Do Fermion-Pseudoparticle Interactions Generate Quark Masses?*

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MASTER

Abstract

We show that pseudoparticle solutions with integer winding number do not generate mass terms or breaking of chiral $SU(N)$ in theories with a non-trivial flavor group.

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An outstanding question, raised by recent studies^{1,2,3} of pseudoparticle⁴ effects in massless Quantum Chromodynamics, has been that of quark mass generation^{3,5} and the implied breaking of chiral $SU(N)$ in theories with a non-trivial flavor group.

't Hooft's¹ analysis in terms of an effective Lagrangian for fermions clearly established the breaking of chiral $U(N)$ down to chiral $SU(N)$. However, some questions of interpretation arise concerning the subsequent use of this effective Lagrangian in higher order perturbation theory or in other dynamical calculations which seek the generation of a quark mass term.⁵ Indeed, we have shown^{6,7,8} that a direct approach via the functional integral enables one to calculate n-point functions without recourse to this effective Lagrangian, correctly taking into account pseudoparticles of all sizes and defining a consistent perturbation expansion.

We shall employ this approach here to show that quark mass generation is not present in any order of perturbation theory about the known pseudoparticle solutions. The demonstration rests on the fact⁹ that there are $|v|N_f$ purely right (left) handed zero energy isospinor fermion modes in a pseudoparticle potential with positive (negative) winding number v , where N_f is the number of flavors. In the course of obtaining our result, we display rather explicit forms for the fermionic generating functional, and establish the Feynman rules of the theory. Using known zero energy solutions,⁹ and known classical propagators,¹⁰ it is a straightforward matter to complete the perturbative calculation of the n-point functions.

We argue, finally, that the failure of mass generation from the $v \neq 0$ paths in the functional integral is an exact result which goes beyond perturbation theory.

To begin, consider the generating functional for paths with winding number v ,

$$W_V(\eta, \eta^\dagger) = \int [DA]_V \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}\psi^\dagger \exp \int d^4x [\mathcal{L}(A, \bar{\psi}) - \bar{\psi}^\dagger \not{D}\psi + \psi^\dagger \eta + \eta^\dagger \psi] \quad (1)$$

where $\mathcal{L}(A, \bar{\psi})$ contains gauge fixing and ghost terms in addition to the purely gauge field Lagrangian, and $\not{D}$ is the covariant derivative on the isospinor fermions ψ ,

$$\not{D} = \gamma^\mu \partial_\mu - ig \not{A}$$

$$\not{A} = \frac{\tau^a}{2} A_\mu^a \gamma^\mu. \quad (2)$$

We split the fermionic integration into parts within and perpendicular to the zero energy solutions,

$$\not{D}_{c\ell} \psi_0 = 0$$

$$\not{D}_{c\ell} \psi_\perp \neq 0. \quad (3)$$

The reason for this separation is twofold. First, we note the field configuration $\{A_{c\ell}, \psi_0\}$ is an extremum of the action, a generalization of the conventional extremum $\{A_{c\ell}, 0\}$, and as such the functional integral is not Gaussian in the ψ_0 direction. Secondly, because these zero modes are finite in number and can be integrated exactly, there is no need to handle their interaction with the vector field perturbatively.

By familiar techniques, the quantum theory developed about a classical solution $A_{c\ell}$ is given by

$$\begin{aligned}
W_V(\eta, \eta^\dagger) = & \int \mathcal{D}A_{c\ell} \mathcal{D}A_{qu} \mathcal{D}\psi_0 \mathcal{D}\psi_0^\dagger \mathcal{D}\psi_\perp \mathcal{D}\psi_\perp^\dagger \mathcal{D}\bar{\Phi} \\
& \exp \int d^4x \left[\mathcal{L}(A, \bar{\Phi}) - \psi_\perp^\dagger \not{D}_{c\ell} \psi_\perp + ig \psi_\perp^\dagger \not{A}_{qu} \psi_\perp + ig \psi_\perp^\dagger \not{A}_{qu} \psi_0 + ig \psi_0^\dagger \not{A}_{qu} \psi_\perp \right. \\
& \left. - \psi_\perp^\dagger \not{D}_{c\ell} \psi_0 - \psi_0^\dagger \not{D}_{c\ell} \psi_\perp - \psi_0^\dagger \not{D}_{c\ell} \psi_0 - ig \psi_0^\dagger \not{A}_{qu} \psi_0 \right]. \quad (4)
\end{aligned}$$

The last four terms in the action vanish, three because of Eq.(3), the last because the zero energy fermion modes are purely right (left) handed for known pseudoparticle (anti-pseudoparticle) solutions. Adding sources $\eta = \eta_0 + \eta_\perp = P_0 \eta + P_\perp \eta$ (where P_0 and $P_\perp$ project on the zero fermion mode subspace, and its complement, respectively) and treating the perpendicular part of the interaction perturbatively, we have

$$\begin{aligned}
W_V(\eta, \eta^\dagger) = & \int \mathcal{D}A_{c\ell} \mathcal{D}A_{qu} \mathcal{D}\bar{\Phi} \exp \int d^4x \left[\mathcal{L}(A, \bar{\Phi}) + ig \frac{\delta}{\delta \eta} P_\perp \not{A}_{qu} P_\perp \frac{\delta}{\delta \eta^\dagger} \right] \\
& \int \mathcal{D}\psi_0 \mathcal{D}\psi_0^\dagger \mathcal{D}\psi_\perp \mathcal{D}\psi_\perp^\dagger \exp \int \left[-\psi_\perp^\dagger \not{D}_{c\ell} \psi_\perp + ig \psi_\perp^\dagger \not{A}_{qu} \psi_0 \right. \\
& \left. + ig \psi_0^\dagger \not{A}_{qu} \psi_\perp + \eta^\dagger \psi_\perp + \psi_\perp^\dagger \eta + \eta^\dagger \psi_0 + \psi_0^\dagger \eta \right] d^4x. \quad (5)
\end{aligned}$$

The integration of the non-zero fermionic modes now yields

$$\begin{aligned}
W_V(\eta, \eta^\dagger) &= \int \mathcal{D}A_{c\ell} \mathcal{D}A_{qu} \exp \int d^4x \left[\mathcal{L}(A, \Phi) + ig \frac{\delta}{\delta \eta} P_\perp A_{qu} P_\perp \frac{\delta}{\delta \eta^\dagger} \right] \\
&\quad \left[\text{Det } \mathcal{D}_{c\ell}^\perp \right] \exp \int d^4x \eta^\dagger \left[\mathcal{D}_{c\ell}^\perp \right]^{-1} \eta \int \mathcal{D}\psi_0 \mathcal{D}\psi_0^\dagger \\
&\quad \exp \int d^4x \left\{ \left(\eta^\dagger + ig \eta^\dagger \left[\mathcal{D}_{c\ell}^\perp \right]^{-1} A_{qu} \right) \psi_0 + \psi_0^\dagger \left(\eta + ig A_{qu} \left[\mathcal{D}_{c\ell}^\perp \right]^{-1} \eta \right) \right\}. \quad (6)
\end{aligned}$$

To facilitate the remaining zero-mode integration, let us write the ψ_0 as

$$\psi_{o\Lambda s}^{\alpha i}(t) = f_{o\Lambda}^{\alpha i}(x) \delta_{st} \theta(s, \Lambda) \quad (7)$$

where $\theta(s, \Lambda)$ are anticommuting c-numbers labeled by a flavor index $s=1, 2, \dots, N_f$ and a remaining degeneracy index $\Lambda=1, 2, \dots, |v|$. $f_{o\Lambda}^{\alpha i}(x)$ is an ordinary c-number function with Dirac spinor α and isospin (i) indices. Doing the θ integration, we then have

$$\begin{aligned}
W_V(\eta, \eta^\dagger) &= \int \mathcal{D}A_{qu} \mathcal{D}A_{c\ell} \exp \int d^4x \left[\mathcal{L}(A, \Phi) + ig \frac{\delta}{\delta \eta} P_\perp A_{qu} P_\perp \frac{\delta}{\delta \eta^\dagger} \right] \\
&\quad \left[\text{Det } \mathcal{D}_{c\ell}^\perp \right] \exp \int d^4x \eta^\dagger \left[\mathcal{D}_{c\ell}^\perp \right]^{-1} \eta \\
&\quad \prod_{s=1}^{N_f} \prod_{\Lambda=1}^{|v|} \left[\int d^4x \left(\eta^\dagger + ig \eta^\dagger \left[\mathcal{D}_{c\ell}^\perp \right]^{-1} A_{qu} \right) f_{o\Lambda}(x) \right]_s \left[\int d^4y f_{o\Lambda}^\dagger(y) \left(\eta + ig A_{qu} \left[\mathcal{D}_{c\ell}^\perp \right]^{-1} \eta \right) \right]_s. \quad (8)
\end{aligned}$$

The Feynman rules for fermions are now manifest. $\psi_\perp$ propagate with $\mathcal{D}_{c\ell}^{-1}$, and $\psi_{o\Lambda}$ with the zero mode projectors $f_{o\Lambda}(x) f_{o\Lambda}^*(y)$. In addition,

there is an interaction term in which a $\psi_{\perp}$ emits a ψ_0 and an A-field quantum. Note that the zero mode part of the interaction does not exponentiate.

From Eq.(8) it is clear that $W_v(0,0)$ vanishes for v and $N_f > 0$ to all orders in perturbation theory; terms which might possibly survive have the form f_0^* (odd number of γ 's) f_0 which vanishes because the f_0 are of a single chirality in a given v -sector. Moreover, we have also

$$\left. \frac{\delta^2}{\delta \eta \delta \eta^\dagger} W_v(\eta, \eta^\dagger) \right|_{\eta = \eta^\dagger = 0} = 0 \quad (9)$$

for $|v|N_f > 1$, to all orders of perturbation theory, by the same reasoning. Thus there is no 2-point function in the instanton sector (with $|v|N_f > 1$), no vacuum expectation value of $\sigma = \bar{\psi}(x)\psi(x)$, no mass generation and no breakdown of chiral SU(N). Of course, higher n -point functions ($n \geq 2|v|N_f$) do exist in the theory, e.g., a scattering amplitude⁸ for $N_f = 2$, $|v| = 1$. The point is that these amplitudes cannot, in any loop order, be converted into a propagator by joining external legs.

The fact that so many correlation functions vanish unless there are a sufficient number of external fermions may suggest that a proper formulation of the vacuum---more general than that considered here---will require it to contain a coherent superposition of soft fermion-antifermion pairs, indefinite in number, with possible far-reaching implications for the infrared structure of the theory.

In Gaussian approximation and lowest order g , our result for the two-point function ($|v| = 1$ sector, $N_f = 1$) is

$$\left. \frac{\delta^2 W_v(\eta, \eta^\dagger)}{\delta \eta(y) \delta \eta^\dagger(x)} \right|_{\eta = \eta^\dagger = 0} = \int \mathcal{D}A_{qu} \mathcal{D}A_{cl} f_0(x) f_0^\dagger(y) [\text{Det } \not{D}_{cl}] \exp \left[\int d^4x \mathcal{L}(A, \Phi) \right] \quad (10)$$

where f_0 depends on the collective coordinates in the $\mathcal{D}A_{cl}$ integration. This is the same interaction previously obtained by the mass limiting technique of Ref.8,

where the one-loop $N_f = 1$ propagator and $N_f = 2$ scattering amplitude have been worked out in detail. The present, more rigorous and easily generalizable technique, does suggest that the zero mass limit of the massive fermion theory is indeed the massless theory we have worked out here.

The demonstration above rests on projections over zero modes of $\not{D}_{c\ell}$ with A and $\psi_\perp$ handled perturbatively. However, we can as well project out the zero modes ψ_0 of the full operator $\not{D} = \not{D}_{c\ell} - ig A$, label orthogonal modes $\psi_\perp$, and do the ψ_0 and $\psi_\perp$ functional integrals exactly. The result is

$$W_V(\eta, \eta^\dagger) = \int \mathcal{D}A \mathcal{D}\bar{\psi} \exp \int d^4x \mathcal{L}(A, \bar{\psi}) [\text{Det } \not{D}^\perp(A)]$$

$$\prod_{\Lambda = n_L, n_R} \int d^4y \eta^\dagger(y) f_{0\Lambda}(y) \int d^4z f_{0\Lambda}^\dagger(z) \eta(z) \exp \int d^4w \eta^\dagger(w) [\not{D}^\perp]^{-1} \eta(w) \quad (12)$$

where $f_0(x)$, now functionals of A , are the c-number parts of the zero mode right hand and left hand solutions, n_R and n_L in number, respectively. This formal result is useful because of the Atiyah-Singer¹² index theorem,

$$n_R - n_L = \nu N_f$$

which, with Eq.(12) implies that the two-point function vanishes for $|\nu| N_f > 1$.¹³ Clearly, what is needed to support a 2-point function when $N_f \geq 2$ is a path or configuration which violates the $n_R - n_L = \nu N_f$ (ν integer) condition. From our first, perturbative, calculation it is clear that such a path cannot be reached by an expansion about the pseudoparticle solution, for which $n_R - n_L = \nu N_f$; our second calculation extends this result to all paths which satisfy $n_R - n_L = \nu N_f$, whether or not they are perturbatively reached from the pseudoparticle solutions.

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