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VARIATIONAL THEORY OF NUCLEAR AND NEUTRON MATTER

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I. INTRODUCTION

In the many-body theory of nuclei, nuclear and neutron-star matter it is assumed that the nucleons are the main degrees of freedom, and their dynamics is given by the Hamiltonian:

$$H = \sum_i -\frac{\hbar^2}{2m} \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk} + \dots \quad (1.1)$$

Here v_{ij} and v_{ijk} are two- and three-nucleon interactions that contain the effects of all mesonic and quark degrees of freedom. In principle we can also have many-body interactions (more than three) between the nucleons, and we will study such a model. This approach to the theory of nuclei and nuclear matter was pioneered in the fifties by Brueckner and Bethe, and many researchers have contributed to its development over the past forty years.

In this theory the properties of nuclei and nucleon matter can be studied from the solutions of the many-body Schrödinger equation:

$$H \Psi_n (1,2,\dots,A) = E_n \Psi_n (1,2,\dots,A), \quad (1.2)$$

where Ψ_n is a wave function of the positions, spins and isospins of all the A nucleons in the system. The $A=2$, two-body problem can be solved by simple numerical integration of the radial Schrödinger equation.¹ The three-body problem can also be solved exactly for the ground state² and low-energy continuum states³ with the Faddeev equations. The four-body ground state can in principle be calculated with the Faddeev-Yakubovsky equations.⁴ The $A=4$ to ∞ problems can be

treated approximately by perturbative methods based on the Brueckner-Bethe theory.^{5,6}

In these lectures we will discuss attempts to solve the $A=3$ to ∞ nuclear many-body problems with the variational method.^{7,8} We choose the form of a variational wave function $\Psi_V(1,2\dots A)$ to describe the ground state. The Ψ_V and the ground-state energy E_V are obtained by minimizing

$$E_V = \langle \Psi_V | H | \Psi_V \rangle / \langle \Psi_V | \Psi_V \rangle \quad (1.3)$$

with respect to variations in Ψ_V . If the form of the variational wave function is chosen properly we can expect $\Psi_V = \Psi_0$ and $E_V = E_0$ where Ψ_0 and E_0 are the exact ground-state wave function and energy. In general $E_V \geq E_0$ in variational calculations.

Some of the advantages of the variational method are as follows.

(i) With the Green's function Monte Carlo method^{9,10,11} it is possible to start from the variational Ψ_V and obtain the exact Ψ_0 and E_0 ; so far however, this has only been done for the $A \leq 4$ nuclei¹¹ due to the complexities of the nuclear forces. (ii) It is simpler to treat both many-body forces and high density matter with the variational method. (iii) It is possible to treat excited states either directly¹³ or by using orthogonal correlated states^{14,15} based on Ψ_V . (iv) It is simple to estimate expectation values of various observables^{11,15} with the Ψ_V . An overview of the variational method is given in section II.

Only the very long-range properties of the interactions v_{ij} and V_{ijk} can be calculated from the known pion-nucleon coupling. Thus both v_{ij} and V_{ijk} have to be determined from experimental observations. Since the two-body problem can be solved exactly the available $N-N$ scattering data can be used to develop realistic models of v_{ij} . The ground state energies of ^3H , ^3He and ^4He and the binding energy and equilibrium density of nuclear matter have been used to construct models of V_{ijk} . However, since the many-body calculations are not exact, their errors influence the parameters of V_{ijk} . The recent models of nuclear forces are discussed in section III, and some of the results obtained for nuclear and neutron matter are given in sections IV-VI.

II. THE VARIATIONAL METHOD

It is most important to choose the form of the variational wave function Ψ_V correctly. For example, nuclei and nuclear matter are not

even bound if Ψ_v is chosen as a Slater determinant Φ , as in Hartree-Fock theory,

$$\Phi = \mathcal{A} \prod_i \phi_i(x_j), \quad (2.1)$$

or as a Jastrow wave function:

$$\Psi_J = \left(\prod_{i<j} f^c(r_{ij}) \right) \Phi, \quad (2.2)$$

with realistic forces. In the above equations x_j denotes position, spin and isospin of particle j , $\phi_i(x_j)$ are single-particle wave functions, $\mathcal{A}$ denotes antisymmetrization and $f^c(r_{ij})$ are central spatial correlation functions.

The tensor force, dominated by the one-pion exchange, provides a crucial part of the attraction with which nuclei are bound. The wave functions (2.1) and (2.2) do not contain tensor and other spin-isospin correlations, and hence are not suitable for variational calculations of nuclei. A reasonable Ψ_v for nuclear systems has the form:

$$\Psi_v = \left(\mathcal{S} \prod_{i<j} F_{ij} \right) \left[\mathcal{S} \prod_{i<j<k} (1+u_{ijk}) \right] \Phi, \quad (2.3)$$

where the pair correlation operator F_{ij} is given by:

$$\begin{aligned} F_{ij} &= f^c(r_{ij}) + f^t(r_{ij}) \tau_i \sigma_j + f^{\sigma}(r_{ij}) \sigma_i \sigma_j + f^{\sigma t}(r_{ij}) \sigma_i \sigma_j \tau_i \sigma_j \\ &\quad + f^t(r_{ij}) S_{ij} + f^{tt}(r_{ij}) S_{ij} \tau_i \sigma_j + f^b(r_{ij}) L \cdot S + f^{bt}(r_{ij}) L \cdot S \tau_i \sigma_j \\ &= \sum_{p=1,8} f^p(r_{ij}) O_{ij}^p. \end{aligned} \quad (2.4)$$

The N-N interaction has strong components associated with the above eight operators. Not much of work has been done on three-body correlations u_{ijk} in nuclear matter. In light nuclei these three-body correlations can reduce the energy by several percent.^{8,16} They are generated by both the two-body forces¹⁷ and three-body forces.¹⁶ The correlation operators F_{ij} and u_{ijk} do not commute with each other, and hence their products have to be symmetrized.

In principle the single particle functions $\phi_i(x)$, the correlation functions $f^{p=1,8}(r)$ and u_{ijk} should be obtained by minimizing the ground state energy. It is difficult to treat this variational problem exactly. The calculations are simpler for light nuclei (^3H , ^3He and ^4He), where $\phi_i(x)$ can be taken as the four spin isospin states $\chi^{\sigma\tau} = n\uparrow, n\downarrow, p\uparrow$ and $p\downarrow$ with no spatial dependence, and for nuclear matter, where $\phi_i(x)$ are plane waves $\exp(ik \cdot r) \chi^{\sigma\tau}$ with $k \leq k_F$ the Fermi momentum. For simplicity and brevity we will consider only these systems, discuss only the parametrization of F_{ij} in nuclear matter and

furthermore ignore the effects of Fermi motion on F_{ij} ; they are given in ref. 18.

One can generally assume that F_{ij} is given by:

$$-\frac{\hbar^2}{m} \nabla_{ij}^2 F_{ij} + v_{ij} F_{ij} = \omega_{ij} F_{ij}. \quad (2.5)$$

This equation serves to define ω_{ij} which can be considered as the change in v_{ij} due to many-body effects. The ω_{ij} is responsible for obtaining the correct long-range behavior of F in nuclear matter; $f^c(r \rightarrow \infty) = 1$ and $f^{p^c}(r \rightarrow \infty) = 0$, and in many ways it is more convenient to vary F_{ij} by varying ω_{ij} . In a commonly used form the parameters of F are denoted by d_p , α_p and β_p ; d_p is the range at which f^p attains its asymptotic value. The ω_{ij} is taken as:

$$\omega_{ij} = \sum_p \left[(1-\alpha_p) v^p(r_{ij}) + \lambda^p \right] o_{ij}^p, \quad (2.6)$$

and λ^p are adjusted to obtain the prescribed d_p 's. The number of parameters can be greatly reduced by assuming that

$$d_t = d_{t\tau}, \quad (2.7)$$

$$d_c = d_\sigma = d_\tau = d_{\sigma\tau} = d_b = d_{b\tau}, \quad (2.8)$$

$$\alpha_c = 1, \text{ and } \alpha_{p^c} = \alpha. \quad (2.9)$$

These approximations are found to be reasonable in several calculations.¹⁹

Let the F obtained from equations (2.5-9) by minimizing the energy with respect to d_t , d_c and α be denoted by $\tilde{F}$. The F used in calculations including three-body forces is defined as:

$$F_{ij} = \tilde{f}^c(r_{ij}) + \sum_{p=2,8} \beta_p \tilde{f}^p(r_{ij}) o_{ij}^p \quad (2.10)$$

with the "reasonable" approximations:

$$\beta_t = \beta_{t\tau}, \quad (2.11)$$

$$\beta_\sigma = \beta_\tau = \beta_{\sigma\tau} = \beta_b = \beta_{b\tau}. \quad (2.12)$$

The β_p parameters are meant to take into account the effect of V_{ijk} on F .¹⁹ When V_{ijk} is zero all β_p 's are close to unity, and it is sufficient to vary only d_c , d_t , and α . The two-pion exchange part of V_{ijk} increases the magnitude of tensor and spin-correlations significantly, thus $\beta_{t\tau}$ and $\beta_{\sigma\tau}$ obtain equilibrium values > 1 .

The expectation value of H has to be calculated and minimized in variational calculations. In the light nuclei $\langle H \rangle$ is calculated exactly by the Monte Carlo method.²⁰ Unfortunately the computational time required to calculate $\langle H \rangle$ with the currently available Monte Carlo

methods increases approximately as $2^A/N!Z!$, where A , N and Z are respectively the number of nucleons, neutrons and protons in the system. Thus this exact method is practical only for nuclei having $A \lesssim 8$, with presently available computers. The expectation value $\langle H \rangle$ in nuclear matter is calculated with chain summation techniques.^{7,19} Even though this method is not exact its accuracy is believed to be very good²¹; for example the error in the calculation of E_v for the ground state of nuclear matter at equilibrium density is expected to be < 1 MeV per nucleon.

Let us denote the ground state of noninteracting Fermi gas by $|0\rangle$. The normalized variational ground state $|0\rangle$ can then be written as:

$$|0\rangle = G|0\rangle / [0|G^\dagger G|0]^{1/2}, \quad (2.13)$$

where G represents the product of correlation operators in eq. (2.3). The G can be used to transform all noninteracting states $|I\rangle$ into correlated states $|I\rangle$. For example a correlated two-particle two-hole state $|p_1 p_2 h_1 h_2\rangle$ is obtained as

$$|p_1 p_2 h_1 h_2\rangle = G|p_1 p_2 h_1 h_2\rangle / [p_1 p_2 h_1 h_2|G^\dagger G|p_1 p_2 h_1 h_2]^{1/2}, \quad (2.14)$$

where $|p_1 p_2 h_1 h_2\rangle$ is a Fermi-gas state with particles p_1 and p_2 and holes h_1 and h_2 . These correlated states have reasonable energies, but they are not orthogonal to each other. They have been used in nonorthogonal basis perturbation theory^{13,22} to calculate various quantities such as corrections to E_v , the real and imaginary parts of the optical potential of a nucleon in nuclear matter, properties of matter at finite temperatures²³, etc.

Recently¹⁴ new methods have been developed to orthogonalize the correlated states while preserving their energies. Let $|I\rangle$ be the orthogonal correlated state that corresponds to the correlated state $|I\rangle$ or Fermi-gas state $|I\rangle$. The orthogonalization preserves energies:

$$\langle I|H|I\rangle = \langle I|H|I\rangle, \quad (2.15)$$

which are determined variationally. A perturbation theory can be easily developed using the orthogonal states $|I\rangle$.

$$H = H_0 + H_I, \quad (2.16)$$

$$\langle I|H_0|J\rangle = \langle I|H|J\rangle \delta_{IJ}, \quad (2.17)$$

$$\langle I|H_I|J\rangle = \langle I|H|J\rangle (1 - \delta_{IJ}). \quad (2.18)$$

It is expected to have good convergence when the correlation operator G

is realistic so that $|I\rangle$ are close to the eigenstates of H and $\langle I|H|J\rangle$ is small when $I \neq J$. This orthogonal correlated functions theory has been used to study the response of nuclear matter,²⁴ spectral functions,²⁵ etc., and it can also be used to study the optical potentials and many other properties.

III. NUCLEAR FORCES

Nuclear forces are the only input in the many-body theory; however, they are not yet well understood. Nucleons are finite objects made up of quarks and gluons, and they interact in a fairly complex manner. Realistic models of two-nucleon-interaction potential v_{ij} are obtained in several towns like Argonne²⁶, Bonn²⁷, Nijmegen²⁸, Paris²⁹, and Urbana³⁰, by fitting the deuteron properties and two-nucleon scattering data at $E_{\text{lab}} \lesssim 400$ MeV. All these models have a one-pion exchange long-range part and more or less phenomenological intermediate and short range parts. The Nijmegen potential gives the best χ^2 fit to the data, while the Argonne model gives the best description³¹ of the deuteron form factors $A(q^2)$ and $B(q^2)$.

The Urbana and Argonne models are simpler to use in many-body calculations. The Argonne potential is a more recent version of the Urbana; the tensor force in the Argonne model is much stronger than that in the Urbana at small r (fig. 1). In other respects these two models are similar. The tensor forces in Argonne and Paris models are similar (fig. 1); the D-states account for 5.2, 5.8 and 6.1% of the Urbana, Paris and Argonne deuteron wave functions. The properties of nuclear matter depend to some extent on the strength of the tensor force. For example, neutron matter has a transition to a phase in which π^0 -condensation appears likely, when the Argonne model is used, but not if the Urbana model is used.¹⁹

Fig. 1. The $v^{TT}(r)$ in Urbana, Paris and Argonne models is compared with the one-pion-exchange potential with a cutoff $\Lambda = 7m_\pi$.

The binding energy of the three-body nucleus ${}^3\text{H}$ can be very accurately calculated for each model from 34-channel Faddeev calculations.^{2,32} The experimental energy is -8.48 MeV, while the Paris, Argonne and Nijmegen potentials give -7.47, -7.67 and -7.62 MeV respectively. The coordinate-space Bonn potential gives a better triton energy^{32,33} of -8.29 MeV, but this result may not be very significant since the improvement in the energy probably comes from the unrealistic energy dependence of the coordinate space Bonn ${}^3\text{S}_1$ - ${}^3\text{D}_1$ mixing phase³⁴ ϵ_1 .

The energies of nuclei having four or more nucleons can not as yet be calculated accurately, i.e. with <1% error. Quite recently Carlson^{11,35} has made significant advances in exact calculations of ${}^4\text{He}$ binding energy with the Green's function Monte Carlo method. Exact calculations have been carried out with the Reid- v_8 interaction³⁵ for which the exact energy (-24.5 MeV) is ~6% below the variational energy of -23 MeV. Currently available variational calculations of ${}^3\text{H}$ also have ~6% error; more recently Wiringa¹⁶ has used better parameterization of the F_{ij} and u_{ijk} to reduce the error to ~3%. Available variational calculations^{8,35} give energies of $-23 \pm .2$ MeV with the Urbana, Argonne and Nijmegen potentials for the α particle.

Brueckner theory³⁶ and the variational method^{36,19} have been used to study nuclear matter (NM) with the Paris, Argonne and Urbana interactions. Day³⁶ obtained almost identical NM energies with the Argonne and the Paris models by summing two-, three- and some four-hole line terms in Brueckner theory. His results are compared with those obtained for the Argonne and Urbana potentials by the variational method¹⁹ in fig. 2. The Brueckner theory energies are ~10% below the variational energies.

Fig. 2. The nuclear matter energy obtained with the Urbana and Argonne models in variational calculations. The vertical lines BB-A show results obtained with the Brueckner-Bethe theory and the Argonne model by Day. The curve FP has correct empirical saturation point.

The results obtained with the Argonne, Nijmegen, Paris and Urbana models can be summarized as follows. All these potentials underbind ${}^3\text{H}$ and ${}^4\text{He}$, they give reasonable value for the binding energy of nuclear matter, but the calculated equilibrium density ($\sim 0.3\text{fm}^{-3}$) has about twice the empirical value of $\sim 0.16\text{fm}^{-3}$. Most importantly the differences between the results of different potential models are smaller than those between experiment and theory. It thus appears likely that the simple model of nucleons interacting with only two-body forces is not realistic for many-nucleon systems.

It is well known that there are three-nucleon forces³⁷ V_{ijk} . Particularly the two-pion exchange $V_{ijk}^{2\pi}$ is theoretically well established³⁸, and it can provide the additional attraction necessary to obtain the correct binding energy of ${}^3\text{H}$ and ${}^4\text{He}$.³⁹ However, if one takes only the attractive $V_{ijk}^{2\pi}$, NM gets overbound and also has too large an equilibrium density. Hence it is necessary to have other terms in V_{ijk} to obtain reasonable NM. Our phenomenological approach⁸ has been to consider a model of V_{ijk} :

$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^R \quad (3.1)$$

in which V_{ijk}^R is a spin-isospin independent repulsive interaction of reasonable range, and to adjust the strengths of $V_{ijk}^{2\pi}$ and V_{ijk}^R to obtain the experimental binding energies of ${}^3\text{H}$, ${}^4\text{He}$ and reasonable nuclear matter properties in variational calculations. The obvious problems with this approach are that the variational calculations are not exact, and the assumed form of V_{ijk}^R , which should contain all the components of V_{ijk} other than $V_{ijk}^{2\pi}$ may be simplistic. Better calculational methods are being developed, and attempts to calculate the binding energies of more nuclei like ${}^6\text{Li}$, ${}^8\text{He}$, ${}^{16}\text{O}$ etc. are underway to address these problems.

The latest model of the type (3.1) is numbered VII; it gives the experimental ${}^3\text{H}$ and ${}^4\text{He}$ binding energies⁸, when used with either Argonne or Urbana interactions, and NM results¹⁹ shown in fig. 3. Arguments suggesting that the parameters of model VII are not unrealistic are given in ref. 40. Unfortunately neither Urbana or Argonne $v_{ij} + V_{ijk}$ (VII) models give the empirical NM energy (~ 16 MeV) or density ($\rho_0 = 0.16\text{fm}^{-3}$), but they come quite close.

One can develop even more phenomenological models that are constructed to explain the equilibrium properties of NM. In a model developed by Lagaris and Pandharipande (LP) the Urbana model of v_{ij} is expressed as a sum of pion exchange v_{ij}^{π} , two-pion exchange v_{ij}^I and short range v_{ij}^S parts:

$$v_{ij} = v_{ij}^{\pi} + v_{ij}^I + v_{ij}^S. \quad (3.2)$$

The v_{ijk}^R essentially makes the attractive v_{ij}^I weaker as the density increases. LP assume that the energy of NM (per nucleon) is given by:

$$E(\rho) = E[v_{ij}(\rho)] + \gamma_2 \rho^2 \exp(-\gamma_3 \rho) \left(3 - 2 \left(\frac{N-Z}{A}\right)^2\right), \quad (3.3)$$

where $E[v_{ij}(\rho)]$ is the energy obtained with the density dependent interaction

$$v_{ij}(\rho) = v_{ij}^{\pi} + v_{ij}^I \exp(-\gamma_1 \rho) + v_{ij}^S, \quad (3.4)$$

The term linear in ρ in $v_{ij}(\rho)$ contains the effects of v_{ijk}^R , while the γ_2 -term in eq. (3.3) represents the contributions of $v_{ijk}^{2\pi}$ and all other three-body interactions. The ρ^2 and higher terms in $v_{ij}(\rho)$ represent four and more body interactions.¹⁸ The parameters γ_1 , γ_2 and γ_3 are varied to obtain $E_0 = -16$ MeV, $\rho_0 = 0.16 \text{ fm}^{-3}$ and compressibility $K=240$ MeV for NM. This model has been used by Friedman and Pandharipande⁴¹ (FP) to study properties of hot and cold nuclear and neutron matter. Their results are also shown in figs. 2 and 3.

Fig. 3. The nuclear matter energy obtained with the three models.

Unfortunately the LP model cannot be used to study the light nuclei. In this respect the models with explicit three-body forces are superior. Secondly the compressibility of nuclear matter is not well known. The values of K extracted from observed breathing mode energies of nuclei range from $\sim 220 \text{ MeV}^{42}$ to 300 MeV^{43} , and it has been suggested that there may be large uncertainties in these extractions.⁴⁴

IV. NEUTRON STAR STRUCTURE

The structure of neutron stars is determined by the equation of state $P(\epsilon)$ of electrically neutral matter at zero temperature. The pressure P and energy density ϵ of matter is related to the energy $E(\rho)$ per nucleon in matter at nucleon density ρ :

$$\epsilon(\rho) = \rho(E(\rho) + mc^2), \quad (4.1)$$

$$P(\rho) = \rho^2 \partial E(\rho) / \partial \rho, \quad (4.2)$$

and the $E(\rho)$ is calculated from chosen nuclear forces. There have been many calculations of neutron star structure in the past thirty years. We will discuss here the most recent results obtained by Wiringa, Fiks and Fabrocini¹⁹ with the Argonne and Urbana v_{ij} and model VII of V_{ijk} , and with the density-dependent interaction model of LP. These models are denoted by A+VII, U+VII and FP respectively. The $\epsilon(\rho)$, $P(\rho)$ and the sound velocity

$$s(\rho) = (\partial P(\epsilon) / \partial \epsilon)^{1/2} |_{\epsilon=\epsilon(\rho)} \quad (4.3)$$

are shown in fig. 4. Both the U+VII and A+VII models give similar results; in these models $s(\rho)$ exceeds c at $\rho \gtrsim 0.8 \text{ fm}^{-3}$ primarily due to the contribution of V_{ijk} . The FP model gives a softer equation of state, and its $s(\rho)$ does not exceed c at least up to $\rho \approx 2 \text{ fm}^{-3}$. The main difference in the U+VIII and FP models is that the latter include many-body (more than three) forces which help keep $s(\rho) \leq c$.

Fig. 4. The $\epsilon(\rho)$, $P(\rho)$ and $S(\rho)$ for the three models. The scale on the right side is for $S(\rho)$.

The A+VII equation of state has a structure at $\rho \sim 0.2 \text{ fm}^{-3}$ that is most visible in the $s(\rho)$. In the density region $\rho = 0.2$ to 0.25 fm^{-3} the tensor and spin correlations change very rapidly¹⁹ in this model of neutron matter, and the contribution of pion exchange interactions suddenly increases as expected in a phase transition to a state with π^0 condensation. This transition does not occur in nuclear matter with the Argonne or Urbana models with or without three-nucleon interactions. It also does not occur in neutron matter with the Urbana model with or without V_{ijk} or with the Argonne model without V_{ijk} . Thus, in the models studied it occurs for only the A+VII. It is very sensitive to the details of the forces, and does not have a large effect on the static properties of neutron stars. It can however have dynamical effects, such as enhanced cooling rates, that can be observed.

The properties of neutron stars calculated with these models are compared with some of the available observational data in fig 5. For comparison the results obtained with only the Urbana v_{ij} are also shown by curves labeled U. Masses of seven binary x-ray pulsars, believed to be neutron stars, have been determined⁴⁵; they are all near $1.4 M_\odot$ as expected from evolutionary considerations. The largest minimum mass is $1.55 M_\odot$ for 4U0900-40 ($M = 1.85^{+0.35}_{-0.3} M_\odot$) as indicated in fig. 5. The mass and radius obtained for the source MXB 1636-536, by assuming that it is a neutron star whose x-ray bursts are due to thermonuclear flashes⁴⁶ in the accreted surface matter, are also shown in fig. 5 by a box. The surface red-shift inferred from the gamma-ray bursts of many stars⁴⁷ suggests that they are neutron stars with $M \sim 1.4 M_\odot$ and $R \sim 10.5 \text{ km}$; consistent with the values assigned to MXB 1636-536. It appears that the neutron stars obtained from models including the three-nucleon interaction V_{ijk} are in better agreement with the observational data than those obtained from models with only two-nucleon interactions.

Fig. 5. The mass-radius relation for neutron stars.

The properties of the neutron star that may have formed in supernova 1987A have been studied by many authors. For example analysis^{48,49} of the neutrinos observed during this supernova indicates that the binding energy of the star could be $2-5 \cdot 10^{53}$ ergs which is consistent with $3 \cdot 10^{53}$ ergs predicted for a $1.4 M_{\odot}$ star by these models. The reported observation of a half millisecond period in SN1987A has generated renewed interest in the maximum rotational velocity of neutron stars. Recent calculations^{50,51} indicate that if the half millisecond period is indeed the period of rotation of the neutron star in SN1987A, and if our present understanding of the equations of state is even approximately correct, the mass of the neutron star in SN1987A is probably greater than the Chandrashekhar mass limit, it is stable due to its large angular momentum, and it will collapse into a black hole on slowing down.

V. MOTION OF NUCLEONS IN NUCLEAR MATTER

The motion of nucleons in nuclear matter is generally described with an optical potential whose real and imaginary parts $U(\rho, e)$ and $W_0(\rho, e)$ give the momentum and mean free path of nucleons of energy e in nuclear matter at density ρ :

$$k^2(\rho, e) = \frac{2m}{\hbar^2} [e - U(\rho, e)], \quad (5.1)$$

$$\lambda(\rho, e) = \hbar^2 k(\rho, e) / [2mW_0(\rho, e)]. \quad (5.2)$$

The optical potential is used to study nucleon-nucleus scattering⁵², and heavy-ion collisions.⁵³

In the variational method the state of a nucleon of momentum p moving in nuclear matter is described by the wave-function:

$$|p\rangle = |p\rangle = G a_p^\dagger |0\rangle / [0|a_p G^\dagger G a_p^\dagger |0\rangle]^{1/2}, \quad (5.3)$$

following the notation of equations 2.13 and 2.14. The variational approximation $U_v(\rho, e)$ to the $U(\rho, e)$ has been calculated^{54,55} from $\langle p|H|p\rangle$,

$$e(p) = \frac{\hbar^2}{2m} p^2 + U_v(\rho, e) = \langle p|H|p\rangle - \langle 0|H|0\rangle. \quad (5.4)$$

The motion of the nucleon is damped by collisions which generate two-particle-one-hole states $|p'p'h\rangle$, and the second-order contributions of $\langle p|H|p'p'h\rangle$ give the imaginary part $W(\rho, e)$ of the single-particle energy as well as a correction ΔU to $U(\rho, e)$. Thus, optical potentials correct up-to second order in correlated basis theories are given by:

$$U_2(\rho, e) = U_V(\rho, e) + \Delta U(\rho, e), \quad (5.5)$$

$$W_0(\rho, e) = m^*(\rho, e) W(\rho, e) / m \quad (5.6)$$

$$\frac{m^*(\rho, e)}{m} = 1 - \frac{dU(\rho, e)}{de}. \quad (5.7)$$

The effective mass factor reduces the $W_0(\rho, e)$ significantly at low energies where $m^* \sim 0.7m$, and it is necessary to obtain the empirical mean-free paths in nuclear matter.^{56,57} The $U(\rho, e)$ and $W_0(\rho, e)$ obtained with the FP model²² are compared with the empirical data at $\rho = \rho_0$ in figures 6 and 7. The U_V ($\rho > \rho_0, e$) obtained with A+VII, U+VII and DDI models are given in ref. 55.

Fig. 6. The real part of the optical potential calculated with the FP model. Full lines give results of variational calculations and dashed lines include second order correlated basis corrections. The sources of empirical data at ρ_0 are given in Ref. 54.

Fig. 7. The imaginary part of the optical potential calculated with the FP model at ρ_0 . The sources of empirical data are given in ref. 56.

The two nucleon scattering cross section $d\sigma(\rho, \theta, \phi)$ in matter at density ρ is important in theoretical simulations of heavy-ion collisions.⁵³ It is given by:

$$\left(\frac{\hbar \vec{p}_1}{m^*(p_1)} - \frac{\hbar \vec{p}_2}{m^*(p_2)} \right) d\sigma(\rho, \theta, \phi) = \frac{2\pi}{\hbar} |\langle p_1 p_2 | H | p_1' p_2' \rangle|^2 \rho_D, \quad (5.8)$$

where the factor multiplying $d\sigma$ is the relative velocity, $|p_1 p_2\rangle$ and $|p_1' p_2'\rangle$ are correlated orthogonal many-body states, and ρ_D is the density of final states $|p_1' p_2'\rangle$. At low energies $d\sigma(\rho, \theta, \phi)$ is approximately half the corresponding cross section for free nucleons. Much of the difference is due to effective masses; ρ_D in matter is smaller than that in free space by a factor of m^*/m , and so, if the interaction matrix elements are similar in vacuum and matter,

$$d\sigma(\rho, \theta, \phi) = (m^*/m)^2 d\sigma_{\text{free}} \quad (5.9)$$

The effective mass $m^*(\rho, e)$ depends upon both the density and energy. At low energies and equilibrium density $m^* \sim 0.7m$ and hence $d\sigma$ is reduced by almost a factor of two from its free space value. At higher densities m^* is even smaller and hence the difference could be larger. At larger energies $m^* \sim m$ and hence differences will be smaller.

VI. THE EQUATION OF STATE

The free-energy

$$F(\rho, T) = U(\rho, T) - TS(\rho, T) \quad (6.1)$$

has been calculated variationally⁴¹ with the DDI model of LP at selected values of ρ, T for nuclear and neutron matter. In principle all thermodynamic quantities like pressure $P(\rho, T)$, entropy $S(\rho, T)$ internal energy $U(\rho, T)$ etc. can be obtained from the derivatives of $F(\rho, T)$. A very useful procedure, developed by Ravenhall⁵⁸, is to use a Skyrme type Hamiltonian density fitted to the variational $F(\rho, T)$ to calculate these thermodynamic quantities. The parameters of the Ravenhall-Skyrme Hamiltonian, whose low- and high-density behavior is quite different from that of the common Skyrme Hamiltonians, are given in ref. 58. The calculated pressure isotherms $P(\rho, T)$ are shown in fig. 8; they have the typical van-der-Waal's shape. At temperatures above the critical temperature $T_c = 17$ MeV the $P(\rho, T)$ increases monotonically with ρ and matter is stable at all densities. Below T_c there exists a density range $\rho_g(T) < \rho < \rho_l(T)$ in which matter separates into gas at

density $\rho_g(T)$ and liquid at $\rho_l(T)$. The densities $\rho_l(T)$ and $\rho_g(T)$ are found by standard Maxwell construction, and their loci form the liquid-gas coexistence curve shown in fig. 9. Matter is unstable against small isothermal density fluctuations in the region where $\partial P/\partial \rho|_T$ is negative. This region is bounded at a given T by isothermal spinodal densities $\rho_g'(T)$ and $\rho_l'(T)$ at which $\partial P/\partial \rho|_T = 0$, and their loci is shown in fig. 9 as the isothermal spinodal curve. Matter at ρ, T in the region between the isothermal spinodal and coexistence curves is metastable, i.e. it is stable against small fluctuations, but will phase separate via nucleation in the presence of large fluctuations. This is the region in which supercooled liquid or gas can exist, and cloud and bubble chambers operate.

Figure 9 also shows the contours for equal energy $U(\rho, T)$ and entropy $S(\rho, T)$. These contours are very useful in estimating the expansion and fragmentation of hot matter formed in heavy-ion collisions.⁵⁹ The curve labeled $P=0$ is the loci of points $P(\rho, T) = 0$. It has a maximum temperature $T_m = 12$ MeV, and one can have liquids cooling by evaporation in vacuum only at $T < T_m$. Matter at ρ, T inside the adiabatic spinodal curve in fig. 9 is unstable against small adiabatic fluctuations.⁵⁸

Fig. 8. The $P(\rho)$ isotherms of the FP model of nuclear matter. The coexistence and spinodal points at $T=10$ MeV are denoted by ρ_g , ρ_g' , ρ_l' and ρ_l respectively.

Fig. 9. The phase diagram of nuclear matter showing coexistence (CE), $P=0$, isothermal (ITS) and adiabatic (AS) spinodal curves. The thin full lines are isentropes labeled with entropy, while the dashed lines are contours of equal internal energy labeled with $U(\rho, T)$ per nucleon.

At high temperatures the non-nucleonic degrees of freedom are excited and contribute to the properties of matter. The two excitations that appear to be most relevant in the $T \sim 50$ MeV region are pion production and $N \rightarrow \Delta$ (nucleon-hole Δ -particle) excitation. The spectrum of noninteracting pions $\omega_F(k)$ and that of $N \rightarrow \Delta$ excitation $\omega_R(k)$ (neglecting the momentum of nucleon hole and Δ -decay width) are shown in fig. 10. At small k the pion excitation has lower energy, while at large k the $N \rightarrow \Delta$ excitation has lower energy. The two modes are coupled with the strong $\pi N \Delta$ coupling and they naturally mix. The coupled low-energy excitation, recently named pisobar⁴⁴, is essentially a pion at small k , a $N \rightarrow \Delta$ excitation at large k , and a mixture of the two at intermediate values of k . The calculation of the spectrum $\omega(k, \rho)$ of this pisobar excitation is discussed by Migdal⁶⁰ and the results obtained in ref. 61 are shown in fig. 10. The $\omega(k, \rho_0)$ at small k can be estimated from the pion-nucleus optical potential, and the values obtained from that of ref. 62 are also shown in fig. 10.

Fig. 10. The curves $\omega_F(k)$ and $\omega_R(k)$ give the dispersion relations for free pions and $N \rightarrow \Delta$ excitations. The curves labeled with density show pisobar dispersion relations, and the + signs show empirical data at $\rho = \rho_0$ from ref. 62.

The effects of pisobar excitations on the equation of state have been estimated⁶¹ by assuming that hot hadronic matter can be treated as a mixture of hot nuclear matter and pisobar gas. The typical results obtained are shown in fig. 11, where E_π is the energy of pisobars per nucleon, and $R_{\pi N}$ is the ratio of pisobars to nucleons. The E_N^* (S_N) and E^* (S) are the nuclear and total excitation energies (entropies) per nucleon ($E^* = E_N^* + E_\pi$ for example). The pisobars are created at $T \sim 30$ MeV and dominate the properties of hot matter at $T \sim 100$ MeV.

The approximations in these calculations are probably reasonable at lower temperature where $R_{\pi N}$ is small. The calculation neglects the interaction between pisobars, which would be important when $R_{\pi N} \sim 1$. At $T \gtrsim 100$ MeV heavier mesons and higher energy baryon resonances may get excited before matter turns into a quark-gluon plasma. At zero baryon density the transition to quark-gluon plasma is expected to occur at $T \sim 155$ MeV.⁶³ We may expect this transition to occur at $T \lesssim 155$ MeV at densities $\sim \rho_0$. At some high density $\rho_{\max}$ matter will become a deconfined quark liquid even at $T=0$. We do not know what $\rho_{\max}$ is, but the many-body theory discussed in these lectures will certainly break down as ρ approaches $\rho_{\max}$.

Fig. 11. the properties of very hot matter at ρ_0 and $2\rho_0$. The energies are shown in units of 100 MeV per nucleon.

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