lambda}^i(r)$, $u_{\Lambda}^j(r)$

$$U_{\Lambda A}(r) = -D_{\Lambda} f(r) + U_{\Lambda A}^{s.o.} \frac{\hbar}{2} \cdot \frac{\hbar}{2} \left(\frac{\hbar}{m\pi c} \right)^2 \frac{1}{r} \frac{df(r)}{dr}$$

$$f(r) = (1 + \exp(r-R)/a)^{-1} = \text{Woods-Saxon form}$$

$$R = r_0(A-1)^{1/3}, \quad a = 0.6 \text{ fm}, \quad r_0 = 1.1 \text{ fm}$$

- 2) Use residual ΛN interaction of form

$$V_{\Lambda N}(r_{\Lambda} - r_N) = -V_0 \delta(r_{\Lambda} - r_N) (1 + \alpha \sigma_{\Lambda} \cdot \sigma_N)$$

$$V_0 = D_{\Lambda} \int dr f(r) / A \quad \alpha \approx -0.05 \text{ to } -0.13$$

- 3) Diagonalize $V_{\Lambda N}$ in a Λ -particle,
neutron hole basis

$$|J\rangle = \sum_{j_1 j_N} \chi^{(j_1 j_N')_J} c_{j_1}^{\dagger} b_{j_N} |0\rangle$$

$$N_{\text{eff}}^J = \left| \sum_{j_1 j_N} [N_{\text{eff}}^{(j_1 j_N')_J}]^{1/2} \chi^{(j_1 j_N')_J} \right|^2$$

Choice of parameters of Λ -nucleus potential:

1) fit large narrow peak in (K, π) spectrum

$$\rightarrow D_{\Lambda} = 28 \pm 3 \text{ MeV}$$

consistent with $D_{\Lambda} = 30 \pm 5 \text{ MeV}$ obtained by extrapolating ground state hypernuclear binding energies for $3 \leq A \leq 15$

2) fit split narrow peak in $^{16}\text{O}_{\Lambda}$
 $(p_{3/2}^{\Lambda} (p_{3/2}^n)^{-1})^{0+}$ and $(p_{1/2}^{\Lambda} (p_{1/2}^n)^{-1})^{0+}$

$$\rightarrow U_{\Lambda A}^{\text{so}} = (0 \pm 0.3) U_{nA}^{\text{so}}$$

Test of "strangeness analog state" concept :

$$\text{Example of } ^{12}\text{C} : |S\rangle = \sqrt{\frac{2}{3}} (P_{3/2}^\dagger (P_{3/2}^\eta)^{-1})_{0+} \\ + \sqrt{\frac{1}{3}} (S_{1/2}^\dagger (S_{1/2}^\eta)^{-1})_{0+}$$

In shell model, we have

$$\Lambda : \begin{array}{ll} 1P_{3/2} & \text{b.e.} \approx 0 \\ 1S_{1/2} & \text{b.e.} \approx -10 \text{ MeV} \end{array} \quad \eta : \begin{array}{l} 1P_{3/2} - 1S_{1/2} \approx 20 \text{ MeV} \end{array}$$

$$\text{so } \varepsilon (S_{1/2}^\dagger (S_{1/2}^\eta)^{-1})_{0+} - \varepsilon (P_{3/2}^\dagger (P_{3/2}^\eta)^{-1})_{0+} \approx 10 \text{ MeV}$$

$$\text{but } \langle S_{1/2}^\dagger S_{1/2}^{-1} | V_{\Lambda N} | P_{3/2}^\dagger P_{3/2}^{-1} \rangle \approx 1.5 \text{ MeV}$$

$$\text{so } \frac{\langle V_{\Lambda N} \rangle}{\varepsilon_{ph}} \sim 0.1$$

so $S_{1/2}^\dagger S_{1/2}^{-1}$ and $P_{3/2}^\dagger P_{3/2}^{-1}$ weakly mixed

$\therefore$ eigenstates are $\approx$ pure configurations

$\therefore |S\rangle$ not eigenstate for light nuclei
perhaps better for heavy nuclei (more collectivity)

ENERGY LEVELS IN $^{12}_{\Lambda}\text{C}$

W. Brückner et al, Phys. Letters (1975)
(Povh group at CERN)

STRUCTURE OF $^{12}\text{C}_\Lambda$

EXPERIMENT		THEORY		
Emulsion exp	Counter exp	Dominant Configuration	Energy and Width	Rate
$E = -11.10 \pm 0.11^a)$	$E = -11 \pm 1$ $\Gamma \leq 6$ $R = (2 \pm 1) \cdot 10^{-4}$	$(\Lambda^{S_{1/2}} \pi^{P_{3/2}})^{-1}_{1^{-}}$	$E = -11.0^b)$ $\Gamma \approx 0.$	$R = 2 \cdot 10^{-4}$
$E = -0.1 \pm 0.2$	$E = 0 \pm 1$	$(\Lambda^{P_{3/2}} \pi^{P_{3/2}})^{-1}_{2^{+}}$	$E = -2.9$ $\Gamma \approx 0.$	$R = 2 \cdot 10^{-4}$
$\Gamma = 0.4$	$\Gamma \leq 6$	$(\Lambda^{P_{3/2}} \pi^{P_{3/2}})^{-1}_{0^{+}}$	$E = -1.7$ $\Gamma \approx 0.$	$R = 3 \cdot 10^{-4}$
$R \approx 3 \cdot 10^{-4}$	$R = (3 \pm 1) \cdot 10^{-4}$	$(\Lambda^{P_{1/2}} \pi^{P_{3/2}})^{-1}_{2^{+}}$	$E = 0.9$ $\Gamma \leq 0.5$	$R = 2 \cdot 10^{-4}$
unobserved		$(\Lambda^{S_{1/2}} \pi^{S_{1/2}})^{-1}_{0^{+}}$	$E \approx +9$ $\Gamma = 5-10$	$R = 0.5 \cdot 10^{-4}$

Hatner, Lee, Weidenmüller, Phys. Letters B (1974)
(binding energy for analog nucleus $^{12}\text{B}_\Lambda$)

STRENGTH FUNCTION FOR $Cr^{55}Ti^{52}$ (A. BEUSSON)

SHELL MODEL CALCULATION OF (K^-, π^-)

A. Bouyssy, MPI Preprint (Heidelberg)

(K^-, π^-)

EFFECTIVE NUMBER OF NEUTRONS

Hypernucleus	Broad Bump		Narrow Peak + Ground State		Whole Spectrum		
		Valence Shell	Deep Shell	Valence Shell	Valence Shell	Deep Shell	Sum
$^{12}_{\Lambda}\text{C}$	$\Lambda < h_{1/2}$	—	.12	.35	.62 (80%)	.15	.77
	$\Lambda > h_{1/2}$	.26	.03	.01			
$^{16}_{\Lambda}\text{O}$	$\Lambda < h_{1/2}$	—	.12	.42	.79 (84%)	.15	.94
	$\Lambda > h_{1/2}$	.35	.03	.02			
$^{32}_{\Lambda}\text{S}$	$\Lambda < h_{1/2}$	—	.16	.36	.69 (73%)	.22	.91
	$\Lambda > h_{1/2}$	.29	.06	.04			
$^{40}_{\Lambda}\text{Ca}$	$\Lambda < k_{1/2}$	—	.13	.36	.72 (81%)	.17	.89
	$\Lambda > k_{1/2}$	.31	.04	.05			

A. BOUYSSY

Average Excitation Energy $\langle E_{\Lambda}^n - E_{\Lambda}^0 \rangle$

Average Energy to change $n \rightarrow \Lambda = \langle \Delta V_{NA} \rangle$

Hypernucleus	$\langle E_{\Lambda}^n - E_{\Lambda}^0 \rangle$		$\langle \Delta V_{NA} \rangle$	
	Experiment	Shell-Model	Experiment	Shell-Model
$^{12}_{\Lambda}\text{C}$	18 MeV	17	27 MeV	26
$^{16}_{\Lambda}\text{O}$	25	24	27	29
$^{32}_{\Lambda}\text{S}$	35	31	30	30
$^{40}_{\Lambda}\text{Ca}$	35	31	30	30

from A. Boussy

(K^-, π^-)

AVERAGE EXC. ENERGY OF QUASIFREE
 Λ PEAK = $\langle E_{\Lambda}^{QF} - E_{\Lambda}^0 \rangle$

AVERAGE EXC. ENERGY OF RECOILLESS
 Λ PEAK = $\langle E_{\Lambda}^R - E_{\Lambda}^0 \rangle$

Hypernucleus	$\langle E_{\Lambda}^{QF} - E_{\Lambda}^0 \rangle$	$\langle E_{\Lambda}^R - E_{\Lambda}^0 \rangle$	$\langle E_{\Lambda}^{QF} - E_{\Lambda}^R \rangle$
$^{12}_{\Lambda}\text{C}$	21	13	8
$^{16}_{\Lambda}\text{O}$	28	18	10
$^{32}_{\Lambda}\text{S}$	33	25	8
$^{40}_{\Lambda}\text{Ca}$	33	25	8

A. BOUYSSY

FRACTION OF RECOILLESS Λ PRODUCTION IN (K^-, π^-) REACTION

Hypernucleus	Broad Bump	Narrow Peak	Whole Spectrum
$^{12}_{\Lambda}\text{C}$	25 %	85 %	50 (14 + 36)
$^{16}_{\Lambda}\text{O}$	16	75	42 (9 + 33)
$^{32}_{\Lambda}\text{S}$	20	54	31 (11 + 20)
$^{40}_{\Lambda}\text{Ca}$	13	49	26 (7 + 19)

A. Bouyssy

Possibility of stable Light Systems with $S=-$,
could be probed with (κ^-, κ^+) reaction

Second order processes:

$B=2$

$B=3$

$B=4$

$B=6$

D. J. Prowse, Phys. Rev. Letters 17, 782 (1966)

He⁶_{ΛΛ} Double Hypernucleus
seen in K⁻ + Emulsion (4-5 GeV)

- $(K^-, K^+) \text{ to make } \Xi^-$
- $\Xi^- + {}^{12}\text{C} \rightarrow \text{Li}^7 + \text{He}_{\Lambda}^6$
- $\text{He}_{\Lambda}^6 \rightarrow \pi^- + p + \text{He}_{\Lambda}^5$
- $\text{He}_{\Lambda}^5 \rightarrow \pi^- + p + \alpha$

Graphs for formation of $\Lambda\Lambda$ hypernuclei:

1) non-strange meson exchange

$$\sim f_{K^- p \rightarrow \pi^0 \Lambda} f_{\pi^0 p \rightarrow K^+ \Lambda}$$

2) strange meson exchange

$$\sim f_{K^- p \rightarrow K^+ \Xi^-} f_{\Xi^- p \rightarrow \Lambda}$$

$\sim$ not product of amplitudes

$$\sigma_{K^- p \rightarrow K^+ \Xi^-} \sim 0.1 \text{ mb}$$

$$\sigma_{K^- p \rightarrow \pi^0 \Lambda} \sim 2-5 \text{ mb}$$

$$\sigma_{\pi^0 p \rightarrow K^+ \Lambda} \sim \frac{1}{2}-1 \text{ mb}$$