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**THE TIME-STEPPED ENERGY SYSTEM
OPTIMIZATION MODEL (TESOM):
OVERVIEW AND SPECIAL FEATURES**

A.S. KYDES AND J. RABINOWITZ

December 1979

DEPARTMENT OF ENERGY AND ENVIRONMENT

**BROOKHAVEN NATIONAL LABORATORY
UPTON, NEW YORK 11973**

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BNL 51095

UC-13

MASTER

[General, Miscellaneous, and Progress
Reports (Nonnuclear)- TID-4500]

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December 1979

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ENERGY DATA AND MODELS**

**NATIONAL CENTER FOR ANALYSIS OF ENERGY SYSTEMS
DEPARTMENT OF ENERGY AND ENVIRONMENT
BROOKHAVEN NATIONAL LABORATORY
ASSOCIATED UNIVERSITIES, INC.**

UNDER CONTRACT NO. DE-AC02-76CH00016 WITH THE
UNITED STATES DEPARTMENT OF ENERGY

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Printed in the United States of America
Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22161
Price: Printed Copy ~~\$4.50~~; Microfiche ~~\$3.00~~

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Abstract

This paper provides an overview of TESOM, a time-dependent energy intergrating model used at Brookhaven National Laboratory for energy systems analysis. Special control theoretic features which act to smooth the intertemporal transitions between time periods are highlighted. These features include a new market penetration algorithm, an explicit representation of the technological and cost behavior of inherited (vintaged) stock, and the incorporation of risk factors in resource pricing.

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1. INTRODUCTION

The intent of this paper is to provide a clear descriptive overview of the Time-stepped Energy System Optimization Model (TESOM), the Brookhaven National Laboratory energy system integrating model.

The scope of energy systems analysis includes not only energy-economy interactions and their implications for policy planning but also social concerns and security requirements and their interactions with energy use and the economy. The quality of an energy system cannot be judged solely by economic considerations. Social concerns and security currently compete with the goal of system cost minimization.¹ It is essential to identify and employ approaches and a systems framework which facilitate the evaluation and characterization of an energy system by multiple criteria.² An optimization framework is particularly well suited to the task of performing trade-off analysis when the criteria are quantifiable.³ One such established framework is the BNL/DJA* approach, which couples macroeconomic driving variables to energy-economy process models to derive internally consistent energy-economy projections.⁴ TESOM is the energy system component of this energy-economy modeling framework.

The combination of time- and device-dependent features permits a comprehensive analytic framework for performing technology assessments and policy analysis related to the role of government in affecting the direction of R&D. The proper role of government in this area is to reduce or eliminate the barriers to R&D for those technologies which may be attractive in the long run but which present too high a risk or initial investment for commercial establishments to initiate the process of development. Those technologies which are clearly cost-effective, or ineffective, will not require government intervention. It is this type of analysis for which TESOM is particularly well suited.

2. OVERVIEW OF TESOM

The Brookhaven Time-stepped Energy System Optimization Model, TESOM, is a single region, time-stepped (sequential) linear programming model that was developed for the quantitative evaluation of national energy technologies and policies within a dynamic system's framework. The model is designed

*BNL/DJA: Brookhaven National Laboratory/Dale W. Jorgensen Associates.

to examine interfuel substitution in the context of time-dependent constraints on the availability of competing resources and technologies (stocks) and their associated costs.

TESOM is structured around the Reference Energy System (RES) that depicts the flow of energy from a resource to the point of actual end use. The RES is a specialized format for representing the detailed technological structure of the energy system along with resource consumption and associated emissions. Conceptually, the RES is a network representation of the flow of energy from resource extraction, through all intermediate conversion processes, to the satisfaction of final energy service needs (e.g., space heating, air conditioning, etc.). The RES provides a complete and thermodynamically consistent energy accounting system, in physical units, for energy flow through energy conversion devices; the extraction or importation of primary energy resources and products; refining and the various stages of energy conversion, transportation, distribution, and storage; and consumption of fuels and electricity by end-use technologies corresponding to particular energy service demands. Within the RES, emphasis is placed on the comprehensive technological structure relating the energy flows which enter the system (e.g., oil) to the nonsubstitutable, functional energy services which are the final products of the flow (e.g., process heat). A sample RES is shown in Figure 1.

The time-stepped model optimally allocates energy resources and products and selects the optimal mix of supply, conversion, and demand technologies according to selected energy, environmental, or economic criteria (usually least cost) to satisfy a set of energy service demands. Energy service demand representations are specified as exogenous requirements by year or as annual growth rates. Resource supply representations are specified as long- or short-term supply curves or fixed prices and availabilities by year. The TESOM model provides a "vintage" representation of the nation's energy system in that the optimal levels of the decision variables for any time period are determined from the following:

- The optimal levels established for previous periods;
- The retirement and deterioration rates, the lifetimes, and the associated costs of vintage capital stocks;
- Economic and technological factors affecting the feasible levels of the decision variables for the period under investigation (e.g., decline rates, supply elasticities, cumulative resource availabilities, market penetration considerations).

Mathematically, the model is formulated as a sequence of expanding linear programming formulations of the RES--one for each time period. For a given time period, the solutions derived for earlier periods along with assumptions regarding retirement rates, average lifetimes, age-dependent conversion efficiencies, plant factors, operation and maintenance costs, and the capital charges for the stocks-in-place are incorporated into the sequenced formulation. Energy demands are satisfied in accordance with the supply expansions and increased penetrations that are attainable for the period, the net resource and stock availabilities from the evolution of the energy system to date, and the objective function criteria selected. In meeting future energy demands, TESOM has a memory of the net availabilities from the past augmented by that which is attainable in the near-term. The updating and sequencing procedures are repeated until solutions for the entire time horizon are determined.

The input information required for the model falls into seven general time-dependent categories: (1) supply and demand efficiencies, (2) costs, (3) environmental impact factors, (4) resource availabilities (entered explicitly or generated by submodels), (5) basic energy demands (entered explicitly or computed with growth rates), (6) permitted optimistic market penetration levels, and (7) demand load characteristics of all electric service demands.

The supply efficiencies, which relate intermediate energy forms to primary resources, include efficiencies for refining, electric conversion, transmission and distribution of electricity and by-product heat, and synthetic fuel manufacturing. A refining efficiency represents a combined extraction, cleaning, and transportation efficiency and is defined as the ratio of the useful energy content of the fuel delivered to the user to the energy content at the minemouth or wellhead. In addition to these supply efficiencies, effective end-use utilization efficiencies must be specified for each utilizing device. Finally, efficiency and electric generation plant factor decay rates must be specified for all conversion equipment as a function of age.

The costs considered in TESOM include charges for investment in capital equipment, operation and maintenance of the stock, transmission and distribution of electricity, and resource consumption. Capital recovery costs are associated with each individual supply category and end-use

device. Also specified are maximum utilization factors for electric generation plant types. The actual capacity factor is determined by the model and is related to the load factor of the demands served by each technology.

Environmental coefficients are associated with each process in the various fuel paths from primary resource supply to end use. The total emissions for each process are calculated and summed.

Six main categories of constraint equations are used in TESOM:

(1) Energy conservation process equations balance the energy content of fuels into a process with the energy content of fuels out of a process including energy losses.

(2) Supply equations limit the amount of a given resource, at a given price, in terms of the energy content of the primary fuel (minemouth and well-head) available from that supply category. For synthetic fuels, the supply constraint equations represent the energy content of the synthetic fuel after manufacturing.

(3) Demand equations specify the level of nonsubstitutable functional end-use energy demands; each basic energy demand is defined as the amount of energy required to support an energy utilizing activity (e.g., residential space heat) assuming that the energy could be used at 100% technical efficiency (after conversion). Basic energy demands are specified by level or by growth rate for each of the demand categories.

(4) Electrical supply and demand load characterization equations limit the electrical generating capacity of plant types as well as accounting for peak electric demands and energy balances. The load characterization equations relate the type and quantity of electric generating capacity to the demand load characteristics of each demand type by season and time of day.

(5) Environmental constraints limit emissions of various types, such as sulfur dioxide or particulates. In most model runs these constraints are nonbinding and serve only to sum up environmental residuals.

(6) Market penetration equations limit the technology penetration of markets and define the range of permissible fuel usage or permitted capacity of a particular type of energy conversion process.

TESOM provides a time-dependent representation of the electric sector. Certain of the energy service demands may be satisfied by electricity. However,

the simple summation of the electricity inputs to these end-use devices does not determine the generating capacity which must be installed. Rather, a set of demand types (e.g., base and intermediate loads, off-peak, heating, cooling, etc.) are defined. Each demand type has its own set of characteristics regarding its stochastic behavior and its seasonal (winter, summer, fall-spring) and daily (day, night) loading. Required capacity is governed by the highest total peak demand which occurs during any time of the year and day. By appropriately loading the electric energy service demands onto the various (or, in some cases, corresponding) demand types and, subsequently, loading these onto the various season-day combinations, the height of the total peak for each season-day is determined. Required capacity is simply the maximum of the individual season-time-of-day peaks with allowances for transmission and distribution losses and reserve margins. This feature permits the introduction of load management considerations into the problem formulation as the load duration curve is, in part, exogenously determined from the detailed demand characteristics and their implications for the electric system.

The model contains a number of features designed to smooth the intertemporal transitions indicated by the sequence of solutions. Among these are the mechanism for pricing and adjusting the availabilities of vintage stocks and an improved market penetration algorithm which avoids the all-or-nothing (bang-bang) characteristic of linear programming models. The annualized capital charges for a technology impose a cost on the system as long as the age of a vintage stock is within its economic lifetime. Thus, fixed costs associated with previous investments are incurred regardless of whether or not it is optimal to operate, partially or fully, an older technology. "Unrealistic" displacements of relatively inefficient vintages are discouraged when the associated variable costs of alternative stocks are combined with the fact that immediate write-offs are precluded.

The procedure for market penetration requires, as input, "optimistic" penetration levels for each technology in each year. The penetration algorithm then incorporates the marginal values, implementation rates, and lag times from previous periods as well as the technological and market characteristics for the current period into the determination of more realistic bounds for the activity levels. The user-specified bounds are adjusted endogenously to account for the previous penetration and attractiveness of the technology.

3. SPECIAL FEATURES

3.1. Electric Sector Formulation

The formulation of the electric sector in TESOM incorporates the essentials of capacity expansion, generation, and transmission of electricity and by-product heat and their relation to the stochastic behavior of electric demands. On the electric conversion side, central station electric power plants deliver their high voltage electricity to a network "pool" from which the demand sectors draw energy to satisfy their needs. Conservation of energy relations link the electric generation side, with associated capacity requirements, to the demand side. Empirical evidence also shows that the demand for electricity is both seasonally and time-of-day dependent, and this has led to the natural partition of the year into six season and time-of-day divisions (winter day, winter night, summer day, summer night, fall-spring day, fall-spring night). On the demand side, the determinants of the load-duration curves which ultimately define capacity needs are "demand types" (base load, intermediate load, etc.) and not demands (space heating). Each of these demand types has a primary load factor and a set of secondary load factors. The primary load factor characterizes the major component of the demand type whereas the secondary load factors describe the demand type's stochastic behavior.

Each type of demand for electricity (e.g., space heating) is fractionally distributed among each of the relevant demand types. These demand types, coupled with the primary and secondary load factors, act to generate required capacity to meet the potential demand for electricity in each season-time-of-day period. Capacity is built to meet the maximum load period.

The key capacity equation which defines this process is described below. A complete list of equations is given by Kydes and Rabinowitz.⁵
Let

DE = the set of end-use energy demand categories, e.g., residential space heating, process heating, etc.;

DT = the set of demand types, e.g., base load, intermediate load, off-peak, etc.;

$E(DE)$ = the amount of electricity utilized by demand DE, and is a decision variable for the optimization process;

$ECS(DE)$ = the amount of centralized electricity utilized by demand DE;

DEC = the set of decentralized electricity generating plant types,
 e.g., total energy system, fuel cell, small wind system;
 E(DEC)(DE) = the amount of electricity generated by decentralized plant
 type DEC utilized by demand DE;
 te = the long distance, high voltage transmission efficiency;
 rmcs = the reserve margin required for the central station pool;
 rm(DEC) = the reserve margin required for decentralized plant type
 DEC;
 α (DE,DT) = the fraction of demand DE which is loaded onto demand type DT
 as specified by the user, e.g., 70% of process heat may be base
 loaded while 30% is intermediate load;
 S = the set of seasons: winter, summer, fall-spring;
 T = the time of day: day, night;
 μ (S,T,DT) = 1 if demand type DT occurs during season S and time-of-day T;
 it equals zero otherwise;
 tl(DT) = the fraction of the year demand type DT occurs, and, therefore,
 acts as the primary load factor;
 pf(DT) = the peak fraction for demand type DT, which defines the fraction
 of the demand that occurs at the secondary load factors;
 specifically, $[1+pf(DT)]^{-1}$ occurs at the primary load factor and
 $pf(DT)/[1+pf(DT)]$ occurs at the secondary load factors;
 f(i,DT) = the fraction of the secondary (peak) demand, associated with
 demand type DT, which is allocated to secondary peak interval i;
 slf(i,DT) = the load factor of the ith secondary (peak) interval associated
 with demand type DT.

The quantity $E(DE)*\alpha(DE,DT)$ represents the proration of demand DE to
 demand type DT; this multiplied by $\mu(S,T,DT)$ distributes each demand type
 component to the appropriate season-time-of-day combinations. Hence,

$$P(DT,S,T) = \frac{E(DE)*\alpha(DE,DT)*\mu(S,T,DT)}{1 + pf(DT)}$$

represents the quantity with a primary load factor of demand type DT con-
 tributed by demand DE during season S time-of-day T. The quantity

$$S(DT,S,T) = \frac{E(DE)*\alpha(DE,DT)*\mu(S,T,DT)}{1 + pf(DT)} * pf(DT)$$

represents the fraction of demand type DT contributed by demand DE occurring
 at the secondary (peaking) load factors.

The translation of the electricity demands to installed generation capacity requires division of each component of demand type by the appropriate load factors. The parameter $tl(DT)$ is the primary load factor. The quantity $S(DT, S, T) * f(i, DT)$ represents the portion of the secondary contribution which has a secondary load factor of $slf(i, DT)$. Capacity required to satisfy electricity demand $E(DE)$ during season S time-of-day T , omitting transmission losses, is

$$\frac{E(DE) * \alpha(DE, DT) * \mu(S, T, DT)}{[1 + pf(DT)] * tl(DT)} + \frac{E(DE) * \alpha(DE, DT) * \mu(S, T, DT)}{[1 + pf(DT)]} * pf(DT) * \sum_i \frac{f(i, DT)}{slf(i, DT)}.$$

Total capacity required prior to transmission losses to satisfy all demands for a particular season S and time-of-day T is

$$\sum_{DE} \sum_{DT} \frac{E(DE) * \alpha(DE, DT) * \mu(S, T, DT)}{[1 + pf(DT)] * tl(DT)} * \left[1 + pf(DT) * tl(DT) * \sum_i \frac{f(i, DT)}{slf(i, DT)} \right].$$

A function which translates a unit of electricity demand into units of installed capacity can be defined as

$$cm(DE, S, T) = \sum_{DT} \frac{\alpha(DE, DT) * \mu(S, T, DT)}{[1 + pf(DT)] * tl(DT)} * \left[1 + pf(DT) * tl(DT) * \sum_i \frac{f(i, DT)}{slf(i, DT)} \right].$$

Transmission and distribution losses can be incorporated by dividing the electricity utilization of each demand into electricity from the central station pool and electricity from each decentralized plant type. In addition, the incorporation of reserve margins yields a season S time-of-day T minimum electricity generating capacity of

$$\sum_{DE} \frac{ECS(DE) * cm(DE, S, T) * (1 + rmcs)}{te} + \sum_{\substack{DE \\ DEC}} E(DEC) (DE) * cm(DE, S, T) * [1 + rm(DEC)].$$

By capturing many of the essential components for building electric capacity, the electric formulation of TESOM represents a characteristic property of simulation models, that of reflecting a causal relationship between the demand for electricity (and its stochastic properties) and the requirements for installed capacity and reserve margins. Further, since capacity is installed to meet the maximum load season-time-of-day combination, this formulation also explicitly captures the reality that substantial idle capacity will exist during portions of the year (mostly in the fall and spring seasons) while the system will operate at nearly full capacity during other parts of the year. The current reality

where differentiated pricing policies do not exist is also reflected. The effect of differentiated pricing policies can easily be captured using the facilities of demand loading and differential "mark-ups" for electricity use by sector developed for TESOM.

3.2. Time-Dependent Adjustments

3.2.1. Inventory Characterizations Over Time. For the initial time period, preferably an historical year, TESOM requires, as input, specifications for each kind of conversion device averaged over all stock in place. These "averaged" specifications include economic lifetime, capital cost, operation and maintenance cost, conversion efficiency, fixed charge rate, and years remaining in the economic lifetime. For following periods, generally future years, similar specifications are required, but they are expected to characterize new stock introduced in that period rather than an average of vintaged stock.

Investment costs are specified in constant dollars and are annualized by using a capital recovery factor which depends on the device's fixed charge rate and economic life. Specifically, if FCR is the fixed charge rate (interest rate) specified by device and year of introduction and LIFE is the economic life also specified by device and year of introduction, then the capital recovery factor is given by

$$\frac{FCR}{1 - (1 + FCR)^{-LIFE}}$$

Investment costs may also be specified as time dependent to reflect learning curves for advanced technologies; that is, initial investments may be high but may decline to some asymptotic value. The model ensures that annualized capital charges for equipment introduced in earlier periods continue to be paid over the economic lifetime of the device. This discourages premature retirement of equipment. Operation and maintenance charges are accrued by installed devices only if they are utilized (at a rate proportional to their usage).

Capital charges cease at the end of the economic life of the device even though some of the devices installed are available for use past their economic life. For a given device type let

f = a fraction such that f*LIFE is the number of years of potential operation beyond the economic lifetime;

t = the set of periods where $t \in \{0,1,\dots\}$;
 Y_t = the year representing period t ;
 $A_{t,t-k}$ = the amount of stock installed in period $t-k$ available in period t ;
 $\Delta T_{t,t-k} = Y_t - Y_{t-k}$;
 α = a scale factor to be used in association with the tanh function.

When $t - k \neq 0$,

$$A_{t,t-k} = \begin{cases} A_{t-k,t-k} & \text{if } \Delta T_{t,t-k} \leq \text{LIFE} \\ 0 & \text{if } \Delta T_{t,t-k} \geq (1+f)*\text{LIFE} \\ A_{t-k,t-k} * \gamma_{t,t-k} & \text{otherwise} \end{cases}$$

where

$$\gamma_{t,t-k} \equiv 1 - \tanh \left[\frac{\alpha * (\Delta T_{t,t-k} - \text{LIFE})}{f * \text{LIFE}} \right]$$

Since the devices "introduced" in the initial period represent an "average" of vintage stock already in place, the availability of these devices over time is computed in a slightly different manner.⁵

To represent the aging of devices, the TESOM characterization of energy conversion technologies incorporates operation and maintenance cost growth rates, efficiency decay rates, and, for electricity generating plants, maximum availability factor decay rates. The maximum availability factor is defined to be the maximum fraction of the year that a given type of device could be in operation. All of these growth/decay rates are assumed to be linear.

3.2.2. Market Penetration Adjustments. Market penetration, the gradual substitution over time of a new commodity for one or more existing commodities in satisfying basic consumption or production needs, is a process which must be captured in some form if the projection framework for evaluating evolving energy systems is to have credibility. Hurter and Rubenstein⁶ provide a general literature search and review of work in this area. Condap and Kydes⁷ discuss market penetration representations in relation to an important new class of energy equilibrium models, of which a typical example is the U.S. Department of Energy Long-range Energy Analysis Program (LEAP)⁸ model. This class of models incorporates market penetration in an integrated framework.

The TESOM formulation permits a natural marriage of the traditional "stand-alone" market penetration algorithms^{9,10} with an integrated energy

system's framework by incorporating cost effectiveness, time lags, and market shares, determined from the TESOM model, as the key coupling mechanisms to link with the traditional approaches.

For each type of energy conversion device (e.g., electricity generating plants, end-use devices) TESOM adjusts the optimistic market penetration rates through time. Marginal values are generated for each technology in the penetration algorithm by imposing small lower bounds for each year. To keep the problem size manageable, those device types which remain at the lower bound are deleted from the inventory rather than carried forward. If two conversion devices are equally attractive, "within noise" (as specified by the user), then the new capacities installed for each technology are proportional to the ratios of capacity expansion potential implied by the optimistic penetration rates. The time lags, marginal values, and current market shares are used to adjust the market penetration trajectory.

To define the market penetration optimistic upper bound adjustment algorithm for a given device, let

- π_t = the marginal value for the technology in period t ;
- U_t = the user-specified optimistic upper bound on fuel consumption (production in the case of processing plants) over all vintage stock in period t ;
- AT_t = the measure of the technology's cost attractiveness for period t ;
- a = the user-specified weight for the importance of cost attractiveness on behavior;
- β = the user-specified weight for the importance of time lag on behavior;
- τ = the upper bound on marginal values above which the effects are unchanged for market penetration (required to normalize the penetration function);
- σ = the value below which marginal values are considered "noise" in the system;
- I_t = the optimistic upper bound on the fuel consumption (production) by stock that can be introduced in period t ;
- I'_t = the adjusted upper bound on fuel consumption (production) by newly introduced (period t) stock;
- $\gamma_{t,t-k}$ = the fraction of capacity still in existence as described in an earlier section;

$E_{t,t-k}$ = the conversion efficiency in period t of period $t-k$ vintage stock;

P = the most recent positive optimistic increment (see below).

The optimistic new stock upper bound is computed as

$$I_t = \begin{cases} U_t & \text{if } t = 0 \\ U_t - \left[\sum_{i=0}^{t-1} \left(I_i * \frac{E_{i,i}}{E_{t,i}} * \gamma_{t,i} \right) \right] & \text{if } t \geq 0. \end{cases}$$

The parameter P , which is initialized to zero before period 0, is set equal to I_t if $I_t \geq 0$.

The cost attractiveness indicator is derived from the technology's marginal value in the previous period as follows:

$$AT_t = \begin{cases} \tau & \text{if } \pi_{t-1} \geq \tau \\ -\tau & \text{if } \pi_{t-1} \leq -\tau \\ 0 & \text{if } |\pi_{t-1}| \leq \sigma \\ \pi_{t-1} & \text{otherwise.} \end{cases}$$

Finally the adjusted incremental upper bound is calculated as

$$I'_t = P * \tanh(a * AT_t + \beta * \Delta I_{t,t-1}).$$

The parameter P rather than I_t is used in the above calculation so that the overall penetration of the technology can continue to grow, up to the optimistic upper bound, even though the optimistic increment may be less than or equal to zero.

By fixing the availability of old stock as described in an earlier section and restricting the fuel consumption (production) by new stock to the adjusted increment upper bound, TESOM controls the market penetration of the technology for each vintage stock.

3.2.3 Supply-Demand Representations. Five supply representation options are currently available in TESOM. The user may specify, for each resource,

- quantities of a given resource at different prices by resource type and year; or
- cumulative supply curves or long-run marginal cost curves; or
- a short-run supply elasticity representation using, as input, an initial marginal price-quantity reference point with a supply elasticity for each resource and year; or
- a short-run supply elasticity representation overlayed on the cumulative supply curve; or
- short-run relative unit price increments between successive pools of resources with the initial "pool" reference price/quantity relationship determined from a cumulative supply curve.

The first option represents individual short-run supply curves (price/quantity relationships) for each year. The curves are represented by pools of resources available at specified prices (step functions).

The second option reflects the long-term effects of resource depletion and associated escalating costs due to declining discovery rates and is represented by using approximated long-run marginal cost curves which are year independent. Implicit is the assumption of that there is sufficient lead time for exploration and capital investment to reduce demand induced rents to zero. In other words, the marginal cost curve represents the replacement cost for the last unit of resource consumed.

The third option, the short-run elasticity representation, is derived from the definition of own-price elasticity; if $\tilde{P}$ is the reference marginal price for reference quantity $\tilde{Q}$ and σ is the short-run own-price elasticity, then the own-price elasticity is defined by

$$\frac{\partial Q/Q}{\partial P/P} = \sigma.$$

Since $C(Q)$, the total cost to the system for using quantity Q , is found by noting that the marginal rate of resource cost is given by

$$\frac{dC(Q)}{dQ} = P(Q) \text{ with } C(\tilde{Q}) = \tilde{P} \cdot \tilde{Q}, \text{ and } \sigma, \tilde{P}, \tilde{Q} \geq 0,$$

$$\text{then } C(Q) = \frac{\tilde{P} \cdot \tilde{Q}}{1+\sigma} \left[\sigma (Q/\tilde{Q})^{1+1/\sigma} + 1 \right].$$

If

q_i = the resource pool size available at a given price p_i where p_i is to be determined, and

Q_i = cumulative quantity of the resource used,

then $q_i = Q_i - Q_{i-1}$, $Q_0 = 0$.

If the relation that the total cost to the system, evaluated at the end-points of these pools of resources, equals the sum of the prices times the quantities used, i.e.,

$$C(Q_i) = \sum_{k=1}^i p_k q_k \text{ for all } i,$$

is enforced, then the price per unit resource for each pool is given by

$$p_k = [C(Q_k) - C(Q_{k-1})] / q_k, \quad k = 1, 2, \dots$$

The q_k are input; the Q_k are computed as

$$Q_k = Q_{k-1} + q_k, \quad k = 1, 2, \dots$$

The fourth alternative simply uses the marginal cost curve to derive the initial marginal price/quantity reference point for the short-run elasticity representation.

The fifth option requires that the user provide differences between the short-run quantity/price pools for a given resource. The cumulative supply curve provides the reference marginal price from which the short-term curve is completely defined.

A measure of uncertainty in future resource prices is introduced in the decision process by adding an adjustment factor which reflects near-term resource "price expectations." If $P(t-1)$ and $P(t-2)$ are the marginal prices of a resource for periods $t-1$ and $t-2$ respectively, and if $\Delta(t-1, t-2)$ is the time span between periods $t-1$ and $t-2$, then the adjusted price for period t is given by

$$P(t) + \frac{P(t-1) - P(t-2)}{\Delta(t-1, t-2)} * W$$

where

$$\frac{P(t-1) - P(t-2)}{\Delta(t-1, t-2)}$$

represents the annual price growth rate in the previous two periods and W is a "look ahead" weighting factor which implies price expectations by consumers.

REFERENCES

1. Hobbs, B. F. and Voelker, A. H., Analytical Multiobjective Decisionmaking Techniques and Power Plant Siting: A Survey and Critique, ORNL 5288, Feb. 1978.
2. Zionts, S. and Wallenius, J., An interactive programming method for solving the multicriteria problem, Management Science 22 (6), 652-63 (1976).
3. Kydes, A. S., The Brookhaven Energy System Optimization Model, Its Variants and Uses, BNL 50873, May 1978.
4. Behling, D. J., Dullien, R., and Hudson, E., The Relationship of Energy Growth to Economic Growth Under Alternative Energy Policies, BNL 50500, 1976.
5. Kydes, A. S. and Rabinowitz, J., The Time-Stepped Energy System Optimization Model, Methodology and Calibration, draft report, March 1980.

6. Hurter, A. P., Jr. and Rubenstein, A. H., Market penetration by new innovations: The technological literature, Technological Forecasting and Social Change, Vol. II, 197-221, Elsevier North-Holland, Amsterdam, 1978.
7. Condap, R. and Kydes, A. S., Modeling Market Penetration with Emphasis on the DFI Energy-Economy System, BNL 50999, April 1979.
8. Adler, R. J., Cazalet, E. G., Haas, S. M., Marshalla, R. A., Nesbitt, D. M. and Phillips, R. L., The DFI Energy-Economy Modeling System, draft report, Decision Focus Inc., Palo Alto, CA, 1978.
9. Fisher, J. C. and Pry, R. H., A simple substitution model of technological change, Technological Forecasting and Social Change, Vol. III, 75-88 (1971).
10. Peterka, V. and Fleck, F., The Dynamics of Energy Systems and the Logistic Substitution Model, Vol. II, Theoretical Part, International Institute for Applied Systems Analysis (IIASA), AR-78-1B, July 1978.

Figure 1. Reference Energy System, Year 1990