PROGRESS REPORT

NONLINEAR DYNAMICS AND PLASMA TRANSPORT

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I. INTRODUCTION AND PROGRAM OUTLINE

This progress report details work done on a program in nonlinear dynamical aspects of plasma turbulence and transport funded by DOE since 1989. This program has been in cooperation with laboratories in the USSR [now Russia and the Confederation of Independent States (CIS)]. The purposes of this program have been:

a. To promote the utilization of recent pathbreaking developments in nonlinear science in plasma turbulence and transport.

b. To promote cooperative scientific investigations between US and CIS in the related areas of nonlinear science and plasma turbulence and transport.

Since July 1990, R. Z. Sagdeev joined the University of Maryland as a distinguished Professor of Physics. Two senior theorists from Moscow Space Institute, Vitali Shapiro and Valentin Shevchenko, came to the University of Maryland in late January 1991 to work here. Two younger scientists, Alexander Chernikov and Daniel Usikov, also came to work here. Recently Andrey Rogalsky came in February 1992 from Space Research Institute. The collaboration between our visitors and Maryland fusion theory group has been active and effective. Particularly noteworthy are the new concepts and methods in nonlinear analysis, such as convection driven flow, that Soviet scientists have introduced into our program.

Several distinguished Russian theoretical physicists have visited the Laboratory for Plasma Research and gave seminars on plasma and fluids physics: Prof. Pitaevskii, holder of Landau chair in the Landau Institute, Prof. Golitsen, Director of the Institute of Atmospheric Physics, and Prof. A. A. Galeev, Director of Space Institute. In addition, several scientists have extended their stays here in the laboratory to do collaborative work: Prof. V. Karpman, senior theorist from the Institute of Terrestrial Magnetism and Ionospheric and Radio Waves (two weeks), Dr. A. Balk, and Prof. Zakharov from Landau Institute. Many other Russian physicists have also visited the laboratory.

Since the dissolution of the Soviet Union, the opportunity to expand our collaboration with CIS colleagues has increased greatly, and we would like to continue our program vigorously to seize this uniquely historical opportunity.
II. PROGRESS REPORT

In this section we summarize the progress made over the last year in three different areas of research:

a. Shear flow generation and reduced transport in fluids and plasma,

b. Nonlinear dynamics and visualization of 3D flows, and

c. Application of wavelet analysis to the study of fractal dimensions in experimental and numerical data.

A. Shear Flow Generation and Reduced Transport in Fluids and Plasmas

1. Peeling of Convective Cells and the Generation of Sheared Flow (Drake, Finn, Guzdar, Shapiro, Shevchenko, Waelbroeck, Hassam, Liu, and Sagdeev)

The generation of shear flow in the edge region of tokamaks is an important aspect of the L-H transitions. Thus understanding the generation mechanism of these flows is fundamental to gaining an understanding of the L-H transition. Our recent 3D simulations of drift resistive ballooning modes have shown that vortices can drive shear flows. The flow generation is governed by the vorticity equation and is intrinsically two-dimensional. Thus to understand the flow generation, over the past year we have studied the stability and nonlinear evolution of a periodic array of vortices to shear flow using a 2D incompressible fluid code. In this study we have developed a comprehensive understanding of the creation of shear flow from a periodic array of convective cells in an incompressible plasma or fluid.¹

We have found both analytically and numerically that a periodic array of vortices is unstable to a shear flow instability. These initial periodic vortices can arise from a wide variety of instabilities. For the tokamak edge plasmas they can arise from unstable resistive ballooning modes. For the atmosphere of Venus, the vortices are generated by thermal convection caused by the nonuniform solar radiation. The instability can be ideal (viscosity $\mu = 0$) or nonideal ($\mu \neq 0$) depending on the elongation of the periodic vortices. Nonlinearly the instability goes through a vortex reconnection or “peeling” phase leading to a state with an island chain. Without source, the flows evolve to a pure one-dimensional shear flow which eventually decays on a much longer time scale due to the viscosity. However, in the presence of a source maintaining the initial two counter-rotating vortices (the source compensates for the viscous damping), the flow evolves to an equilibrium with islands, i.e., having vortex flow plus shear flow and having only one vortex.
A simple four-wave truncation model captures many of the features obtained in the 2D nonlinear simulations. For the case without a source it shows that the final state is one with pure shear flow, slowly decaying with time. For the case with source, the strength of the vortex flow compared to the shear flow scales as $\mu^{1/2}$.

The present work also qualitatively explains experiments involved in the study of an array of driven counter-rotating vortices. In the simulations, as in the experiments, the flow develops to a state in which there are half as many vortices all having the same sign of vorticity, if the Reynolds number is high enough.

In this study the initial vortex flow was prescribed and its stability to shear flow was studied. The next problem addresses the generation of the vortices by the Rayleigh Taylor instability and the subsequent generation of shear flow. The effect of the shear flow on the anomalous transport is an important issue that is studied in this work.

2. Reduction of Transport Due to Rayleigh-Taylor Convection by Development of Shear Flow (Finn, Drake, Guzdar, Hassam, Shapiro, and Shevchenko)

The study of the stability of a given periodic array of vortices to the shear flow instability, discussed above, is an important step in understanding the flow generation. However, it lacks the self-consistency aspect of the instability generating the vortices, the vortices in turn giving rise to shear flow, which in turn affects the instability. The question remains as to what happens in the nonlinear phase. Does it all go into shear flow or does it evolve into a state in which shear flow and vortices coexist? Limited work on the 3D resistive ballooning modes indicate that both scenarios are possible, depending on the choice of parameters. However, a study of the nonlinear states for varying parameters is prohibitive in the 3D case because of limited computer resources.

The 2D Rayleigh-Taylor instability in a finite box serves as a simple model which captures the self-consistency aspects of the problem of the interplay between instability and shear flow. In this case, however, one can do a very systematic study of the nonlinear states as the dimensionless parameters are varied.

In the past year we have begun such a systematic study of the development in two dimensions of shear flow from the convection vortices produced by the Rayleigh-Taylor instability in an incompressible fluid. In the plasma context, the incompressible assumption is based on the existence of a large unsheared magnetic field in the third direction, whereas in the fluid dynamics context the assumption is based on the flow being much smaller than the sound speed. This study, like that in Sec. A, was initiated in order to help understand the generation of sheared rotation in tokamaks and its relation with the H-mode transition. Specifically, this model was constructed in order to capture the essential physics found in our nonlinear resistive ballooning simulations, in which it was found that the ballooning modes lead to
convective vortices, which in turn can lead to the development of sheared rotation and a reduction of the level of the turbulence.

The results obtained to date indicate that a shear instability can develop, depending on the elongation of the vortices. For certain parameters, it has been found that the shear flow can dominate for long times, and that the level of the amplitude of the Rayleigh-Taylor vortex motion can be sufficiently low that the transport is nearly purely classical, i.e., does not have an appreciable convective component. We have also shown that if the shear flow is driven by an external source, so that it does not require the presence of density fluctuations to persist in the presence of viscous decay, the Rayleigh-Taylor mode is completely stabilized. In other areas of the parameter space, the vortex flow and the shear flow can oscillate in time, producing a flow with islands whose width oscillates in time. In such cases the transport is determined by the chaotic motion of the flow trajectories caused by the time dependence of the stream function. In between these two areas of the parameter space there is an area in which the time dependence of the flow has a bursty nature due to the following: the vortex flow leads to the development of sheared velocity, as described in Sec. A. This shear flow completely stabilizes the Rayleigh-Taylor instability which initially led to the vortices. The shear flow then decays very slowly by viscous dissipation until it can no longer stabilize the Rayleigh-Taylor mode. This mode grows rapidly and quickly generates shear flow, and the cycle repeats.

As in the studies of externally driven vortex motion described in Sec. A, the insights developed by these studies have helped to guide the directions of the three-dimensional ballooning studies. For example, the first unequivocal demonstration of vortex stabilization, both linearly and nonlinearly, was shown in these simulations, and the bursty nature of the motion, suggestive of ELM's, was first seen in this much simpler model.

B. Nonlinear Dynamics and Visualization of 3D Flows
   (Chernikov, Finn, Guzdar, Lau, and Usikov)

1. (Flow Visualization and Nonlinear Dynamics of Fluid Trajectories)

We have studied the nonlinear dynamics and transport associated with divergence free flows having a vortex-like structure with stagnation points. The motivation for this work is to apply what is learned about the nonlinear dynamics of the flow trajectories to plasma transport in the presence of three-dimensional convective vortices.

In the tokamak context, a flow $\mathbf{v} = \nabla \phi \times \mathbf{b} + \eta || \mathbf{b}$ can be set up having convective vortices associated with drift resistive ballooning turbulence. In this context, the flow need not be divergence free. Indeed, if the density, satisfying $\partial \rho / \partial t + \nabla \cdot$
(ρv), remains bounded away from zero and infinity, then the equations for the flow
\[ \frac{dx}{dt} = v \] can be made formally divergence free by a change of variables. (This
is analogous to finding canonical variables for the field line equations in a tokamak
\[ \frac{dx}{dz} = B/B_z \].)

The first step we have taken is to continue the work on the nonlinear dynam-
icity of streamlines of steady divergence free flows with stagnation points. In this
work, we took the flow v(x) to correspond to a single convective vortex. The model
studied has a two-dimensional parameter space and has a pair of nulls in a region
of this parameter space. The nonlinear dynamics was found to be very complex
in this range, exhibiting behavior which is qualitatively different from that in a
system in which a component (say v_x) never changes sign. (A system of the latter
type, such as the field lines in a tokamak, is essentially a 1 - 1/2 degree-of-freedom
Hamiltonian system.) There are several bifurcations, of both local and global char-
acter, separating regions having different dynamical characteristics. In one range,
there is a volume of toroidal KAM surfaces bounded by a region connected to the
nulls in which the streamlines can escape to infinity. That is, these outer stream-
lines can exhibit nonhyperbolic chaotic scattering. This behavior corresponds to
a convective vortex in which the separatrix connected to the nulls is destroyed by
a symmetry breaking perturbation. In another range, the KAM surfaces are com-
pletely destroyed. The flow locally looks like a convective vortex, but globally (i.e.,
following the streamlines for a long time) all points within the vortex can leak away
to infinity. The degree to which the KAM surfaces are destroyed determines how
well the convective vortex confines particles and heat.

We have also begun a study of the nonlinear dynamics of the streamlines of
a class of divergence free flows which are periodic in all three directions. These
flows, which are related to the Chandrasekhar and Arter flows, are a Galerkin
approximation to the flow in hexagonal Benard flow. These flows have pairs of
stagnation points and have dynamics qualitatively similar to that of the aperiodic
flows of Ref. 5. However, because of the spatial periodicity, the dominant chaotic
feature is the existence of stochastic webs rather than chaotic scattering.

2. Three-Dimensional Visualization Software — Development and
Application

Three-dimensional graphics application software has been developed for the
DEC 5000/200 Workstation. This software has been designed to be used as modules
in the object-oriented programming package AVS (Advanced Visualization System.)
For many applications, AVS has modules which provide very useful visualization
tools. The two main modules we developed involved visualization of incompressible
flows v(x, t) [or magnetic fields B(x, t)] with diagnostics which involve the nonlinear
dynamical properties of the streamlines of the flow.

The first of these modules plots the streamlines of a flow v(x, t) the compo-
nents of which are given on a rectangular grid. Typical commercial software for such applications does not have the flexibility to allow the user to determine certain nonlinear dynamical properties of the flow, such as finding unstable periodic orbits and their stable and unstable manifolds and computing the time delay diagnostic of chaotic scattering. This module takes the components \( u_i(x, t) \), interpolates to arbitrary points off the grid, and integrates the orbits by a choice of several numerical schemes.

The second module computes Poincaré sections through an arbitrary plane in the space. The input data is again flow components on a grid, and interpolation to an arbitrary point is performed as above. The streamlines are integrated by a fourth order Runge-Kutta. This scheme has been found to preserve the volume preserving property adequately for fairly long orbits, for example to show the existence of invariant KAM surfaces when they exist. Progress is being made toward developing an exactly volume preserving integrator using data from the grid so that arbitrarily long orbits can be studied.

These modules have been used to study the nonlinear dynamics of divergence free flows and magnetic fields given on a grid with and without stagnation points, similar to the analytic models of Sec. B1. In particular, we have computed 3D Beltrami flows, satisfying \( \nabla \times \mathbf{v} = \lambda \mathbf{v} \) as models for convective vortices. These vortices show very complex nonlinear dynamical structure, including chaotic scattering which allows leakage of particles from the vortex, and bifurcations leading to new unstable closed streamlines where vortex reconnection can occur.

C. Applications of Wavelet Analysis (D. Uskov, R. Z. Sagdeev)

Wavelet analysis is a method of analyzing data which has proved useful in investigating the fractal nature of fluid turbulence. The method is based on the wavelet transform, an integral transform related to the Fourier transform, but with more general, and typically localized, kernels, which can be modified to deal with each specific problem. This method is particularly useful in investigating processes that have a self-similar or, more generally, a fractal nature, either in space or in time. Specifically, it provides a very efficient method of computing the fractal dimension associated with a physical system exhibiting chaos or turbulence.

In the past year, we have continued to develop codes to perform wavelet analysis, and begun to apply these codes to physical problems, with the ultimate goal of applying this type of data analysis to experimental and simulation results relevant to tokamak transport. One problem we have completed in the past year is the application of wavelet analysis to a study of the local fractal dimension of turbulent flows in the inertial range. The experimental data analyzed consisted of the turbu-
lent velocity fields in a wind tunnel. The scaling exponent was found to be close to
1/3, which is predicted by the Kolmogorov-Obukov theory. A detailed analysis of
the limitation of using the wavelet transform in determining the scaling exponents
has also been begun.6,7

D. Kinematic Dynamo (Chernikov, Rogalsky, Finn)

The three-dimensional kinematic dynamo problem was under study for the class
of flows which, in general, have chaotic streamlines and thus are the candidates
for the fast dynamos. An algorithm was developed to compute the solution of the
induction equation for the magnetic field for the limit of zero resistivity \( \eta \). Special
efforts were necessary, because of the short length scales obtained for \( \eta \rightarrow 0 \), to get
accurate results for the magnetic flux and derivative of the flux. Specifically, the field
was computed on a variable grid which was generated by a "fractal grid" algorithm.
The structure of algorithm is such that it is possible to get very high parallelism on
the different levels and, on the other hand, to get very high spatial resolution with
reasonable amount of computation for the high performance computer. The code for
this algorithm was implemented for the parallel computer system based on MIMD
architecture and for the Cray series of vector supercomputers. The algorithm was
not based on the specific features or symmetry of the flow and thus can be applied
for any particular flow under study.

The preliminary results were obtained for the ABC flow and Q-flows with three-
fold symmetry.8,9 These results are based on simulations carried out on the moderate
(up to 20M flops) parallel system.

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