DESIGN OF A 30 HP SOLAR STEAM POWERED TURBINE

Topical Report

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WORK PERFORMED FOR THE SOLAR HEATING AND COOLING
RESEARCH AND DEVELOPMENT BRANCH, CONSERVATION AND
SOLAR APPLICATIONS, U.S. DEPARTMENT OF ENERGY

UNDER CONTRACT AC03-78ET20110 "CHILLER DRIVEN
BY A SOLAR STEAM-POWERED RANKINE ENGINE (SSPRE)"

PRINCIPAL INVESTIGATORS: Noam Lior and Hsuan Yeh

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ABSTRACT

The type-selection and complete design of a 30HP turbine which operates on superheated steam are described. The steam obtains most of its energy (the latent heat of vaporization) from solar energy at approximately 100°C (design) and is then superheated by a fuel energy source to 600°C (design). The turbine is designed for an exit pressure of 10.086 kPa(abs) corresponding to a condenser temperature of 46°C (115°F), as required for air-cooled condensers.

The turbine selected is of counter-rotation radial outflow design, employing 10 rows of reaction-type blading, and it operates at 15300 rpm (design).

The calculated turbine efficiency at design conditions is 75%, well above the efficiency of existing steam turbines of same power and steam conditions. Extensive calculations of the turbine efficiency at off-design conditions indicate a very small deterioration in efficiency, of the order of a few percent, over the practical range of operations.

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Nomenclature

a	acoustic velocity (m/sec)
A	flow area through the blading; in eqn (25), (27), cross-sectional area of the blade
b	radial width of blade row (mm)
c	tip clearance, i.e., the distance between blade tip and opposite wall
d	wheel diameter at blade path exit (mm)
f	fill factor, eqn (29)
h	specific enthalpy (kJ/kg)
I	blade moment of inertia
k	c_p/c_v
K	coefficient used in tip clearance correction, eqn (20)
l	blade height (mm)
m	mass flow (kg/sec)
M	bending moment on a blade
n	number of blade rows, also polytropic exponent as in $p/p^n = \text{constant}$
N	rpm
Q	volumetric flow (m^3/sec)
p	pressure (kPa)
P	power output (kJ/sec)
r	radius to any point (mm)
Re	Reynolds number
$\mathcal{R}$	reheat factor
U	wheel speed at any radius (m/sec)
U_∞	fluid velocity far from the wall
v	specific volume (m^3/kg)

V	absolute velocity (m/sec)
w_t	turbine work performed per unit mass (kJ/kg)
W	relative velocity (m/sec)
x	distance along the wall in the direction of flow
Z	blade section modulus

Greek Symbols

α	ratio of inlet to outlet diameter
α_1	angle between absolute flow entering rotor and tangential direction
β	ratio of clearance between rows to blade width
γ	dimensionless value, eqn (12)
δ_1	wall boundary layer displacement thickness
ΔA	tip leakage area
ϵ	ratio defined by eqn (10)
η	stage efficiency
η_t	overall turbine efficiency
ν	kinematic viscosity, also velocity ratio defined in eqn (30)
ρ	density
σ	bending stress of a blade
ξ	blade profile loss coefficient

Subscripts

e	turbine exit
i	turbine inlet
j	after j-th blade row
1,2,3	location after 1st, 2nd, 3rd row

Superscripts

*	design conditions
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1. Introduction

Under Contract with the Department of Energy (DOE Contract No.AC03-78ET 20110) we at the University of Pennsylvania have completed the design of a turbine to drive a chiller. This turbine is powered by superheated steam which receives the major portion of its heat from solar energy, hence the name SSPRE (Solar Steam Powered Rankine Engine) [1]. The complete system includes the collectors, the storage, the turbine, the condenser, several heat exchangers, as well as controls and piping. In the initial phase of our study, we have run through computers, numerous temperature and pressure ranges in the total Rankine Cycle System in order to optimize the output, while keeping the cost down. As a consequence of such calculations, we have finally selected the inlet and exit conditions for the turbine: inlet temperature 600°C (1112°F), inlet pressure atmospheric (14.7 psi) and exit pressure 1.463 psi (corresponding to condenser temperature of 115°F). The rated horsepower of the turbine is to be 30. The efficiency of the turbine, including an estimated leakage of 2.5% through the labyrinth seals at steam inlet should be at least 75%. Another requirement is that the efficiency should be kept reasonably well at off-design conditions, since much of the time the turbine inlet pressure and temperature may be quite different from design. Last but not least, the turbine should have a 10-20 year life and be inexpensive to build in quantity.

Early in the design phase, we decided to select a counter-rotation radial outflow design for the following reasons:

1. For this small power output, having an inlet diameter smaller than the outlet diameter results in inlet blade heights that are acceptable from the standpoint of efficiency. (In an axial flow full admission design, the blade height at inlet would be prohibitively small.)
2. Reaction-type blading of moderate turning angle, proven to be highly efficient in gas turbine design, can be used.

3. Radial flow permits operation with small tip clearances; perhaps even "wearing in" the runners is possible. With the small blading which is essential for this small power output, small tip clearances are extremely important for turbine efficiency.
4. The use of counter rotation is very advantageous. With the same turbine diameter and the same number of stages, the wheel speed for counter rotation is $1/2$ of that for single rotation. Since centrifugal stresses are proportional to the square of the wheel speed (for the same diameter), it is possible, with counter rotation, to go to much more favorable aspect ratios (height/width) for the blading.

It is gratifying that the type of turbine selected appears to be confirmed in a recent German paper [2].

In the preliminary design phase of the work, design parameters were systematically varied to achieve an optimum design. The number of blade rows (n) was varied, as well as the ratio of inner to outer diameter (α) and the outside diameter (d) of the rotor.

From the standpoint of turbine efficiency, a large aspect ratio (l/b) is favorable. This would point to a small rotor diameter and small blade sections. However, the smaller the sections, the more difficult it is to obtain accurate blade shapes. It was judged that a blade width of 6 mm was the smallest to be considered.

As explained later, bending stresses in the blading must also be taken into account. The smaller the diameter, the higher these stresses are for the same optimum work output per stage. The optimum work output per stage is determined by maximizing the stage efficiency. A design with 10 blade rows, and a ratio of inner diameter to outer diameter of 0.35 proved to be the optimum choice.

In the beginning, computations were made without a diffuser following the blade path. However, the use of a diffuser to reduce the kinetic energy leaving the rotor is obviously advantageous and has been incorporated in the final design.

In the following pages, we will present the major features of the design, followed by the methodology that we used in the design. The last section described the estimation of part-load performance which is important in optimizing the control strategy in the whole system.

2. Design Parameters

Major features of the final design of a radial, counter rotation turbine are as follows:

Turbine inlet temperature	600°C (1112°F)
Turbine inlet pressure	101.33 kPa (14.7 psi)
Steam inlet enthalpy	3704.62 kJ/kg
Steam inlet specific volume	3.98 m ³ /kg (63.76 ft ³ /lbm)
Pressure at exit of last row	9.562 kPa
Temperature at exit of last row	318.2°C
Enthalpy at exit of last row	3113.07 kJ/kg
Specific volume at exit of last row	29.14 m ³ /kg
Condenser temperature	46°C (114.8°F)
Turbine exit pressure, after diffuser	10.086 kPa (1.463 psia)
Turbine exit temperature, after diffuser	333°C (631.4°F)
Turbine exit specific volume	27.81 m ³ /kg (445.5 ft ³ /lbm)
Isentropic enthalpy drop	729.43 kJ/kg
Turbine work	565.31 kJ/kg
Rated horsepower	30 (computed horsepower = 32.8)
Steam mass flow	0.0445 kg/sec (0.0993 lbm/sec)
Number of rows	10
Outside diameter of last row	228.22 mm
Diameter at diffuser exit	269.24 mm
RPM	15,300
Wheel speed at last row exit	182.8 m/sec
Fluid relative velocity at last row exit	311.1 m/sec
Acoustic velocity at last row exit	608.8 m/sec

Fluid relative velocity at exit of 9th row	545.1 m/sec
Acoustic velocity at exit of 9th row	609.9 m/sec
Mach number at exit of 9th row	0.894
Blade width	6.053 mm
Blade height at entry	8.074 mm
Blade height at last row	7.95 mm
Smallest blade height (5th row)	4.24 mm
Profile efficiency (assumed)	0.82
Aspect ratio correction	0.947
Tip clearance correction (tip clearance 0.2 mm)	0.95
Stage efficiency	0.738
Reheat factor	1.065
Leaving loss correction	0.98
Turbine efficiency without seal leakage	0.770
Seal leakage	2.5%
Turbine efficiency with seal leakage	0.751

The dimensions of each blade row are given in Table 1 on the next page. Other design features are shown in Figures 1-4. Fig. 1 is the complete turbine layout. Fig. 2 shows the drive train. Fig. 3 and 4 are blade profiles.

Blade Radial Width $b=r_b-r_a$
 $b = 0.2383''$ (all rows)

BLADE ROWS	NO. OF BLADES	OPENING "o"	RADIUS "r" (INCHES)	"l" (inches)	pressure (psi)
1	40	0.119"	a 1.5731	0.331	14.70 inlet
			b 1.8114		14.57
			c 1.8412		
2	82	0.068"	a 1.8710	0.260	13.52
			b 2.1093		
			c 2.1391		
3	94	"	a 2.1698	0.210	12.17
			b 2.4072		
			c 2.4370		
4	106	"	a 2.4668	0.172	10.60
			b 2.7051		
			c 2.7349		
5	118	"	a 2.7647	0.169	8.87
			b 3.0030		
			c 3.0328		
6	130	"	a 3.0626	0.167	7.10
			b 3.3009		
			c 3.3307		
7	140	"	a 3.3605	0.175	5.37
			b 3.5900		
			c 3.6286		
8	152	"	a 3.6584	0.196	3.81
			b 3.8967		
			c 3.9265		
9	164	"	a 3.9563	0.243	2.50
			b 4.1946		
			c 4.2244		
10	146	0.151"	a 4.2542	0.326	1.49
			b 4.4925		
			c 5.3000		
		diffuser exit		0.506	1.36
					1.463

l = blade length
 l = distance between discs minus clearance

TABLE 1: Blade Row Dimensions

Figure 1 Turbine Layout

FIGURE 2

BASIC DRIVE TRAIN SCHEMATIC

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SECTIONS ARE DRAWN TO DIFFERENT SCALES

FIG. 3: BLADE CROSS SECTIONS FOR ROWS 2 THROUGH 10
(FOR DIMENSIONS SEE TABLE 1)

FIG. 4: BLADE CROSS SECTION FOR ROW 1
(FOR DIMENSIONS SEE TABLE 1).

3. Impeller Geometry

The relation between blade radial width b , wheel diameter at last blade exit d , number of blade rows n , ratio of inlet to outlet diameter α , and ratio of clearance between rows to blade width β is a matter of simple geometry:

$$\frac{b}{d} = \frac{(1-\alpha)/2}{n + (n-1)\beta} \quad (1)$$

provided b and β remain the same for all blade rows. A value of $\beta = 0.25$ has been assumed for this design as being typical, allowing for flow adjustment between blade rows. With $\beta = 0.25$, the above equation becomes

$$\frac{b}{d} = \frac{1-\alpha}{2.5n - 0.5} \quad (2)$$

4. Work Output and Mass Flow

With given turbine inlet pressure p_i , inlet temperature, T_i , and exit pressure p_e , the isentropic enthalpy drop Δh_s can be calculated from steam tables. The turbine work output w_t is then

$$w_t = \eta_t (\Delta h_s) \quad (3)$$

where η_t , the turbine efficiency, is calculated on the basis of considerations to be presented in a later section of this report.

With P being the power output of the turbine, the mass flow is

$$m = \frac{P}{w_t} \quad (4)$$

The volumetric flow at any blade row is then

$$Q = mv \quad (5)$$

where v is the specific volume corresponding to the pressure and temperature at that blade row.

5. Velocity Triangles and Turbine Output

Fig. 5: Blade Rows j and (j-1)

Except for the first and last row, the angles β_1 and β_2 are taken to be the same for all rows. Using here the subscripts e for exit, i for inlet to each row, the velocity triangles at r_{j-1} are:

Fig.6: Velocities leaving row j-1

Fig.7: Velocities entering row j

Since $V_{j-1,e} = V_{j,i}$, we have the following trigonometric relation:

$$(V_{u_{j,i}} + U_{j-1}) \tan \beta_2 = (V_{u_{j,i}} - U_{j-1}) \tan \beta_1 \quad (6)$$

and therefore

$$V_{u_{j,i}} = U_{j-1} \left(\frac{\tan \beta_1 + \tan \beta_2}{\tan \beta_1 - \tan \beta_2} \right) \quad (7)$$

At the exit of row j the velocity triangle is:

Fig.8: Velocities leaving row j

It can be concluded that:

$$V_{u_{j,e}} = U_j \left(\frac{\tan \beta_1 + \tan \beta_2}{\tan \beta_1 - \tan \beta_2} \right) \quad (8)$$

The work done per unit mass by row j is then according to Euler's equation:

$$w_j = (U_{j-1}^2 + U_j^2) \frac{\tan \beta_1 + \tan \beta_2}{\tan \beta_1 - \tan \beta_2} \quad (9)$$

We introduce the expression

$$c = \frac{\tan \beta_1 + \tan \beta_2}{\tan \beta_1 - \tan \beta_2} \quad (10)$$

and can now determine the total output of the turbine, noting that the inlet

velocity to the first row and the exit velocity of the last row are to be radial

$$w_t = U_n^2 \left(\frac{r_1^2 + r_2^2 \dots + r_{n-1}^2}{r_n^2} \right) 2\epsilon \quad (11)$$

We can form a dimensionless expression γ

$$\gamma = \frac{r_1^2 + r_2^2 \dots + r_{n-1}^2}{nr_n^2} \quad (12)$$

γ is plotted on Fig. 9 in terms of α and n . With this the turbine output per unit mass is

$$w_t = 2n\gamma\epsilon U_n^2 \quad (13)$$

After some study, the values $\beta_1 = 60^\circ$, $\beta_2 = 25^\circ$ were selected. With these $\epsilon = 1.736$, and the relative velocity at the end of row j is for $j \leq n-1$:

$$w_{j,e} = 3.019 U_j \quad (14)$$

For the last row:

$$w_{n,e} = 1.621 U_n \quad (15)$$

and the velocity component in radial direction is

$$v_{rj,e} = 1.276 U_j \quad (16)$$

The relative velocity at the exit of the row can be compared with the acoustic velocity at this point, which can be found from

Fig. 9: Dimensionless value γ (eqn 12) as a function of ratio of inner to outer radius of blading, α

$$a_j = \sqrt{k p_j v_j} \quad (17)$$

where k is the ratio of specific heats.

Continuity determines the blade height ℓ at exit of each row:

$$\ell_j d_j = \frac{Q_j}{\pi V_{rj,e}} \quad (18)$$

6. Profile Efficiency

The actual flow pattern through turbines is extremely complex, and recourse has been taken to segregating the various factors which contribute to energy losses. It is customary to start out with a "profile efficiency", which could presumably be achieved by very long blades (no end effects) and without considering leaving losses. With the kind of velocity transfer chosen here, profile efficiencies of 0.92 for the stage have been achieved at high Reynolds numbers. Since the blading for this radial turbine is small, and the Reynolds number low, we undertook to take a closer look at the "Reynolds number effect" on blade efficiency (viscous losses due to boundary layer build-up and the effect of wakes on subsequent rows).

The degree to which subsequent blade rows can handle strong wakes depends much on details of the blade shapes. It is not surprising, therefore, that various authors report wide variation among the losses experienced. At the end of this report we list some of the references we have studied [3-9].

The simplest way in which the profile loss coefficient has been stated is in relation to the loss coefficient ξ_0 obtained at some standard Reynolds number Re_0 (usually taken at $Re = 2 \times 10^5$), in the form

$$\xi = \xi_0 \left(\frac{Re_0}{Re} \right)^x \quad (19)$$

Here Re is the Reynolds number at which the turbine is to be operated. The exponent x has been given as varying from 0.03 to 0.4, which shows a large variation indeed. A representative value is 0.2. The Reynolds number for our turbine is somewhat below 10^4 so the "Reynolds number effect" is very important.

Several small turbines on which data have been reported, have been studied in this connection. Fig. 10 shows the profile efficiency plotted against Re based on the data from several different sources. On this basis we have chosen a profile efficiency of 0.82 for our turbine. It is felt that the blade profiles selected for this turbine are relatively insensitive to different angles of attack due to the influence of wall boundary layer and wakes from the preceding blade row, thus the chosen value for the profile efficiency should be on the conservative side.

Figure 10 Effect of Reynolds No. on Profile Efficiency

7. Aspect Ratio Correction

Even without clearance, the rotor walls induce a modification in the profile flow, which is the more detrimental the shorter the blades in relation to their chord. For convenience the "aspect ratio" is here expressed as ℓ/b where ℓ is the blade height and b the radial width of the blade row.

Some experimental data, obtained from axial flow tests (on wider blades) was available, and also there are data reported in the literature. Fig. 11 shows this information in graphical form. Based on the average aspect ratio of 0.96 for all blade rows, the aspect ratio correction for this turbine is 0.947.

Figure 11 Aspect Ratio Correction Coefficient

8. Tip Clearance Correction

The presence of a clearance between the blade tips and the walls means that a portion of the steam does not flow through the blade path. Reliable information was available from tests on large axial flow reaction blading at high Reynolds numbers.

From these tests it was found that a tip clearance correction could be determined, as

$$1 - K \frac{\Delta A}{A} \quad (20)$$

in which ΔA is the tip leakage area, A the flow area through the blading (the gauge area). The coefficient K is approximately constant and can be taken to equal to 1.5. If c designates the clearance, it will be seen that for short blades (considering cylindrical passages)

$$\frac{\Delta A}{A} = \frac{c}{\ell \sin \beta_2} \quad (21)$$

This result was applicable only for large blading at high Reynolds numbers, since it was based on the assumption that the flow through the tip clearance is not retarded at all by the wall boundary layers. In the present case of small blading and low Reynolds numbers, the wall boundary layer thickness δ is relatively large. A rough estimate is provided by the displacement thickness δ_1 :

$$\delta_1 = 1.7208 \sqrt{\frac{\nu x}{U_\infty}} \quad (22)$$

where ν is kinematic viscosity and x is the distance along the wall in the

direction of flow. In our case, the average x is of the order of 4 cm at the mean radius, and the average δ_1 is therefore of the order of 0.1 mm. Since the tip clearance in our design is to be less than 0.2 mm, it is clear that the flow in the tip clearance is reduced to only a small fraction of what it would be for large Reynolds numbers. The ratio of average c/l in our case is 0.034, and we have therefore selected a tip clearance correction of 0.95 as a conservative estimate.

9. Leaving Losses

At the exit of the last blade row, the absolute leaving velocity V_n is designed to be radial. The flow then passes through a vaneless diffusing passage before it leaves the turbine. Part of the kinetic energy associated with V_n , i.e. $V_n^2/2$, is thus recovered in the diffuser, resulting in an exit velocity $V_e < V_n$ leaving the turbine (see Fig. 12).

Fig. 12: Steam Expansion through Turbine, and Compression through Diffuser

p_e = pressure leaving turbine	w_t = turbine work
p_n = pressure at exit of last blade row	Δh_s = overall isentropic enthalpy
V_e = velocity after the diffuser	$\Delta h_{e'-n}$ = enthalpy drop e' to n
V_n = velocity entering the diffuser	

To be consistent with our method of calculation of other losses, a leaving loss correction is formed as follows:

$$\text{Leaving Loss Correction} = \frac{w_t}{w_t + \left(\frac{V_n^2}{2} - \Delta h_{e'-n} \right)} \quad (23)$$

In our final design, $w_t = 565.31$ kJ/kg, $V_n^2/2 = 26.24$ kJ/kg, $\Delta h_{e'-n} = 14.70$ kJ/kg. These values give a "Leaving Loss Correction" of 0.980.

10. Reheat Factor

In a multistage turbine the entropy increases from stage to stage, and the available enthalpy drop for the combined stages exceeds the isentropic enthalpy drop computed from inlet conditions and turbine exit pressure. This effect is taken into account by a "reheat factor"

$$\mathcal{R} = \frac{\eta_{\text{turbine}}}{\eta_{\text{stage}}} \equiv \frac{\eta_t}{\eta} \quad (24)$$

Assuming, for simplicity, that the stage efficiency is the same for all stages, and that the number of stages is large (both reasonable assumptions), the reheat factor can be computed. For convenience, it is plotted on Fig. 13.

For our case, with pressure ratio of 10.05, η_t of 0.738, the reheat factor is 1.065.

The various efficiencies and correction factors are related in the following way:

Stage efficiency η = profile efficiency x aspect ratio correction
x tip clearance correction

$$(0.738 = 0.82 \times 0.947 \times 0.95)$$

Turbine efficiency without seal leakage = stage efficiency x reheat factor
x leaving loss correction

$$(0.770 = 0.738 \times 1.065 \times 0.98)$$

$$R = \frac{1}{\eta} \frac{1 - \left(\frac{P_n}{P_i}\right)^{\eta \frac{k-1}{k}}}{1 - \left(\frac{P_n}{P_i}\right)^{\frac{k-1}{k}}}$$

where $k = 1.291$

Design:

P_i/P_n	EFFDS(η)	RF(R)
10.0426	0.7389	1.0650

Graph:

P_i/P_n	EFFDS(η)	RF(R)
5.0000	0.6600	1.0603
10.0000	0.6600	1.0854
20.0000	0.6600	1.1097
40.0000	0.6600	1.1334
5.0000	0.6900	1.0548
10.0000	0.6900	1.0774
20.0000	0.6900	1.0994
40.0000	0.6900	1.1207
5.0000	0.7200	1.0493
10.0000	0.7200	1.0696
20.0000	0.7200	1.0892
40.0000	0.7200	1.1081
5.0000	0.7500	1.0439
10.0000	0.7500	1.0618
20.0000	0.7500	1.0791
40.0000	0.7500	1.0958
5.0000	0.7800	1.0385
10.0000	0.7800	1.0541
20.0000	0.7800	1.0692
40.0000	0.7800	1.0836

FIGURE 13 Reheat Factor (R) as a Function of Pressure Ratio (P_i/P_n) for different turbine stage efficiencies η

11. Blade Centrifugal Stresses in a Radial Turbine

With A the cross sectional area of the blade section and ρ the mass density of the blade material, the bending moment due to centrifugal forces about the base of the blade, is

$$M = \frac{A\rho U^2 \ell^2}{2r_d} \quad (25)$$

where r_d is the radius to the center of mass of the blade of the row in question. For this design, the 9th row has the highest bending stress.

Figure 14: Blade Profile and Moments of Inertia

With $Z_{\min}$ the minimum section modulus and $Z_{\max}$ the maximum section modulus (Fig. 14), the maximum bending stress due to centrifugal force is:

$$\sigma = M \left(\frac{\sin \phi}{Z_{\min}} + \frac{\cos \phi}{Z_{\max}} \right) \quad (26)$$

For a typical blade shape, $A \propto b^2$ and $Z_{\min} \propto b^3$, $Z_{\max} \propto b^3$.

The expression

$$\sigma = \frac{\rho U^2 \ell^2}{2r_d} A \left(\frac{\sin \phi}{Z_{\min}} + \frac{\cos \phi}{Z_{\max}} \right) \quad (27)$$

can then be evaluated for the blade section selected, for which we find

$$\sigma = 8.33 \frac{\rho U^2 \ell^2}{r_d b} \quad (28)$$

The highest bending stress was found to be 1475 kg/cm^2 , which seems quite conservative even at the operating temperature of this turbine.

12. Row-by-Row Calculations

While the formulas listed above permit an evaluation of size and performance, it is necessary, in order to determine the blade length ℓ for each row, and to find the pressure throughout the turbine, to perform a row-by-row calculation.

Steam, under the conditions encountered in the turbine, is fairly close to saturation and cannot be taken to act like a perfect gas. To facilitate the computations, the actual steam properties, taken from the steam tables, were expressed in mathematical form.

With assumed flow angles, the enthalpy drop for each row can be found. For a given efficiency, the isentropic enthalpy drop is then also known and the pressures between the rows can be determined. The specific volume at each point can then also be computed and finally the necessary blade height for each row, necessary to pass the desired flow, can be found. In this manner, Table 1 on page 8 was constructed.

13. Off Design Performance

Of interest for the overall operation of the equipment are power output, efficiency and mass flow of the turbine when the steam conditions and the turbine rpm are different from the design conditions.

The control system envisaged is to keep the turbine back pressure constant. Thus, the chief variables to be considered are the inlet pressure, inlet temperature and the turbine rpm.

In the industry, one procedure used to compute off-design performance is to start out with available multistage test data on turbine efficiency at off-design conditions (characterized by the ratio of mean blade speed and isentropic velocity per row). By taking into account a large number of parameters in the blade configuration and the steam conditions, the mass flow can be determined. Having the efficiency and the steam flow, the power output of the turbine can then be calculated. A paper by one of the authors [10] described one such method.

A simpler approach that has proven successful in practice will now be developed here. The method is based on an earlier work by one of the authors (C. A. Meyer). Consider a typical velocity triangle for a symmetrical stage (i.e., 50% reaction) in an axial-flow multi-stage turbine (see Figure 15).

Fig. 15: Velocity and expansion diagrams for off-design calculations

Also shown is the h-s diagram for the stage.

Let A be the annular area, ρ the fluid density, f the fill factor, and α_1 the angle between the absolute flow entering rotor and the tangential direction, the mass flow

$$m = A f \rho V_1 \sin \alpha_1 \quad (29)$$

Following conventional turbine practice, we define two velocity ratios, v and v_s , in the following manner

$$v = \frac{U}{V_1} \quad v_s = \frac{U}{C} \quad (30)$$

where C is defined by $C^2/2 = \Delta h_s$.

$$\text{or} \quad C = \sqrt{2\Delta h_s} \quad (31)$$

This newly defined velocity C is related to V_1 through the stage efficiency η in the following way: (Note $V_1^2 - V_2^2 = 2V_1U \cos \alpha_1 - U^2$)

$$\eta = \frac{\Delta h}{\Delta h_s} = \frac{V_1^2 - V_2^2}{C^2} = \frac{V_1^2}{C^2} v(2 \cos \alpha_1 - v) \quad (32)$$

But, from the definition of v and v_s

$$\frac{V_1^2}{C^2} = \frac{v_s^2}{v^2} \quad (33)$$

Hence

$$\eta = \frac{v_s^2}{v} (2 \cos \alpha_1 - v) \quad (34)$$

or, solving for $\frac{v_s}{v}$

$$\frac{v_s}{v} = \left(\frac{\eta}{v_s} + v_s \right) \left(\frac{1}{2 \cos \alpha_1} \right) = \frac{V_1}{C} \quad (35)$$

Now solving for V_1

$$V_1 = \left(\frac{\eta}{v_s} + v_s \right) \left(\frac{1}{2 \cos \alpha_1} \right) \sqrt{-2\Delta p / \rho} \quad (36)$$

The minus sign is there because Δp is a pressure drop, hence is negative.

Substituting into $m^2 = A^2 f^2 \rho^2 V_1^2 \sin^2 \alpha_1$, the result is

$$\frac{m^2}{A^2} = f^2 \left(\frac{\eta}{v_s} + v_s \right)^2 \frac{\tan^2 \alpha_1}{2} (-\rho \Delta p) \quad (37)$$

The above expression is for one stage. We now sum up these same expressions for all stages and consider that η , v_s , α_1 are the same for all stages. Then, if the number of stages is large, the summation $\Sigma (-\rho \Delta p)$ may be taken as an integration $\int (-\rho dp)$ between inlet p_i and exit p_e .

$$-\int \rho dp = \left(\frac{n}{n+1} \right) p_i \rho_i \left[1 - \left(\frac{p_e}{p_i} \right)^{(n+1)/n} \right] \quad (38)$$

where n is the exponent in the polytropic relation $p/\rho^n = \text{constant}$.

It can be shown from the definition of η that

$$\frac{n}{n+1} = \frac{1}{2-\eta \frac{k-1}{k}} \quad (39)$$

where k is the ratio of specific heats. Thus, the final expressions is:

$$\frac{m\sqrt{RT_i}}{p_i} = \sqrt{\frac{1}{2(2-\eta\frac{k-1}{k})}} \sqrt{\frac{1}{\sum \frac{1}{A^2}}} \sqrt{1 - \left(\frac{p_e}{p_i}\right)^{\frac{2-\eta\frac{k-1}{k}}}} \left(\frac{\eta}{v_s} + v_s\right) f \tan \alpha_1 \quad (40)$$

Many tests have shown that " f " is constant and close to unity over the entire unstalled operating region of the blading ($0.3 < v_s < 1.5$). For a multi-stage turbine, the parameter v_s in the above expression should be an averaged value over all stages,

$$v_s = \sqrt{\frac{\sum (u^2/2)}{(\Delta h_s) \mathcal{R}}} \quad \text{for single-rotation turbine} \quad (41)$$

$$v_s = \sqrt{\frac{\sum (2u)^2/2}{(\Delta h_s) \mathcal{R}}} \quad \text{for counter-rotation turbine} \quad (42)$$

where Δh_s is now the overall isentropic enthalpy drop through the turbine and $\mathcal{R}$ is the reheat factor. The second expression is the one used here and yields a design value $v_s^* = 0.663$.

The above result is a modification, and we believe, an improvement of the well known "Stodola ellipse" since the influences of rpm and stage efficiency are included here in addition to inlet and exit conditions.

It enables us to calculate the change in mass flow under different inlet and exit conditions, i.e., when p_i , T_i or p_e varies. It also shows the influence of v_s which in turn is dependent on rpm and overall isentropic enthalpy drop; the latter again is a function of p_i , T_i and p_e . Furthermore, η in general varies with v_s . From a large number of experimental data (cf. [11,12] as well as some 35 internal reports by C.A. Meyer at the Westinghouse Electric Corp.), we have been successful in correlating the variation of stage efficiency η with v_s for well designed reaction type blading such as ours when v_s is different from its design value v_s^* . The result in terms of ratios with the design value η^* and v_s^* is shown in Table 2:

v/v_s^*	η/η^*
0.6818	0.9255
0.7955	0.9691
0.9091	0.9936
1.0000	1.0000
1.1364	0.9926
1.2500	0.9766
1.3636	0.9532
1.4773	0.9170

Table 2: Off-design stage efficiency ratio as a function of the off-design velocity ratio.

After mass flow m and stage efficiency η are obtained, the power output is then

$$\frac{P}{P^*} = \frac{m}{m^*} \frac{R}{R^*} \frac{\Delta h_s}{\Delta h_s^*} \frac{\eta}{\eta^*} \quad (43)$$

In summary, the above formulation enables us to calculate the dependent variables at various off-design values of the independent variables:

Dependent variables: mass flow m
stage efficiency η
turbine efficiency η_t
power output P

Independent variables: inlet pressure p_i
inlet temperature T_i
exit pressure p_e
rpm

Based on the above formulation, the off-design performance of our designed turbine was calculated. The results are shown in Figures 16-29. Fig. 16, 17, and 18 show the turbine efficiency η_t , the mass flow, and the power output for various values of speed and inlet pressure, while keeping the inlet temperature constant at 600°C. Figures 19-29 show the same dependent variables for various inlet temperatures while keeping the inlet pressure constant (19-21), the rpm constant (22-24), and both inlet pressure and rpm constant (25-29). In Figures 19-29, the lowest temperature of each curve (marked by a black circle) is the lowest temperature to retain a dry turbine after expansion.

It is noteworthy that in our design the turbine efficiency is well maintained at vastly different operating conditions.

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FIGURE 16

TURBINE EFFICIENCY
INLET TEMPERATURE: 600°C
CONDENSING TEMPERATURE: 46°C
VARIABLE INLET PRESSURE

FIGURE 17

MASS FLOW AT OFF - DESIGN
INLET TEMPERATURE: 600°C
CONDENSING TEMPERATURE: 46°C
VARIABLE INLET PRESSURE

FIGURE 18

POWER AT OFF-DESIGN
INLET TEMPERATURE: 600°C
CONDENSING TEMPERATURE: 46°C
VARIABLE INLET PRESSURE

FIGURE 19

EFFICIENCY
DESIGN INLET PRESSURE (101.33 kPa)
CONDENSING TEMPERATURE: 46°C
VARIABLE SPEED

FIGURE 20

FIGURE 21

POWER
DESIGN INLET PRESSURE (101.33 kPa)
CONDENSING TEMPERATURE: 46°C
VARIABLE SPEED

FIGURE 22

EFFICIENCY
 DESIGN SPEED (15300 RPM)
 CONDENSING TEMPERATURE: 46°C
 VARIABLE INLET PRESSURE

FIGURE 23

MASS FLOW
DESIGN SPEED (15300 RPM)
CONDENSING TEMPERATURE: 46°C
VARIABLE INLET PRESSURE

FIGURE 24

POWER
DESIGN SPEED (15300 RPM)
CONDENSING TEMPERATURE: 46°C
VARIABLE INLET PRESSURE

FIGURE 25 EFFICIENCY, MASS FLOW & POWER AS A FUNCTION OF TURBINE INLET TEMPERATURE AT SPEED 13114 RPM

FIGURE 26 EFFICIENCY, MASS FLOW AND POWER AS A FUNCTION OF TURBINE INLET TEMPERATURE AT DESIGN SPEED 15300 RPM

FIGURE 27 EFFICIENCY, MASS FLOW & POWER AS A FUNCTION OF TURBINE INLET TEMPERATURE AT SPEED 17486 RPM

FIGURE 28 EFFICIENCY, MASS FLOW & POWER AS A FUNCTION OF TURBINE INLET TEMPERATURE AT DESIGN SPEED 15300 RPM & $P_i = 47.36$ kPa

FIGURE 29 EFFICIENCY, MASS FLOW & POWER AS A FUNCTION OF TURBINE INLET TEMPERATURE AT DESIGN SPEED 15300 RPM & $P_i = 198.54$ kPa