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**RESERVOIR PERFORMANCE IN VISCOELASTIC
POROUS MEDIA**

TOPICAL REPORT

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ABSTRACT

The mass balance equations for a two-phase two-component fluid system are written for viscoelastic porous media. The resulting equations are approximated by finite differences and the resulting numerical simulator is used to conduct a sensitivity study on the effects of uniaxial viscoelastic deformation in geopressured aquifers. Results of this study indicate that viscoelastic deformation may have considerable influence on the pressure maintenance of these aquifers. A numerical model of the geopressured aquifer in Brazoria County, Texas, is constructed and the numerical simulator is used to predict the ultimate recovery of solution gas from this viscoelastic geopressured aquifer.

INTRODUCTION

The reservoir mechanics operative in geopressured-geothermal aquifers have been published in an earlier paper [5]. The authors explain that fluid expansion and reservoir rock compaction will play a more dominant role in these reservoirs than in normally pressured petroleum reservoirs. They proceed to derive the mass balance equations for a two-component two-phase water and methane gas system in compacting porous media. The mass balance equations so derived were approximated by finite differences and the resulting finite difference geopressured-geothermal reservoir simulator was used to describe the process of fluid withdrawal from a geopressured aquifer in south Texas.

The simulator designed in [5] is capable of describing the process of fluid withdrawal from geopressured aquifers only when the reservoir rock behaves like an ideal elastic solid. However, testing of rock samples taken from the Austin Bayou geopressured aquifer in Brazoria County indicates that the material has a tendency to creep [9]. This phenomenon is characterized by time-dependent deformation of the rock matrix as opposed to the instantaneous deformation associated with elastic materials. Time-dependent deformation is associated with the "dirty" portions of the reservoir, which, because of their clay content, display viscous flow characteristics. As a first approximation this time dependency may be considered as a linear viscoelastic behavior.

This paper describes the procedure by which the geopressured-geothermal simulator was modified to account for uniaxial viscoelastic deformation of the rock matrix. The simulator is then employed in an effort to quantify the significance of transient rock compaction in geopressured aquifers.

GOVERNING DIFFERENTIAL EQUATIONS

Fluid Mass Balance Equations. The reader is referred to reference [5] for the derivation of the governing differential equations which describe the mass balance of a two-component two-phase system, consisting of water and methane gas, where it is assumed that the vapor phase consists of only methane gas and the liquid phase consists of water and dissolved gas. These assumptions are similar to those made for black oil simulators. In addition the reservoir is assumed to be of large areal extent relative to reservoir thickness; therefore, the porous medium is presumed to deform predominantly in the vertical direction [1,2] (uniaxial compaction). The resulting differential equations are then expressed as

for the water component

$$\frac{d}{dt} \left(\frac{\phi S_\ell}{B_\ell} \right) - \left(\frac{\phi S_\ell}{B_\ell} \right) \frac{d\varepsilon_z}{dt} = \nabla \cdot (\tilde{T}_\ell \nabla \Phi_\ell) - q_w \quad (1)$$

for the gas component

$$\frac{d}{dt} \left(\frac{\phi S_v}{B_v} + \frac{R_s \phi S_\ell}{B_\ell} \right) - \left(\frac{\phi S_v}{B_v} + \frac{R_s \phi S_\ell}{B_\ell} \right) \frac{d\varepsilon_z}{dt} = \nabla \cdot (\tilde{T}_v \nabla \Phi_v + R_s \tilde{T}_\ell \nabla \Phi_\ell) - q_g \quad (2)$$

where the derivation has been carried out from the point of view of an observer moving with the deforming solid phase, and, thus, the conventional partial time derivatives have been replaced with the total time derivative. S represents the saturation, B the formation volume factor, $\tilde{T}$ the transmissibilities and Φ is the flow potential. The subscripts ℓ and v indicate liquid and vapor phases respectively. q_w and q_g represent the sink terms for the water component and gas component respectively. R_s is the solution gas-water ratio and ϕ the porosity. Finally ε_z represents the state of strain of the porous medium, here assumed to be uniaxial in the z direction with compression taken as positive. These equations

differ from conventional mass balance equations by inclusion of the terms involving the strain rate, $\frac{d\epsilon_z}{dt}$.

Elastic Deformation. The conservation of the solid phase (rock matrix) in a uniaxially compacting porous medium, [5], is simply restated here as

$$\frac{d}{dt} (\rho_r (1-\phi)) - \rho_r (1-\phi) \frac{d\epsilon_z}{dt} = 0 \quad (3)$$

where ρ_r is the rock matrix density, ϕ the porosity and ϵ_z the uniaxial state of strain with compression taken as positive. Defining the rock matrix compressibility, C_{Rm} , and the pore compressibility, C_ϕ as

$$C_{Rm} = \frac{1}{\rho_r} \left(\frac{d\rho_r}{dP} \right)_T, \quad C_\phi = \frac{1}{\phi} \left(\frac{d\phi}{dP} \right)_T \quad (4)$$

and expanding the first term in equation (3) and substituting the relations of (4) into (3) yields

$$(1-\phi)C_{Rm} \frac{dP}{dt} - C_\phi \phi \frac{dP}{dt} - (1-\phi) \frac{d\epsilon_z}{dt} = 0 \quad (5)$$

Solving (4) for C_ϕ yields

$$C_\phi = \frac{(1-\phi)}{\phi} \left(C_{Rm} - \frac{\frac{d\epsilon_z}{dt}}{\frac{dP}{dt}} \right) \quad (6)$$

C_ϕ as given in (6) is undetermined until the stress-strain relationship for the rock matrix is specified. For porous media assumed to deform elastically, the concept of effective stress and the linear theory of elasticity are combined to form the relationship between stress and strain, i.e.,

$$\epsilon_z = -C_{me} \Delta P \quad (7)$$

where C_{me} is the uniaxial elastic compaction coefficient and where once again compression is taken as positive. Differentiating (7) with respect to time (C_{me} assumed constant) yields the strain rate term needed to evaluate C_ϕ for elastic porous media,

$$C_\phi = \frac{1-\phi}{\phi} (C_{Rm} + C_{me}) \quad (8)$$

Viscoelastic Deformation. For viscoelastic materials the relationship between stress and strain is complicated by the time-dependent properties of the rock. For these materials the stress-strain relations may be prescribed by the behavior of analagous mechanical systems which allow formulation of the transient rock behavior. The volumetric deformation of cores taken from the Brazoria County geopressed well indicate that the modified Kelvin substance (Figure 1) can describe the viscoelastic deformation of that geopressed aquifer [8]. The modified Kelvin substance consists of the parallel arrangement of a dashpot and spring in series with a second spring. In this mechanical system the spring models the elastic component of strain while the dashpot models the viscous component of strain. The differential equation relating the state of stress to the state of strain which result from analyzing the behavior of the modified Kelvin substance is here presented without derivation (the reader is referred to reference [3] for a more thorough derivation.

$$C_{me} \dot{\sigma}(t) + \frac{\sigma(t)}{\eta} \left(\frac{C_{me} + C_{mv}}{C_{mv}} \right) = \dot{\epsilon}_z(t) + \frac{\epsilon_z(t)}{C_{mv}\eta} \quad (9)$$

where C_{me} and C_{mv} are the elastic and viscous uniaxial compaction coefficients corresponding to the two springs, η is the viscosity modulus which relates the state of stress to the strain rate for the viscous element, σ and ϵ are the total stress and strain of the model. The superposed dot is used to indicate the time derivatives. To apply equation (9) to geopressed aquifer compaction problems, the stress is

Modified Kelvin Substance

$$C_{me} \sigma'_{(t)} + \frac{\sigma_{(t)}}{\eta} \left(\frac{C_{me} + C_{mv}}{C_{mv}} \right) = \epsilon'_{(t)} + \frac{\epsilon_{(t)}}{C_{mv} \eta}$$

Figure 1. Modified Kelvin Substance Strain Response for Constant Stress

replaced by the effective stress at reservoir conditions given by the difference between constant overburden pressure σ_{ob} and instantaneous fluid pressure P (i.e., $\sigma(t) = \sigma_{ob} - P(t)$).

Recalling that our present task is to arrive at the relationship between stress and strain for viscoelastic materials in order to complete the rock mass balance equation, we now obtain the solution to equation (9).

The method we choose to solve (9) is to determine the rock's differential response to a unit impulse stress and then to use the principle of superposition to determine the rock's complete response. The unit impulse stress can be viewed physically as a sharp blow to a rock. It can be represented mathematically by the delta function $\delta(t)$ which has the following properties

$$\begin{aligned}\delta(t) &= 0 & t &\neq 0 \\ \int_{-\infty}^{\infty} \delta(t) dt &= 1 \\ \int_{-\infty}^{\infty} \delta(t) f(t) dt &= f(0)\end{aligned}$$

Letting $\sigma(t) = \delta(t)$ (unit impulse at $t = 0$) and solving (9) via a Laplace transform technique yields the desired impulse response of the rock

$$\epsilon_z(t) = C_{me} \delta(t) + \frac{1}{\eta} e^{-t/C_{mv}\eta} \quad (10)$$

and after defining the rocks time constant $\tau = C_{mv}\eta$ equation (10) becomes

$$\epsilon_z(t) = C_{me} \delta(t) + \frac{C_{mv}}{\tau} e^{-t/\tau} \quad (11)$$

If the unit stress acts at a time $t = a \neq 0$ then the impulse response of the rock material is given by

$$\epsilon_z(t) = C_{me}\delta(t-a) + \frac{C_{mv}}{\tau} e^{-(t-a)/\tau} \quad (12)$$

When a stress is present both at $t = 0$ and $t = a$ the state of strain will be given by

$$\epsilon_z(t) = C_{me} [\delta(t) + \delta(t-a)] + \frac{C_{mv}}{\tau} \left[e^{-\frac{t}{\tau}} + e^{-\frac{(t-a)}{\tau}} \right] \quad (13)$$

and in the limit when the pulses become continuous

$$\epsilon_z(t) = \int_0^t \left[C_{me} \delta(t-\xi) + \frac{C_{mv}}{\tau} e^{-\frac{(t-\xi)}{\tau}} \right] d\xi \quad (14)$$

In general the stress will not be of unit amplitude and in that case equation (14) becomes

$$\epsilon_z(t) = \int_0^t \sigma(\xi) \left[C_m \delta(t-\xi) + \frac{C_{mv}}{\tau} e^{-\frac{(t-\xi)}{\tau}} \right] d\xi \quad (15)$$

Equation (15) is alternatively written

$$\begin{aligned} \epsilon_z(t) &= \int_0^t \sigma(t-\xi) h(\xi) d\xi \\ &= \sigma(t) * h(t) \end{aligned} \quad (16)$$

where $h(\xi) = C_{me}\delta(\xi) + \frac{C_{mv}}{\tau} e^{-\xi/\tau}$ and is simply the unit impulse response of the viscoelastic material. The integral in equation (16) is known as the convolution integral; thus, the state of strain at any time, t , is given by the convolution of the stress history with the unit impulse response. Defining $h_v(t) = \frac{C_{mv}}{\tau} e^{-t/\tau}$ and taking the time derivative of equation (16) yields the desired strain rate relationship for viscoelastic materials

$$\frac{d \epsilon_z}{dt} = - C_{me} \frac{dP}{dt} + \dot{\sigma}(t) * h_v(t) \quad (17)$$

Substituting equation (17) into equation (6) yields the condition for rock mass balance in viscoelastic porous media

$$C_\phi \frac{(1-\phi)}{\phi} (C_{Rm} + C_{me} - \frac{\dot{\sigma}(t) * h_v(t)}{\frac{dP}{dt}}) \quad (18)$$

NUMERICAL APPROXIMATION

Discrete Fluid Mass Balances. For a detailed description of the simulator the reader is referred to references [5,6]; however, a brief description is in order.

The simulator can be used for areal (x,y) simulations or cross-sectional simulations (radial or Cartesian). For areal studies the domain is discretized into N_x by N_y grid blocks. For cross-sectional studies the domain is discretized into N_r by N_z grid blocks for radial models or N_x by N_z grid blocks for (x,z) models.

The differential equations (1 and 2) are initially integrated over each grid volume. The integrals are then replaced by their average values via application of the mean value theorem for integrals, and, then, derivatives are approximated by finite differences where the diffusion terms are replaced by centered differences and the time derivatives are replaced with backward time differences. As a consequence of this procedure the following difference equations have been generated for each grid block, J,

$$\frac{\Delta v_{wJ}^{n+1}}{\Delta t} = \sum_i \{ \lambda_{\ell, J+i/2}^{n+1} \Delta \phi_{\ell, J+i/2}^{n+1} - \lambda_{\ell, J-i/2}^{n+1} \Delta \phi_{\ell, J-i/2}^{n+1} \} - Q_{wJ}^{n+1} \quad (19)$$

$$\begin{aligned}
\frac{\Delta_t V_{gJ}^{n+1}}{\Delta t} = \sum_i \{ (R_s \lambda_\ell)^{n+1}_{J+i/2} \Delta_i \Phi_{\ell,J+i/2}^{n+1} - (R_s \lambda_\ell)^{n+1}_{J-i/2} \Delta_i \Phi_{\ell,J-i/2}^{n+1} \\
+ \lambda_{v,J+i/2}^{n+1} \Delta_i \Phi_{v,J+i/2}^{n+1} - \lambda_{v,J-i/2}^{n+1} \Delta_i \Phi_{v,J-i/2}^{n+1} \} - Q_{gJ}^{n+1}
\end{aligned} \quad (20)$$

where Δt is the time step size, Δ_t is the backward time difference and Δ_i is the centered space difference in the i -th axis direction,

$$\Delta_i \Phi_{\ell,J+i/2} = \Phi_{\ell,J+i} - \Phi_{\ell,J}$$

and the following definitions have been used

$$\begin{aligned}
V_{wJ} &= \left(\frac{V \phi S_\ell}{B_\ell} \right)_J ; \quad V_{gJ} = \left(\frac{V \phi S_v}{B_v} + \frac{R_s V \phi S_\ell}{B_\ell} \right)_J \\
\lambda_{\ell,J+i/2} &= \left(\frac{A_i T_{\ell i}}{\Delta x_i} \right)_{J+i/2} \quad \lambda_{v,J+i/2} = \left(\frac{A_i T_{vi}}{\Delta x_i} \right)_{J+i/2}
\end{aligned}$$

where V_J is the grid block volume and $A_{iJ+i/2}$ is the area of the grid interface perpendicular to the i -th direction. The subscript J is a two component vector identifying a grid block location. The index $J+i$ refers to the grid block adjacent to grid block J in the positive i -th axis direction. The grid block interface between nodes J and $J+i$ is denoted by $J+i/2$.

Recalling the apriori integration the parameters in equations(19) and (20) should be interpreted as averaged values over the grid block volumes or interfaces. The functional dependence of the rock properties and fluid properties on the phase pressures and saturations make equations (19) and (20) non-linear difference equations. They are treated in the simulator by a semi-implicit procedure where the non-linear terms at a new time level are approximated by a linear expansion about

the values of the old time level. For example the term involving the water volume is approximated by

$$v_{wJ}^{n+1} = v_{wJ}^n + v_{wJ}^{(P)} \Delta_t P_{\ell,J}^{n+1} + v_{wJ}^{(S)} \Delta_t S_{\ell,J}^{n+1} \quad (21)$$

where the slopes $v_{ws}^{(P)}$ and $v_{ws}^{(S)}$ (with respect to pressure and saturation) are given by

$$v_{wJ}^{(P)} = \frac{v_w(P_{\ell,J}^{n+1}, S_{\ell,J}^n) - v_w(P_{\ell,J}^n, S_{\ell,J}^n)}{P_{\ell,J}^{n+1} - P_{\ell,J}^n}$$

$$v_{wJ}^{(S)} = \frac{v_w(P_{\ell,J}^{n+1}, S_{\ell,J}^{n+1}) - v_w(P_{\ell,J}^{n+1}, S_{\ell,J}^n)}{S_{\ell,J}^{n+1} - S_{\ell,J}^n}$$

In this manner the non-linear equations are linearized and are then solved for changes in liquid pressures and saturations by Gaussian elimination. The first solution is obtained by using chord slopes approximated by the gradients at the beginning of each time step. The chord slopes are updated by using the first solution to generate the improved solution for the new time level. The iteration is repeated until convergence is obtained.

Treatment of Formation Compaction. The problem of modifying the geopressured-geothermal simulator to account for compaction effects reduces to the problem of determining the pore volume as a function of time. In conventional simulators the rock matrix compressibility, C_{Rm} , and the compaction coefficients are assumed negligible in comparison with the fluid compressibilities; thus, pore volumes are assumed constant during depletion. For geopressured aquifers pore volume reduction may constitute an additional driving force.

Equation (18) applied to each grid block provides the necessary constitutive relationship between pore compressibility and the state of strain for viscoelastic reservoirs. This equation includes the effects of the transient period. Noting that $C_{\phi J} = \left(\frac{1}{\phi} \frac{d\phi}{dP} \right)_J$, then the porosity change in finite difference form can be written

$$(\Delta_t \phi)_J = \left[(1 - \phi) (C_{Rm} + C_{me} - \frac{\dot{\sigma}^{eff}(t) * h_v(t)}{\frac{\Delta_t P}{\Delta t}}) \right]_J \Delta_t P_J \quad (22a)$$

$$= (1 - \phi)_J (C_{Rm} + C_{me}) \Delta_t P_J + (1 - \phi)_J (\dot{P}(t) * h_v(t))_J \Delta t \quad (22b)$$

The convolution integral in (22) is easily calculated after noting that

$$\begin{aligned} & \left[\int_0^{t+\Delta t} \dot{P}(t+\Delta t-\xi) \frac{C_{mv}}{\tau} e^{-\xi/\tau} d\xi \right]_J \\ &= \left[\int_0^{\Delta t} \dot{P}(t+\Delta t-\xi) \frac{C_{mv}}{\tau} e^{-\xi/\tau} d\xi \right]_J \\ &+ \left[\int_{\Delta t}^{t+\Delta t} \dot{P}(t-(\xi-\Delta t)) \frac{C_{mv}}{\tau} e^{-\xi/\tau} d\xi \right]_J \end{aligned} \quad (23)$$

Making a change of variable $\xi' = \xi - \Delta t$ for the second integral in (23) results

$$\begin{aligned} & \left[\int_0^{t+\Delta t} \dot{P}(t+\Delta t-\xi) \frac{C_{mv}}{\tau} e^{-\xi/\tau} d\xi \right]_J \\ &= \left[\int_0^{\Delta t} \dot{P}(t+\Delta t-\xi) \frac{C_{mv}}{\tau} e^{-\xi/\tau} d\xi \right]_J \\ &+ \left[e^{-\Delta t/\tau} \int_0^t \dot{P}(t-\xi') \frac{C_{mv}}{\tau} e^{-\xi'/\tau} d\xi' \right]_J \end{aligned} \quad (24)$$

Since $\dot{P}(t) = (\Delta_t P / \Delta t)_J = \text{constant}$ for each time step equation (24) becomes

$$\begin{aligned} & \left[\int_0^{t+\Delta t} \dot{P}(t+\Delta t-\xi) \frac{C_{mv}}{\tau} e^{-\xi/\tau} d\xi \right]_J \\ &= \left[C_{mv} \frac{\Delta_t P}{\Delta t} (1 - e^{-\Delta t/\tau}) \right]_J \\ &+ \left[e^{-\Delta t/\tau} \int_0^t P(t-\xi) \frac{C_{mv}}{\tau} e^{-\xi/\tau} d\xi \right]_J \end{aligned} \quad (25)$$

Defining $H^n = \int_0^{T^n} \dot{P}(t^n-\xi) \frac{C_{mv}}{\tau} e^{-\xi/\tau} d\xi$, equation (25) displays the

convenient iterative relationship

$$H_J^{n+1} = C_{mv} \frac{\Delta_t P}{\Delta t} (1 - e^{-\Delta t/\tau}) + e^{-\Delta t/\tau} H_J^n \quad (26)$$

and therefore equation (22) can be written for each block as

$$\phi_J^{n+1} = \phi_J^n + (1-\phi_J^n) (C_{Rm} + C_{me})_J (P^{n+1} - P^n)_J + (1-\phi_J^n) (t^{n+1} - t^n) H_J^{n+1} \quad (27)$$

The contraction in reservoir thickness, ℓ_z , is calculated via

$$\ell_z^{n+1} = \ell_z^n (1 - \Delta \epsilon_z) \quad (28)$$

where

$$\Delta \epsilon_z = C_{me} (P^{n+1} - P^n) + (t^{n+1} - t^n) H^{n+1} \quad (29)$$

PRODUCTION PERFORMANCE OF VISCOELASTIC RESERVOIRS

In order to investigate the effects of the viscous time constant, τ , on reservoir performance as well as the effects of the viscous uniaxial compaction coefficient,

C_{mv} , a sensitivity study was conducted for an ideal reservoir with a square drainage area and a single well at its center.

The reservoir is homogeneous and isotropic and of constant thickness 100 feet. The initial permeability and porosity are 100 md and 30% respectively. The reservoir is presumed to be at a depth of 14,600 feet. The reservoir fluid is water with dissolved methane gas in a saturated condition. Finally, the reservoir is perforated over the entire 100 foot interval and the fluid withdrawal is held constant at 10,000 STBW/day until bottom hole well pressure reaches 7,000 psi, at which point the simulator maintains a constant bottom hole pressure of 7,000 psi. Table 1 summarizes the reservoir properties.

Compaction Parameters. In all simulations the elastic uniaxial compaction coefficient, C_{me} , is held constant at $5.0 \times 10^{-7} \text{ psi}^{-1}$. This value corresponds to an experimentally determined value for the compaction of clean sandstone in the Pleasant Bayou #1 and #2 geopressed wells in Brazoria County, Texas [4]. The viscous uniaxial compaction coefficient, C_{mv} , and the viscous time constant, τ , are the only variables in these simulations with C_{mv} ranging from $1.0 \times 10^{-5} \text{ psi}^{-1}$ to $1.0 \times 10^{-7} \text{ psi}^{-1}$ and τ ranging from infinity (the purely elastic reservoir with equivalent compaction coefficient C_{me}) to $\tau = 0$ (the purely elastic reservoir of equivalent compaction coefficient $C_{meq} = C_{me} + C_{mv}$). These values were chosen to display the general pressure maintenance trends of viscoelastic reservoirs. For values of τ such that $0 < \tau < \infty$ the reservoir exhibits viscoelastic behavior. In order to display only the viscoelastic compaction effects the rock matrix compressibility, C_{Rm} is identically zero for all simulations. Table 2 summarizes the simulations made for this study.

Fluid Properties. The fluid properties were estimated based upon the assumption

Table 1

Basic Reservoir Parameters and Conditions

Type Simulation	Areal
Drainage Area	16 square miles
Length in x-direction	24,000 feet
Length in y-direction	24,000 feet
Well Radius	.292 feet
Reservoir Depth	14,600 feet
Initial Pressure	11,500 psi
Temperature	300°F
Porosity	30%
Permeability	$K_x = 100\text{md}$ $K_y = 100\text{md}$
Elastic Uniaxial Compaction Coefficient	$C_{me} = 5 \times 10^{-7} \text{psi}^{-1}$
Viscous Uniaxial Compaction Coefficient	$C_{mv} = 1.0 \times 10^{-7} \text{psi}^{-1} \sim 1.0 \times 10^{-5} \text{psi}^{-1}$
Viscous Time Constant	$\tau = 0 \sim \infty \text{ (days)}$
Initial Gas Saturation	0.0%
Critical Gas Saturation	0.0%
Flow Rate	10,000 STBW/day
Minimum Bottom Hole Pressure	7,000psi

Table 2

Summary of Compaction Parameters Used in Simulations

Case No.	C_{me} (psi ⁻¹)	C_{mv} (psi ⁻¹)	η (psi-day)	τ (day)	C_{Rm} (psi ⁻¹)
1	5.0×10^{-7}	1.0×10^{-5}	∞	∞	0
2	5.0×10^{-7}	1.0×10^{-5}	6.0×10^8	6000	0
3	5.0×10^{-7}	1.0×10^{-5}	3.0×10^8	3000	0
4	5.0×10^{-7}	1.0×10^{-5}	1.0×10^8	1000	0
5	5.0×10^{-7}	1.0×10^{-5}	0	0	
6	5.0×10^{-7}	1.0×10^{-6}	∞	∞	0
7	5.0×10^{-7}	1.0×10^{-6}	6.0×10^9	6000	0
8	5.0×10^{-7}	1.0×10^{-6}	3.0×10^9	3000	0
9	5.0×10^{-7}	1.0×10^{-6}	1.0×10^9	1000	0
10	5.0×10^{-7}	1.0×10^{-6}	0	0	0
11	5.0×10^{-7}	1.0×10^{-7}	∞	∞	0
12	5.0×10^{-7}	1.0×10^{-7}	6.0×10^{10}	6000	0
13	5.0×10^{-7}	1.0×10^{-7}	3.0×10^{10}	3000	0
14	5.0×10^{-7}	1.0×10^{-7}	1.0×10^{10}	1000	0
15	5.0×10^{-7}	1.0×10^{-7}	0	0	0

that the water was saturated with methane, 26 SCF/STBW at the initial reservoir pressure of 11,500 psi and at an initial temperature 300°F. This value of gas-water ratio approximates the value of 30 SCF/STBW as measured in the Pleasant Bayou wells. The gas-water ratio property as well as formation volume factors (B_g and B_v) and fluid viscosities (μ_g and μ_v) were input as piecewise linear functions of pressure (at the assumed aquifer temperature of 300°F). Table 3 summarizes the fluid properties.

Viscoelastic Reservoir Behavior. Figure 2 is a comparison plot of the first five simulations where $C_{mv} = 1.0 \times 10^{-5} \text{ psi}^{-1}$ and τ takes on the values ∞ , 6000, 3000, 1000 and zero days. Case 1, the base case, is the expected pressure decline when $\tau = \infty$. In this case the minimum bottom hole pressure of 7000 psi is reached after 17 years of production. Case 2 ($\tau = 6000$) through Case 5 ($\tau = 0$) exhibits the expected improved pressure maintenance as the viscous time constant approaches zero.

Figure 3 is a comparison plot of Case 6 through Case 10. In these cases C_{mv} has been reduced to $1.0 \times 10^{-6} \text{ psi}^{-1}$. τ once again has the values ∞ , 6000, 3000, 1000 and zero days. As before performance improves as τ decreases.

Figure 4 compares the results of Case 11 and Case 15. In these cases $C_{mv} = 1.0 \times 10^{-7} \text{ psi}^{-1}$ and is of the same order of magnitude as C_{me} . For this value of C_{mv} only the cases with $\tau = \infty$ and $\tau = 0$ are presented. Reservoir improvement, although small, is once again observed as τ decreases.

Collectively Figures 2, 3, and 4 display relationships from which some very general conclusions can be made. Noting that for all cases the $\tau = 1000$ days is not very much different from $\tau = 0$ days, one can treat viscoelastic reservoirs for which $\tau = 1000$ days as an elastic reservoir with $C_{meq} = C_{me} + C_{mv}$. It is also observed that the $\tau = 0$ and $\tau = \infty$ cases form an envelope which encompasses all modes of

Table 3
Fluid Properties

Pressure (psi)	B_{ℓ} (bbl/STB)	R_S (SCF/STB)	μ_{ℓ} (CP)	B_v (bbl/MCF)	μ_v (CP)
5000	1.0685	15.000	.1781	.849	.02245
5500	1.0675	16.000	.1788	.783	.02326
6000	1.0660	17.000	.1794	.741	.02406
6500	1.0648	17.500	.1801	.699	.02484
7000	1.0635	18.800	.1808	.667	.02562
7500	1.0623	19.800	.1818	.637	.02639
8000	1.0610	20.500	.1824	.609	.02715
8500	1.0596	21.500	.1828	.586	.02790
9000	1.0585	22.500	.1833	.568	.02865
9500	1.0573	23.300	.1841	.550	.02939
10000	1.0560	24.000	.1849	.534	.03013
10500	1.0548	25.000	.1856	.519	.03686
11000	1.0536	26.000	.1863	.505	.03157
11500	1.0523	27.200	.1870	.492	.03226
12000	1.0510	28.000	.1877	.481	.03290
12500	1.0497	29.200	.1889	.458	.03470
13000	1.0485	30.000	.1902	.436	.03630
13500	1.0473	31.300	.1915	.413	.03800

Figure 2. Viscoelastic Reservoir Performance

Figure 3. Viscoelastic Reservoir Performance

Figure 4. Viscoelastic Reservoir Performance

viscoelastic behavior. As C_{mv} decreases the envelope decreases. For viscoelastic materials with $C_{mv} \leq .2 C_{me}$ one might approximate the viscoelastic reservoir as an elastic reservoir with $C_{meq} = C_{me}$.

Non-linear Viscoelastic Performance. It has been observed that the viscous time constant, $\tau = C_{mv}$, is less sensitive to changes in stress than is either C_{mv} or η when considered separately. It may therefore be possible to consider τ as constant during the depletion of a geopressed aquifer. However, rocks which have the same constant, τ , but differing in their values of C_{mv} will display unique behavior. Figure 5 compares Case 2, Case 7 and Case 12. The time constant is 6000 days for each case but C_{mv} is $1.0 \times 10^{-5} \text{ psi}^{-1}$, $1.0 \times 10^{-6} \text{ psi}^{-1}$ and $1.0 \times 10^{-7} \text{ psi}^{-1}$, respectively. As expected when C_{mv} decreases so does reservoir performance. However it is not possible to make any simplifying convergence assumptions. In view of the non-linear nature of viscoelastic materials Figure 5 indicates that predicted reservoir performance based upon a linearly viscoelastic model can be in significant disagreement with actual reservoir rock. In cases where "strain hardening" occurs (C_{mv} decreases as compaction increases) the linearly viscoelastic model will give results which are optimistic. Note that experimental data on geopressed-geothermal cores suggest that this strain hardening does occur [9].

AUSTIN BAYOU GEOPRESSED AQUIFER

Reservoir Description. In a final application, the geopressed simulator as hitherto developed was used to quantify the effect of viscoelastic rock compaction in the geopressed test well located in Brazoria County, Texas. Suffice it to say at this time that the producing interval consists of 100 feet of gross sand, of which 60 feet is clean sandstone and is determined to deform predominantly via elastic deformation. Forty feet of sand contains appreciable amounts of clay and these lobes have been determined experimentally to deform in a viscoelastic manner.

Figure 5. Viscoelastic Reservoir Performance
(Constant τ)

These lobes (C_1 and C_3) are separated from the thick clean lobe (C_2) by 20-foot shale breaks. Figure 6 illustrates the sand shale sequence which comprises the C-sand in the Pleasant Bayou #2 test well. The permeability of the central lobe has been estimated from pressure build-up data to be 200 md, while the permeability of lobes C_1 and C_3 is estimated to be somewhat lower due to the clay content and is assumed to be 70 md for this study. The C-sand, of deltaic origin, is approximately 30 miles long and 5 miles wide in area extent. Dissolved methane gas exists within the geopressured waters. It is for this reason that the C-sand has received considerable attention in the petroleum literature. To produce economically this resource of methane gas at present gas-water ratios of 30 SCF/STBW requires a production rate of 40,000 STBW/day over a 15-year life. The results of the sensitivity study indicate that the viscoelastic effects will increase the life of geopressured wells. Two simulations were made for a reservoir model of the C-sand interval in order to estimate viscoelastic compaction effects upon the life of the well.

The fluid properties of the geopressured waters are the same as those used in the sensitivity study and the reader is referred to Table 3. The compaction parameters have been estimated from cores taken from the C-sand interval [8]. These data indicate that the central lobe, C_2 , has a uniaxial elastic compaction coefficient, C_{me} , of $5 \times 10^{-7} \text{ psi}^{-1}$ and behaves like an elastic solid. In contrast lobes C_1 and C_3 show viscoelastic deformation. For these intervals C_{me} is estimated to be $2.2 \times 10^{-6} \text{ psi}^{-1}$ while C_{mv} is estimated as $1.5 \times 10^{-6} \text{ psi}^{-1}$. The 100% shale zones are also viscoelastic and are assigned the same compaction parameters as for lobes C_1 and C_3 . Large pressure gradients between the sands and shales will tend to enhance the shale-dewatering effect (explained in a prior publication [5]), and, thus also contribute to the lengthening of the life of the geopressured well. Table 4

Figure 6. The C-Sand Interval

IR = Induction Resistivity Curve

IC = Induction Conductivity Curve

SP = Spontaneous Potential Curve

Table 4
Formation Rock Properties

Rock Type	C_{me} (psi ⁻¹)	C_{mv} (psi ⁻¹)	τ (days)	K (md)	l_z (feet)
Shale	2.2×10^{-6}	1.5×10^{-6}	.51	10^{-4}	100
Dirty Sand	2.2×10^{-6}	1.5×10^{-6}	.51	70	40
Clean Sand	5.0×10^{-7}	0.0	0.0	200	60

summarizes the rock properties for the sand-shale sequence of Figure 6.

For this study a cylindrical coordinate system was used. A schematic representation of the model geometry is shown in Figure 7. For this study it was assumed that the reservoir is symmetric about the well and the shale sand sequence could be described in 20-foot intervals. Figure 7 shows the well perforated over the entire sand interval.

The well is assumed to have a drainage area of 16 square miles which corresponds to a drainage radius of 12,000 feet. The initial pressure was taken as 11,500 psi at a depth of 14,600 feet. Table 5 summarizes the reservoir parameters and conditions.

C-Sand Reservoir Performance. Previous studies of the C-sand [5,7] have investigated the effects of shale-dewatering and gas buildup around the wellbore when the reservoir is assumed to be elastic. The reader is referred to references [5,7] for the results of these studies. It is the intent of the following simulations to display solely the effect on recovery when the media are treated as viscoelastic rather than elastic.

Case 16, the base case simulates the C-sand performance when the shales and sands are treated like only elastic materials. Case 17 simulates C-sand behavior when the shales and dirty sands are represented by their viscoelastic properties. Figure 8 is a comparison plot of bottom hole pressure versus time for cases 16 and 17. It indicates that after 5 years of production viscoelastic reservoir production is considerably better than that of the elastic reservoir. Extrapolation of the curves to the point where the bottom hole pressure reaches 7000 psi gives the expected life of the well. For the elastic case the life of the well (at 40,000 STBW/day production rate) is about 7 years whereas for the viscoelastic case the life of the well approaches 10 years, a 40% improvement in the life of the well. In addition, after 5 years of production, the viscoelastic case indicates that water recoveries will be

Figure 7. Cross-section View of C-Sand Interval

Table 5
C-Sand Parameters and Conditions

Drainage Area	16 square miles
External Radius	12,000 feet
Well Radius	.292 feet
Reservoir Depth	14,600 feet
Reservoir Facies	60 feet clean sand 40 feet dirty sand 100 feet shale
Initial Pressure	11,500 psi
Reservoir Temperature	300°F
Salinity	96,000 ppm
Porosity	$\phi_{\text{snd}} = 18\%$ $\phi_{\text{shale}} = 30\%$
Permeability	$K_{\text{c. snd}} = 200 \text{ md}$ $K_{\text{d. snd}} = 70 \text{ md}$ $K_{\text{shale}} = 10^{-4} \text{ md}$
Uniaxial Compaction	$C_{\text{me c. snd}} = 5 \times 10^{-7} \text{ psi}^{-1}$
Coefficient	$C_{\text{me d. snd}} = 2.2 \times 10^{-6} \text{ psi}^{-1}$ $C_{\text{me shale}} = 2.2 \times 10^{-6} \text{ psi}^{-1}$
Viscous Compaction	$C_{\text{mv c. snd}} = 0.0$
Coefficient	$C_{\text{mv d. snd}} = 1.5 \times 10^{-6} \text{ psi}^{-1}$ $C_{\text{mv shale}} = 1.5 \times 10^{-6} \text{ psi}^{-1}$
Initial gas saturation	0.0%
Critical gas saturation	5.0%
Maximum Allowable Rate	40,000 BWPD
Minimum Bottom Hole Pressure	7,000 psi

Figure 8. C-Sand Reservoir Performance

1.98% of the initial in place resulting in a per well recovery of 1.83% of the initial gas in place. These compare to a 1.5% recovery for the purely elastic reservoir.

Figure 9 is a comparison plot of gas flow rate versus time for cases 16 and 17. It is recognized that the gas flow rate for the viscoelastic reservoir is greater than that for the elastic case. This results from the always higher pressure in the viscoelastic reservoir resulting in a relatively constant produced gas-water ratio of 30 SCF/STBW. This effect may be of importance because previous studies [7] have shown that free gas in the reservoir, particularly close to the wellbore, will tend to decrease the relative permeability to the liquid (water) phase for non-zero values of critical gas saturations, resulting in a lower productivity index.

CONCLUSIONS

The conclusions of this paper are restated.

- 1) Viscoelastic reservoirs can be characterized by their unit impulse response which yields values for the viscous time constant, τ , and the viscous uniaxial compaction coefficient, C_{mv} .
- 2) The complete response of a viscoelastic reservoir is given by the convolution of the stress history with the unit impulse response.
- 3) Viscoelastic reservoirs with time constants less than 1000 days can be viewed as elastic reservoirs with $C_{meq} = C_{me} + C_{mv}$.
- 4) Viscoelastic reservoirs with $C_{mv} \leq 0.2 C_{me}$ can be approximated by the elastic reservoir with $C_{meq} = C_{me}$.
- 5) The expected life of a single well in the Austin Bayou geopressured prospect in Brazoria County, Texas, may be increased by 40% by viscoelastic compaction effects.

Figure 9. C-Sand Reservoir Performance

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