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by Relativistic Charge Bunches**

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**Michael J. Moran
Britton Chang**

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Basic Features of Coherent Radiation UCRL-JC--105079
Generated by Relativistic Charge Bunches DE91 017656

Michael J. Moran and Britton Chang
University of California
Lawrence Livermore National Laboratory
P.O. Box 808 (L-41), Livermore, CA 94550, USA

ABSTRACT

Radiation generated by relativistic charges can be analyzed and described in exquisite detail. One reason that such detailed analysis is possible is because the phases of radiated photons often are determined completely by the initial conditions of the relativistic charges and the radiating system. The phase relationships between the initial charges and the radiated photons represent coherence in the emitted radiation. A previous paper described how this coherence could affect the spatial and spectral distributions of radiation generated by a single charge in a periodic radiator. The present paper discusses a complementary issue; namely, how the temporal shape of a relativistic charge bunch can emphasize specific features of the radiation generated at a single interaction site.

Coherent emission of photons by relativistic electron beams has been demonstrated dramatically by systems such as free-electron lasers (FEL) with wavelengths from the microwave to the u.v. spectral regions. The technology of electron-beam driven radiation sources is being pursued with the goal of generating coherent photon beams with ever higher efficiencies and ever shorter wavelengths. One key feature that makes such sources possible is that the radiated intensity can become proportional to the square of the number of electrons in a pulse. Thus, while the intensity radiated by a single electron might be quite low, the flux radiated by a pulse containing more than, say 10^{10} electrons, can represent a significant fraction of the total kinetic energy in the pulse. Radiation generated in such a situation is said to be "coherent" because the collection of charge can be considered to be radiating as a single large unit of charge, rather than as a collection of independent "incoherent" sources.

Radiation generated by relativistic electrons often can be described in detail, given a few basic parameters of the mechanism of interest. For example, the spatial and temporal phase of a photon radiated by a single electron may depend only on the position of the scattering system relative to the electron trajectory, and on the time at which the electron passes through it. Diffraction Radiation (DR) from a linear slit is an example of just such a mechanism that can act as a source of photons having well defined phases.

Because of the well defined phases, sources of this type can be configured in arrays that take advantage of constructive interference to increase dramatically specific aspects of the

radiation distribution. A previous paper has discussed how a linear periodic array of slits can act as a source of DR that features intense narrow-band spectra at specific angles of emission (Fig. 1)¹. In this case, because contributions to the overall emission distribution from the individual slits have well defined mutual phase relationships, the entire radiating structure can be said to radiate coherently.

The present paper addresses a complementary situation where the radiation generated by a temporal distribution of charge has well defined coherence with respect to the individual charges in the distribution. In this case, the previous linear array of scattering systems illuminated by a single charges is replaced by a single scattering system illuminated by a temporal (spatial) distribution of charge (Fig. 2). The paragraphs below discuss how the coherence of radiation generated in this new case also can lead to radiation distributions having dramatic increases in characteristic spatial or spectral features, compared to the emission expected from isolated charges.

The basic approach in the paragraphs below will be rather ad hoc, and is not intended to be a thorough theoretical study of coherent radiation generated by relativistic particles. Rather, the emphasis will be to seize on a few basic features that are common to many radiation mechanisms and to see how they might apply to the generation of coherent radiation. The intent here is to identify key parameters that can assist in the design of experiments to study some of the phenomena that are discussed below.

The analysis below assumes the existence of a well defined radiation field $E_0(x,y,z,t)$ that describes the response of a radiating system to a single incident relativistic electron having relativistic speed $\beta = v/c$ in the z direction, where v is the electron speed, and c is the speed of light (Fig. 3). $E_0(x,y,z,t)$ could represent Ter-Mikaelian's description of diffraction radiation from a single slit,² or any of a large number of other types of interaction. Synchrotron radiation is another example that also would apply well to the present analysis. Depending on additional considerations (such as elastic collisions), mechanisms such as parametric x-rays, transition radiation or channeling radiation might also be appropriate.

The radiation field $E_0(x,y,z,t)$ can be written in terms of its Fourier transform, $E_0(k_x, k_y, k_z, \omega)$, where ω is the photon radial frequency, $k = \omega/c = \sqrt{k_x^2 + k_y^2 + k_z^2}$:

$$E_0(k_x, k_y, k_z, \omega) = \int_V \int_{-\infty}^{\infty} e^{i(k \cdot r - \omega t)} E_0(x, y, z, t) dt d^3R \quad (1)$$

The spatial-spectral intensity of the radiated fields, I_0 , is given by:

$$\frac{d^2N}{d\Omega d\omega} = \frac{1}{\hbar\omega} \left(\frac{ck^2}{4\pi} \right) |E_0|^2 \equiv I_0 \quad (2)$$

Now, the field generated by a second electron that passes along the same trajectory, but follows the first electron by a time τ (in the lab frame) will be given by $E_\tau(x,y,z,t) = E_0(x,y,z,t - \tau)$, will have the Fourier transform:

$$\begin{aligned}
 E_t(k_x, k_y, k_z, \omega) &= \int_V \int_{-\infty}^{\infty} e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} E_0(x, y, z, t - \tau) d\tau d^3R \\
 &= E_0(k_x, k_y, k_z, \omega) \cdot e^{-i\omega t} .
 \end{aligned} \tag{3}$$

If, instead of a single charge, a pulse containing Q electrons (Fig. 4) and having time dependence $\rho(t)$, where $Q = \int \rho(t) dt$, passes along the same trajectory then the radiated field $E_p(x, y, z, t)$ will have a Fourier transform:

$$E_p(k_x, k_y, k_z, \omega) = E_0(k_x, k_y, k_z, \omega) \cdot \int_{-\infty}^{\infty} \rho(t) e^{-i\omega t} dt . \tag{4}$$

Equation (4) describes the radiation field generated by a distribution of charges in terms of the field generated by a single charge. This result, together with Eq. (2), shows that the spatial-spectral intensity of the radiation distribution is proportional to the square of the Fourier transform of the charge distribution $\rho(t)$.

The consequences of this result can be described best by considering a particular example. Let the charge distribution $\rho(t)$ be a single pulse with a Gaussian distribution having full $1/e$ width Δt :

$$\rho(t) = \left(\frac{Q}{\pi 2 \Delta t} \right) e^{-(4t^2/\Delta t^2)} \tag{5}$$

Inserting this form into Eq. (4) and using Eq. (2) gives for the radiated intensity I_G :

$$I_G \equiv \frac{1}{\hbar\omega} \left(\frac{ck^2}{4\pi} \right) |E_G|^2 = I_0 * Q^2 e^{-(\omega\Delta t/4)^2} \quad (6)$$

This result can be discussed in terms of the wavelength $\lambda = c/2\pi\omega$ and the pulse length $l = c\Delta t$. If $l \lesssim \lambda/2$ then the intensity will be given roughly by $I_0 Q^2$, ie, the pulse is radiating "coherently", or together as a single charge with magnitude Qe . The radiated intensity is Q times greater than would be obtained from the single-particle description.

On the other hand, if $l > \lambda/2$ then the intensity will decrease rapidly as the pulselength l increases. For example, if $Q = 10^{10}$ and $l = 2.5\lambda$ then $I_G \approx I_0 Q$ - which is the result that would be expected from a simple single-particle description. Further lengthening of the pulse would decrease I_G rapidly. Thus, the coherent nature of the radiation mechanism can either enhance or reduce the radiated intensity, depending on the width Δt .

As another example, consider the emission from a train of N Gaussian pulses having a full $1/e$ width of Δt and separated by a time T :

$$p_N(t) = \left(\frac{Q}{N\pi\Delta t} \right) * \left(\sum_{j=-(N-1)/2}^{(N-1)/2} e^{-(4(t+jT)/\Delta t)^2} \right). \quad (7)$$

with N an odd positive integer. Again, Eq.(2) and Eq.(4) are used to obtain:

$$I_N = I_0 * Q^2 \left(e^{-2(\omega \Delta t / 4)^2} \right) * \left(1 + 2 * \sum_{j=1}^{(N-1)/2} \cos(j \omega T) \right) \quad (8)$$

and, when $\omega T = r\pi$ (r is any integer):

$$I_N = I_0 * Q^2 * e^{-2(\omega \Delta t / 4)^2} \quad (9)$$

Equation (9) says once again that the radiated intensity is proportional to the square of the charge contained in the entire train of pulses (as long as $l \lesssim \lambda/2$). This result depends crucial assumptions (such as $\omega T/r\pi$) but is important for some practical situations. For example, in a conventional "undulator" visible-wavelength FEL, an initially "bunched" electron pulse (length $\approx 3\text{mm}$) is further bunched by an optical wave into a "train" of pulses having a period of the order of $5000 \text{ \AA}$. Equation (9) shows that such a "bunch of bunches" can radiate coherently with an effective charge Q , rather than as N separate bunches, each radiating coherently with charge Q/N . The latter case would give intensities smaller by a factor of N (about 6000 in this case).

Obviously, there are limitations on the applicability of Eq. (8). This result relies on an integration over time that extends from $-\infty$ to $+\infty$. In practice, any physical system will have a finite length that will limit duration over which the integration should be performed. The result is that Eq. (8) will be valid only if the distance between the slit and the detector is greater than NcT , the length of the train of electron pulses. Also, in order for

contributions from the individual bunches to add coherently, they must have a length that is small compared to $\lambda/2N$, rather than $\lambda/2$.

Finally, the results above have not addressed the finite transverse size of the electron beam. When the electrons have a finite distribution about the ideal electron trajectory their individual emissions may not add coherently. This issue can be discussed in terms of the inherent transverse coherence radius R_c of photons radiated by relativistic electrons³:

$$R_c = \frac{\sigma\lambda}{2\pi} . \quad (10)$$

When the electron beam radius is less than R_c the photons will be radiated essentially with total transverse coherence. With the low-emittance beams that are becoming available it now is possible to satisfy this condition for photon energies that extend well into the x-ray spectral region.

These ideas can be applied to a variety of topics relevant to coherent photon generation. The results above describe two kinds of restrictions that make possible coherent radiation of photons with specific wavelengths. For single pulses, Eq. (6) suggests that photons having wavelengths longer than the electron pulse itself will be radiated coherently by the entire charge in the pulse. For extended charge distributions, Eq. (9) and its discussion suggest that coherent radiation can take place for wavelengths that correspond to strong components of the Fourier transform of the charge distribution. This result is somewhat more general than the commonly accepted requirement that beam pulses be "bunched" to

within lengths that are smaller than the radiated wavelength of interest. This work was performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract number W-7405-ENG-48.

References:

- ¹ M. J. Moran, *Proceedings of the 12th International Conference on the Penetration of Charged Particles in Matter*, San Sebastian, Basque Country, Spain (September 4-7, 1989), p. 537.
- ² M. L. Ter-Mikaelian, *High-Energy Electromagnetic Processes in Condensed Media*, John Wiley & Sons, New York (1972), p. 382.
- ³ W. K. Panofsky and M Phillips, *Classical Electricity and Magnetism*, Addison-Wesley Co., Cambridge, Mass. (1955), p. 295.

Figure Captions:

Figure 1 This sketch illustrates the geometry for coherent photon generation by an electron passing through a periodic-slit array.

Figure 2 Here, a periodic distribution of charge is shown generating radiation at a single interaction site.

Figure 3 This diagram illustrates the situation where a single charge experiences a radiating interaction and produces a specific radiation field $E_0(x,y,z,t)$.

Figure 4 Here, an arbitrary line charge $\rho(t)$ is shown producing a radiation field $E_p(x,y,z,t)$ from interactions at a single site.

Figure 1

Figure 2

Figure 3

Figure 4