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MODELING BEAM-FRONT DYNAMICS
AT LOW GAS PRESSURES

R. J. Briggs and S. Yu

MAY 13, 1982

Lawrence
Livermore
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MODELING BEAM FRONT-DYNAMICS AT LOW GAS PRESSURES

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Modeling Beam Front Dynamics at Low Gas Pressures*

R. J. Briggs and S. Yu

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Lawrence Livermore National Laboratory, University of California
Livermore, California 94550

ABSTRACT

The dynamics of space charge neutralization at the front of an intense self-focused electron beam pulse exhibits important differences in different gas pressure regimes. At very low pressures, the beam front is in the so-called ion-focused regime (IFR) where all secondary electrons are expelled from the beam region by the radial electric field without causing significant additional ionization. We estimate the upper pressure boundary of this regime by considering the distance scale length for cascade (avalanche) ionization. Data from the FX-25 diode experiments indicate a critical "transition pressure" (P_c) that agrees with this estimate and with its scaling among various gas types. Normal mobility-limited treatments (local conductivity models) of the secondary electrons at the beam front are not justified until the gas pressure is 10 - 50 times higher than P_c , due to "runaway" of these secondary electrons in the strong space-charge electric field at the lower pressures.

The main conclusion of this report is that a non-local phase space (Boltzmann) treatment of the secondary electrons is required to accurately describe these different beam front regimes and the transitions between them; such a code model is currently under development.

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I. INTRODUCTION AND SUMMARY

Self-focusing of an intense electron beam in a gas arises through neutralization of the radial space-charge electric field by the beam-generated gas ionization. The front part of a propagating beam pulse where this space charge neutralization occurs is often very short in duration (a fraction of a nanosecond at pressures of several torr or higher), but this short piece is nonetheless of great significance in determining the equilibria, stability, and trajectory of the entire pulse. The main goal of this paper is to discuss the important differences that arise in beam front dynamics over the pressure ranges encountered in beam propagation experiments, and to estimate the conditions for various limiting models to apply.

Two simple limiting regimes of beam front charge neutralization dynamics have been recognized for a long time:

A. The "local conductivity approximation." Here the mean free path of the plasma electrons is very short, and their motion is described by a collisionally-limited drift velocity. This leads to the usual "macroscopic" local conductivity $\sigma \propto n_e / v_m$, with plasma electron density (n_e) determined by direct impact ionization plus avalanche. The charge neutralization timescale in this model is of order ϵ_0 / σ . We note that the direct ionization process is usually treated as "local and instantaneous" in current computer models, even though about 2/3 of the ionization events (in N_2) are due to cascade ionization processes by energetic secondaries, tertiaries, etc., which can have significant mean free paths even at high pressure.¹ This approximation could, of course, be removed while still retaining the local conductivity modeling for the bulk of the secondary electron population.

B. Ion-focused regime (IFR), or "low pressure window." In this regime, secondary electrons produced by primary impact ionization events are expelled radially from the beam region by the strong space charge electric field

without causing significant additional ionization.^{2,3} The ions produced by the primary ionization accumulate within the beam, and complete charge neutralization occurs when the ion density equals the beam density. As the charge neutralization approaches completion, the secondary electrons are retained within the beam and local conductivity models can become applicable once again. Interest in this regime is due in no small part to the fact that stable propagation of the 5 MeV, 300 - 500 amp, 360 nsec Astron beam pulse was observed in this ion-focused regime many years ago,⁴ typically at pressures of 20 - 50 microns in hydrogen.

The main focus of our attention in this paper is on the following question: what are the requirements on gas pressure (with a given set of beam parameters) for either one of these simple descriptions to be a reasonable approximation to reality? We emphasize that our concern here is only with the front part of the beam pulse, where the charge neutralization process is going on. (The body of the pulse is generally well described by the local conductivity model over a broad range of conditions.) The radial electric field of a partially-neutralized, multi-kiloampere electron beam is very intense, and the response of the secondary electrons to this electric field in the presence of the gas collision processes is the determining factor setting the limits of validity of the simple beam front models.

In Section II, we present and discuss the various collision processes (in nitrogen) as a function of electron energy and E/p . For $E/p > 1$ keV/cm-torr, we show that secondary electrons (born at any energy) are in a "runaway" state and the steady-state mobility description of their motion is not valid. This runaway condition applies, for example, at pressures of order 50 torr or less for 10 kA, 1 cm radius beams. The required E/p to produce a "runaway" state is reduced for secondary electrons that are energetic at birth (delta rays). Hence, at still higher pressures (even up to one atmosphere) while the bulk of secondary electrons are mobility-limited, there is a small, but potentially significant population of energetic electrons which can be accelerated freely by the fields, and thereby contribute significantly to the overall plasma current.

In the ion-focused regime, all the secondary electrons are assumed to leave the beam region rapidly without causing any significant additional ionization. For most regimes of interest, we believe that the upper pressure limit to the IFR is set by comparison of an appropriately-defined effective mean free path for cascade ionization (λ_i) with the beam radius "a." If $\lambda_i \gg a$, secondary electrons will not cascade inside the beam, and the simple IFR model applies. If, on the other hand, $\lambda_i \ll a$, each individual secondary electron creates a large number of new secondaries on its way out of the beam region. Localized cascade (or avalanche) ionization inside the beam results in the second case, and the beam region should contain an appreciable density of secondary electrons throughout the charge neutralization process. (The significance of having this population of electrons inside the beam is the possibility of creating a high degree of current neutralization, two stream interactions with the beam, etc.)

Although secondary electrons are essentially in a state of "free acceleration" for $E/p \gg 1$ keV/cm-torr, we find that avalanche ionization is still quite localized up to considerably higher E/p values. The effective mean free path for avalanche ionization (λ_i) is found to increase from its minimum value to very large values with small increases in E/p above a "threshold" of 10 - 20 keV/cm-torr (in nitrogen). A sharp transition pressure for the IFR to occur is therefore indicated. These considerations are presented in Section III.

The question of whether the beam-generated magnetic field can contain the secondary electrons (in the IFR regime) during the neutralization process is addressed in Section IV. With beam currents of 10 kA or less, we conclude that free escape of the secondary electrons is a reasonable model for fractional neutralizations of order 0.5 or less even without considerations of how gas scattering effects the motion.

In Section V, using a "typical" average value of E_p (in r and t), assuming a "mature" nose with $I_B \approx$ constant and a trumpet-shaped (time-dependent) radius equal to its constant-emittance equilibrium value, and using the consideration of Section III, we estimate a transition into the IFR in nitrogen when

$$p a_0 \lesssim \frac{I_B}{10 \text{ kA}} \text{ torr-cm,} \quad (1.1)$$

where a_0 is the fully-pinchd beam radius. The scaling of this transition pressure in various gases is also derived, and in Section VI we compare the experimental indications of a sharp transition pressure in the FX-25 experiments⁵ (the bottom end of the stable propagation window) with the predicted IFR transition pressure. Good agreement is indicated, with a variation of over an order of magnitude in the observed critical pressures in various gases.

Implications of these considerations for beam front dynamics modeling are discussed in Section VII.

II. COLLISION PROCESSES AND RUNAWAY CONDITIONS IN STRONG ELECTRIC FIELDS

The electric fields at a beam front, before charge neutralization occurs, are extremely large. Our main purpose in this section is to discuss the motion of secondary electrons in these "unusual" E/p regimes, and to deduce criteria for validity of the mobility-limited description of electron motion.

In Figure 1, we present the collision rate coefficients for momentum exchange ($\tilde{\nu}_m$), energy loss ($\tilde{\nu}_u$), and ionization ($\tilde{\nu}_i$) as a function of electron energy (w) in Nitrogen. The momentum transfer rate per molecule (in $\text{cm}^3 \text{ sec}^{-1}$) is given by

$$\tilde{\nu}_m = v \sigma_m \quad (2.1)$$

where v is the speed of the electron, and the momentum transfer cross section is the usual weighted average of the differential cross section for scattering $\left(\frac{d\sigma}{d\Omega}\right)$;

$$\sigma_m = \int d\Omega (1 - \cos \theta) \frac{d\sigma}{d\Omega} . \quad (2.2)$$

The lower energy end of the curve ($w \leq 50$ eV) was constructed by Englehardt, Phelps and Risk⁶ to fit swarm data. The momentum transfer rate in the higher energy range was constructed by M. Berger⁷ from the differential cross sections for elastic scattering measured by Shyn et al.⁸ and Bromberg.⁹ Corrections to $\tilde{\nu}_m$ from inelastic collisions in 10 to 60 eV range was shown by Cartwright to be about 20%.⁽¹⁰⁾ On the basis of total cross section and inelastic cross section measurements, we expect similar corrections at higher energies ($\frac{\sigma_{inel}}{\sigma_{tot}} \sim \frac{1}{4}$). In the 10 to 60 eV range, Berger's values (Figure 1) are roughly a factor of two lower than those obtained by D. Cartwright¹⁰ and Srivastava et al.¹¹

A key observation concerning $\tilde{\nu}_m$ is its fast fall-off at high energies. The (non-relativistic) asymptotic dependence of $w^{-3/2}$ is readily obtained from simple considerations of Coulomb's Law, independent of the details of the molecular structure. Hence, the degree of frictional drag experienced by an electron is strongly dependent on its energy.

The ionization rate per molecule

$$\tilde{\nu}_i = \nu \sigma_i \quad (2.3)$$

was measured by Rapp and Englander-Golden.¹² The asymptotic behavior of $\tilde{\nu}_i$ is given by the Born limit, $w^{-1/2} \ln w$. In the energy range below 100 keV, $\tilde{\nu}_i$ is quite insensitive to the electron energy. Thus, an energetic electron which is relatively free of frictional drag can nevertheless be quite effective in creating secondaries.

We have also plotted in Figure 1 the frequency $\tilde{\nu}_u$, which determines the rate of energy loss via

$$\frac{dw}{dt} = -\nu_u w \equiv -N_g \tilde{\nu}_u w. \quad (2.4)$$

This frequency is related to the usual loss function by

$$\tilde{\nu}_u = \nu L(w)/w \quad (2.5)$$

where the loss function is given by

$$L(w) = \sum_k \epsilon_k \sigma_k \quad (2.6)$$

with the sum performed over all inelastic channels. The data plotted is from the work of Dalgarno et al.¹³ It is consistent to within a factor of 2 with the results from Green et al.¹⁴ and Phelps¹⁵. We note that $\tilde{\nu}_u$ is slightly lower in value than $\tilde{\nu}_m$, but otherwise exhibits the same high energy behavior as $\tilde{\nu}_m$. This trend is consistent with the ratio of elastic to inelastic collisions discussed earlier.

In a heuristic "single particle" picture, the drift velocity ($\bar{v}_d$) of an individual electron in the presence of the electric field ($\bar{E}$) is governed by

$$m \frac{d\bar{v}_d}{dt} + m \nu_m \bar{v}_d = e\bar{E} \quad (2.7)$$

where $\nu_m = N_g \tilde{\nu}_m$. If this electron has an instantaneous speed v , corresponding to a kinetic energy $1/2 m v^2 = ew$ (with w the energy in electron volts) we note that the macroscopic drift velocity concept is valid only when

$$|\bar{v}_d| < v. \quad (2.8)$$

The collision frequency ν_m is, of course, a function of the speed v , as shown in Figure 1.

The local conductivity model discussed in Section I is applicable when the electron inertia is negligible, with an instantaneous balance maintained between the acceleration force (eE) and the frictional drag, i.e.

$$\bar{v}_d \approx \frac{e}{\nu_m m} \bar{E} \quad (2.9)$$

and

$$\begin{aligned}\bar{J} &= e n_e \bar{v}_d = \frac{e^2 n_e}{v_m m} \bar{E} \\ &= \sigma \bar{E} .\end{aligned}\quad (2.10)$$

The drift velocity of a (typical) electron will increase in time when $eE > m v_m \bar{v}_d$. Noting the limitation on $\bar{v}_d$ in Equation (2.8), we see that the friction drag is insufficient to restore balance against continued acceleration when

$$E > m v_m v/e . \quad (2.11)$$

A plot of $v_m \sqrt{w}$ is presented in Figure 1; we see that a maximum in $(v_m v)$ occurs around $w \approx 50 - 100$ eV in N_2 and it drops off beyond that point. We therefore can conclude that a runaway in electron drift motion in the direction of the electric field will occur, limited only by electron inertia, when $E > E_m$ with

$$E_m = \text{Max} (m v_m v/e) . \quad (2.12)$$

In N_2 , from Fig. 1 and the STP gas density relation $N_g = p(3.54 \times 10^{16}/\text{cm}^3)$ with p the pressure in torr, we have

$$\frac{E_m}{p} \approx 1.25 \text{ keV/cm-torr} . \quad (2.13)$$

An alternate point of view to use in estimating this runaway condition is to note that an electron undergoing free acceleration from rest in a uniform constant electric field E will achieve a velocity v (energy w) in a time

$$\tau_{\text{acc}} \approx \frac{mv}{eE} . \quad (2.14)$$

The timescale for momentum exchange (90° scattering) is of course

$\tau_m = v_m^{-1}$, so

$$\frac{\tau_m}{\tau_{\text{acc}}} = \frac{eE}{m v_m v} . \quad (2.15)$$

Therefore, inequality (2.11) is basically the condition for acceleration to an energy w on a timescale faster than one momentum exchange time; with electric fields much larger than E_m , the electrons undergo collisionless "free acceleration" from rest to arbitrarily large kinetic energies. An argument based on energy loss collisions can be invoked to yield a qualitatively similar conclusion.

As we will show in Section V, inequality (2.11) is reached at pressures of 50 torr or less for a 10 kA beam. In these runaway regimes, the dynamics of electron transport is entirely different from the mobility-limited regime. The plasma current, instead of depending on the instantaneous electric field through a local conductivity, now depends on the history of the fields. In general, both electric and magnetic effects are important. Details of electron transport are rather complicated and will require a nonlocal time-dependent Boltzmann treatment. Such a code is being developed.

At higher pressures, (up to full atmospheric density) while the bulk of the plasma electrons are mobility-limited, a small population of electrons remain in the runaway state. These electrons are the ones that are sufficiently energetic at birth (delta rays). Due to the sharp fall-off of the momentum-transfer rate with increased energy (see Figure 1), these electrons can accelerate freely in the fields. Hence, for example, the critical E/p for runaway of a 10 keV electron is

$$\frac{E}{p} \approx 0.125 \text{ keV/cm-torr}, \quad (2.16)$$

a factor of 10 lower than the condition for bulk runaway in Eq. (2.13). Although the number density of deltas is in general much smaller than the total plasma electron density, they can have a potentially significant effect on the overall plasma current because of their high velocity.

III. DISTANCE SCALE LENGTHS FOR AVALANCHE IONIZATION

In this section, we estimate the distance scale lengths for avalanche ionization processes at very high E/p . For simplicity, we consider the case of a uniform electric field when describing the secondary electron collision processes.

The ionization mean free path at a given electron energy is

$$\lambda_{ion} = \frac{1}{N_g \sigma_i(w)} \quad (3.1)$$

The cross section for ionization in nitrogen is shown in Figure 2; the dropoff at high energy is roughly as $1/w$, consistent with the above discussion and with the data in Figure 1. The minimum ionization mean free path is $(N_g \sigma_0)^{-1}$ with σ_0 the maximum of the cross section ($\sim 2.5 \times 10^{-16} \text{ cm}^2$), so in N_2

$$p(\lambda_{ion})_{min} \approx 0.11 \text{ torr-cm} \quad (3.2)$$

We might be tempted to use this minimum mean free path for estimating the ionization scale length at high electric fields. However, the following considerations show that it grossly underestimates the actual avalanche scale length at very high electric fields. The physical reason is that a rapid enough acceleration rate of the electrons makes them "pass over" the cross section peak so quickly that the integrated ionization probability stays much less than one up to very high energies (i.e. - over very long path lengths).

To quantify the distance scale length of the avalanche ionization, we consider a steady-state, one-dimensional system, where electrons are continuously "born" at $x = 0$ and accelerated in the direction $x > 0$ by a uniform electric field E . We consider the regime $E \gg E_m$, where we can assume that every electron undergoes free acceleration after it is born, and we also assume for simplicity that all the electrons produced by ionization events are created at zero initial energy. With these assumptions, we can

write the following simplified Boltzmann equation for the secondary electron distribution function, $f(v, x)$:

$$v \frac{\partial f}{\partial x} + \frac{eE}{m} \frac{\partial f}{\partial v} = \delta(v) \int_0^{\infty} N_g \sigma_i(w) v' f(v', x) dv' \quad (3.3)$$

where $\delta(v)$ is the impulse function, normalized to unit area.

A formal solution of this integral equation for given initial conditions at $x = 0$ can be developed using Laplace transforms. One can show that the following homogeneous solution of Eq. (3.3) dominates at large x :

$$f(v, x) = A\theta(v) e^{-\eta(w-Ex)} \quad (3.4)$$

where A is a constant (that depends on the initial conditions), $\theta(v)$ is the Heavyside step function ($\theta = 1$ for $v > 0$, $\theta = 0$ for $v < 0$), and the constant η is determined by the condition

$$\int_0^{\infty} \frac{N_g \sigma_i(w)}{E} e^{-\eta w} dw = 1 \quad (3.5)$$

(Condition (3.5) follows from matching the discontinuity in the derivative $\partial f / \partial v$ at $v = 0$ with the right hand side of Eq. (3.3).)

The secondary electron density corresponding to the solution for f in Eq. (3.4) has the form

$$\begin{aligned} n(x) &= \int_0^{\infty} f(x, v) dv \\ &= (\text{const}) \cdot e^{\eta Ex} \end{aligned} \quad (3.6)$$

The exponential growth rate of $n(x)$ is therefore governed by the rate constant

$$\alpha \approx \eta E \quad (3.7)$$

The quantity α can be directly compared with the usual first Townsend ionization coefficient,¹⁶ although the classical usage of this parameter has not included the runaway regime considered here. Similarly, η corresponds to the ionizations per volt in the Townsend definitions.

We use a model cross section for $\sigma_i(w)$ to determine η as a function of E/p from Equation (3.5). A good fit to the ionization cross sections of most gases can be obtained using the form

$$\sigma_i(w) = 4 \sigma_0 \frac{w/w_0}{(1 + w/w_0)^2} \quad (3.8)$$

This form is in error at low energies, but matching the cross section there is not important for our purposes since we must have E/p much larger than the runaway threshold for the theory to be meaningful. A fit to the nitrogen data (of Kapp et al) is shown in Figure 2, using

$$\begin{aligned} w_0 &= 100 \text{ eV} \\ \sigma_0 &= 2.5 \times 10^{-16} \text{ cm}^2 \end{aligned} \quad (3.9)$$

With the cross section model of Equation (3.8), we can obtain a "universal" relation between $\tilde{\eta} = \eta w_0$ and $\tilde{E} = E/4N_0 \sigma_0 w_0$ as

$$\tilde{E} = \int_0^{\infty} e^{-\tilde{\eta} u} \frac{u du}{(1 + u)^2} \quad (3.10)$$

This relation can be put in the alternate form

$$\tilde{E} = e^{-\tilde{\eta}} (1 + \tilde{\eta}) \left[-\text{Ei}(-\tilde{\eta}) \right] - 1 \quad (3.11)$$

where the exponential integral is defined as

$$-Ei(-x) = \int_x^{\infty} \frac{e^{-v}}{v} dv \quad (3.12)$$

In Figure 3, we present

$$\frac{N_g \sigma_0}{\alpha} \equiv (4\tilde{n}E)^{-1} \quad (3.13)$$

as a function of $\tilde{E}$. Note that $N_g \sigma_0 / \alpha$ is basically the ratio of the effective ionization mean free path ($1/\alpha$) to the minimum mean free path $(N_g \sigma_0)^{-1}$. We see from Figure 3 that the ionization scale length starts to increase exponentially with electric field as E increases above the critical value $4N_g \sigma_0 w_0$. (Note that the runaway condition in momentum discussed earlier is the order of unity times $N_g \sigma_0 w_0$ in nitrogen, as can be seen from Figure 1.)

It is instructive to plot this "universal" normalized result for a couple of specific cases, and this is done in Figures 4 and 5 for nitrogen and hydrogen. (In hydrogen¹⁶, the cross section fit gave $w_0 = 70$ eV and $\sigma_0 = 0.97 \times 10^{-16}$ cm².) For comparison, data¹⁶ on the first Townsend ionization coefficient in both gases is also shown. We see from Figure 4 that our "free acceleration" model overlaps the nitrogen data within 50% or so at the runaway onset, which is quite good for such a simple model. The sign of the error is also understandable; the effects of gas scattering (in the data) reduces the mean free path in the direction of E , as it should.

The main point for our purpose, of course, is the dramatic increase of p/α (the mean free path) as E/p is raised from 10 to 25 KeV/cm-torr (in nitrogen)). Factors of 2 or 3 change in pressure, therefore, can carry us from a short mean free path situation to very long mean free paths, with other things held constant.

IV. MOTION OF SECONDARY ELECTRONS IN THE COLLISIONLESS LIMIT

The simple picture of beam space charge neutralization in the ion-focused regime assumes rapid radial expulsion of the secondary electrons. As the beam space charge becomes compensated by the positive ions, we might ask whether the magnetic field of the beam will be strong enough to confine the secondary electrons inside the beam well before complete electrical neutralization is obtained, since E_p is less than B_{gc} . We estimate the effects of the magnetic field in this section; for beam parameters of interest ($I \sim 10$ kA) we conclude that "free" escape of the secondaries from the beam region is a reasonable assumption through much of the charge neutralization phase at pressures below the transition pressure estimated in Section V.

At pressures well below the transition pressure, the secondary electrons (having very few collisions) might accumulate enough density in an "oscillating cloud" surrounding the beam to be a potential worry as far as two-stream type instabilities are concerned. The main point we emphasize in the present context is that typical orbits place the average position of these electrons far from the beam region, so they cannot contribute significantly to the cascade ionization inside the beam.

(a) Constant radius beam.

We first consider the fields that would result with a Bennett beam profile and a constant beam radius, as for example, near the injection point into the gas cell. The fractional neutralization f_e is then a constant in radius, and since

$$\frac{\partial n_i}{\partial \tau} = v n_B, \quad (4.1)$$

we have

$$f_e = \frac{n_i}{n_B} = v\tau \quad (4.2)$$

with ν the primary ionization rate ($\nu = 10^9$ p in air) and $\tau = t-z/c$.
The fields are

$$E_r = (1 - f_e) \frac{c\mu_0 I_B r}{2\pi(a^2 + r^2)} \quad (4.3)$$

$$B_\theta = \frac{\mu_0 I_B r}{2\pi(a^2 + r^2)} \quad (4.4)$$

We consider a secondary electron born at rest at a radius $r = r_i$. The motion is treated in the collisionless limit, the fields are assumed constant in time during the motion, and we neglect the E_z fields due to the time-dependence of the neutralization fraction.³ All these neglected factors will generally enhance the escape of the secondaries at low pressure although collisions will ultimately retard the escape rate at sufficiently high pressures, where mean free paths become smaller than the beam radius.

The equations of motion with $v_\theta \equiv 0$ are

$$m_0 \frac{d(\gamma v_z)}{dt} = e v_r B_\theta \quad (4.5)$$

$$m_0 \frac{d(\gamma v_r)}{dt} = e (E_r - v_z B_\theta) \quad (4.6)$$

$$m_0 c^2 \frac{d\gamma}{dt} = e v_r E_r \quad (4.7)$$

with

$$\gamma = (1 - v_r^2/c^2 - v_z^2/c^2)^{-1/2}.$$

Using $v_r = dr/dt$, from Equations (4.3) and (4.7) we have

$$\gamma - 1 = (1 - f_e) \frac{I_B}{I_0} \ln \frac{r^2 + a^2}{r_i^2 + a^2}, \quad (4.8)$$

with the initial condition $\gamma = 1$ at $r = r_i$; and the definition

$$I_0 = \frac{4\pi m_0 c}{e \hbar} = 17,000 \text{ amps} . \quad (4.9)$$

From Equation (4.5) we have

$$\frac{\gamma v_z}{c} = \frac{I_B}{I_0} \ln \frac{r^2 + a^2}{r_i^2 + a^2} . \quad (4.10)$$

From Equation (4.6), or the definition of γ , we obtain

$$\left(\frac{\gamma v_r}{c}\right)^2 = \frac{2I_B}{I_0} \left\{ \left(1 - f_e\right) - \frac{f_e I_B}{I_0} (1 - f_e/2) \ln \frac{r^2 + a^2}{r_i^2 + a^2} \right\} \ln \frac{r^2 + a^2}{r_i^2 + a^2} . \quad (4.11)$$

The solution for the orbit $r(t)$ from Equation (4.11) is complicated, but we are mainly interested in the outer limit of the orbit, r_M (see Figure 6).

This is where $v_r = 0$, which requires

$$\ln \frac{r_M^2 + a^2}{r_i^2 + a^2} = \frac{I_0 (1 - f_e)}{I_B f_e (1 - f_e/2)} . \quad (4.12)$$

The smallest value of r_M is obtained with particles starting near the axis ($r_i \ll a$); their turning radius is explicitly given by

$$\frac{r_M^2}{a^2} = \exp \left[\frac{2 I_0 (1 - f_e)}{I_B f_e (2 - f_e)} \right] - 1 . \quad (4.13)$$

A plot of r_M/a vs f_e for $r_i = 0$ is presented in Figure 7 for beam currents of 5 kA and 10 kA. As $f_e \rightarrow 1$, $r_M \rightarrow r_i = 0$. We see from Figure 7 that r_M is larger than a for all $f_e < 0.9$ with $I_B = 5$ kA, and all $f_e < 0.8$ with $I_B = 10$ kA.

Our model with constant fields and no gas scattering predicts a periodic return to the point of "birth," $r = r_i$, as sketched in Figure 6. With an outer turning radius several times "a" or larger, however, the electron will not contribute significantly to the average charge density inside the beam.

For completeness, we note that the maximum axial velocity occurs at $r = r_M$ and is given by

$$\left(\frac{v_z}{c}\right)_{\max} = \frac{2(1 - f_e)}{1 + (1 - f_e)^2} \quad (4.14)$$

With $f_e \rightarrow 0$, $(v_z/c)_{\max} \rightarrow 1$; with $f_e = 0.5$, $(v_z/c)_{\max} = 0.8$.

(b) Trumpet-shaped beam.

A propagating beam develops a "trumpet" shape,³ and in this case the ion density at a given time τ is broader than the electron beam density. Recent ETA experiments in the IFR regime indicate reasonable agreement with the simple constant-emittance equilibrium.

If we assume a Bennett beam profile,

$$n_B = \frac{I_B}{ec\pi a^2 (1 + r^2/a^2)^2} \quad (4.15)$$

and a variation of the scale radius $a(\tau)$ as (following a constant emittance equilibrium)

$$a^2(\tau) = \text{const.}/\tau, \quad (4.16)$$

then the solution of Equation (4.1) for $n_i(r, \tau)$ is

$$n_i = \frac{v\tau a^2 I_B}{ec\pi r^4} \left[\ln(1 + r^2/a^2) - \frac{r^2}{r^2 + a^2} \right]. \quad (4.17)$$

The radial electric field is now

$$E_r = \frac{c \mu_0 I_B}{2\pi r} \left\{ -\frac{r^2}{a^2 + r^2} - v\tau \left[1 - \frac{a^2}{r^2} \ln(1 + r^2/a^2) \right] \right\}. \quad (4.18)$$

The solution of Equation (4.7) for an electron starting at $r = r_i$ with $\gamma = 1$ is

$$\gamma - 1 = \frac{I_B}{I_0} \left\{ \ln \frac{r^2 + a^2}{r_i^2 + a^2} - v\tau \left(\ln \frac{r^2 + a^2}{r_i^2 + a^2} + \frac{a^2}{r^2} \ln(1 + r^2/a^2) - \frac{a^2}{r_i^2} \ln(1 + r_i^2/a^2) \right) \right\}. \quad (4.19)$$

The solution of Equation (4.5) is the same as before. (See Equation (4.10).) Combining Equations (4.19) and (4.10), we can again solve for the outer limit of the orbit $r = r_M$ where $v_r = 0$. For calculations, it is simpler to express τ in terms of r_M/a ; for the case $r_i = 0$ this solution is

$$v\tau = \frac{1 + y - (1 + y^2)^{1/2}}{(1 + a^2/r_M^2) y - I_B/I_0} \quad (4.20)$$

where

$$y = \frac{I_B}{I_0} \ln(1 + r_M^2/a^2). \quad (4.21)$$

Calculations of r_M/a vs. $v\tau$ are presented in Figure 8 for beam currents of 5 kA and 10 kA.

In the present model, with $a^2 = \text{const}/\tau$, from an expansion of Equation (4.18), we can show that the electric field near $r = 0$ has the form

$$E_r = \frac{c u_0 I_B r}{2 \pi a^2} (1 - v\tau/2) . \quad (4.22)$$

Therefore, a beam length of $v\tau = 2$ is required before complete neutralization of the beam charge density on axis is obtained. In fact, with a "top-hat" beam profile and $a = a_0 / \sqrt{f_e}$, f_e is constant in r and equal to $v\tau/2$.³ We must use caution in extending the present IFR model up to $v\tau = 2$, however. Beyond $v\tau = 1$, the ion charge per unit length exceeds the beam charge per unit length, and E_r reverses sign at a radius $r > a(\tau)$. The electric field at the outer wall will, therefore, tend to draw electrons from the wall into beam region. Also, Caporaso¹⁷ has shown that the appropriate radial average of f_e to use in an envelope description goes to unity when $v\tau \approx 1.4$ (with a Bennett profile).

In summary, these calculations indicate that secondary electrons should be expelled far from the beam region if we are in a regime where $v\tau < 0.3-0.4$ but beyond that time (or pressure) secondary electrons may oscillate in a cloud near enough to be a potential problem.

V. CONDITIONS FOR IFR TO OCCUR

To estimate the critical upper pressure for "pure" IFR to occur at a beam front, we need an estimate of the electric field magnitude. We concentrate on the radial field in making this estimate, but we should emphasize that the axial electric fields can also be quite strong.³ We use the Bennett profile in r described by Equation (4.3), with a time-dependent scale radius following a constant-emittance profile in τ , and constant beam current through the trumpet-shaped nose,³ so that

$$a^2 = a_0^2 / f_e(\tau) \quad (5.1)$$

with a_0 the fully-pinched beam radius.

The maximum value of E_r at a given τ is at $r = a(\tau)$, given by

$$(E_r)_{\max \text{ in } r} = \sqrt{\frac{10}{E_0}} \frac{I_B}{4\pi a(\tau)} (1 - f_e(\tau)) \quad (5.2)$$

Combining Equations (5.1) and (5.2), we have

$$(E_r)_{\max} = \frac{30 I_B}{a_0} f_e^{1/2} (1 - f_e) \quad (5.3)$$

with I_B in amps. A plot of $(E_r)_{\max}$ vs. f_e is presented in Figure 9; a broad maximum occurs at $f_e = 1/3$, and is given by

$$\text{MAX in } \tau \text{ of } (E_r)_{\max} = E_0 = \frac{11.5 I_B}{a_0} \quad (5.4)$$

Considering the profile of E_r in r and τ , the beam front should be "solidly" in the ion-focused regime if $1/3 E_0$ exceeds the threshold value needed for λ_i to become very long. Therefore, we will use a characteristic field strength of

$$\langle E_r \rangle = \frac{1}{3} E_0 = \frac{4 I_B}{a_0} \quad (5.5)$$

as a "mean" value of E_r during the charge neutralization phase. From Figure 4, we see that an E/p of order 30 keV/cm-torr or greater will have very large λ_i in N_2 . This requires

$$\frac{4 I_B \text{ (kA)}}{p a_0} > 30 \quad (5.6)$$

or

$$p a_0 < \left(\frac{I_B}{10 \text{ kA}} \right) \text{ torr-cm} \quad (5.7)$$

for the whole beam front to be in the "pure" ion-focused regime (negligible population of secondary electrons inside the beam during the charge neutralization phase).

This estimate of the critical pressure for IFR can be put in a more graphic form by using the characteristic field strength $\langle E_p \rangle$ in Eq. (5.5) to determine $\lambda_i/a_0 = 1/\alpha a_0$ (the ratio of the ionization avalanche mean-free-path to the beam radius) as a function of pa_0 , for a given I_B . This is easily done using the normalized curve in Figure 3: picking a value for pa_0 determines $\langle E_p \rangle/p$, and we can then read off $\alpha/p = \alpha a_0/pa_0$. The result for N_2 (see Eq. (3.9)) at $I_B = 10$ kA is displayed in Fig. 10. This presentation shows the dramatic dependence of λ_i/a_0 on pa_0 ; a factor of 2 increase in pressure can result in a reduction in the effective avalanche ionization mean free path by 4 orders of magnitude! This steep dependence of λ_i on pressure suggests that the onset of the IFR at the beam front should be "sharp and decisive."

The scaling of this critical pressure among various gases and different beam parameters is clear from the normalization of E in Fig. 3, namely

$$p_{\text{crit}} \sim I_B (w_0 \sigma_0 a_0)^{-1} \quad (5.8)$$

where w_0 and σ_0 must be determined from a fit of the model cross section in Eq. (3.8) with data.

VI. DISCUSSION OF FX25 DATA

In the FX25 propagation experiments,⁵ a sharp transition in pressure between two very different "states" of the beam-gas interaction was clearly evident. The reader should consult the reports listed in Reference 5 for details (and for any caveats on the experimental uncertainties) - here we will only highlight the key observations with a few reproduced data sheets. For example, the net current measured near the injection point is shown in

Figure 11 as a function of pressure for a number of gases.⁵ It is tempting to suggest that the critical pressure (p_c) where net current drops so abruptly is related to the critical pressure where λ_i/a abruptly drops, and we shall attempt to correlate this data with the IFR boundary predictions.

The first question that arises is whether any sort of quantitative comparison with the model calculation of Section V can be done, since E_r was estimated for a "fully-developed" trumpet-shaped nose. Around one torr in air (or N_2), direct primary ionization timescales are about a nanosecond, and avalanche breakdown time scales (several times the $1/(v_{ion})_{max}$ e-folding time) are comparable. Therefore, it is not unreasonable to use $I_B \sim 5-10$ kA at the front since the risetime (20%-80%) of the injected current to 10 kA is about 2-3 nsecs, and it steepens as it propagates.

Using the experimental radius $a_0 \sim 1$ cm, we conclude that the predicted pressure in N_2 (or air) for λ_i/a to abruptly drop below one (see Figure 10) is the right order of magnitude compared to the experimental P_c for air, namely about one torr.

The scaling of P_c among the various gases with the same beam parameters is a much better test of the correlation of P_c with the IFR predictions. As shown in Section V, the scaling among various gases of the IFR boundary pressure should follow $(w_0 \sigma_0)^{-1}$ for fixed I/a (see Eq. (5.8)).

Fitting the cross section data in references 12 and 16 with our model cross section (Eq. (3.8)), we obtain the model parameters σ_0 and w_0 listed in Table I. To test the prediction $P_c \sim (w_0 \sigma_0)^{-1}$, we normalize the cross section parameters $(w_0 \sigma_0)^{-1}$ to N_2 in the right hand column of Table I. In Figure 12, we plot the observed P_c for the various gases, (defined as the midpoint of the abrupt drop in net current shown in Figure 11) against the cross section parameter $(w_0 \sigma_0)^{-1}$. We see that the scaling of this experimental transition pressure among different gases does follow the predicted scaling very closely.

As has been emphasized in previous publications,¹⁸ many other measured quantities also undergo an abrupt change at P_c . For example, the peak plasma density in air drops by almost two orders of magnitude as pressure is increased from one to two torr (see Figure 13). The plasma density in the pressure range below P_c also continued to rise after the beam pulse was over and its decay time is much longer. We have previously interpreted this strong plasma heating in the $P < P_c$ regime to a two-stream interaction,^{18,19} and the present authors remain in complete accord with this interpretation. A reasonable hypothesis is that P_c is indeed a boundary of having the beam front in the IFR, and that two-stream is somehow made more violent when this is the case. For example, we might expect that after $f_e = 1$ occurs (about one nanosecond at around 1 torr in N_2), the plasma electron density builds up to $n_e = n_B$ and extremely strong two-stream interaction occurs, responsible for the violent plasma heating. A violent two-stream phenomenon was, of course, observed in the Astron experiments in the IFR regime, with current multiplication accompanying the strong ionization of the gas after $f_e = 1$ occurred.

Our previous interpretation¹⁹ placed emphasis on the stabilization of the two-stream interaction by the increased strength of the secondary electron-gas collisions above one torr. Collisional stabilization of the two-stream instability over the full electron density range is predicted in this general pressure range,¹⁹ and this aspect of the complex phenomenology is undoubtedly a factor in the overall picture. Stability could be further enhanced in the "modified" view presented here by having n_e/n_B remain larger than one through the entire electrical neutralization phase when p is above P_c .

A full theoretical modeling of this beam-gas interaction, including two-stream instability for a finite beam and rapidly varying n_e and n_B , is probably close to prohibitive. Some attempt to quantify the considerations presented here is clearly warranted, however, to increase our confidence in the general picture. We also note that the "current multiplication" effects that were observed so clearly in the Astron experiments could also play a role in the observed net current behavior in these FX25 experiments.

	$\sigma_0/0.88 \times 10^{-16} \text{ cm}^2$	$w_0(\text{eV})$	$\frac{(\sigma_{0w_0})_{N_2}}{\sigma_{0w_0}}$
Nitrogen	2.84	100	1
Neon	0.9	160	2.0
Helium	0.42	115	5.9
Argon	3.2	100	0.9
SF ₆	8	130	0.27

Table I Ionization cross section parameters for various gases

VII. DISCUSSION AND CONCLUSIONS

Evolution of the beam-generated plasma channel at the front of an electron beam passing through a gaseous medium is strongly pressure dependent. In the previous sections, we have attempted to quantify the conditions for transition from the ion-focused regime on the one hand, and from the simple mobility-limited regime on the other.

The physics in the low pressure regime is relatively simple. Ejection of secondary electrons by the radial electric field and the self-consistent evolution of the beam head are relatively well understood. One aspect we have not discussed in this paper is the role of ion motion in the channel.² The ions generated by beam ionization can be accelerated to hundreds of keV's at low pressures. These ions have large cross sections for ionization and charge exchange. History of these ions would probably have to be included for quantitative descriptions of many phenomena in IFR propagation. On the other hand, the boundary pressure of the IFR as described in Section V should be relatively independent of ion dynamics, since the entire charge neutralization phase near the transition pressure is too short for significant ion acceleration to occur. At much lower pressures, ion dynamics and ionization by the energetic ions can certainly become important processes, as discussed by Olsen.²

We focused attention here on the scale length of the avalanche ionization compared to the beam radius as the determining factor in where the IFR regime will occur. One must, in a given physical situation, also compare the timescales for avalanche ionization with other characteristic times (like the beam current risetime) to be sure that these considerations apply directly.

We have presented in Section III arguments for an abrupt transition in the avalanche ionization scale length at a critical E/p of about 10 to 25 keV/cm-torr in nitrogen. Above this critical pressure, strong avalanching sets in. However, the mobility-limited description for the bulk of the plasma is not valid until E/p falls below 1 keV/cm-torr (Eq. 2.13). Hence, there is a decade range in E/p for which rapid breakdown occurs, and yet the entire population of plasma electrons is in a runaway state. At higher pressures (all the way up to full atmospheric density), there remains a smaller (but potentially significant) population of high energy runaways. The behavior of these electrons is quite complex. In addition to inertial effects, a proper treatment must include nonlocal electron ionization, magnetic field effects (Hall currents, $\vec{E} \times \vec{B}$ drifts etc.) as well as molecular collisional processes.

To model the development of the beam-generated channel over the full range of pressures, one of us (S. Y.) has been developing a nonlocal Boltzmann code (NUTS). The Boltzmann treatment (in contrast to fluid approach) is necessary to model the strongly energy-dependent collision processes. To make the problem computationally tractable, we have adopted a dipole approximation in velocity space. The treatment is otherwise exact (i.e., all inertial, diffusive, and field effects together with collision processes are retained). The fields are calculated self-consistently from the evolving plasma currents and an assumed beam profile.

In the high pressure limit, this treatment reduces to the simple mobility-limited description for the bulk. Although the high energy "deltas" are treated on the same footing as the low energy plasma electrons, their distinct runaway character falls out naturally as a consequence of the rapid fall-off in collision rates at high energies. The transition from mobility-limited regime into bulk runaway is described by the code.

The dipole approximation has reasonable physical basis in the high pressure regime, and is least justifiable in the IFR limit. However, even here, many of the bulk properties of the secondary electrons can be retrieved to a good approximation. Preliminary results from the code indicate a critical E/p for the onset of avalanche ionization derived from this two-term expansion which is similar to the results of Section III.

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FIGURE-CAPTIONS

1. Collision rate coefficients in N_2
 $\tilde{\nu}_i$ - ionization rate coefficient
 $\tilde{\nu}_m$ - momentum transfer rate coefficient
 $\tilde{\nu}_u \equiv \frac{v}{w} \sum_k \epsilon_k \alpha_k$ - energy loss rate coefficient
2. Ionization cross section in N_2 , and fit of data with analytical model
3. Normalized ionization scale length vs. electric field for the model cross section
4. Ionization scale length (P/α) vs. E/p for N_2 , and comparison with data on first Townsend ionization coefficient
5. Ionization scale length vs. E/p for H_2
6. Sketch of secondary electron orbit, assumed born at rest at $r = r_i$
7. Outer limit of orbit for the case $E_r = (1 - f_e) B_\theta c$ and $r_i = 0$
8. Outer limit of orbit for "trumpet-shaped" propagating beam, $r_i = 0$
9. Plot of maximum radial electric field vs. f_e
10. Plot of λ_i/a_0 vs. pa_0 assuming an E_r given by Eq. (5.5) and $I = 10$ kA
11. FX-25 data on net current vs. pressure (Fig. 5 of UCIR-979)

12. Comparison of the observed critical pressure vs. the cross section parameter $(\sigma_0 w_0)^{-1}$, normalized to N_2
13. Dependence of peak plasma density and net current on fill pressure in air