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HIGH ENERGY BREMSSTRAHLUNG IN AN INTENSE LASER FIELD

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ABSTRACT

The cross section for bremsstrahlung emission and absorption by electrons in an intense laser field has been calculated in the Born approximation for the electron-ion potential. Typical numerical results are presented as a function of the ratio of the electron quiver energy to its energy and the ratio of the bremsstrahlung energy to the electron energy. The intense field correction factor for the rate of bremsstrahlung emission and absorption for electrons with a Boltzmann distribution of energies has been calculated. Numerical results for the correction factor are presented for the Boltzmann case as a function of the ratio of the electron quiver energy to its thermal energy and the ratio of the bremsstrahlung energy to the thermal energy. For typical laser fusion parameters, this correction factor which is the ratio of the thermal bremsstrahlung emission rate in the intense laser field to the rate at zero field can be quite significant. For a laser of wavelength $1.06 \mu\text{m}$ at an intensity of $3 \times 10^{15} \text{ W/cm}^2$ and an electron temperature of 1 keV, the correction factor varies from 0.98 at a bremsstrahlung energy of 100 eV to greater than 5 at a bremsstrahlung energy of 10 keV.

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I. INTRODUCTION

For the laser intensities now used and those planned in the Livermore Laser Fusion Program, the quiver energy of electrons in the laser field is a significant fraction of the electron total energy. In this situation the electron collision rates and bremsstrahlung rates depend on the strength of the laser field. The theory of the intense field effects has been developed in a number of papers (Refs. 1,2,3,4,5). In a previous work (Ref. 6) that was performed for Livermore, we used that theory to calculate the correction factor to the rate at which electrons with a Boltzmann energy distribution absorb energy from the laser field.

In this paper we apply that theory and the techniques developed in Reference 6 to calculate the rate at which an electron in a strong laser field emits or absorbs bremsstrahlung of all energies (in Born approximation in the electron-ion potential).

The bremsstrahlung rate for a monoenergetic electron energy distribution (i.e., a single electron), and form appropriate to numerical evaluation and incorporation into the Livermore code LASNEX are derived. Typical, but not extensive, numerical results are presented for this case.

Averages over the Boltzmann distribution are performed to give the radiative properties of an ionized plasma in an intense laser field as a function of temperature and laser parameters. The numerical results for the thermal-averaged case are presented as a correction factor which is defined as the ratio of the emission (absorption) rate in the intense field to the rate at zero field. The numerical results cover the range of physical conditions appropriate for most of the Livermore laser fusion applications. These results will probably have to be curve fit with simple functions to be

incorporated into LASNEX. For strong fields the effects calculated here will be significant in the prediction of the radiative properties of a plasma and in the correct determination of the plasma temperature from the measured bremsstrahlung spectrum.

This paper is organized as follows. In Section II we derive the expression for the bremsstrahlung rate for a monoenergetic electron energy distribution, the expression for the radiation rate for electrons with a Boltzmann energy distribution and the expression for the intense field correction factor. In Section III.1 we reduce the expression for the single electron rate to a form more appropriate for numerical calculation. Since the evaluation of this form is straightforward we present only typical results for this case. In Section III.2 we describe several methods for the evaluation of the expression for the intense field correction factor for the Boltzmann electron distribution. Because this expression is somewhat difficult to evaluate we present numerical results for a wide range of parameters of interest for laser fusion applications. The major results of this work are Equations (3.3) and (3.8) and Figures 1 and 2 for the monoenergetic electron bremsstrahlung rate and Equation (2.27) and Figures 3 and 4 for the thermal bremsstrahlung intense field correction factor.

II. DERIVATION OF BREMSSTRAHLUNG CROSS SECTION, ABSORPTION AND EMISSION RATE

The theory for the inclusion of intense field effects on electron scattering and radiative processes has been developed and presented in detail in several places (Refs. 1,2,3,4,10). In the following, we use that theory to account exactly for the effects of the strong laser field while the electron-ion Coulomb interaction and bremsstrahlung emission or absorption are treated in the Born approximation. We use the techniques and normalization of Reference 6.

The transition rate for an electron to go from a state of momentum $\vec{p}_1$ to a state of momentum $\vec{p}_2$ in the Coulomb field of an ion of charge Ze and to emit (absorb) a photon of energy $\hbar\omega$ per photon is (the upper sign corresponds to emission and the lower to absorption)

$$T = \frac{Z^2}{\pi^2} \alpha r_e^2 c^5 \frac{(\vec{Q} \cdot \vec{\epsilon})^2}{Q^4 (\hbar\omega)^3} \cdot \sum_{n=-\infty}^{\infty} \delta \left(\frac{p_1^2}{2m} - \frac{p_2^2}{2m} - n\hbar\omega_L \mp \hbar\omega \right) \cdot \left[J_n \left(\frac{ea_L}{m\hbar c} \frac{\vec{Q} \cdot \vec{\epsilon}_L}{\omega_L} \right) \right]^2 \quad (2.1)$$

Here α is the fine structure constant, c is the velocity of light, $\hbar$ is Planck's constant, m is the electron mass, and r_e is the classical electron radius. We have used the dipole approximation for the laser field and $J_n(x)$ is the n^{th} order Bessel function.

$$\vec{Q} = \vec{p}_1 - \vec{p}_2$$

$\vec{\epsilon}_l$ = laser polarization vector

ω_l = laser circular frequency

a_l = laser vector potential strength

$\vec{\epsilon}$ = bremsstrahlung polarization vector

ω = bremsstrahlung circular frequency.

The electric field of the laser is given by

$$E(\text{laser}) = \frac{\omega_l \vec{\epsilon}_l}{c} a_l \sin \omega_l t. \quad (2.2)$$

In the following we will use units where $m = c = \hbar = 1$, and we define

$$v = \frac{e^2 a_l^2}{mc^2}$$

so that $\frac{v}{4}$ is the electron quiver energy.

Summing over bremsstrahlung polarizations and averaging over bremsstrahlung direction yields

$$T = \frac{2Z^2 \alpha r_e^2}{3\pi^2} \frac{1}{\omega Q^2} \sum_{n=-\infty}^{\infty} \delta(\vec{P} \cdot \vec{Q} - n\omega_l \mp \omega) J_n^2 \left(\frac{\sqrt{v} Q x}{\omega_l} \right) \quad (2.3)$$

where

$$\vec{P} = \frac{\vec{p}_1 + \vec{p}_2}{2}$$

and

$$x = \frac{\vec{Q} \cdot \vec{\epsilon}_l}{Q}.$$

The rate of photon absorption per unit volume per frequency interval by n_e electrons per unit volume with momentum distribution $f(p)$ if there are n_i ions per unit volume and $n_\gamma(\omega)$ photons per unit volume of phase space is given by

$$\frac{dR_a}{d\omega} = n_i n_\gamma(\omega) n_e \frac{dI_a}{d\omega}. \quad (2.4)$$

The rate of photon emission per unit volume per frequency interval is

$$\frac{dR_e}{d\omega} = n_i n_e \frac{dI_e}{d\omega} \quad (2.5)$$

where

$$\frac{dI}{d\omega} = \frac{8\pi\hbar^3 \omega^2}{c^3} \int d^3 p_1 \int d^3 p_2 f(p_1) T \quad (2.6)$$

and emission corresponds to the upper sign in Equation (2.3) and absorption to the lower sign.

In the following we calculate the quantity $\frac{dI}{d\omega}(\omega)$ from Equation (2.6) for a monoenergetic electron distribution with energy E ,

$$f_1(p) = \frac{1}{4\pi p^2} \delta(p - \sqrt{2mE}); \quad (2.7)$$

and for a Boltzmann distribution of electron energies,

$$f_2(p) = \frac{1}{(2\pi mkT)^{3/2}} \exp\left(-\frac{p^2}{2mkT}\right), \quad (2.8)$$

where k is Boltzmann's constant and T the temperature. (We also set $k=1$ in the following.)

To proceed in evaluating Equation (2.6), we change variables from $\vec{p}_1, \vec{p}_2$ to $\vec{P}$ and $\vec{Q}$ and integrate over the azimuthal angles of $\vec{P}$, $\vec{Q}$ and the angle between $\vec{P}$ and $\vec{Q}$. Then Equation (2.6) in dimensionless form becomes

$$\begin{aligned} \frac{dI}{d\omega} = & \frac{32\pi}{3} \frac{Z^2 \alpha r_e^2}{\omega} \cdot \sum_{n=-\infty}^{\infty} \int_{-1}^1 dx \int_0^{\infty} \frac{dQ}{Q} \int_0^{\infty} P dP \cdot f(p_1) \\ & \cdot J_n^2\left(\frac{\sqrt{v_x Q}}{\omega_\ell}\right) \theta(PQ - n\omega_\ell \mp \omega) \theta(PQ + n\omega_\ell \pm \omega) \end{aligned} \quad (2.9)$$

where

$$\theta(x) = 0 \quad x < 0$$

$$\theta(x) = 1 \quad x > 0.$$

and

$$p_1 = \sqrt{P^2 + Q^2/4 + n\omega_\ell \pm \omega}. \quad (2.10)$$

We have found that the sum over n is very slowly convergent. To eliminate it, we first use the identity (Ref. 7)

$$J_n^2(\rho) = \frac{1}{\pi} \int_0^\pi J_0(2\rho \sin \psi) \cos(2n\psi) d\psi \quad (2.11)$$

and then the identity

$$\sum_n g(n) = \sum_m G(m) \quad (2.12)$$

where

$$G(m) = \int_{-\infty}^{\infty} \exp(2i\pi nm) g(n) dn. \quad (2.13)$$

As in Reference 6, the sum is then replaced by an integral and Equation 2.6 becomes

$$\frac{dI}{d\omega} = \frac{32}{3} \frac{z^2 \alpha I^2}{\omega} \int_{-1}^1 dx \int_0^\infty \frac{dQ}{Q} \int_{-\infty}^{\infty} d\psi J_0\left(\frac{2\sqrt{Q} x \sin \psi}{\omega_\lambda}\right) G(\psi) \quad (2.14)$$

where

$$G(\psi) = \text{Re} \int_{-\infty}^{\infty} dn \exp(2in\psi) \int_0^\infty p dP f(p_1) \theta(PQ - n\omega_\lambda \mp \omega) \cdot \theta(PQ + n\omega_\lambda \pm \omega). \quad (2.15)$$

Re denotes the real part and p_1 in Equation (2.15) is given by Equation (2.10).

For the monoenergetic distribution, Equation (2.15) is

$$G^{(1)} = \text{Re} \frac{i \exp(\mp 2i\psi\omega/\omega_l)}{8\pi\psi\sqrt{2E}} \left\{ \exp \frac{-i\psi}{\omega_l} (Q^2 + 2\sqrt{2EQ}) - \exp \frac{-i\psi}{\omega_l} (Q^2 - 2\sqrt{2EQ}) \right\} \quad (2.16)$$

and

$$\frac{dI^{(1)}}{d\omega} = \frac{4Z^2 \alpha r_e^2}{3\pi\sqrt{2E}\omega} \int_{-1}^1 dx \int_{-\infty}^{\infty} \frac{dQ}{Q} \int_{-\infty}^{\infty} \frac{d\psi}{\psi} \cdot i \exp -i\psi [Q^2 + 2Q \pm \omega/E] \cdot J_0 \left(2Qx\sqrt{2E}v \frac{\sin(\omega_l\psi/2E)}{\omega_l} \right). \quad (2.17)$$

In Equation (2.17) we have changed the scale of the integration variables Q and ψ . In the weak field case ($v \rightarrow 0$), Equation (2.17) reduces to the standard Born approximation result (restoring units)

$$\frac{dI^{(1)}}{d\omega} (v \rightarrow 0) = \frac{16Z^2 \alpha r_e^2}{3} \frac{c}{\omega} \left(\frac{c}{v} \right) \ln \left(\frac{(\sqrt{E} + \sqrt{E \mp \hbar\omega})^2}{\hbar\omega} \right) \theta(E \mp \hbar\omega) \quad (2.18)$$

where v is the electron velocity.

For the Boltzmann distribution, Equation (2.15) is

$$G^{(2)}(\psi) = \text{Re} \frac{Q}{2\pi\omega_l} \exp \left(\mp \frac{2i\psi\omega}{\omega_l} - \frac{i\psi Q^2}{\omega_l} - \frac{2Q^2 T \psi^2}{\omega_l^2} \right) \quad (2.19)$$

and

$$\frac{dI^{(2)}}{d\omega} = \frac{16}{3\pi} \frac{\alpha r_e^2}{\omega \omega_l} \int_{-1}^1 dx \int_0^\infty dQ \int_{-\infty}^\infty d\psi \cdot J_0\left(\frac{2\sqrt{vQx} \sin \psi}{\omega_l}\right) \cdot \exp\left(\mp \frac{2i\psi\omega}{\omega_l} - \frac{i\psi Q^2}{\omega_l} - \frac{2Q^2 \pi \psi^2}{\omega_l^2}\right). \quad (2.20)$$

Using the identity (Ref. 3)

$$\int_{-\infty}^\infty \exp(-\alpha Q^2) J_0(\beta Q) dQ = \sqrt{\frac{\pi}{\alpha}} \exp(-\beta^2/8\alpha) I_0(\beta^2/8\alpha) \quad (2.21)$$

We can write Equation (2.20) as (restoring units)

$$\frac{dI^{(2)}}{d\omega} = \exp\left(\mp \frac{\hbar\omega}{2kT}\right) \cdot \frac{8c}{3\pi} \frac{z^2 \alpha r_e^2}{\omega} \left(\frac{\pi m c^2}{2kT}\right)^{\frac{1}{2}} \int_{-1}^1 dx \int_{-\infty}^\infty dy \cdot \frac{1}{\sqrt{y^2 + 1}} \exp(-\xi) I_0(\xi) \exp\left(\mp \frac{i\hbar\omega}{2kT} y\right) \quad (2.22)$$

where

$$\xi = \frac{v}{4kT} \frac{x^2 \sin^2[\varepsilon(y-i)]}{\varepsilon^2(y^2 + 1)} \quad (2.23)$$

and

$$\varepsilon = \hbar\omega_p/4kT. \quad (2.24)$$

At $v \rightarrow 0$, Equation (2.22) is

$$\frac{dI^{(2)}}{d\omega} = \exp\left(\mp \frac{\hbar\omega}{2kT}\right) \cdot \frac{32}{3\pi} \frac{c^2 z_{ar}^2}{\omega} \left(\frac{\pi mc^2}{2kT}\right)^{\frac{1}{2}} \cdot K_0\left(\frac{\hbar\omega}{2kT}\right). \quad (2.25)$$

This agrees with the standard Born approximation result (Ref. 9) and corresponds to a Boltzmann-averaged Gaunt factor for emission of

$$\langle g \rangle = \frac{\sqrt{3}}{\pi} \exp\left(\frac{\hbar\omega}{2kT}\right) K_0\left(\frac{\hbar\omega}{2kT}\right). \quad (2.26)$$

The correction factor for the Boltzmann average case is then the ratio of Equations (2.22) and (2.25)

$$F = \frac{1}{4K_0(\hbar\omega/2kT)} \cdot \int_{-1}^1 dx \int_{-\infty}^{\infty} \frac{dy}{\sqrt{y^2 + 1}} \exp(-\xi) I_0(\xi) \exp\left(\mp \frac{iy\hbar\omega}{2kT}\right) \quad (2.27)$$

where ξ is given in Equation (2.23).

In terms of the maximum value of the electric field of the laser and the laser wavelength, λ , the variable v/kT is

$$\frac{v}{kT} = \frac{r_e \lambda^2}{4\pi^2 kT} \cdot E^2. \quad (2.28)$$

For a traveling wave the intensity I is related to E^2 by

$$E^2 = 8\pi I/v_g \quad (2.29)$$

where v_g is the group velocity. For a plasma with $v_g = c\sqrt{1 - \omega_p^2/\omega_l^2}$ and ω_p the plasma frequency we have

$$\frac{v}{kT} = \frac{1.12}{\sqrt{1 - \omega_p^2/\omega_l^2}} \left(\frac{I}{3 \times 10^{15} \text{ watts/cm}^2} \right) \left(\frac{\lambda}{1\mu\text{m}} \right)^2 \left(\frac{1}{\text{Kev}} \right). \quad (2.30)$$

Reference 10 contains a semi-classical derivation of the cross section and correction factor, but no numerical results are presented. We have not quantitatively compared our results to those of Reference 10.

III. EVALUATION OF THE BREMSSTRAHLUNG EMISSION AND ABSORPTION RATES

In this section we evaluate the expressions for the bremsstrahlung rates for a monoenergetic distribution, Equation (2.17), and for a Boltzmann energy distribution, Equation (2.27). In Section III.1 we reduce the rates for a monoenergetic distribution to a form which, although complicated, can be evaluated easily. We present typical numerical results for this case. In Section III.2 we discuss the numerical evaluation of Equation (2.27) for the Boltzmann average. This integral is more difficult, and we give several methods for its evaluation as well as numerical results that cover the range of parameters of interest for laser fusion applications.

1. EVALUATION OF THE BREMSSTRAHLUNG RATE FOR A MONOENERGETIC DISTRIBUTION

For most conditions of interest the laser photon energy is very small compared to the electron energy; i.e.,

$$\frac{\hbar\omega_l}{E} \ll 1.$$

With this approximation, $\sin\left(\frac{\omega_l \psi}{2E}\right) \approx \frac{\omega_l \psi}{2E}$ in Equation (2.17) and the expression to evaluate becomes

$$\frac{dI^{(1)}}{d\omega} = \frac{16}{3} \frac{Z^2 \alpha r_e^2 c}{\omega} \frac{c}{v} \cdot H\left(\frac{\hbar\omega}{E}, \frac{v}{E}\right) \quad (3.1)$$

where

$$H\left(\frac{\hbar\omega}{E}, \frac{v}{E}\right) = \frac{1}{2\pi} \int_0^1 dx \int_{-\infty}^{\infty} \frac{dQ}{Q} \int_{-\infty}^{\infty} \frac{d\psi \sin \psi}{\psi} \left[\psi \left(Q^2 + 2Q \pm \frac{\hbar\omega}{E} \right) \right] J_0 \left(2\sqrt{\frac{v}{2E}} Qx\psi \right). \quad (3.2)$$

In this approximation the rate is independent of the laser frequency. Equation (3.2) may be evaluated using the identity (Ref. 8)

$$\int_{-\infty}^{\infty} \frac{\sin a \psi}{\psi} J_0(\psi) d\psi = \pi \theta(a-1) + 2 \sin^{-1} a \theta(1-a) \theta(1+a) - \pi \theta(-a-1),$$

to integrate over ψ , then integrating over x and Q . The results are different for emission and absorption.

For emission

$$\begin{aligned} H_e\left(\frac{\hbar\omega}{E}, \frac{v}{E}\right) &= \theta(V-1) \left[\ln\left(V + \sqrt{V^2 - 1}\right) - \theta(1-U) I_2 \right] \\ &+ \theta(U-1) \left[\ln\left(U + \sqrt{U^2 - 1}\right) + I_3 \right] \\ &- \theta(U-1) \left[\ln\left(-U + \sqrt{U^2 - 1}\right) - I_1 \right] \end{aligned} \quad (3.3)$$

where

$$U = \frac{\sqrt{E} - \sqrt{\frac{v}{2}}}{\sqrt{\hbar\omega}}, \quad V = \frac{\sqrt{E} + \sqrt{\frac{v}{2}}}{\sqrt{\hbar\omega}} \quad (3.4)$$

and

$$I_1 = \frac{2}{\pi} \sqrt{\frac{v}{2}} \int_{\frac{\sqrt{\hbar\omega} + \sqrt{E}}{\sqrt{2}}}^1 \frac{dy}{\left[-\hbar\omega + \left(\sqrt{E} - \sqrt{\frac{v}{2}} y\right)^2\right]^{\frac{1}{2}}} \left[\sin^{-1} y - \frac{y}{2} \ln \left(\frac{1 - \sqrt{1-y^2}}{1 + \sqrt{1-y^2}} \right) \right], \quad (3.5)$$

$$I_2 = \frac{2}{\pi} \sqrt{\frac{v}{2}} \int_{\frac{\sqrt{\hbar\omega} - \sqrt{E}}{\sqrt{2}}}^1 \frac{dy}{\left[-\hbar\omega + \left(\sqrt{E} + \sqrt{\frac{v}{2}} y\right)^2\right]^{\frac{1}{2}}} \left[\sin^{-1} y - \frac{y}{2} \ln \left(\frac{1 - \sqrt{1-y^2}}{1 + \sqrt{1-y^2}} \right) \right], \quad (3.6)$$

$$I_3^{\pm} = \frac{2}{\pi} \sqrt{\frac{v}{2}} \int_{-1}^1 \frac{dy}{\left[\pm \hbar\omega + \left(\sqrt{E} - \sqrt{\frac{v}{2}} y\right)^2\right]^{\frac{1}{2}}} \left[\sin^{-1} y - \frac{y}{2} \ln \left(\frac{1 - \sqrt{1-y^2}}{1 + \sqrt{1-y^2}} \right) \right]. \quad (3.7)$$

For absorption

$$H_a \left(\frac{\hbar\omega}{E}, \frac{v}{E} \right) = \left\{ \ln \left[\left(U + \sqrt{1+U^2} \right) \left(V + \sqrt{1+V^2} \right) \right] + I_3^{\pm} \right\}, \quad (3.8)$$

In Figure 1 we plot H_e as a factor of $\hbar\omega/E$ for a range of values of v/E . In Figure 2 we plot H_a as a factor of $\hbar\omega/E$ for a range of values of v/E .

2. EVALUATION OF THE BREMSSTRAHLUNG RATE FOR A BOLTZMANN ENERGY DISTRIBUTION

For this case we assume that $\frac{\hbar\omega_l}{4kT} \ll 1$. This approximation, which is quite good for most conditions of interest, renders the correction factor independent of the laser frequency. With this approximation, Equation (2.27) for the correction factor remains the same but ξ given in Equation (2.2) becomes

$$\xi = \frac{v}{4kT} x^2 \cdot \frac{(y-i)}{(y+i)}. \quad (3.9)$$

The problem in integrating Equation (2.27) is that for large values of ω/kT the integrand oscillates rapidly and it is difficult to do the integral accurately. The methods to overcome this difficulty are different for absorption and emission.

For absorption (the lower sign), the integral may be performed by deforming the contour of integration for y from $-\infty$ to ∞ to an integral along the branch line from $y = +i$ to $y = \infty$. Thus, for absorption the integral becomes

$$F_a\left(\frac{\hbar\omega}{kT}, \frac{v}{kT}\right) = \frac{1}{K_0(\hbar\omega/2kT)} \int_0^1 dx \cdot \int_0^\infty dz \exp(-W) I_0(W) \cdot \exp\left(\frac{-\hbar\omega}{2kT} \cosh z\right) \quad (3.9)$$

where

Figure 1. H_e for emission vs $\hbar\omega/E$ for several values of ν/E .

Figure 2. H_a for absorption vs $\hbar\omega/E$ for several values of ν/E .

$$W = \frac{v x^2}{4kT} \left(\frac{\cosh(z) - 1}{\cosh(z) + 1} \right). \quad (3.10)$$

In this form the integral in Equation (3.9) is easily evaluated. Values of $F_a \left(\frac{\hbar\omega}{kT}, \frac{v}{kT} \right)$ are plotted in Figure 3 as a function of $\frac{\hbar\omega}{2kT}$ for various values of $\frac{v}{kT}$. The intense field effects do not significantly reduce the bremsstrahlung absorption rate.

For emission (the upper sign), we must close the contour in the lower-half y plane. In this case, however, if one tries to deform the contour to run along the branch line from $y = -i$, to $y = -\infty$, there is a contribution to the integral from the integration around the branch point at $y = -i$. We have not been able to calculate that contribution in any simple way. Consequently, we evaluate the correction factor for emission by integrating along the contour running from $y = -a - i\infty$ to $y = -a$ along a line parallel to the imaginary axis, from $y = -a$ to a along the real axis and from $y = a$ to $a - i\infty$ along a line parallel to the imaginary axis. The choice of a is, of course, arbitrary. In practice it is chosen small enough so that the integrand along the real axis does not oscillate too many times as y goes from a to $-a$, and it is chosen large enough so that the path parallel to the imaginary axis does not come too close to the singularity at $y = -i$. We also evaluated the integral along the contour from 0 to ∞ at an angle of $-\phi$ with respect to the real axis. We varied ϕ from $\frac{\pi}{3}$ to $\frac{\pi}{6}$ to check convergence. Either method renders the integral tractable.

Using these techniques, the expression for the emission correction factor can be evaluated. In Figure 4 we plot $F_e \left(\frac{\hbar\omega}{kT}, \frac{v}{kT} \right)$ as a function of $\frac{\hbar\omega}{2kT}$ for various values of $\frac{v}{kT}$. It is clear from the figure that the correction factor is significant when $\left(\frac{\hbar\omega}{kT} \right) \cdot \left(\frac{v}{kT} \right) \gtrsim 1$.

Figure 3. Correction factor for absorption vs $\hbar\omega/2 kT$ for several values of ν/kT for a Boltzmann energy distribution.

Figure 4. Correction factor for emission vs $\hbar\omega/2kT$ for several values of ν/kT for a Boltzmann energy distribution.

REFERENCES

1. Brown, L. S., and Kibble, T.W.B., *Phys. Rev.*, Vol. 133, No. A705, 1964.
2. Fried, Z., and Eberly, J. H., *Phys. Rev.*, Vol. 136, No. B871, 1964.
3. Kibble, T.W.B., *Phys. Rev.*, Vol. 130, No. B740, 1965.
4. Kroll, N. M., and Watson, K. M., *Phys. Rev.*, Vol. A8, No. 804, 1973.
5. Brown, L. S., and Goble, R. I., *Phys. Rev.*, Vol. 173, No. 1505, 1968.
6. Schlessinger, L., and Wright, J., "Inverse-Bremsstrahlung Absorption Rate in an Intense Laser Field," *Phys. Rev.*, Vol. A20, No. 5, 1979.
7. Abramowitz, M., and Stegun, I. A., *Handbook of Mathematical Functions* Dover, New York, 1970, p. 485.
8. Gradshteyn, I. S., and Ryzhik, I. M., *Table of Integrals, Series and Products*, Academic Press, New York, 1965, pp. 710-744.
9. Karzas, W. J., and Latter, R., *Astrophys. J. Suppl. Ser.*, Vol. 6, No. 55, p. 167 (1967).
10. Brueckner, K.A., *Laser Induced Fusion, The Physics of Quantum Electronics*, Vol. 3, ed. by S. Jacobs, M. Sargent, III, M. Scully, and C. Contrell, III, Addison-Wesley Publishing Co., Massachusetts, 1976, pp. 89-ff.

REFERENCES

1. Brown, L. S., and Kibble, T.W.B., *Phys. Rev.*, Vol. 133, No. A705, 1964.
2. Fried, Z., and Eberly, J. H., *Phys. Rev.*, Vol. 136, No. B871, 1964.
3. Kibble, T.W.B., *Phys. Rev.*, Vol. 130, No. B740, 1965.
4. Kroll, N. M., and Watson, K. M., *Phys. Rev.*, Vol. A8, No. 804, 1973.
5. Brown, L. S., and Goble, R. I., *Phys. Rev.*, Vol. 173, No. 1505, 1968.
6. Schlessinger, L., and Wright, J., "Inverse-Bremsstrahlung Absorption Rate in an Intense Laser Field," *Phys. Rev.*, Vol. A20, No. 5, 1979.
7. Abramowitz, M., and Stegun, I. A., *Handbook of Mathematical Functions* Dover, New York, 1970, p. 485.
8. Gradshteyn, I. S., and Ryzhik, I. M., *Table of Integrals, Series and Products*, Academic Press, New York, 1965, pp. 710-744.
9. Karzas, W. J., and Latter, R., *Astrophys. J. Suppl. Ser.*, Vol. 6, No. 55, p. 167 (1967).
10. Brueckner, K.A., *Laser Induced Fusion, The Physics of Quantum Electronics*, Vol. 3, ed. by S. Jacobs, M. Sargent, III, M. Scully, and C. Contrell, III, Addison-Wesley Publishing Co., Massachusetts, 1976, pp. 89-ff.