MASTER

Level Densities in Nuclear Physics

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I. Introduction

In the independent particle model which forms the basis for the nuclear level density expressions of Bethe¹⁾, Van Lier and Uhlenbeck²⁾ and Bloch³⁾ nucleons move independently in a central potential. There is a well-defined set of single-particle orbitals, each nucleon occupies one of these orbitals subject to Fermi statistics, and the total energy of the nucleus is equal to the sum of the energies of the individual nucleons. The basic question is what is the range of validity of this Fermi gas description and, in particular, what are the rôles of the residual interactions and collective modes. In this talk I will try to provide some insight into these questions through a detailed examination of experimental level densities in light mass systems.

Most experimental information on nuclear level densities has been obtained from analyses of s-wave neutron resonance spacings⁴⁾, of low-lying excited states^{5,6)} and of evaporation spectra.⁷⁾ I will discuss (Section III) level densities over the first 10 MeV or so in excitation energy as deduced from neutron and proton resonances data and from spectra of low-lying bound levels. To exhibit some of the salient features of these data comparisons will be presented (Section IV) to independent particle (shell) model calculations. Shell structure is predicted to manifest itself thorough discontinuities in the single-particle level density at the Fermi energy⁸⁾ and through variations in the occupancy of the valence

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orbitals^{3,8,9)}. These predictions will be examined through combinatorial calculations¹⁰⁾ performed using the Grover¹¹⁾ odometer method. Before discussing the experimental results statistical mechanical level densities for spherical nuclei will be reviewed (Section II). Deformed nuclei will be discussed in Section IV, and concluding remarks will be made in Section V.

II. Statistical Mechanical Level Densities (Bethe Formulae)

To obtain the statistical mechanical state density $\omega(E, N)$ of the form given by Bethe¹⁾ we introduce the grand partition function

$$Q_G(\beta, \alpha) = \sum_i e^{\alpha N_i - \beta E_i} = \int_0^\infty \int_0^\infty dN dE \omega(E, N) e^{\alpha N - \beta E} \quad (1)$$

where E is the total energy, N is the number of nucleons, and β and α are Lagrange multipliers corresponding to E and N , respectively. The state density is obtained by inversion

$$\omega(E, N) = \left(\frac{1}{2\pi i}\right)^2 \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} d\beta d\alpha e^{\Phi(\beta, \alpha) - \alpha N + \beta E} \quad (2)$$

where $\Phi(\beta, \alpha) \equiv \ln Q_G(\beta, \alpha)$. We can then evaluate Eq. (2) using^{2,6)} the method of Darwin-Fowler. The result is the basic relation

$$\omega = \frac{e^S}{2\pi D^{1/2}} \quad (3)$$

The entropy S is

$$S = \Phi(\beta_0, \alpha_0) + \beta_0 E - \alpha_0 N \quad (4)$$

the determinant D is

$$D = \begin{vmatrix} \frac{\partial^2 \phi}{\partial \beta^2} & \frac{\partial^2 \phi}{\partial \beta \partial \alpha} \\ \frac{\partial^2 \phi}{\partial \alpha \partial \beta} & \frac{\partial^2 \phi}{\partial \alpha^2} \end{vmatrix}_{\beta=\beta_0, \alpha=\alpha_0} \quad (5)$$

and the saddle point (β_0, α_0) conditions are

$$\left. \frac{\partial \phi}{\partial \beta} \right|_{\beta=\beta_0} = E, \quad \left. \frac{\partial \phi}{\partial \alpha} \right|_{\alpha=\alpha_0} = N \quad (6)$$

The central feature of Eq. (3) is the exponential dependence of the state density upon the entropy giving, upon evaluation of ϕ , the dominant energy and mass dependence.

To evaluate $\phi(\beta, \alpha)$ we first note that for fermions $Q_G(\beta, \alpha) = \prod_i [1 + \exp(\alpha - \beta \epsilon_i)]$ where ϵ_i are the fermion single-particle energies. We then make the continuous approximation to the discrete single-particle spectrum. That is, we introduce a continuous density of single-particle levels $g(\epsilon)$ enabling us to express ϕ in integral form

$$\phi(\beta, \alpha) = \int_0^\infty d\epsilon g(\epsilon) \ln(1 + e^{\alpha - \beta \epsilon}) \quad (7)$$

Noting that $\alpha_0 = \beta_0 \mu$ where β_0^{-1} is the thermodynamic temperature and μ is the chemical potential we expand $g(\epsilon)$ about $\epsilon = \mu$ in a Taylor's series and retain terms

to second order. We then solve for E and N using the saddle-point conditions Eq. (6), and find that

$$S = \frac{\pi^2}{3\beta_0} g(\mu) \left[1 + \frac{7\pi^2}{30\beta_0^2} \frac{g''(\mu)}{g(\mu)} \right] \quad (8)$$

and

$$D = \frac{\pi^2}{3\beta_0^4} g^2(\mu) + \text{higher-order terms} \quad (9)$$

To determine under what conditions higher-order terms may be neglected in Eqs. (8) and (9) we consider a potential of uniform depth $g(\epsilon) \sim \epsilon^{1/2}$. The corresponding chemical potential is related to the Fermi energy μ_F by $\mu = \mu_F \left[1 - \frac{\pi^2}{12} (\beta_0 \mu_F)^{-2} \right]$ and we can replace μ by μ_F . The total number of nucleons A and the ground-state energy E_0 are defined by the expressions $A = \int_0^{\mu_F} g(\epsilon) d\epsilon$ and $E_0 = \int_0^{\mu_F} \epsilon g(\epsilon) d\epsilon$. The equation of state is

$$U = \frac{\pi^2}{6} g(\mu_F) \beta_0^{-2} \quad (10)$$

where $U = E - E_0$ is the excitation energy. We find that the second term in S is on the order $\beta_0^{-3} g \mu_F^{-2}$ and may be neglected provided that $U \ll \mu_F A^{1/3}$. Typical higher-order terms in D are on the order $\beta_0^{-6} g^2 \mu_F^{-2}$ and may be neglected provided that $U \ll \mu_F A$.

Upon inserting the leading-order expressions for S and D into Eq. (3) and employing Eq. (10), we obtain the formula for the state density originally derived by Bethe¹⁾,

$$\omega(U) = \frac{1}{\sqrt{48} U} \exp \left[2 \left(\frac{\pi^2}{6} g(\mu_F) U \right)^{1/2} \right] \quad (11)$$

Eq. (11) is usually derived assuming an equally spaced single particle spectrum. From the above we see that it is also valid for a single-particle spectrum corresponding to a potential well of uniform depth provided U is not too high.

The specification for spherical nuclei of the additive quantum number M , the space-fixed axis projection of the total angular momentum I , gives rise to the level density $\rho(U, M)$. This quantity is proportional to the state density evaluated at an excitation energy $U - E_{\text{rot}}(M)$ ^{3,12)} where $E_{\text{rot}}(M) = \hbar^2 M^2 / 2\mathcal{I}$ and $\mathcal{I}$ is moment of inertia:

$$\rho(U, M) \propto \omega(U - E_{\text{rot}}(M)) \quad (12)$$

For E_{rot} small compared to the excitation energy we can write $\omega(U - E_{\text{rot}}(M))$ as $\omega(U) \exp[-E_{\text{rot}}(M)/t]$ where we have replaced the nuclear temperature $\tau = [\partial \ln \omega / \partial E]^{-1}$ by the thermodynamic temperature t . Following Bethe¹⁾ we obtain the level density, $\rho(U, I)$, by taking the difference between $\rho(U, M=I)$ and $\rho(U, M=I+1)$, and keeping only the leading order terms. The result is

$$\rho(U, I) = \rho(U) \frac{2I+1}{2\sigma^2} e^{-(I+1/2)^2 / 2\sigma^2} \quad (13)$$

where

$$\rho(U) = \frac{\omega(U)}{\sqrt{2\pi}\sigma} \quad (14)$$

The spin-cutoff factor σ is related to the moment of inertia and the thermodynamic temperature by $\sigma^2 = (\mathcal{I}/\hbar^2)t$, and $\sqrt{2\pi} \sigma = \langle 2I+1 \rangle$ is the normalization constant for Eq. (12).

The quantity $a = \frac{\pi^2}{6} g(u_F)$ occurring in the level density and equation of state is the level density parameter. In general the state density can be written as a product of a pre-exponential term $C(U)$ and $\exp[2(aU)^{1/2}]$. Thus, from Eq. (11) $C(U) = \sqrt{48} U^{-1}$ corresponds to the grand canonical ensemble given in Eq. (1) while C is a constant for a microcanonical ensemble and $C(U) = \pi^{1/2} / 12a^{1/4} U^{5/4}$ for the grand partition function $\tilde{Q}_G = \sum_i \exp[\alpha_1 N_i + \alpha_2 Z_i - \beta E_i]$. It is these latter expressions for the state density, suitably parametrized, which are most often employed in data analyses.

III. Experimental Level Densities

1) Level Spacings

Information on the structure of states populated in a specific reaction is contained in the distribution of nearest-neighbor spacings S about the mean spacing D_{obs} . Information on whether levels are missing or misidentified is also provided by these distributions. It has been shown¹³⁾ that spacings between levels populated in s-wave neutron resonance reactions involving medium and heavy mass even-even target nuclides obey a Wigner distribution. This is the expected distribution of spacings between levels of a particular spin and parity and is characterized by an absence of small spacings

$$p(x) = \frac{1}{2} \pi x R(x) = \frac{1}{2} \pi x e^{-\frac{1}{4} \pi x^2} \quad (15)$$

where x represents the ratio S/D_{obs} .

The distribution of spacings between levels of mixed spin and parity has been derived¹⁴⁾ assuming that (i) levels of different spin and parity are not in any way correlated in position and (ii) levels of a particular spin and parity obey a Wigner distribution. The resulting distribution $P(x)$ has a shape intermediate between a Wigner distribution and an exponential distribution. The

general form for $\underline{P}(x)$ is

$$\underline{P}(x) = \Pi(x) \left\{ \sum_i q_i \frac{2P(q_i x)}{D(q_i x)} + \left[\sum_i q_i \frac{R(q_i x)}{D(q_i x)} \right]^2 - \sum_i \left[q_i \frac{R(q_i x)}{D(q_i x)} \right]^2 \right\}$$

where

(16)

$$\Pi(x) = \prod_k D(q_k x), \quad D(x) = 1 - \left(\frac{2}{\pi}\right)^{1/2} \int_0^{x(\frac{\pi}{2})^{1/2}} \exp\left(-\frac{1}{2} y^2\right) dy$$

The label i denotes the i^{th} J^π combination occurring in the sample and q_i are the corresponding fractional spin populations.²⁰⁾

The most extensive source of information on variations with mass number of level spacings of excited nuclei has been s-wave neutron resonance reactions^{4,15,16)}. For nuclides below A~65 Coulomb barriers are sufficiently low and level spacings are sufficiently large to enable the extraction of reliable mean spacings from proton resonance data as well. In s-d shell nuclei several low ℓ -values typically contribute. Energies, spins and parities of proton resonance levels and low-lying bound levels have been tabulated and conflicting assignments evaluated by Endt and van der Leun¹⁷⁾. In fp shell nuclei extensive high resolution proton resonance measurements have been performed by the TUNL group; much of this data has been summarized in Ref. 18. The fp shell s- and p-wave data can yield reliable mean spacings depending upon the behavior of the proton strength function.

Useful level density information also can be obtained from stripping and pickup reactions in many light mass systems. In s-d shell nuclei levels populated by low ℓ -values have been used up to excitation energies of about 10 MeV. In fp-shell nuclei the useful range of data is restricted to lower excitation energies. This data is limited to levels with large concentrations of fragmented spectroscopic strength populated by corresponding transfer reactions of low, easily identifiable ℓ -values, and is used primarily to supplement and examine statistical properties of low-lying bound levels.

The distribution¹⁹⁾ of s-wave neutron resonance spacings in Ca, Ti, Cr and Fe is displayed in Fig. 1a and the distribution of $\ell=0$ to 3(4) proton resonance spacings in Na, Al, P and Cl is displayed in Fig. 1b. The level repulsion effect is clearly exhibited by the $1/2+$ level spacings and the experimental distribution is in good agreement with the Wigner distribution. The proton resonance spacing distribution contains a mixture of $1/2\pm$ to $7/2\pm$ levels; the distribution is peaked at small spacings and is in good agreement with that calculated using Eq. (16).

Spacing distributions have also been used to test statistical properties of low-lying bound levels²¹⁾. As pointed out in Ref. 21, in such analyses it is essential to take into proper account the experimental energy resolution.

2) Level Densities

Neutron resonance spacing analyses have established that (i) the observed spacings are consistent with the predicted exponential increase in level density with increasing energy and mass ($g(\mu_F) \sim A/\mu_F$); and (ii) there are substantial departures from the overall trend which are correlated with shell structure. In the level density analyses Eq. (13) is used to relate the observed mean level spacings to the level densities:

$$\frac{1}{\rho(E_x)} = \frac{D_{\text{obs}}}{4\sigma^2} \sum_{(J^\pi)_{\text{obs}}} (2J^\pi + 1) \exp \left(- (J^\pi + \frac{1}{2})^2 / 2\sigma^2 \right) \quad (17)$$

where E_x is the mean excitation energy of the energy range in which D_{obs} is determined and the sum extends over all spin-parity (J^π) combinations occurring in the sample. A level density expression of the form presented in Section II is then employed for $\rho(E_x)$ and the level density parameter is deduced.

The level density information extracted in neutron resonance analyses is restricted to a narrow excitation energy range just above the neutron binding energy.

Furthermore, in those mass regions where the s-wave neutron strength function is near or at a minimum (see Ref. 4) it is difficult to extract s-wave resonance spacings. More extensive and reliable information can be obtained by requiring that the deduced level densities describe the low-lying bound levels as well as the resonance levels and determining in addition to the exponential level density parameter either an energy scale normalization²²⁾ or a preexponential level density normalization^{19,20)}.

In the level density analyses the quantities t , σ , a and Δ ²²⁾ or C ²⁰⁾ are simultaneously determined. The moment of inertia entering into the definition of σ is usually taken as that of a rigid-body calculated with a radius parameter r_0 in the range 1.2 to 1.25 fm. Experimental information on σ (or $\mathcal{I}$) has been obtained from measurements of isomer ratios²³⁾ and from measurements of angular distribution of evaporation particles²⁴⁻²⁶⁾. For light mass nuclides the experimental spin-cutoff factors are consistent with an $\mathcal{I}_{\text{rig}}$ calculated using $r_0 \sim 1.2$ fm while for heavier ($A > 100$) nuclides the experimental values are smaller than this $\mathcal{I}_{\text{rig}}$ ²⁷⁾. The results presented herein were obtained using the above-mentioned $\mathcal{I}_{\text{rig}}$. Some indications of the sensitivity of the resulting level densities to the moment of inertia is given in Ref. 22. Shortcomings of the statistical treatment of σ and of the parity distribution have been discussed by S. Grimes in this conference.

Level densities in representative light mass nuclides are displayed in Figures 2a, b. Open symbols denote counting data and filled symbols denote level densities deduced from transfer and resonance data using Eq. (17) with $\rho(E) = C \exp[2\sqrt{aU}]$. The smooth curves represent $\rho(E)$; the corresponding values for t , σ , a and C are listed in Reference 20 for $A \leq 40$ and in the Appendix for $A > 40$. The following features are exhibited by the data.

(i) There are odd-even differences; such differences are well-known and are one manifestation of the residual interactions. They are usually incorporated into Bethe formulae by introducing a pairing energy shift to the excitation energy.

The energy shifts observed in the mass range $25 \leq A \leq 40$ are on the order of 1.5 MeV between even-even and odd mass nuclides and 1.3 MeV between odd and odd-odd mass nuclides. The net average shift of ~2.8 MeV may be compared to the pairing shift²⁸⁾ $\Delta = \delta/A^{1/2} - \mu/A$ with $\delta=12.77$ and $\mu=29.34$ giving an average $\Delta \sim 3.6$ MeV.

(ii) There is an additional lowering of the effective excitation energy by ~1.5 MeV in the even-even symmetric ($N=Z$) nuclides. This symmetry energy shift may be compared to the Wigner symmetry term of the form²⁹⁾ $\delta_{\text{sym}}(N-Z)^2/A$ with $\delta_{\text{sym}}=23.3$. It is difficult, however, to see any influence of this term on level densities of nuclides other than doubly even ones.

(iii) To exhibit the underlying structure in the level densities over the entire mass range examined, namely $A=23$ to 57 , the mass dependence was removed. First the $\rho(E)$ were normalized to $A=40$ to remove the A dependence entering in from the level density parameters. Then the density of spin zero levels $\rho_0(E)$ were determined in order to remove the A dependence originating from the spin-cutoff factors. Odd-even differences and the symmetry energy were taken into account by shifting the excitation energies as discussed in (i) and (ii) above. The resulting ρ_0 's are plotted in Figure 3 at a representative excitation energy of 5 MeV. Broad structure is clearly observed superimposed upon the Bethe formula predicted mass dependence. There is a gentle minimum in the mid to upper s-d shell. While the level density for ^{40}Ca is low there is a sharp maximum in the calcium region. The level densities rapidly decrease as A increases further reaching a minimum for those nuclides having filled or nearly filled proton and neutron $1f_{7/2}$ orbitals.

The energy dependence of the level density variations is displayed in Figure 4. In this figure values of the spin-zero spacings $D_0=1/\rho_0$ at 1.5 MeV and at 8 MeV are shown; the former are representative of the spectra of low-lying levels while the latter are representative of the resonance region. There are two types of variations present. First, there are the systematic variations discussed in reference to Figure 3, and second, there are isobaric variations

found principally in the vicinity of closed shells. The systematic variations, or structure of the first kind, are observed with comparable magnitude in both 1.5 and 8 MeV data. Several interrelated characteristics of the calcium region, where the maximum in ρ_0 (minimum in D_0) occurs, are worth noting. In this region one finds many unnatural parity levels described³¹⁾ by couplings to an excited core³²⁾ as well as numerous fragmented single-particle levels. Changes in single-particle order and spacings occur as the $1f_{7/2}$ shell fills, and ^{48}Ca and ^{56}Ni are better closed-shell nuclei than ^{40}Ca . The isobaric variations, or structure of the second kind, are reduced in magnitude at 8 MeV. These variations are usually associated with ground-state properties such as degeneracy. Finally, it should be noted that statistical behavior rapidly sets in^{5,21,30)} as the level density increases over the first few MeV. While the increases in level density are nearly exponential, descriptions of nuclear spectra over the first 10 to 20 MeV in terms of simple Bethe formulae can only be accomplished using energy dependent level density parameters.

IV. Shell Structure/Deformations

1) Shell Model Level Densities

There are two aspects to the shell structure in the level densities predicted^{3,8,9,33)} within the framework of the independent particle (shell) model. First, level densities are expected to be modulated by variations in the single particle level density. Discontinuities in the single particle level density in the vicinity of the Fermi energy are expected to have a pronounced effect upon the level density at low excitation energies, where only a few orbitals are occupied due to the high degeneracy of the Fermi gas. Second, there is a predicted dependence upon the number of nucleons in the valence orbital. Level densities of midshell nuclei are expected to be considerably larger than those of near closed shell nuclei due to the increased number of ways nucleons can be rearranged within an orbital without expending energy.

To exhibit these predictions in comparison with the experimental results we calculate numerically level densities starting with a set of shell model orbitals. A number of methods for doing numerical calculations of this type have been developed^{11, 27, 34-36}). We perform the calculations using the Grover odometer method¹¹). In this combinatorial approach configurations are cycled and sorted out by energy, particle number and angular momentum. Odometers are used to select only those configurations which obey the exclusion principle. The excitation energy of a particular configuration is the sum of the energies of the occupied orbitals minus the ground state configuration energy, and the angular momentum of each configuration is calculated using vector coupling rules. To make the comparisons as clear as possible shell model orbitals characteristic of a spherical well were used. Those orbitals in the vicinity of the Fermi energy were determined from stripping and pickup reactions as spectroscopic factor weighted energy centroids as described in Reference 20, the BCS interactions were turned off, and no parameter adjustments were made.

Level densities calculated for A=51 isobars are shown in comparison with the experimental results in Figure 5. The influence of degeneracy is apparent in the calculated level densities. The lowest calculated results are for the closed neutron shell nucleus ^{51}V while the highest calculated results are for the most nearly midshell nucleus ^{51}Mn . At the higher excitation energies there is no evidence for any degeneracy influence (structure of the second kind) upon the experimentally deduced level densities and, compared to the predictions, only a slight indication at the lowest excitation energies. Additional shell model combinatorial calculations are compared to experimental results in Figure 6. Underestimates in predicted level densities for nuclides in the vicinity of ^{40}Ca are observed; these undershoots are, on the average, somewhat larger at 10 MeV than at 5 MeV. (The rises in calculated level densities for the higher mass Ar isotopes reflect the influence of degeneracy.)

Turning on the BCS interactions will not resolve the above-mentioned discrepancies. For mid-shell nuclides the energy dependence resulting from adjusting calculations to fit low energy data is too shallow¹¹⁾, and for closed-shell nuclides the BCS interactions cannot remove undershoots. It is clear that fragmentation of the single-particle strength must be considered in model calculations; the fragmentation arises from collective couplings and from residual interactions.

2) Deformations

In addition to fragmenting the single-particle strength collective modes may give rise to an enhanced level density³⁷⁻³⁸⁾. Estimates of the importance of collective contributions to the nuclear level density can be obtained using the statistical mechanical approach. Let us consider an axially deformed nucleus. The level density $\rho(U, I)$ is obtained by summing over the intrinsic states with $|k| \leq I$ where k denotes the angular momentum projection along the body-fixed (symmetry) axis and $k = \Omega$, the intrinsic spin, is assumed to be a good quantum number. Then^{37,39)}

$$\rho(U, I) \propto \sum_{k=-I}^I \omega_{\text{intr}}(U - \epsilon_{\text{rot}}(k, I)) \quad (18)$$

where ω_{intr} is the intrinsic state density and ϵ_{rot} is the sum of the intrinsic rotational energy $E_{\text{intr}}^k = \hbar^2 k^2 / 2\mathcal{I}_{\parallel}$ and $E_{\text{rot}}(k, I) = \hbar^2 [I(I+1) - k^2] / 2\mathcal{I}_{\perp}$. $\mathcal{I}_{\parallel}$ denotes the moment of inertia with respect to the symmetry axis and $\mathcal{I}_{\perp}$ denotes the moment of inertia with respect to an axis perpendicular to the symmetry axis. For rotational energies small compared to the excitation energy we can write an expression analogous to Eq. (13), namely

$$\rho(U, I) = \rho(U) \zeta(I) e^{-(I+1/2)^2 / 2\sigma_{\perp}^2} \quad (19)$$

where

$$\zeta(I) = \frac{1}{2} \sum_{k=-I}^I \exp[-\hbar^2 k^2 / 2\mathcal{I}_{\text{eff}}]$$

occurs in place of the factor $(2I+1)$. In the above $\mathcal{Q}_{\text{eff}} = \mathcal{Q}_I \mathcal{Q}_{II} / (\mathcal{Q}_I - \mathcal{Q}_{II})$ and $\sigma_I^2 = (\mathcal{Q}_I / \hbar^2) t$. If we assume that the total state density contains distinct contributions from intrinsic and rotational states then

$$\rho(U) = \frac{\omega(U)}{(2\pi)^{1/2} \sigma_{II}} \quad (20)$$

where $\sigma_{II}^2 = (\mathcal{Q}_{II} / \hbar^2) t$. For low I , $\zeta(I) \rightarrow (2I+1)/2$ and $\rho(U, I)$ is a factor of σ^2 larger than that of Eqs. (13), (14). If, on the other hand, we assume that all states are contained in ω_{intr} then

$$\rho(U) = \frac{\omega(U)}{(2\pi)^{1/2} \sigma_{II} \sigma_I^2} \quad (21)$$

and there is no enhancement.

Comparisons have been reported⁴⁰⁾ between numerical shell model calculations, which incorporate collective rotational enhancements of Eqs. (19), (20), and the deduced^{4,15,16)} s-wave neutron resonance spacings for medium and heavy mass nuclides. For nuclides known to be strongly deformed better agreements between calculated and experimental spacings was obtained using enhancements than was obtained without them.

The question of collective enhancements brings me to the last point of this talk. That is that analyses of low-lying states and resonance spacings as well as light-ion induced evaporation processes only provide information on level densities of nuclei at or near their ground-state deformation. Heavy-ion induced fusion reactions, in contrast, can produce nuclei with spins up to $\sim 100\hbar$ and thereby provide information on level densities in highly deformed nuclei. This point is illustrated in Figure 7 in which predicted rotating liquid drop equilibrium and saddle point shapes, as calculated by Cohen, Plasil and Swiatecki⁴¹⁾, are shown at representative angular momenta for an $A=75$ nucleus and an $A=150$ nucleus. The (triaxial) saddle-point

deformed nuclei are dumbbell shaped and at low angular momenta have lower rotational symmetry than the corresponding equilibrium deformed nuclei. The equilibrium deformations increase with increasing angular momenta becoming triaxial at high angular momenta and eventually equalling those of the saddle-point. Measurements of heavy-ion induced of fission and evaporation residue excitation functions and decay product characteristics provide basic information on fission barriers, particle transmission coefficients and level densities of highly deformed nuclei (at high excitation energies)^{42,43}.

V. Concluding Remarks

Two kinds of structural variations are present in light mass nuclear level densities. First, there is a modulation of the smooth Bethe formulae predicted mass dependence which is of the same magnitude at resonance energies as at low energies. Second, there are some isobaric differences principally near closed shells which are relatively minor at resonance energies. The combinatorial shell model calculations, in contrast, predict substantial isobaric differences due to degeneracy and underestimate level densities near closed shells.

This confrontation of the independent particle model with data provides strong evidence for the influence of fragmentation of the single-particle strength upon nuclear level densities. Fragmentation arises from collective couplings and from residual interactions; it is noteworthy that in the calcium region, where the maximum level densities occur, many fragmented single-particle levels and states described by excited core couplings are found.

Due to the smoothing aspect of fragmentation the continuous approximation introduced in the statistical mechanical approach to obtain Bethe formulae is better than one might expect. When temperatures are comparable to excitation energies the saddle-point approximation is expected to fail and it is not surprising that departures from Bethe formulae-like behavior are observed at low excitation

energies. Finally, level density parameters experimentally deduced using simple Bethe formulae have a limited range of validity namely that spanned by the data used in their determination.

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Appendix

Table 1.

Summary of statistical quantities characterizing
odd mass nuclides for A=41 to 57. Range of
validity is approximately from 2 to 10 MeV

nuclide	a	C	$\bar{E}$	σ
⁴¹ Ar	2.485	.0972	1.39	3.06
⁴¹ K	2.446	.100	1.56	3.24
⁴¹ Ca	2.645	.0421	1.45	3.13
⁴³ Ca	2.450	.129	1.51	3.32
⁴³ Sc	2.515	.197	1.53	3.34
⁴⁵ Ca	2.800	.0611	1.36	3.27
⁴⁵ Sc	2.509	.138	1.49	3.43
⁴⁷ Ti	2.519	.0953	1.47	3.52
⁴⁷ V	2.743	.0554	1.38	3.41
⁴⁹ Ti	3.043	.0403	1.36	3.50
⁴⁹ V	2.455	.135	1.56	3.76
⁵¹ V	3.300	.0216	1.39	3.67
⁵¹ Cr	2.719	.0633	1.48	3.79
⁵¹ Mn	2.570	.0650	1.42	3.71
⁵³ Cr	2.865	.0565	1.35	3.73
⁵³ Mn	2.735	.0635	1.43	3.85
⁵⁵ Mn	2.670	.0934	1.54	4.11
⁵⁵ Fe	2.789	.0575	1.51	4.07
⁵⁵ Co	3.642	.0146	1.19	3.62
⁵⁷ Fe	2.958	.0807	1.33	3.93
⁵⁷ Co	2.893	.0631	1.39	4.03

Table 2.
Summary of statistical quantities characterizing
odd-odd mass nuclides for A=46 to 58. Range
of validity is from 2 to 6 MeV

nuclide	a	C	$\bar{E}$	σ
^{46}Sc	2.000	.644	1.32	3.24
^{48}Sc	2.526	.154	1.22	3.26
^{50}Sc	2.941	.0293	1.33	3.25
^{48}V	2.087	.281	1.34	3.42
^{52}Mn	2.737	.0953	1.17	3.42
^{56}Mn	2.947	.214	1.13	3.57
^{56}Co	3.294	.0911	1.06	3.47
^{58}Co	2.762	.396	1.16	3.73

Table 3.
Single-particle (quasiparticle) energies* used
in combinatorial calculations for A=51 isobars
deduced (PW) from stripping and pickup data

orbital	protons	energy (MeV)	orbital	neutrons	energy (MeV)
$1d_{3/2}$		-5.2	$1d_{3/2}$		-2.3
2s		-5.1	2s		-2.2
$1f_{7/2}$		0.	$1f_{7/2}$		0.
$2p_{3/2}$		3.8	$2p_{3/2}$		5.3
$1f_{5/2}$		5.5	$2p_{1/2}$		6.9
$2p_{1/2}$		6.2	$1f_{5/2}$		7.5
$1g_{9/2}$		8.2	$1g_{9/2}$		9.5

*Energies are given relative to the $1f_{7/2}$ orbital. These orbitals were supplemented by Seeger's¹¹⁾ values away from the Fermi surface for a total of 32 active orbitals.

Figure Captions

- Fig. 1. Distribution of nearest-neighbor spacings. Histograms represent the experimental results and smooth curves represent the theoretical distributions normalized to the total area of the histograms (from Refs. 19, 20).
- Fig. 2. Experimental level densities²⁰⁾ for A=26 isobars (a) and A=40 isobars (b). Open symbols denote level counting data. Filled symbols denote direct reaction, proton resonance and neutron resonance data. Smooth curves represent the corresponding parameterized Bethe formula level densities.
- Fig. 3. Density of spin-zero levels, at 5 MeV, normalized to A=40. The horizontal dashed line represents the Bethe formulae predictions; the solid line is a visual guide and is not a fit to the data, represented by open and filled symbols. Filled circles denote odd mass nuclides from Ref. 20; open circles denote odd mass nuclides from the present work and Ref. 19. Open squares and triangles represent even-even and odd-odd mass nuclides from Ref. 20; and filled triangles represent odd-odd mass nuclides from Ref. 30.
- Fig. 4. Experimental spin-zero level spacings¹⁹⁾, adjusted to A=47. Dashed curves are visual guides and are not fits to the data.
- Fig. 5. Experimental and combinatorial level densities for A=51 isobars. Open and filled symbols have the same meaning as in Fig. 2. The calculations were performed using the Hillman-Grover code¹¹⁾ with the shell model orbitals tabulated in the Appendix. A 3 MeV averaging interval was used to smooth the calculated results.
- Fig. 6. Ratios of combinatorial to experimental level densities for nuclides in the vicinity of ⁴⁰Ca. Calculations were performed using the Hillman-Grover code¹¹⁾ with shell model orbitals taken from Ref. 20. Open

symbols denote ratios at 5 MeV and closed symbols denote ratios at 10 MeV. A 4 MeV averaging interval was used to smooth the calculated results.

Fig. 7. Rotating liquid drop⁴¹⁾ saddle-point (Pik-Pichak) and equilibrium (Hiskes and Beringer-Knox) nuclear shapes.

SADDLE-POINT & EQUILIBRIUM SHAPES (FROM COHEN, PLASIL & SWIATECKI)

$A=75$

$A=160$