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Research and Development Report:
INVESTIGATION OF THE RESISTIVE PHASE
IN HIGH POWER GAS SWITCHING

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1. INTRODUCTION

Certain pulsed power systems such as pulsed gas lasers require high pulse repetition rates with peak power of the pulses in the 10^8 to 10^{11} W range. Such high powers cannot be handled with the usual laboratory high voltage power supplies. On the other hand, systems such as the Marx generator capable of handling such high powers are plagued with many problems whenever a high pulse repetition rate is required. Many Marx generator designers consider a rate of 1Hz a high repetition rate, although the actual requirements in some applications are for repetition in the 1KHz to 100KHz range.

In what follows we propose a system which can handle high powers as well as high repetition rates.

II. STATEMENT OF OBJECTIVES

1. Theoretical study of the resistive phase in high pressure gas switching; the regime of interest is (10-50)KV from (1J, 10ns, 100KHz) to (100J, 10 μ s, 1KHz).
2. The resistive phase is to be examined as a function of applied field, gap spacing, inductance, gas type and pressure, and electrode material.
3. Integrate into the study a description of the initiating and quenching phases as regards system performance (e.g., the jitter problem).
4. Consider the cooling and electrode debris removal effects of the vortex gas flow on the operating characteristics of the system.

III. SHORT REVIEW OF THE LITERATURE

A capable review of the available literature up to 1975 related to high pressure gas switching can be found in the Früngel's book⁽¹⁾. To our knowledge, since the pioneering experimental work by J.C. Martin⁽²⁾ and the author⁽³⁾ of this report, the only relevant work addressing the problems of the resistive phase has been done at the University of Toronto by V.M. Ristic⁽⁴⁾ and Sorensen and Ristic⁽⁵⁾. The work done in Reference (4) has resulted in real hardware⁽⁶⁾. The work done in Reference (5) is an extension of Martin's⁽²⁾ work and experimental postulation of a new law for the resistance of the resistive phase in the spark discharge. In the following we discuss several aspects of the work done by Martin, O'Rourke, Ristic and Sorensen.

IV. DESCRIPTIONS OF ARC PHASES

When a high voltage pulse is applied to the spark gap as shown in Figure IV.1, several phenomena occur in succession. These phenomena are usually classified in the following way.

- (a) Statistical Phase, $\langle t_s \rangle$ statistical phase or lag time is governed to a large degree by the field emission properties of the cathode. The statistical lag time is measured from the point at which the pulse charge attains its maximum value (nominally 10ns) to the time at which the gap breaks down. To determine this value, statistical analysis requires between 30 to 50 samples at a given electrical field value.
- (b) Formative Phase, t_f is governed by the rate of ionization buildup and is describable by the processes taking place within the gas medium. For pressures required, the formative phase is small compared to the statistical phase.
- (c) Resistive Phase, t_r occurs after the formative phase and is characterized by the space-charge effects or the streamer mechanism, or both. Because of the increase in current, Joule heat is released. A larger current liberates a larger amount of heating, and the cumulative process sets in.
- (d) Quenching Phase, t_q or $\langle t_q \rangle$ is defined as time required for the gap to reach the non-conductive state from the conductive state. By conductive state we mean a single channel resistive discharge. In a properly designed gap the quenching phase is not a statistical variable. It depends on (i) the recombination of the electrons and ions (which is thought to be on the order of a millisecond in hydrogen); (ii) the ambipolar diffusion of the charge particles into electrodes (for hydrogen the time constant for ambipolar diffusion is of the order of 0.1 μ s for gap lengths of about 10 μ m); and (iii) the circuitry of the power supply. Before the quenching phase can begin a certain value of current must be reached, which is termed the minimal current necessary for existence of the arc⁽¹²⁾.

Figure IV.1 - Arc Phases: $\langle t_s \rangle$ Statistical Phase, t_f , Formative Phase, t_r , Resistive Phase and t_q , Quenching Phase

V. EXPERIMENTAL WORK RELATED TO RESISTIVE PHASE

We begin this study with a description of the most recent published experimental results of T. Sorensen and V. Ristic⁽⁵⁾ on the "Rise-Time and Time Dependent Spark Gap Resistance in Nitrogen and Helium". Figures V.1, 2 and 3 were supplied by Professor Ristic and provide a description of the μ -wave circuit technique used in their work along with the equivalent circuit used in the reduction and analysis of the experimental data. The spark gap is located between the center conductors of the charged coaxial transmission line with the cathode on the left and the anode on the right being grounded locally to the outer conductor. The center conductor can be repetitively charged with the charging currents flowing through a resistor and bringing the voltage across the spark gap up to the desired overvoltage level for gas breakdown. The electrodes have approximate Rogowski profiles so that the electric field in the gap volume is initially rather uniform prior to gas breakdown. Since these single pulse experiments were designed to measure the basic temporal characteristics of the resistive phase, it was not necessary to control the statistical and formative time lags by illuminating the cathode. During the resistive phase one obtains a fast rising "reflected" current pulse $I_R(t)$ propagating back along the charged transmission line. As indicated in Figure V.1, a properly designed pick-up loop (B_ϕ probe) is inserted into the transmission line near the spark gap; its output voltage pulse $V_o(t) = k_1 \frac{d}{dt} I_R(t)$ contains the lumped circuit basis for the "whole story" of the spark gap resistance as a function of time as will be shown in the following sections of the report as we review this work of Sorensen and Ristic.

Figure V.1 - A Setup Consisting of the Charging Resistor, Microwave Cavity with Low Loss Dielectric, a Pick-Up Loop and a Spark Gap used for Measuring the Properties of the Resistive Phase

Figure V.2 - The Equivalent Circuit of the System Shown
in Figure V.1

Figure V.3 - Experimental Setup used for Measuring the Resistance of the Resistive Phase Versus Time

Later on in the report some open issues will be addressed which relate to the "non-lumped circuit" aspects of the problem where one tries to understand the structure of the electromagnetic field during the breakdown of the gas in real time (statistical, formative and resistive phases looked at from a more microscopic electrodynamic viewpoint). This output voltage pulse $v_0(t)$ is to be delivered (perhaps after passing through a pulse forming network) to the final load impedance of interest in the L^2 program.

V.1 THE RESISTIVE RISE-TIME

J.C. Martin⁽²⁾ presented his observations of the resistive rise-time for a variety of gases in the following form

$$\tau_R(\text{nsec}) = \frac{35(\frac{\epsilon_0}{\epsilon_{\text{air}}})^{\frac{1}{2}}}{(\epsilon_0(\frac{V_0}{Z_0})^{4/3} \rho_0^{1/3}(\text{cm})} \quad (5.01)$$

where $\epsilon_0 = \frac{V_0}{E_0}$; V_0 is the measured breakdown voltage in MV; l_0 is the gap length in cm; ρ_0 is the ambient gas density in gm/cm³; ϵ_{air} is the STP density of air 1.29×10^{-3} gm/cm³; Z_0 is the characteristic impedance of the circuit driving the spark gap, in Ohms.

Some years ago the author⁽³⁾ noted the obvious fact that (5.01) is dimensionally incorrect and tried to correct the situation using a hydrodynamic model of the resistively heated spark channel. It was found that one could obtain Martin's $\epsilon_0^{-4/3} \rho_0^{-1/3}$ result in this way, but not the $\epsilon_0^{\frac{1}{2}}$ dependence; in addition, a weak but bothersome explicit dependence on ϵ_0 beyond that contained in ϵ_0 was obtained. We will look again at the problem using only the pure method of dimensional analysis because Martin found no explicit dependence on ϵ_0 , and as we will see later, neither did Sorensen and Ristic a decade later.

We begin by assuming we have recognized the "complete set" of dimensional variables in $(\tau_R, \epsilon_0, V_0, l_0, Z_0)$ and write (using e.s.u.):

$$\begin{aligned} \text{CONSTANT} \times \tau_R &= \frac{\epsilon_0^a \rho_0^b}{V_0^c Z_0^d} = \frac{(\text{cm})^{a-3b} (\text{gm})^b}{(\text{ga}^{\frac{1}{2}} \text{cm}^{\frac{1}{2}} \text{sec}^{-1})^c (\text{sec cm}^{-1})^d} \\ &= \text{sec}^i \end{aligned}$$

One then finds that

$$b = \frac{c}{2}; \quad c = a - 1; \quad d = a - 2$$

and therefore

$$\tau_R = \frac{k_1 \epsilon_0^{\frac{a-1}{2}} \omega_0^{\frac{a-1}{2}}}{V_0^{a-1} \omega_0^{a-2}}$$

where k_1 is a pure number. Now in both Martin's work and in that of Sorensen and Ristic as well as many other spark gap investigations, there is general agreement that $\tau_R = \omega_0^{-1/3}$ so that one can with some confidence now choose $a = 7/3$ and thereby obtain

$$\tau_R = \frac{k_1 \epsilon_0^{2/3} \omega_0^{2/3}}{\epsilon_0^{4/3} \omega_0^{1/3}}; \quad \epsilon_0 = \frac{V_0}{\epsilon_0} \quad (5.32)$$

where V_0 is the measured breakdown voltage level. In recent discussions between Martin and the author it was agreed that his original data reduction effort was not of sufficient accuracy to have distinguished between $\omega_0^{1/2}$ dependence and the present $\omega_0^{2/3}$ dependence. The explicit dependence of τ_R on ϵ_0 has, as noted above, not been observed in any experiment to date however, and seriously casts doubt on the above assumption that the correct "complete set" of dimensional variables includes ϵ_0 . Nevertheless, it is clear that we must continue to search for some fundamental physical length in the problem, which will be called x_0 in cm. For example, one might say that the important electric field strength in the problem is not the initial uniform field $E_0 = \frac{V_0}{\epsilon_0}$, but rather $E_c = \frac{V_0}{x_0}$, where x_0 is some effective

radius of curvature of the observable "whisker-tips" on the cathode surface.

The above result could then be written in the form

$$r_0 = \frac{k_2 x_0 \phi_0^{2/3}}{E_c^{4/3} z_0^{1/3}} = \frac{k_3 x_0 \phi_0^{2/3}}{E_c^{4/3} z_0^{1/3}} \quad (5.03)$$

where $k_3 x_0 = k_2 z_0$. In the latter form one can then begin to "see" the possibility that $k_3 x_0$ may in fact be some kind of intrinsic local length in the problem which is independent of or not observably dependent upon the spark gap geometry, the gas, the electrodes, etc. The method of dimensional analysis obviously cannot help in the resolution of such a "heavy" physics "possibility". The physics of " x_0 " seem to be partially contained in the well known theory of Fowler and Nordheim for the pure field emission of electrons from the surface of the cathode. Their theory is represented in the formula for electron emission

$$j \left(\frac{\text{Amps}}{\text{cm}^2} \right) = \alpha E_c^2 e^{-B/E_c} \quad (5.04)$$

where

$$\alpha = \frac{6.2 \times 10^{-6}}{z_F + z_M} \left(\frac{z_F}{z_M} \right)^{1/2} \frac{\text{Amps}}{(\text{Volt})^2}$$

$$B = 6.2 \times 10^7 z_M^{3/2}$$

and the Fermi energy z_F and the work function z_M are both in (e.v.) units; E_c is the actual electric field strength at the cathode surface in $\left(\frac{\text{Volts}}{\text{cm}} \right)$.

In practice one does not measure j or E_c directly, but rather proceeds to (1) assume a cathode emission area A_c ; (2) measure the total current I

flowing in a closed circuit; (3) assume that $jA_c = 1$; (4) use measured values of a and b for a given cathode material; and (5) solve (5.23) for E_c and relate this result to $E_0 = \frac{V_0}{\xi_0}$; typically, one finds $E_c \sim 600E_0$.

One then makes models of the shapes of the microscopic whiskers on the cathode surface in order to qualitatively understand the distribution of the charge density over such real materials where one expects a concentration of charge at the tips of the whiskers on the basis of potential theory.

In such a phenomenological approach, one is led to the idea of pure field enhancement at the cathode surface and one expresses this idea in the form $E_c = g_c \frac{V_0}{\xi_0}$ where g_c is called the field gain factor. There are a small number of geometric shapes for which the classical potential theory yields exact descriptions of the equipotential lines and their corresponding stream-lines (see the theory of lightning arrestors) in the E_c field of interest here. The result of such investigations is contained in the statement $g_c = \xi_0 \frac{h_0}{r_0}$ where h_0 is an effective whisker height above the ideal cathode equipotential plane and r_0 an effective whisker thinness or diameter and ξ_0 is a dimensionless geometric factor of order unity. All such arguments are equivalent to writing $E_c = \frac{V_0}{\xi_0 r_c}$ as discussed above and from this viewpoint one imagines the ensemble of real whiskers to be replaced by an ensemble of tiny spheres of effective radius $\langle r_c \rangle$ connected circuit-wise to the underlying ideal plane of cathode material at potential $-V_0$. In this easy way, one can picture the distribution of circuit charge over a real cathode surface in a rather qualitative fashion. In this way we begin to see that our search for the fundamental length x_0 in this resistive rise-time problem leads us directly to the deeper problem of the field gain factor $g_c = \frac{E_c}{E_0}$ and the surface physics of the electrode materials.

In practice, if one uses the Fowler-Nordheim formula one must proceed with some caution since one cannot measure j or E_c and the assumption that A_c is known is especially hard to accept during the statistical time delay when one is creating the small number of initiating electrons to begin the gas breakdown process. As an example, one can take $\phi_H = 4\text{eV}$, $\phi_F = 10\text{eV}$ and assume the value $E_c = 3.10^7 \frac{\text{Volts}}{\text{cm}}$ to obtain $j = 8.2 \frac{\text{amps}}{\text{cm}^2}$ and then note that if $E_c = 2 \times 10^7 \frac{\text{Volts}}{\text{cm}}$, one obtains $j = 4.3 \times 10^{-4} \frac{\text{amps}}{\text{cm}^2}$. In this easy way one is quickly convinced that it is not a simple matter in practice to use this approach based on (5.04) to design the kinds of spark gaps of interest here. There is vast experimental literature on this subject which demonstrates that there is indeed an absolute statistical time delay involved in the problem of reaching a critical enhanced field strength E_c at the cathode surface after the "sudden turning-on" of V_0 across the gap of length l_0 . For the high rep-rate command triggered and jitter controlled spark gaps of interest here it is quite clear that a strong effort must be made with both theory and new experiments to understand what is happening everywhere in the circuit during this statistical time delay. If possible, one wants to design the switching system in such a way that both the statistical and formative time delay periods are pulse "over-driven" and thereby shorter than all other characteristic times in the problem. Said another way, it is essential to establish the resistive time delay period as quickly as possible with the highly over-volting command trigger pulses. One of the goals of this study is to discover how this is to be accomplished in practice in the high rep-rate regime.

Continuing with the more recent data of Sorensen and Ristic⁽⁵⁾ we first note their final expressions for τ_R in the case of N_2 and He ; namely, for N_2 they find that

$$\tau_R(\text{nsec}) = \frac{44 \rho_{At}^{1/2}}{E_0 \left(\frac{10^4 \text{ V}}{\text{cm}} \right) Z_0^{1/3} (\Omega)} \quad (5.05)$$

and for He they find that

$$\tau_R(\text{nsec}) = \frac{6.4 \rho_A^{1/3}}{E_0 \left(\frac{10^4 \text{ V}}{\text{cm}} \right) Z_0^{1/3} (\Omega)} \quad (5.06)$$

which are to be compared to J.C. Martin's result for many gases

$$\tau_R(\text{nsec}) = \frac{88 \left(\frac{\rho_0}{\rho_{STP}} \right)^{1/2}}{E_0^{4/3} \left(\frac{10^4 \text{ V}}{\text{cm}} \right) Z_0^{1/3} (\Omega)} \quad (5.07)$$

Martin's work was done for longer rise times and lower E_0 fields than in the work of Sorensen and Ristic. It is readily apparent that all three of these results are dimensionally incorrect if one accepts them at face value (i.e., if the multiplicative numerical coefficients are taken to be dimensionless pure numbers). Also, if one assigned some appropriate dimensions to say the factor (44) in $\tau_R(N_2)$ it is not self evident why the factor (6.4) in $\tau_R(He)$ should have different dimensions as it clearly must have according to the formula. We have seen that pure dimensional analysis reasoning yields the result

$$\tau_R = \frac{k_2 x_0 \rho_0^{2/3}}{E_0^{4/3} Z_0^{1/3}} \quad (5.08)$$

and every effort should be made to fit all the experimental data available to this functional form in order to bring some order into our understanding of

the problem. In several recent conversations with Ristic, this author learned that there is no current explanation for these basic problems with dimensions. For N_2 the following parameter ranges were reported

$$2.3 \leq p_{At} \leq 6.39 ; \quad 5 \times 10^{-3} \leq z_0(\text{cm}) \leq 5 \times 10^{-2} ;$$

$$7.6 \leq z_0(n) \leq 29.4 ; \quad 10 \leq E \left(\frac{10^4 \text{ V}}{\text{cm}} \right) \leq 75 ; \quad (5.09)$$

$$0.1 \leq \frac{R_0(t)}{Z_0} \leq 100 .$$

At a value of $E_0 = 8 \times \frac{10^4 \text{ V}}{\text{cm}}$ the numerical result of Martin for τ_R is about the same as that of Sorensen and Ristic for $\tau_R(N_2)$. The value of $\tau_R(N_2)$ in these experiments ranged from 0.66 nsec to 2.3 nsec. Again, no explicit dependence of τ_R or E_0 other than that in E_0 was observed. It is also to be noted that the range $10^5 \frac{\text{V}}{\text{cm}} \leq E_0 \leq 750 \frac{\text{kV}}{\text{cm}}$ is rather far below the field emission threshold value of $E_c = 3 \times 10^7 \frac{\text{V}}{\text{cm}}$ and since the gap was not illuminated (say by an external U-V light source), one must eventually investigate the surface physics of the cathodes to understand how the breakdown processes were actually initiated (again, the surface physics secret lies hidden in the mysterious enhancement or pure field gain factor g_c).

One of the more important contributions in this Sorensen/Ristic paper is the demonstration that in the time domain the dimensionless resistance $\frac{R_0(t)}{Z_0}$ of the spark gap very accurately follows a " t^{-3} law" over three decades rather than J.C. Martin's presumed e^{-t/τ_R} form if one ignores the time-dependent inductance of the spark channel during the times of interest during the resistive phase. After a careful analysis of their circuit and data

these authors concluded that the actual spark channel resistance for N_2 is given by

$$\frac{R_0(t)}{Z_0} = 2 \times 10^4 \left[\frac{p_{At}^{1/2}}{E(10^4 \text{ V/cm}) Z_0^{1/3} (\Omega) t(\text{ns})} \right]^3 \quad (5.10)$$

over the range $0.1 \leq \frac{R}{Z_0} \leq 100$. Introducing their above result for τ_R , we obtain

$$R_0(t) = Z_0 2 \times 10^4 \left(\frac{\tau_R}{44 t} \right)^3 = 0.23 \left(\frac{\tau_R}{t} \right)^3 Z_0 \quad (5.11)$$

for the real time dependence of the spark channel resistance.

In a similar way they report for He that (to $\pm 6\%$)

$$\frac{R_0(t)}{Z_0} = 100 \left[\frac{p_{At}^{1/3}}{E^{1/2} Z_0^{1/3} t} \right]^3 = 100 \left(\frac{\tau_R}{6.4 t} \right)^3 \quad (5.12)$$

so that

$$R_0(t) = 0.38 \left[\frac{\tau_R}{t} \right]^3 Z_0 ; 7.76 \leq p_{At} \leq 21.32; 22 \leq E(10^4 \text{ V/cm}) \leq 60 . \quad (5.13)$$

Such a t^{-3} law receives partial lumped circuit from our above dimensional analysis because one could just as well have written

$$t = k_2 \frac{x_0 \rho_0^{2/3}}{\epsilon_0^{4/3} R_0^{1/3}(t)}$$

where any time dependence in our fundamental length x_0 , in the density ρ_0 of

the plasma spark channel or in the driving trigger voltage V_0 are assumed to be "secular" compared to the rapidly changing resistance of the spark channel itself. In this manner one can write the expression

$$R_0(t) = \left[\frac{k_2 x_0 p_0^{2/3}}{Z_0^{1/3} E_0^{4/3} t} \right]^3 Z_0 = k_3 \left(\frac{\tau_R}{t} \right)^3 Z_0 \quad (5.14)$$

where k_3 would be obtained from the experimental data. In Figure V.4 we reproduce the shape factor $\frac{R_0(t)}{Z_0}$ for N_2 to illustrate the remarkable three decade fit to this " t^{-3} law" which Sorensen and Ristic have discovered.

In the next section we will review the lumped circuit theory used by these authors to see if such a " t^{-3} law" has other consequences which can be observed in such experiments.

Figure V.4 - Experimental Postulation of " $t^{-3/2}$ law"

VI. THE TIME DEPENDENT SPARK GAP RESISTANCE

In Figure V.2 we present the proposed equivalent circuit underlying the Sorensen-Ristic experiment. The equations for the currents are seen to be the following

$$I_1(t) = I_2(t) + I_3(t) = I_I(t) - I_R(t) = \frac{V_I(t) - V_R(t)}{Z_0}$$

and

$$I_2(t) = C_0 \frac{d}{dt} [V_I(t) + V_R(t)]$$

with

$$I_3(t)R_0(t) + \frac{d}{dt} [L_0(t)I_3(t)] = V_0 + V_I(t) + V_R(t)$$

where V_0 is the static voltage on the transmission line; $V_I(t)$ and $V_R(t)$ are the incident and reflected voltage waves supplied by the charging circuit to the spark gap region which is treated as a lumped element characterized by C_0 , $L_0(t)$ and $R_0(t)$. If the inductive voltage drop across the spark channel is neglected, one immediately obtains

$$[V_0 + V_I(t) + V_R(t)] + C_0 R_0(t) \frac{d}{dt} [V_I(t) + V_R(t)] = \frac{R_0(t)}{Z_0} [V_I(t) - V_R(t)] \quad (6.01)$$

which is the operative circuit equation used by these authors in their investigation of $R_0(t)$, the so-called spark gap resistance. In this experiment one can choose the initial time $t = 0$ to correspond with the time at which the voltage across the capacitor reaches the measured breakdown strength of the gas. One then can take $V_0 + V_I(0)$ to be this measured breakdown strength which is still labeled V_0 in their paper. Also for $t > 0$ when one is trying to deduce $R_0(t)$ it is shown that $\frac{d}{dt} V_I(t)$ is small compared to $\frac{d}{dt} V_R(t)$ and in addition, the prescribed slowly varying $V_I(t)$ is small compared to $V_R(t)$. With these approximations, which are quite accurate for this experiment, the authors then obtain the expression

$$-\frac{R_0(t)}{Z_0} = \frac{[V_0 + V_R(t)]}{V_R(t) + L_0 \frac{d}{dt} V_R(t)} \quad (6.02)$$

It is interesting to note that in such a derivation one could have written the following

$$I_3(t)R_0(t) + L_0(t) \frac{d}{dt} I_3(t) + I_3(t) \frac{d}{dt} L_0(t) = V_0 + V_I(t) + V_R(t) .$$

Then one notes that even if the $L_0(t) \frac{d}{dt} I_3(t)$ can be neglected in this specific experiment, one is still left with the result

$$I_3(t) [R_0(t) + \frac{d}{dt} L_0(t)] = V_0 + V_I(t) + V_R(t) .$$

One can of course combine $R_0(t) + \frac{d}{dt} L_0(t)$ into the symbol $R_0(t)$ used by these authors but it is clear that one has no a-priori reason to assume that one is dealing with a purely resistive load impedance. It may well turn out that we

are seeing a purely inductive phenomena during the breakdown of the spark gap channel and that we should use only $\frac{d}{dt} L_0(t)$ to describe the "resistive phase". Then again, we could be dealing with a mixture of purely resistive and purely inductive phenomena where both $R_0(t)$ and $\frac{d}{dt} L_0(t)$ have physical meaning. One of the future goals of studies of the kind being reported here will certainly be to resolve this matter once and for all; there is a deep connection here to the well known problem of Landau-Damping in plasma physics. There is perhaps an even deeper connection to the correct understanding of the role of time and time-delays in such problems dealing with pulses in the electromagnetic field associated with closed circuits.

Postponing such questions, we proceed to introduce the following dimensionless variables

$$\frac{V_0(t)}{Z_0} = \tilde{V}(t), \quad V_R(t) = V_0 f(t), \quad t = C_0 Z_0 \tau \quad (6.03)$$

to obtain

$$-\tilde{V}(\tau) = \frac{1 + f(\tau)}{f(\tau) + \frac{df(\tau)}{d\tau}} \quad (6.04)$$

Then the output voltage of the pick-up loop is given by

$$V_0(t) = k_1 \frac{d}{dt} I_R(t) \quad (6.05)$$

where k_1 is a dimensional coupling coefficient associated with the area and number of turns in the pick-up loop (a mutual induction coefficient obtained by calibration). Since $V_0(t) = Z_0 I_R(t)$ at the location of the loop we write

$$v_0(z) = \frac{k_1}{z_0} \frac{d}{dt} v_2(z) \quad (6.65)$$

and

$$v_0(\tau) = \frac{k_1}{c_0 z_0} \frac{d v_2(\tau)}{d \tau} = E \frac{d f(\tau)}{d \tau} \quad (6.67)$$

As in Sorensen and Ristic's paper, one chooses a time $t = 0$ to correspond to the beginning of the breakdown of the gas so that V_0 is identified with the breakdown voltage strength of the gap which is measured with an electrostatic probe technique. One also chooses a maximum time of interest, t_0 after which the output voltage $v_0(t)$ for $t \geq t_0$ is taken to be zero to some experimental precision (this assumes the charged transmission line has sufficient length to record the full $v_0(t)$ pulse shape without reflections being present to complicate the analysis). One can then introduce the fluxlike expression

$$\bar{E} f(\tau) = \int_0^\tau d\tau' v_0(\tau') \quad (6.68)$$

and obtain

$$\bar{E} = -v_0(\tau) \bar{E}(\tau) - \left(1 + \bar{E}(\tau)\right) \int_0^\tau d\tau' v_0(\tau') \quad (6.69)$$

Since $\bar{E}$ is a constant, one can take $\tau = t_0$ where $v_0(t_0) = 0$ and obtain

$$\bar{E} = - \left(1 + \bar{E}(t_0)\right) \int_0^{t_0} d\tau' v_0(\tau') \quad (6.70)$$

Then one has the result

$$\bar{R}(\tau) = \frac{\left[1 + \bar{R}(V_0)\right] \int_0^{\tau_0} dt' v_0(t') - \int_0^{\tau} dt' v_0(t')}{v_0(\tau) + \int_0^{\tau} dt' v_0(t')} \quad (5.11)$$

which yields $R(t)$ in terms of the measured pulse shape $v_0(t)$. In the Sorensen/Ristic experiment it was possible to let the reflected voltage wave $V_R(t)$ reflect at the charging resistor and return back to the pick-up loop to be recorded a second time. In this way it was demonstrated that $R(t_0) \approx 0.02 Z_0$ of $\bar{R}(t_0) \approx 1$ and it is this case which we proceed to examine, wherein one has

$$\bar{R}(\tau) = \frac{\int_0^{\tau_0} dt' v_0(t') - \int_0^{\tau} dt' v_0(t')}{v_0(\tau) + \int_0^{\tau} dt' v_0(t')} \quad (5.12)$$

where

$$\int_0^{\tau_0} dt' v_0(t') = -\bar{R}$$

which states that the total area under the output voltage pulse is proportional to V_0 , the breakdown voltage which is measured with an electrostatic probe technique. If one has a fast (broadband) integrating circuit, one can record $\int_0^{\tau} dt' v_0(t')$ as well as $v_0(\tau)$ and in this way deduce $\bar{R}(\tau)$ directly from the experimental data. With an on-line computer one could display $\bar{R}(\tau)$ in real time; to this author's knowledge, such real-time data processing has not been attempted in this spark gap resistive rise-time problem.

Sorensen and Ristic did of course perform "by hand" the data reduction and analysis of this same integral equation for $V_0(\tau)$ and deduced the " τ^{-3} " law for $R(t)$ which is under discussion here.

To examine the effect of the " τ^{-3} " law on the pulse shape $V_0(\tau)$, we can introduce the fluxlike quantity

$$z(\tau) = \int_0^\tau d\tau' V_0(\tau') \quad (5.13)$$

which can be measured with an integrating circuit as noted above. We can then introduce the normalized flux quantity

$$v(\tau) = \frac{z(\tau)}{z(\tau_0)} \quad (5.14)$$

to obtain

$$R(\tau) = \frac{1 + R(\tau_0) - v(\tau)}{v(\tau) + \frac{d}{d\tau} v(\tau)} \quad (5.15)$$

where $v(\tau_0) = 1$ and $V(\tau_0) = \left(\frac{dz}{d\tau}\right)_{\tau_0} = 0$ to some experimental level of precision.

We can also hide the measured constant $[1 + R(\tau_0)]$ in the normalization of $v(\tau)$ by defining

$$\bar{v}(\tau) = \frac{1}{[1 + R(\tau_0)]} \frac{z(\tau)}{z(\tau_0)} \quad (5.16)$$

so that

$$R(\tau) = \frac{1 - \bar{v}(\tau)}{\bar{v}(\tau) + \frac{d}{d\tau} \bar{v}(\tau)} \quad (5.17)$$

is our basic equation with $\bar{\phi}(\tau_0) = [1 + \bar{K}(\tau_0)]^{-1}$. Now with regard to the pulse shape of $V(\tau)$ in relation to the " τ^{-3} " law, we can assume that

$$\bar{K}(\tau) = \frac{K(\tau)}{\tau_0} = \frac{\alpha_0}{\tau} \quad (6.18)$$

where α_0 is to be determined from the data. We then obtain the equation

$$\frac{d\bar{\phi}}{d\tau} + \left(1 + \frac{\alpha_0}{\tau}\right) \bar{\phi} = \frac{\tau^3}{\alpha_0} \quad (6.19)$$

whose solution is

$$\bar{\phi}(\tau) = e^{-\left(\tau + \frac{\tau_0^4}{4\alpha_0}\right)} \int_0^\tau d\tau' \frac{\tau'^3}{\alpha_0} e^{+\left(\tau' + \frac{\tau'^4}{4\alpha_0}\right)} \quad (6.20)$$

where, in the vicinity of $\tau = 0$, $\bar{\phi}(\tau) = \frac{\tau^4}{4\alpha_0}$. In order to carry out such integrations one must know the value of α_0 which can be obtained from the transcendental equation

$$\begin{aligned} \bar{\phi}(\tau_0) &= \frac{1}{1 + \bar{K}(\tau_0)} = \frac{1}{1 + \frac{\alpha_0}{\tau_0}} \\ &= e^{-\left[\tau_0 + \frac{\tau_0^4}{4\alpha_0}\right]} \int_0^{\tau_0} d\tau' \frac{\tau'^3}{\alpha_0} e^{+\left(\tau' + \frac{\tau'^4}{4\alpha_0}\right)} \end{aligned} \quad (6.21)$$

by an appropriate computer program since τ_0 is measured. The voltage profile would then be given by

$$V_0(\tau) = \frac{\tau^3}{\alpha_0} + \left(1 + \frac{\tau_0^3}{\alpha_0}\right) \bar{\phi}(\tau) \quad (6.22)$$

and its peak value would occur at the time τ_p where $\frac{d}{dt} v_0(\tau) = 0$; i.e., one could also solve the equation

$$\frac{3\pi^2}{2} (1 - \bar{v}(\tau_p)) = (1 + \frac{\tau_p^3}{z_0^3}) \left\{ \frac{2}{z_0} - (1 + \frac{\tau_p^3}{z_0^3}) \bar{v}(\tau_p) \right\} \quad (6.23)$$

for z_0 in terms of the measured value of τ_p . If one can measure the initial slope of $v_0(\tau)$ at small values of τ one would obtain z_0 immediately or a first approximation to its value. Once z_0 is established, one can proceed to calculate the rise-time and width parameters of the pulse whose actual shape is illustrated in Figure VI.1 (kindly provided by Professor Ristic). Since the electrical parameters C_0 , V_0 , Z_0 and the calibration coefficient k are all contained in our dimensionless time variable τ , one could examine the experimental data for the case that only the ambient density n_0 in the spark gap is allowed to vary. One may in this way be able to show in the future that n_0 varies as $z_0^{2/3}$ as predicted earlier.

It is clear from the structure of the first order differential equation for $\bar{v}(\tau)$ in terms of $\bar{n}(\tau)$ that other functional choices of $\bar{n}(\tau)$ can be tried in the future (e.g., $\bar{n}(\tau) = \frac{z_0}{\tau^n}$ where one would try to show that $n = 3$ gives the best fit to the data once z_0 is established with sufficient precision). All such exercises clearly require appropriate programs to process the experimental data.

Sorensen and Ristic also analyzed the "width" of their output voltage waveforms and report that

$$\tau_M (\text{nsec}) = 53 \frac{P_{Ar}^{1/2}}{E_0 \left(\frac{10^4 \text{ V}}{\text{cm}} \right) z_0^{1/3} (\text{A})} \quad (\text{for } H_2) \quad (6.26)$$

Figure VI.1 - The Actual Pulse Shape Coming out from the Spark when a Single Channel Resistive Phase is Present

and

$$\tau_R \text{ (nsec)} = 7.7 \frac{p_{At}^{1/3}}{\epsilon^{1/2} \left(\frac{10^4 \text{ V}}{\text{cm}} \right) Z_0^{1/3} \text{ (}\Omega\text{)}} \quad (\text{for He}) \quad (6.25)$$

where the width was taken as the time required for $V(t)$ to rise and then fall a factor $1/7$ of its peak value; the pulses are clearly not symmetric functions of time.

Again, such expressions are dimensionally incorrect if one is to take the numerical coefficients as pure numbers as is being suggested. The measured values of τ_R range from 1 to 2 nsec for the pressure range $7.76 \leq p_{At} \leq 21.32$ and the pulse shape was the same for He and N_2 , which again is most interesting and worthy of additional study later on. These authors also report that excessive jitter was found in the case of He at pressures below 4.35 Atmospheres which is most tantalizing and will require further study.

It should be remembered that the inductance of the spark gap has been ignored in the above discussion and it would appear that by first calculating τ_R in the above manner one will then be able to study the role of this spark gap inductance in the problem. For example, one would expect to see absolute time delays in the problem in the sense that the output voltage will peak later in time than one would calculated in the above manner. The pulse shape $v_0(\tau)$ would also differ from that given above if this inductance is significant during the resistive phase. Since the Sorensen/Ristic data fits the " t^{-3} law" so extremely well ($\pm 8\%$) over some three decades in $\frac{R(t)}{Z_0}$ one can expect that such inductance effects will be rather small for small gap lengths; one must of course, eventually know the quality specifications expected for the output waveform $v_0(t)$ in the L^3 Program in order to justify any further effort at

this time in the direction of this inductance problem. The above discussion of the lumped circuit basis of spark gap switching should be adequate to begin the task of designing and testing a prototype switching system and identifying some of the important practical problems which can be resolved in the laboratory in a simple extension of the work of Sorensen and Ristic to the high rep-rate reg. A number of other basic physics and practical questions about the temporal, spatial and electrodynamic structure of the spark channel (single and multiple) itself will be addressed later in the report after the more practical hardware discussion has been presented.

VII. SOME ENGINEERING CONSIDERATIONS RELATED TO THE REQUIRED SPECIFICATIONS

In what follows we consider a system capable, in principle, to meet the required specifications. The system consists of an electronic pulsed power supply, radial type cavity coupled by a loop to a transmission line, and a radial capacitor, C , as shown in Figure VII.1. During each period of time (which is inversely proportional to pulse repetition rate) the energy from the power supply, is stored in the capacitor C in much the way that the spark gap in the cavity is overvolted. When the gap ignites, the capacitor C will resonate with the cavity inductance, spark-gap inductance and the inductance of the charging capacitor C . At lower frequencies (less than 200MHz), the total inductance is approximately equal to the cavity inductance.

The total losses of the resonant circuit are given by

$$R = R_w + R_0 \phi^2 + R_s, \quad (7.1)$$

where R_0 ($\approx 50\Omega$) is the impedance of the transmission line, R_w are the wall losses and R_s is the spark-gap resistance. At lower frequencies R_w can be neglected, and R_s is typically on the order of 1Ω . The coupling loop is represented by an ideal transformer of transformer ratio ϕ .

The cavity inductance can be evaluated by calculating the admittance, y , seen at the inner radius, R_1 , of a radial transmission line shorted at the outer radius, R_2 . This results in

$$y = \frac{j 2\pi R_1}{h \sqrt{\frac{\mu_0}{\epsilon_0}}} \left(\frac{Y_0(kR_2)J_1(kR_1) - Y_1(kR_1)J_0(kR_2)}{Y_0(kR_1)J_0(kR_2) - Y_0(kR_2)J_0(kR_1)} \right), \quad (7.2)$$

Figure VII.1 - Radial Cavity with Power Supply

Figure VII.2 - Simplified Equivalent Circuit of Radial Cavity Shown in Figure VII.1. L is the Cavity Inductance, C is the Radial Capacitor Capacitance, R_g is the Internal Resistance of the Power Supply, R_S is the Spark Gap Resistance, and ϕ is the Ideal Transformer Ratio of the Coupling Loop

where k is the wavenumber equal to $2\pi/\lambda = \omega/v$. Since the dimensions of the cavity are small compared to resonance wavelength, we have $kR_1 \ll 1$ and $kR_2 \ll 1$, and equation (7.2) can be written as

$$Y \approx \frac{-j^{2\pi v}}{\omega h \sqrt{\frac{\mu_0}{\epsilon_0}} \ln\left(\frac{R_2}{R_1}\right)} = \frac{-j}{\omega L} \quad (7.3)$$

Thus, we have

$$L \approx \frac{h \sqrt{\frac{\mu_0}{\epsilon_0}} \ln\left(\frac{R_2}{R_1}\right)}{2\pi v} \quad (7.4)$$

where $v = 1/\sqrt{\epsilon_0 \mu_0}$. The resonance will occur where

$$\frac{-j}{\omega_0 L} + j\omega_0 C = 0, \quad (7.5)$$

where ω_0 is the resonant frequency.

Depending on the cavity loading (i.e., the adjustment of the coupled loop transformer ratio, ϕ) different kinds of waveforms can be obtained from the cavity. This is shown in Figure VII.3.

In what follows we perform a very rough calculation in order to estimate the circuit elements required to meet the specifications. We assume that the capacitance C is charged to the full voltage available from the power supply, its energy being $CV^2/2$. For our purposes the waveform to be used is the one shown in Figure VII.3a. The peak energy in half cycle will be given by

$$E_{\text{peak}} = \frac{CV^2}{2} \left(\frac{T/2}{L/R_1} \right), \quad (7.6)$$

Figure VII.3 - Possible Outputs from the Cavity into the Transmission Line of Impedance R_0

where $R_1 = R_0 \rho^2$. The maximum obtainable energy will then be when $U/2 = \pi \sqrt{LC} = U/2 \rho^2$, i.e., all the energy stored in the capacitor will be delivered to the transmission line.

VII.1 THE 1000 OSCILLATOR

For $C U^2/2 = 10$ and $U = 100V$ the required capacitance is $C = 20nF$, and for $U = 500V$ the required capacitance is $0.4nF$. The frequency of the oscillation of the cavity is related to the pulse duration of $10ns = U/2$. Thus, $f_0 = 1/T = 50MHz$, which results in the inductance of the cavity L equal to $0.5nH$ and $25nH$ for voltages of $100V$ and $500V$ respectively. The actual dimensions of the cavities can be computed from equation (7.4). A requirement is imposed on resistance R_0 of the power supply, i.e., the time constant $\tau = CR_0$ associated with the charging of the capacitance must be smaller than the period of the repetition rate required, thus $\tau < 1/100MHz = 10ns$. For $C = 20nF$, $R_0 < 500\Omega$. The minimum peak current required from the power supply is then $I = 100V/500\Omega = 20A$. For $C = 0.4nF$ we have $R_0 < 25k\Omega$ and $I > 2A$.

VII.2 THE 100 OSCILLATOR

There $C U^2/2 = 1000$. For $U = 100V$ we have $C = 2nF$ and for $U = 500V$, $C = 40nF$. The oscillation frequency of the cavity is now $f_0 = 1/20ns = 50MHz$. The cavity inductances are now $20nH$ and $250nH$ for voltages of $100V$ and $500V$ respectively. The constraints imposed by the repetition rate on the resistance of the power supply are now $R_0 < 500\Omega$ for $U = 100V$ and the minimum peak current $I > 20A$. For $U = 500V$, $R_0 < 25k\Omega$ and the minimum peak current $I > 2A$. The minimum average current is found as $I_{av} = I_{peak}$ (pulse duration times pulse repetition rate). All these requirements can be met with a single electronic power supply.

VII.3 PRESENTIZATION OF THE CAPACITOR AND QUENCHING TIME

Under DC breakdown conditions the performance of the spark gap switch improves with increasing gas pressure until a certain breakdown electric field intensity is reached. The use of higher electric fields than this by increasing the gas pressure further does not improve the switch performance. The level of electric field intensity in the gas above which no further improvement is obtained seems to be approximately the same in the three gases considered (Ar, He, IF_6). Thus nitrogen, with a higher breakdown strength than helium, reaches this limiting condition at a lower pressure than helium, and IF_6 with the lightest breakdown strength reaches it at a still lower pressure.

The advantage of using one of the three gases listed in preference to another from a short time performance point of view is marginal. However, the choice of gas will depend on such factors as corrosive effects of the gas on the electrodes, quantity of debris, quenching time of the spark for high pulse repetition rate applications, and also gas pressure required to give good performance. For instance, if the quenching time is to be considered alone, hydrogen seems to be a good choice since pulse repetition rates of 1000 for this gas were reported⁽¹⁷⁾. In our opinion however, the requirement of a gas pressure for the good performance is to be equated with the single channel resistive phase spark gaps, since this type of discharge yields a low jitter and repeatedly good voltage waveforms⁽¹³⁾. The influence of the pressure, gas type, applied field and gap spacings for this type of the discharge were considered in Section V.

VIII. THE COOLING AND ELECTRODE GEOMETRIC REMOVAL EFFECTS

Plasma and electrode interactions in short gap discharges in air were studied by von Engel and Robson⁽⁸⁾, Fignard⁽⁹⁾, and more recently by Augus, et al.⁽¹⁰⁾. These studies suggest that a high pressure gas and vaporous sheath exist during the arc discharge in front of the cathode. The sheath is compressed by positive ions drifting in the field of cathode fall.

When the arc strikes in the cathode surface, the high current density causes the electrode surface to melt. The high pressure in this region, due to positive ion bombardment, forces the molten metal out of the spot region. A crater is thus formed with 1% rim made up of some of the molten metal. When the arc is extinguished, the ion pressure is removed, and a wave, resulting from the internal pressure of the compressed sheath, moves across the gap pushing the cathode vapor with it. For a particular arc and crater size it seems reasonable to assume that the amount of cathode material thus ejected into the gap remains constant for each crater, and is much less than the amount of material displaced. It is to be noted that the role of the anode in short duration arcs is only that of an electron collector, thus the contribution (factor $\approx 10^{-2}$) of the anode to debris generation is nil. The size of the cathode craters are on the order of 1mm for an arc current density of 10^7 A/cm² and arc duration of 20ns. The cathode material ejected into the gap space is measured with the so called erosion rate⁽¹¹⁾. Some typical data are provided in the following table.

<u>Material</u>	<u>Gas</u>	<u>Pressure (Torr)</u>	<u>Erosion Rate (miles/hr)</u>	<u>Test Duration (hours)</u>
Cu	air	760	2500	6
10W3	air	760	700	10
10W3	air	760	600	10
10W3	argon	760	150	20
10W3	helium	475	30	100
10W3	helium	10	---	100

Elkonite is a trade name for a copper-tungsten composite manufactured by P.R. Mallory, Inc. The 10W3 Elkonite consists of 57% W, 43% Cu by volume and 10W3 Elkonite consists of 45% W and 55% Cu by volume.

The noble gases reduce the erosion rate by eliminating surface chemical reaction (primarily oxidation). Because of its lower atomic weight, the sputtering rate for helium is lower than for argon under these test conditions and erosion is further reduced in this gas. Furthermore, experiments show that 10W3 Elkonite provides reproducible data when spark phenomena are studied.

The maintenance of a high degree of gas purity in the repetitive spark gap applications is important because of (i) the maintenance of the same breakdown strength of the gas; and (ii) the maintenance of the constant quenching phase. It is therefore important to purge the gas in front of the cathode as soon as possible after each spark occurrence. For high repetition rates this can be done only by continuous purging, and the vortex gas flow appears to be a good approach for continuous purging. In this connection it is proposed to introduce the gas by a cylindrical tube surrounding the cathode with the gas velocity vector pointing out of the cathode. The vortex gas flow could be generated by a segmented section port around the wall of the cavity. Through a continuously purging system the gas should be filtered, cooled and then again

reintroduced at the cathode. The continuously purging system should be dimensioned according to pressure required, cathode erosion rate and the gas temperature required. The segmentation of the cavity wall would only slightly perturb the cavity current, thus not interfering with the operation of the cavity.

It is anticipated that in the long run there will be an accumulation of debris on the cavity walls. This debris will cause an increase in wall losses due to (i) reduced smoothness of the cavity walls; and (ii) reduced conductivity of the cavity walls. The latter will largely depend on the material used for the cathode. These losses will not be serious since in the regime required, the cavity is operating at rather low frequencies (less than 50MHz). If, however, these losses become substantial they can always be compensated by the readjustment of the cavity coupling (transformer ratio ϕ). For a stable operation the cooling of the anode will be required in all cases. However, this is not critical since the average powers involved are at most 2kW. The cooling can be done with standard methods already developed for high power microwave tubes.

IX. CONCLUSIONS AND RECOMMENDATIONS

This report has underlined several important issues related to high pressure gas switching. First, a state-of-the-art of the current resistive phase research has been given. Then enough engineering information has been supplied so that a meaningful laboratory work can be initiated. It would appear that the required specifications could be met with the present technology of the spark gaps. However, more theoretical and experimental work has to be done before the actual construction of the system is undertaken. On the theoretical side, the following topics should be considered:

- (a) Computer simulation of the resistive phase in the single channel and in the multi channel regime.
- (b) Engineering design of the system with a special emphasis on the load matching (a load of the order of 0.01 is assumed) to the cavity. Consideration of different types of cavities.
- (c) Engineering design of the system assuming that the cavity and the connecting transmission line are filled with water.
- (d) Computation of the heat transfer characteristics of the coupling loop and the transmission line.

On the experimental side, it is suggested that an electronic power supply be produced and that the experimental setup suggested in this report be tested. Further experimental investigation is also necessary along the lines of the Ristic-Sorensen experiment.

X. APPENDICES I, II AND III

APPENDIX I. BACKGROUND REMARKS ON GAS/PLASMA SWITCHES

We will briefly review here a typical circuit used in pulsed power technology in order to illustrate the role of the gas/plasma switch (which is a nonlinear active element) as it has been appreciated in the past.

In many typical applications, C_0 is large, L_0 is small so that one's interest is in the effect of $R_S(t)$ and $L_S(t)$ on the current $i(t)$ for given voltage breakdown strength V_0 and load impedance Z . According to J.C. Martin, there are the following cases in which (1) $L_S(t)$ is ignored compared to $R_S(t)$; and (2) $R_S(t)$ is ignored compared to $L_S(t)$. For the former, one simply presumes that

$$i_R(t) = \frac{V_0}{Z} \left[1 - e^{-\frac{t}{\tau_R}} \right]$$

where τ_R is the so-called resistive rise time and was thought to be given by

$$\tau_{RS} = k \left(\frac{\rho_0 Z_0}{E_0^2 Z} \right)^{1/3}$$

on the basis of Martin's data (see text), where $E_0 = \frac{V_0}{\rho_0}$, Z_0 = length of spark channel, ρ_0 = ambient density and k is an empirical constant with dimensions.

The spark channel inductance $L_S(t)$ is proportional to the $\ln r_S(t)$ where $r_S(t)$ is taken to be the radius of the spark channel and in this case (1) this weak time dependence was ignored. For case (2) one then presumes that

$$I_{L_S}(t) = \frac{V_0}{Z} \left[1 - e^{-\frac{t}{\tau_{L_S}}} \right]$$

where

$$\tau_{L_S} = \frac{L_S}{Z}$$

For the actual case in which both $R_S(t)$ and $L_S(t)$ are involved, Martin recommended that all one could do is to presume that

$$I(t) = \frac{V_0}{Z} \left[1 - e^{-\frac{t}{\tau_T}} \right]$$

where

$$\tau_T = \tau_{R_S} + \tau_{L_S} = \left(\frac{R_0^2 L_0}{E_0^4 Z} \right)^{1/3} + \frac{L_S}{Z}$$

It is all too clear that this picture is a gross oversimplification of the actual situation and demonstrates the urgent need for a better understanding of the current flowing in such pulsed circuits in real (observable) time. This situation is characteristic of the state-of-the-art prior to the work of Ristic. In many current applications, three-electrode spark gaps are chosen, in this case the actual values of E_0 and Z to be used are unfortunately not known in the context of the above kind of circuit. In addition to the main circuit voltage and impedance, one has a separate trigger circuit with

quite different voltage and impedance values to drive the spark channel breakdown process. It should be abundantly clear from the above discussion that a much deeper basic understanding of the simpler two-electrode spark gap is certainly of first priority in this study of ultra-fast high rep-rate switching systems.

The above discussion on the vacuum radial transmission line is most instructive in regard to the physics of high pressure spark gaps since it allows one to acquire an orientation and appreciation of the nature of the converging trigger pulse (the "Lark") $V_1(r,t)$ in the time domain in the kinds of geometries of ultimate interest in this study. From a practical viewpoint one must deal with real materials for the electrode boundaries, and of course we always have a large number of ambient gas molecules per cm^3 in the best of vacuum vessels. The net effect is that "vacuum breakdown" is experienced at some value of r where $E_2(r,t)$ exceeds a certain threshold value in the vicinity of $r \rightarrow 0^+$ and we observe an outgoing reflected voltage pulse $V_R(r,t)$ which can be monitored [via $\frac{d}{dt} I_R(r,t)$] at some convenient location $r = R$. By increasing the ambient density p_0 of the gas, one tries to time delay the onset of the breakdown process in such a way that when it does occur, we obtain a very fast rise-time for $I_R(t)$ or $\frac{d}{dt} I_R(t)$. Loosely speaking, if one monitors the ingoing and reflected pulses in the coaxial feed section of the transmission line circuit where we have a constant characteristic impedance Z_0 , we can talk about the ingoing power pulse $\frac{V_1^2(t)}{Z_0} = P_I(t)$ and the reflected power pulse $\frac{V_R^2(t)}{Z_0} = P_R(t)$ at this location. Some authors use the terminology "Power Acceleration" to describe the increase of $\frac{d}{dt} P_R(t)$ over $\frac{d}{dt} P_I(t)$ or $Z_0 \frac{d}{dt} I_R^2(t)$ over $Z_0 \frac{d}{dt} I_I^2(t)$. Such language also conveys the very qualitative idea of pulse compression if one invokes conservation

of energy and writes for location R:

$$\hat{P}_R \tau_R = \hat{P}_I \tau_I$$

and

$$\tau_R = \frac{\hat{P}_I}{\hat{P}_R} \tau_I = \frac{\overline{V_I^2}}{\overline{V_R^2}} \tau_I < \tau_I$$

where $(\hat{})$ is the time average. It is the contention of such authors that the gas/plasma spark channel allows one to handle and transform very high power levels $P_I(t) \rightarrow P_R(t)$ in the required very short times τ_R of interest. In fact, it is claimed with perhaps some justification that no other switching technique can be considered to be anywhere near being competitive at the high power levels of interest herein.

Until we have a theory of the $\underline{j} \cdot \underline{E}$ interactions operative in the vicinity of the singularity on the z-axis of symmetry ($r \rightarrow 0^+$), it is not an easy matter to test or to put such a qualitative picture of pulse compression on a quantitative basis. It is rather important to note that most high pressure spark gap experiments have been conducted with relatively long trigger pulses $V_I(t)$ where one is almost encouraging all possible flashover-shunting processes to become prematurely operative in the time domain. Said another way, it is not obvious that we have even begun to tap the nonlinear structure of the electromagnetic field per se, let alone the $\underline{j} \cdot \underline{E}$ interactions in the vicinity of the singularity. It is not out of the realm of possibilities that given a short enough trigger pulse [measured say by $\frac{d}{dt} V_I(t)$], that a vacuum switch with appropriate electrodes (say highly polished Carbon as now used in the

production of electron beams in vacuum diodes) could well become a viable switch candidate. Such a possibility receives some preliminary support from the fact that the resistive time τ_R is apparently independent of the spark channel length λ_0 and directly proportion to $\rho_0^{2/3}$. If one pursued such an idea "all the way" with the appropriate $V_I(t)$ it is clear that the nonlinear active medium for switching would at first glance be an electron beam per se. Now it would be clearly self-defeating if all or most of the power $P_I(t)$ was absorbed in the kinetic energy of a conventional electron beam which would be deposited in the anode where severe damage would occur; in fact, one would not have a switch of the kind required where we want to deposit minimal energy in the switching volume per se and deliver a maximum amount of power $I_R(t)V_R(t)$ to a final load impedance via the pick-up loop technique. If, however, one can learn how to shape the electrodes and the radial transmission line feeding such "electron beams", there may be techniques as yet undiscovered in the laboratory whereby one can in fact exploit such a vacuum switch concept. The basic idea is to create an extremely high inductance in the nonlinear active medium (not unlike a super-pinch electron beam where in particle electrodynamic language $\beta_{\perp} \gg \beta_{\parallel}$). In such a situation one would expect that such a driven nonlinear active medium could be locally characterized by coherent (long range ordered) time varying surface currents collectively driving, in turn, the vacuum radiation impedance. Such a nonlinearly polarized medium would become more and more inductive as $r \rightarrow 0^+$ in the geometry of interest here. In any case, it will require a most interesting future theory to show that such driven "electron beams" will radiate most of their inductively stored power (leaving very little for kinetic energy at later times) and yield the macroscale power $I_R(t)V_R(t)$ in the transmission

line circuit where all of the fields are of course defined. It will require future proof but it now appears that the same basic mechanisms are responsible for the observed response of a high pressure gas to $P_I(t)$. The electronic structure of the gas molecules is subjected to strong incoming time varying driving induction fields $B_\phi(r,t)$ of TEM pulses. The full nonlinear response of such a collection of electrons to such driving fields (pulsed flux) is most difficult to characterize in the conventional language of plasma physics or classical electrodynamics. It may be possible to invent physical models (of the nonlinear active medium) however, if one is working in close concert with relevant experiments.

A careful study of the linear theory of the time domain response of a biconical antenna to pulsed excitation both in transmission and reception is most relevant to this future theoretical program on nonlinear switching techniques (see for example C.W. Harrison's basic work in IEEE Transactions on Antennas and Propagation, March 1965 and references therein). The physics of such linear biconical antennas is best described in J.C. Slater's "Micro-Wave Transmission", wherein one will find an excellent critique of the pioneering work of Shelnkoff, Stratton and Chu and others. The relevant relationships are in the geometry of both problems. In conventional switches one normally uses electrodes with Rogowski profiles (based on equipotentials or electrostatics concepts) to produce spatially uniform initial electric fields in the spark gap volume. Actually one does not want uniform fields, but rather one wants to tap the real singularity (built into Maxwell's equations) on the symmetry axis as discussed above and this singular "EMF" region is most manifest in the biconical antenna geometry. We therefore can learn from linear antenna theory that wide angle biconical transmission line

geometries (fed by coaxial lines) may play a fundamental role in the problem of ultra-fast switching. In such geometries, one has coaxial "room" in the vicinity of the symmetry axis for small holes in the electrodes for the high speed flow of the gas in the high rep-rate mode.

APPENDIX II. THE VACUUM RADIAL TRANSMISSION LINE

It is clear that we will eventually have to study the difficult problem of a prescribed command triggering pulse $V_I(t)$ propagating down the coaxial transmission line and then reaching the junction where it must "turn the corner" and begin to propagate radially inward to the system axis of symmetry where the gas switch volume is located. In this appendix it is presumed that one has learned how to shape the junction geometry in such a way that one obtains an ingoing TEM pulse in a radial transmission line. In the general μ -wave theory of transmission lines, this means that in the Fourier Integral representation of $V_I(t)$ one can determine the characteristic frequencies $\{\omega\}$ of such a pulse and the associated wavelengths $(\lambda = \frac{1}{\sqrt{\epsilon\mu}} \frac{2\pi}{\omega})$. Then for those wavelengths which are large compared to the characteristic size of the junction (discontinuity in shape) one can show that such spectral components "turn the corner" and one can resurrect a TEM pulse $V_I(t)$ which then propagates radially inward when it is several characteristic wavelengths from the junction. Experimentally, with $\hat{B}_\phi(t)$ sensors, one can of course measure both pulse shapes $[\frac{d}{dt} I_I(t)]$ in the coaxial line and in the radial transmission line. In any case, we are only interested here in the description of the electromagnetic field near the z-axis of symmetry for such ingoing TEM pulses.

We therefore analyze the important problem of a pulse propagating in the vacuum radial transmission line in real time. To make the problem two-dimensional, we can take the inhomogeneous line to have an infinite radius. Then we assume a current density source is located in the line and that it can be described (prescribed) by

$$\frac{4\pi}{c} j_z(r_0, t_0)$$

in a cylindrical coordinate system (r, ϕ, z) . We have to then solve the following wave equation, in Coulomb Gauge,

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A_z}{\partial r} \right) + \frac{\partial^2 A_z}{\partial z^2} = -\frac{4\pi}{c} j_z(z, t)$$

where

$$B_\phi(r, t) = -\frac{1}{r} \frac{\partial A_z}{\partial \phi}(r, t)$$

$$E_z(r, t) = -\frac{1}{c} \frac{\partial A_z}{\partial t}(r, t)$$

To solve this problem one uses the Green's function method where

$$\nabla^2 G + \frac{1}{c^2} \frac{\partial^2 G}{\partial t^2} = -\delta(\mathbf{r} - \mathbf{r}_0) \delta(t - t_0)$$

with both G and $\frac{\partial}{\partial t} G$ vanishing for $t < t_0$ being the initial conditions.

Then we have

$$A_z(r, t) = \int_{t_0}^t dt_0 \int_{V_0} dV_0 G(\mathbf{r}, t | \mathbf{r}_0, t_0) \frac{4\pi}{c} j_z(\mathbf{r}_0, t_0)$$

where we integrate over the source volume V_0 and $\frac{4\pi}{c} j_z(\mathbf{r}_0, t_0)$ is the source density for the volume element dV_0 . For our two-dimensional problem it is well known that

$$G(r, \phi, t | r_0, \phi_0, t_0) = \frac{1}{2\pi} \frac{c [c(t - t_0) - |\mathbf{R}|]}{\sqrt{c^2 (t - t_0)^2 - |\mathbf{R}|^2}}$$

where

$$|K| = \sqrt{(r_0 - r_0')^2 + 4r_0' \ln^2 \left(\frac{r - r_0}{2} \right)}$$

and

$$U(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x > 0 \end{cases}$$

is the Heaviside step function. The following picture shows the geometry

Now for a circular (azimuthally uniform) distribution of sources we can introduce

$$G(r, \phi | r_0, \phi_0) = \int_0^{2\pi} d\phi_0 G(r, \phi | r_0, \phi_0, \phi_0')$$

which we proceed to evaluate in closed form by letting

$$k = \frac{r_0 - r}{2}; \quad k = \frac{[c^2 R - r_0^2 - (r - r_0)^2]^{1/2}}{(R + r_0)^{1/2}}$$

We find

$$G(r, z | r_0, z_0) = \frac{2c}{\pi} \left[c^2 (R - z_0)^2 - (r - r_0)^2 \right]^{-1/2} \times$$

$$\times \int_0^{z/z_0} d\theta \left[1 - \frac{1}{k^2} \sin^2 \theta \right]^{1/2} U \left[R^2 - \sin^2 \theta R^{1/2} \right]$$

and for $k > 1$

$$U \left[(R^2 - \sin^2 \theta)^{1/2} \right] = U \left[c(R - z_0) - |r - r_0| \right]$$

and

$$G(r, z | r_0, z_0) = \frac{2c}{\pi} \left[c^2 (R - z_0)^2 - (r - r_0)^2 \right]^{1/2}$$

$$U \left[c(R - z_0) - |r - r_0| \right] \approx K \left(\frac{1}{k} \right),$$

where $K \left(\frac{1}{k} \right)$ is the standard elliptic integral for $\frac{1}{k} < 1$. The second case is that in which $k < 1$, which yields

$$G(r, z | r_0, z_0) = \frac{2c}{\pi} \frac{1}{\sqrt{4\pi r_0}} U \left[c(R - z_0) - |r - r_0| \right]$$

$$\times \int_0^k d\xi (R - \xi^2)^{-1/2} (R^2 - \xi^2)^{-1/2}$$

$$= \frac{2c}{\pi} \frac{1}{\sqrt{4\pi r_0}} U \left[c(R - z_0) - |r - r_0| \right] K(k)$$

by the substitution $t = t_0 + \tau$. From (10) result we have

$$A_2(r,t) = \int_0^t dt_0 \int_0^{2\pi} d\tau_0 G(r,t|\tau_0,t_0) \frac{4\pi}{c} j_2(\tau_0,t_0)$$

and can in principle proceed to calculate $A_1(r,t)$ and $A_2(r,t)$ for any prescribed $j_2(r_0,t_0)$. There is a most interesting problem which arises when we examine the field at the origin $r = 0$; we proceed to illustrate by going back to

$$G(r,0,t|\tau_0,0,t_0) = \frac{c}{2\pi} \frac{U(c(t-t_0) - |R|)}{\sqrt{c^2(t-t_0)^2 - |R|^2}}$$

with

$$|R| = \sqrt{(r-r_0)^2 + 4r_0 \sin^2 \frac{\theta - \theta_0}{2}}$$

We calculate

$$\begin{aligned} \frac{\partial G}{\partial t} &= \frac{c}{2\pi} \left\{ \frac{|R| U(c(t-t_0) - |R|)}{c^2(t-t_0)^2 - |R|^2} - \delta(c(t-t_0) - |R|) \right. \\ &\quad \left. + \frac{\frac{\partial |R|}{\partial t}}{\sqrt{c^2(t-t_0)^2 - |R|^2}} \right\} \end{aligned}$$

and

$$\frac{1}{c} \frac{\partial G}{\partial r} = - \frac{c}{2\pi} \left\{ \frac{c(R-t_0) U(c(R-t_0) - |R|)}{c^2(R-t_0)^2 - |R|^2} \right. \\ \left. - \delta(c(R-t_0) - |R|) \right\} \pi \\ \pi \frac{1}{\sqrt{c^2(R-t_0)^2 - |R|^2}}$$

hence,

$$\frac{\partial R}{\partial r} = \frac{1}{|R|} \left[r - r_0 + 2r_0 \sin^2 \left(\frac{\phi - \phi_0}{2} \right) \right]$$

and now set $r = 0$ for which case $R = r_0$; this yields

$$\left[\frac{\partial |R|}{\partial r} \right]_{r=0} = -1 + 2 \sin^2 \left(\frac{\phi - \phi_0}{2} \right) = -\cos(\phi - \phi_0)$$

Finally,

$$B_0^*(0, t) = - \frac{\partial G}{\partial r} \Big|_{r=0} = - \frac{c}{2\pi} \left\{ \frac{r_0 U(c(R-t_0) - r_0)}{c^2(R-t_0)^2 - r_0^2} \right. \\ \left. - \delta(c(R-t_0) - r_0) \right\} \pi \\ \frac{\pi \cos \left(\frac{\phi - \phi_0}{2} \right)}{\sqrt{c^2(R-t_0)^2 - r_0^2}}$$

and

$$E_2^*(0, z) = -\frac{1}{c} \frac{\partial G}{\partial t} \Big|_{r=0} = -\frac{c}{2\pi} \left\{ \frac{(cR - z_0) U[(cR - z_0) - r_0]}{c^2 (R - z_0)^2 - r_0^2} - \delta[(cR - z_0) - r_0] \right\} = \frac{1}{\sqrt{c^2 (z - z_0)^2 - r_0^2}}$$

In the future one can write a computer program for interesting initial pulse shapes and study the resulting fields near $r \rightarrow 0^+$ where E_2^* and B_1^* contain the dimensions of the source strength. These are the fields at the origin for a Dirac delta function source and we see that we must choose $z = z_0$ and $c(z - z_0) = r_0$ to make $E_2^*(0, z) = B_1^*(0, z)$; i.e., to say the pulse is TEM. Now what does it mean to choose $z = z_0$? In the above calculations we let $z = \frac{z_0 - \delta}{2}$ and integrated over δ_0 which gave a result independent of δ ! Now we can see that when we took the limit $r \rightarrow 0^+$, we should have said which angle ϕ we had used to approach the origin at $r = 0^+$. Suppose therefore we choose $r \rightarrow 0^+$ and $z = 0$ in our definition of $z = \frac{z_0 - \delta}{2}$, then if we postulate that the pulse is TEM in the vicinity of the singularity (z -axis of symmetry) then both ϕ and δ_0 vanish and $\cos \frac{\phi - \phi_0}{2}$ is still unity as required! Now what does this mean physically? We will in the future examine this most basic question to explore the tantalizing possibility that the only possible basic explanation is that the vacuum state of the electromagnetic field has a unique structure whose microscopic quantized coherence characteristics are actually manifest in this very old macroscopic inhomogeneous transmission line problem where there is a real physical "singularity" on the axis of symmetry in the electromagnetic field (vacuum *per se* and electrode materials).

APPENDIX III. PHYSICS CONCLUSIONS AND RECOMMENDATIONS

If one objectively studies the material in the previous sections of this report and the literature available on ultra-fast spark gap switching, one can identify a number of fundamental issues which require further investigation in the laboratory with the close support of new theoretical efforts to more fully understand the nonlinear electrodynamic processes manifest in the so-called statistical, formative and resistive time phases of the problem. These unresolved issues emerge when one examines Maxwell's equations for the fields and its sources and in particular when one tries to describe what is happening in spark gap circuits in the time domain.

In circuit language the fundamental issues can be seen to center around the proof of validity of any known equivalent circuit representation of such a field theoretic problem; specifically the characterization of the spark channel evolution in real time (relative to the prescribed overvoltage pulse waveform) in terms of such lumped circuit parameters as $C_0(t)$, $L_0(t)$ and $R_0(t)$ requires careful justification in each experimental situation on the basis of Maxwell's equations. It is without doubt convenient to think of the spark channel as an active nonlinear load impedance $Z_L(t)$ in a transmission line circuit but one must then proceed to design experiments which can reveal what is actually happening in the electromagnetic field and its sources (as well as the $\underline{J} \cdot \underline{E}$ interaction) which leads to any particular functional specification of such a $Z_L(t)$. Even in the linear μ -wave theory of transmission lines, W.M. Hansen once said "the first thing to learn about the theory of μ -waves is that the idea of impedance cannot be used as a substitute for thought". In a similar vein, J.C. Slater commented: "When ideas of transmission lines lead to the same results as Maxwell's equations they are right, though superfluous;

when they lead to different results they are wrong, an incorrect analogy".

In the present case of nonlinear active load impedances such as our spark channel, the message is clear we must try to work solely with pulsed solutions of Maxwell's field equations in the time-domain in the design and execution of new experiments.

In this study we have seen that Sorensen and Ristic have been able to deduce an effective resistance $R_0(t)$ proportional to t^{-3} on the basis of the measured value of $\frac{d}{dt} I_R(t)$, the pulse reflected from the spark channel already in its resistive phase. As discussed earlier, it was assumed that the spark channel inductance $L_0(t)$ played a minor role in the problem compared to $R_0(t)$ in controlling the observed $\frac{d}{dt} I_R(t)$. With regard to the equivalent circuit used by these authors it was noted that there is no a-priori reason to introduce a pure Ohmic resistance $R_0(t)$ in favor of the alternative time-derivation of the inductance $\frac{d}{dt} L_0(t)$. It is important to note that by definition $L_0(t)$ has an internal part $L_0^{int}(t)$ of unknown microscopic character and an external part $L_0^{ext}(t)$ which is usually taken to be proportional to $\ln \frac{b}{r_s(t)}$ in the coaxial geometries normally used where $r_s(t)$ is the time-dependent spark channel radius. Even if $L_0^{ext} \frac{dI}{dt}$ can be shown to be small compared to $I R_0(t)$ as in the Sorensen-Ristic experiment, it does not follow that $i \frac{d}{dt} L_0^{int}(t)$ is to be forgotten and in fact it is not out of the question that $R_0(t)$ is in fact very much like or even equivalent to $\frac{d}{dt} L_0^{int}(t)$ in many of these problems. Of course we cannot expect for the sign (=) at this stage readily resolve the basic difference between the positive $R_0(t) \propto t^{-3}$ and $L_0(t) \propto t^{-2}$ because we have no physical understanding of the internal electronic structure and motion of the gas which has been transformed from its initial dielectric state to some kind of rather ill-defined "plasma state" by the overvoltage pulse. In principle we would

have to know the self-consistent solution of Maxwell's equations for the fields $\underline{E}(\underline{x}, t)$, $\underline{B}(\underline{x}, t)$ and the sources $\rho(\underline{x}, t)$, $\underline{j}(\underline{x}, t)$ everywhere in the circuit (in or on the surfaces of the real material boundaries and in the gas in the spark gap volume) and such things must be known in the time domain wherein we can only measure time relative to the prescribed initial pulses in the circuit. Since we are dealing, in addition, with nonlinear electrodynamic processes in the so-called breakdown of the gas, we could easily abandon any further theoretical effort to understand such a field theoretic problem. All of our linear dispersion theoretic techniques based on the superposition of states are quite useless, even the perturbation theories of Nonlinear Optics and Quantumelectrodynamics (carried out to some low order) will not help us very much in understanding the internal structure of the fields and particles (nor their strong $\underline{j} \cdot \underline{E}$ interaction) in a spark channel of the kind we must try to create in the laboratory in the future. When one is faced with such an awesome field theoretic problem it is perhaps wise to simply admit that searching for such fundamental self-consistent solutions in space and time is practically impossible and begin to refine our experiments while searching for phenomenological approaches to the problem; i.e., look for source theories as they are referred to in modern field theory. In simpler language, we must learn to use our physical insight and construct models of the spark channel medium which can guide relevant future experiments designed to test such source models and hopefully find at least one that works in practice in such programs as that for L^3 which is the goal of this study.

In order to acquire some orientation on the kinds of experiments to be required we will consider a radial transmission line or a biconical transmission line with a tiny cylindrical container of gas of volume $\pi r_0^2 l_0$ near the origin

of coordinates (spherical and cylindrical respectively). We will presume that a very short ingoing TEM pulse can be introduced into such a line, with effective width Δt_0 as measured [via $\frac{d}{dt} I_1(t)$] at some large radius r_0 where $r_0 \gg \Delta t_0$. Since such a pulse travels with the speed of light, we know when it should arrive at the gas container at $r = r_0$, and from the solutions given earlier for such transmission lines we can calculate for all $r < r_0$, $\vec{B}_0(r, t)$ and its time derivative $\dot{\vec{B}}_0(r, t)$ if we know their values at $r = r_0$ from measurement of $\frac{d}{dt} I_1(r_0, t)$ with the pick-up loop sensor described in previous sections. Since $B_0(r, t)$ and $E_2(r, t)$ are very large near $r = 0^+$, we know that at some radius $0 < r < r_0$ the stress $\frac{V_1(r, t)}{r_0}$ will reach a threshold value and then proceed to exceed the breakdown strength of the gas. We would like to experimentally record the absolute time delays involved in such a situation where one expects, on field theoretic grounds, that gas (as a dielectric medium) to act like a nonlinear active inductance in which strong driven polarization currents are flowing inside the gas.

From the Poynting Theorem we know that power is flowing in the electromagnetic field (in the pulse) and entering the gas to "fill the proposed nonlinear inductive store". Experiments on the spectral characteristics (e.g., with photoelectric techniques or streak spectrographs) of the light emitted by the gas during this inductive "filling time" (with the exploitation of Zeeman and/or Stark effects) would be most useful if such results can be time-tied to the pulse shape of both $\frac{d}{dt} I_1(r_0, t)$ and $\frac{d}{dt} I_R(r_0, t)$ as recorded at the same radial location r_0 in the transmission line circuit. In the conventional picture one says that the gas is transformed to a "plasma state" of rather high conductivity and with such a conducting link between the cathode and anode the voltage across the gap begins to collapse in real-time.

The lumped circuit theory for this model has been discussed in earlier sections where the spark channel total resistance $R_0(t)$ controls the measured pulse shape $\frac{d}{dt} I_R(t)$. In this model one has a spark channel in driven radial hydrodynamic motion as a result of Ohmic heating [locally described by $j_z^2(r,t)\eta(r,t)$ where the current density $j_z(r,t)$ and the plasma resistivity are not however, as noted earlier, directly observed]. In any case, one assumes that the total effective cross-sectional area of the spark channel is increasing in real-time as a result of the hydrodynamic motion and in this way one obtains a decreasing total resistance $R_0(t)$ and an increasing total current $I_R(t)$ (or a voltage collapse across the gap). In simple spark gap circuits where one can identify the interelectrode capacitance C_0 and one knows that it was charged to say a voltage of V_0 (the breakdown voltage of the gas), one then assumes that the capacitive store $\frac{1}{2} C_0 V_0^2$ is depleted in driving the spark channel and that this happens in such a short time that the store cannot be recharged by the primary charging current supply. The circuit under consideration here with the ingoing TEM pulse in a radial transmission line geometry is of course rather more complex than such a simple circuit but in both cases one would like to know what the fields and its sources are doing in real-time and to avoid, as long as possible, the introduction of lumped circuit concepts. In the inductive store model $L_0(t)$ it would seem to be most natural for the coherent current flowing through the gas to increase in time as the field continues to deliver coherent energy or power via the incoming overvoltage TEM pulse. From an experimental viewpoint, the key difference between the $L_0(t)$ model and the $R_0(t)$ model is that no hydrodynamic motions of any kind need occur in the former. It would therefore be most exciting if some kind of high time resolution Doppler experiment (μ -wave or optical) could

resolve this basic question: is there actual hydrodynamic motion during (not after) the so-called resistive phase of the evolution of the spark channel? Such questions will become more and more pertinent and the answers will be of great importance in the regime of ultra-fast spark gap switching in the sub-nsec time regime. It should be noted again that we are interested in this study in the situation where the incoming overvolting pulse $V_I(t)$ is of sufficient coherence and strength to drive the gas through the so-called statistical and formative phases very rapidly in times comparable to the overall time resolution available (say less than 50 picoseconds). With regard to the output voltage waveform $v_o(t) = k \frac{d}{dt} I_R(r_o, t)$ we also require that it be of high quality and highly reproducible in the high rep-rate modes of interest here (the latter problem is to be addressed with a high speed gas flow system).

In parallel with such an experimental program, it will be necessary to develop a fundamental theory which will provide a microscopic electrodynamic picture of the coherent internal polarization currents in the electronic structure of the gas molecules and in sufficient detail as to how this leads to the observed rapidly growing total current $I_R(r_o, t)$ flowing in the macrocircuit. This author has been working on such a fundamental theory for the last several years and finds that the $L_o(t)$ picture is slowly evolving as a more basic foundation for switching in general, than the $R_o(t)$ picture; only clever diagnostic experiments, however, will settle the matter to everyone's satisfaction. This new theory of electrodynamics is based on the conservation of quantized flux and is most useful in designing such new experiments for the time regime of interest. Progress on such matters will be reported in the future as the new experimental programs unfold.

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