Acceleration of Hydrogen Ions and
Conic Formation along Auroral Field Lines

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Electrostatic ion cyclotron turbulence and the formation of ion conics at low altitudes (≈ 1500 km) along auroral field lines have been investigated analytically and by plasma numerical simulations. Ion cyclotron waves are assumed to be driven unstable by the up-going cold ionospheric electrons associated with the downward auroral current. When the electron drift speed is comparable to the electron thermal speed, it is found that the large amplitude, $e\phi/T_e \approx 1$, coherent, $\omega = \Omega_i$, ion cyclotron waves should exist along auroral field lines at low altitudes extending ≈ 500 - 1000 km. Ion conics are associated with the cyclotron turbulence and the ion bulk temperature is found to increase a factor of 10 of the initial ionospheric temperature, while the temperature of the high energy tail can be as much as a factor of 100 of the ionospheric temperature. Theory and simulations agree well.

DISCLAIMER

I. Introduction

The S3-3 observations have revived interest in the physics of the auroral field lines, and have added a body of new significant observations. To note but a few, observations have included large amplitude ion cyclotron waves ($e\phi/T_e \approx 1$) associated with upward moving ion beams at high altitudes above 6000 km (Kintner et al., 1979), strong field aligned currents of $\approx 1-5 \mu A/m^2$ (Mozer et al., 1980) and the acceleration of ions and the formation of conic distributions (Gorney et al., 1981) at lower altitudes. Such ion conic distribution represents ions with pitch angles between 90° and 130° which are moving upward, away from the ionosphere. Analysis of the phase space distribution of these ions shows them to be of ionospheric origin, which indicates that some mechanism is responsible for accelerating cold ionospheric ions perpendicular to the geomagnetic field (Croley, 1978). This energization could be due to acceleration by the electric field perpendicular to the magnetic field that has strong spatial gradients (Cole, 1976) or either spatially confined or time varying electric fields (Swift, 1975; Mozer et al., 1980) or due to heating by resonant wave particle interactions. The S3-3 data indicate that acceleration by the electric field cannot produce the observed ion conics because the spatial distributions of electrostatic shocks and conics are quite different. In addition it is clear that wave-particle interactions must contribute to the total ion energization, since energies of observed particles are in excess of the measured perpendicular electric potential across the auroral zone.

Kindel and Kennel (1971) have shown that for currents carried by a Maxwellian electron distribution, ion cyclotron waves have the lowest threshold among the current driven instability in an isothermal plasma ($T_e \approx T_i$), and the observed current levels are well above the threshold for the electrostatic ion

cyclotron waves. The electrostatic ion cyclotron waves have been seen at high altitudes ≈ 5000 km, associated with ion beams (Kintner et al., 1979). Unfortunately it is difficult to identify the waves at low altitude due to a Doppler shift of the wave frequency seen from the satellite where it is going at a maximum speed.

There has been a great deal of interest in the acceleration of ions along auroral field lines by ion cyclotron waves. Following the pioneering work of Drummond and Rosenbluth (1962) different authors (Dakin et al., 1976; Okuda et al., 1981; Pritchett et al., 1981) considered the ion heating within the quasilinear theory in an infinite homogeneous plasma. It was found that, for $v_{de} \approx v_e$ where v_{de} is the electron drift speed along the magnetic field and v_e is the thermal speed, the saturation level was lower ($e\phi/T_e \ll 1$) due to the formation of a plateau on the electron velocity distribution. This low amplitude at saturation resulted only in modest heating of the ions.

Palmadesso et al. (1974) argued that processes such as pitch angle scattering operative in the environment of the ionosphere and magnetosphere could inhibit electron plateau formation. Amplitudes of ion cyclotron waves much larger than those predicted by Drummond and Rosenbluth (1962) had already been observed in Q machines (Buchel'nikova and Salimov, 1969).

Okuda and Ashour-Abdalla (1981) developed a simulation model for the ion cyclotron waves in which plateau formation is inhibited by a continuous flow of fresh electrons with an initial drifting Maxwellian (uniform recycling model). It was found that, for $v_{de} \approx v_e$, the saturation level of the ion cyclotron waves is high $e\phi/T_e \gtrsim 1$, resulting in the intense heating of the ion perpendicular temperature, $T_{i\perp} \gg T_{i\parallel}$.

In a more recent attempt, Dusenberry and Lyons (1981) considered ion heating due to ion cyclotron waves excited by the upgoing thermal electrons in

the low altitude auroral zone. Assuming a wave spectrum for the ion cyclotron turbulence, they calculated the heating of test particles using a quasilinear diffusion coefficient. However, the theory developed is not self-consistent, since the wavespectrum is given and, in particular, the heating of ions depends on the interaction time in which the ions are accelerated while passing through the low altitude auroral zone. Dusenberry and Lyons (1981) took the length of the interaction so as to give a heating rate in reasonable agreement with the S3-3 observations.

In this paper we shall assume that ion cyclotron waves are excited by drifting electrons associated with the return flow in the auroral zone and attempt to explain the acceleration of ionospheric ions and the resultant formation of the conic distributions at low altitudes. ≈ 1500 km. We present results obtained from a uniform recycling model as well as from a newly developed simulation model in which the plasma source at the ionosphere is incorporated realistically through the boundary conditions. Since the source is located at one end of the field lines only, the plasma is inhomogeneous along the magnetic field so that the convective nature of the instability as well as the ion heat loss along field lines are included in the new model.

Following this introduction we present a theoretical analysis of ion cyclotron waves and the associated heating of ions (Sec. II). In Sec. III we describe two different simulation models and discuss their applicability. The simulation results are presented in Sections IV and V. Finally in Sec. VI we conclude by discussing the heating of the ions and the evolution of the ion cyclotron waves on auroral field lines and the magnetosphere.

II. Theoretical Analysis

1. Linear Theory

Since the linear theory has been studied extensively both analytically and numerically (Drummond and Rosenbluth, 1962; Kindel and Kennel, 1971; Ashour-Abdalla and Thorne 1978; Okuda et al., 1981) for an isotropic ion species as well as for an anisotropic Maxwellian distribution for the ions (K.F. Lee, 1972; Okuda and Ashour-Abdalla, 1981), we shall discuss briefly the consequences obtained from the linear theory.

The dispersion relation for an anisotropic bi-Maxwellian ions and a drifting Maxwellian electrons along a uniform external magnetic field may be given by (K.F. Lee, 1972; Okuda and Ashour-Abdalla, 1981)

$$1 + \frac{1}{k^2 \lambda_{e\parallel}^2} \left[1 + \frac{\omega - k_{\parallel} v_{de}}{k_{\parallel} v_{te\parallel}} Z \left(\frac{\omega - k_{\parallel} v_{de}}{k_{\parallel} v_{te\parallel}} \right) \right] + \frac{1}{k^2 \lambda_{i\parallel}^2} \left[1 + \sum_{n=-\infty}^{\infty} \frac{\omega - n \Omega_i (1 - T_{i\parallel}/T_{i\perp})}{k_{\parallel} v_{ti\parallel}} \Gamma_n Z \left(\frac{\omega - n \Omega_i}{k_{\parallel} v_{ti\parallel}} \right) \right] = 0, \quad (1)$$

where v_{de} is the electron drift speed along the magnetic field, Ω_i is the ion gyrofrequency, $v_{ti\parallel} = (2T_{i\parallel}/m_i)^{1/2}$, $v_{ti\perp} = (2T_{i\perp}/m_i)^{1/2}$, $v_{te\parallel} = (2T_{e\parallel}/m_e)^{1/2}$ are the ion parallel, perpendicular and electron parallel thermal speeds, and $\Gamma_n(\mu_i) = \exp(-\mu_i) I_n(\mu_i)$, $\mu = k_{\perp}^2 v_{ti\perp}^2 / 2\Omega_i^2 \equiv k_{\perp}^2 \rho_i^2$ where I_n is the modified Bessel function of order n , $\rho_i = (T_{i\perp}/m_i \Omega_i^2)^{1/2}$ is the ion gyroradius, $\lambda_{e\parallel} = v_{te\parallel} / \sqrt{2} \omega_{pe}$, $\lambda_{i\parallel} = v_{ti\parallel} / \sqrt{2} \omega_{pi}$ are the Debye lengths of electrons and ions, and Z is the plasma dispersion function.

Near the fundamental frequency, $\omega \approx \Omega_i$, and for $T_{e\parallel}/T_{i\perp}$ not too small, the real part of Eq. (1) is reduced to

$$\frac{\omega - \Omega_i}{\Omega_i} = \frac{T_{i\parallel}}{T_{i\perp}} \frac{\Gamma_1}{T_{i\parallel}/T_{e\parallel} + (1-\Gamma_1)T_{i\parallel}/T_{i\perp} - (1-\Gamma_0)/\mu_1 (T_{i\parallel}/T_{i\perp})} \quad (2)$$

For $T_{i\parallel}/T_{i\perp} = 1$ and $T_{e\parallel}/T_{i\parallel} \approx 1$, we have

$$\frac{\omega - \Omega_i}{\Omega_i} = \frac{\Gamma_1}{T_i/T_e + 1 - \Gamma_1 + (1-\Gamma_0)/\mu} \quad (3)$$

in agreement with the result of Kindel and Kennel (1971). For $T_{i\parallel}/T_{i\perp} < 1$, Eq. (2) approaches

$$\frac{\omega - \Omega_i}{\Omega_i} = \frac{T_{i\parallel}}{T_{i\perp}} \frac{T_{e\parallel}}{T_{i\parallel}} \Gamma_1 \quad , \quad (4)$$

indicating that the wave frequency approaches the cyclotron frequency for anisotropic ions where $T_{i\perp} \gg T_{i\parallel}$.

It is important to recognize that the group velocity of the ion cyclotron waves is small and is a fraction of ion thermal speed. This can be seen either from the numerical solutions of dispersion relation for different wavenumbers (Okuda et al., 1981) which indicate the frequency varies very little with respect to both $k_{\perp}$ and $k_{\parallel}$. For long wavelengths the group velocity may be calculated from Eqs. (3) and (4). For $k_{\perp} \rho_i < 1$,

$$\frac{\partial \omega}{\partial k_{\perp}} \approx (k_{\perp} \rho_i) v_{ti\parallel} < v_{ti\parallel} \quad (5)$$

or

$$\frac{\partial \omega}{\partial k_{\parallel}} \approx 2 \frac{T_{e\parallel}}{T_{i\perp}} (k_{\perp} \rho_i) v_{ti\parallel} < v_{ti\parallel} \quad . \quad (6)$$

The group velocity for $k_{\perp} \rho_i \approx 1$ and the parallel group velocity $\partial\omega/\partial k_{\parallel}$ must in general be calculated from the dispersion relation numerically using Eq. (1) (Okuda et al., 1981). The smallness of the group velocity of the electrostatic ion cyclotron waves must be taken into account when calculating the turbulence level and production of energetic ions on the low altitude auroral regions associated with the upstream electrons.

We have shown earlier that, in the absence of quasilinear plateau formation on the electron velocity distribution, the nonlinear saturation of ion cyclotron instabilities takes place as a result of the formation of a conic distribution of ions in which $T_{i\perp}$ is much greater than $T_{i\parallel}$ (Okuda and Ashour-Abdalla, 1981). The saturation level was found to be $e\phi/T_e \approx 1$, which is much greater than that obtained from plateau formation. The resultant anisotropy of temperature $T_{i\perp}/T_{i\parallel} \approx (10 \sim 20)$ is in good agreement with the particle data taken from S3-3 spacecraft (Gorney et al., 1981).

As is seen from Eq. (4), the wave frequency ω approaches Ω_i as the ion temperature anisotropy $T_{i\perp}/T_{i\parallel}$ increases thereby increasing the ion cyclotron damping on the waves. As the wave amplitude grows, ions are heated perpendicularly as a result of cyclotron damping. The instability saturates and the ion heating stops at a point where the ion temperature anisotropy is large enough to bring the system toward the marginal stability for the given electron drift speed v_{de} . Such a critical temperature anisotropy or equivalently the maximum perpendicular ion heating for a given electron drift may be calculated from the marginal stability analysis obtained from the imaginary part of Eq. (1) for real frequency ω . For $T_e/T_{i\perp} \approx 1$, this is given by

$$\frac{v_{de}}{v_{te\parallel}} = \frac{\omega}{k_{\parallel} v_{te\parallel}} + \frac{T_{e\parallel}}{T_{i\parallel}} \frac{\omega - \Omega_i (1 - T_{i\parallel}/T_{i\perp})}{k_{\parallel} v_{ti\parallel}} \Gamma_1 \exp\left[-\left(\frac{\omega - \Omega_i}{k_{\parallel} v_{ti\parallel}}\right)^2\right] \quad (7)$$

for a bi-Maxwellian distribution. For $T_e/T_{i\perp} \ll 1$, it is necessary to include the ion Z function since the large argument expansion may not hold (K.F. Lee 1972). Inspection of Eq. (7) reveals that the critical drift speed increases as the ion temperature anisotropy $T_{i\perp}/T_{i\parallel}$ increases.

Figure 1 shows the numerically calculated maximum ion temperature anisotropy obtained from the marginal stability analysis for a given electron drift. It is seen from Fig. 1 a large temperature anisotropy $T_{i\perp}/T_{i\parallel} \approx 10$ can be obtained for a reasonable electron drift speed $v_{de} \approx v_{te}$ seen on the low altitude auroral lines for $T_e \approx T_{i\parallel}$. Since the ion parallel temperature $T_{i\parallel}$ changes much less than $T_{i\perp}$ as a result of ion cyclotron instability, because of the fact that the ion cyclotron waves are destabilized exclusively for nearly perpendicular propagation, the increase of $T_{i\perp}/T_{i\parallel}$ represents the perpendicular heating $T_{i\perp}$ of ions.

2. Ion Heating on Auroral Field Lines

We have shown that the current-driven ion cyclotron waves heat the ions perpendicularly causing a conic velocity distribution which by itself shuts off the instability. The observed conic distributions are in good agreement with the theoretical analysis given here and earlier simulation results (Okuda and Ashour-Abdalla, 1981).

Let us consider here in some detail the ion heating on the auroral field lines due to the ion cyclotron waves destabilized by the upstreaming ionospheric electrons whose drift speed exceeds the marginal drift speed. The model is sketched in Fig. 2 in which the source of drifting electrons, the

ionosphere, is located at $x < 0$ and the upstreaming electrons propagate along the magnetic field, $x > 0$.

As the electrons stream through the ions, the electron velocity distribution develops a plateau at $x = l$ by exciting the ion cyclotron waves. For $x > l$ there is no instability and therefore no heating of ions takes place. Since the ionosphere is sending the electrons continuously, the ion perpendicular temperature for $0 < x < l$ keeps on increasing until it reaches a point where the ion cyclotron waves are stable for the given electron drift. Beyond that time, the ionospheric electrons can reach $x = l$ without forming a plateau since the ion cyclotron waves are now stable for $0 < x < l$ and the same process repeats for $x > l$. We have assumed that the ion heat loss along and across field lines as well as the energy loss by wave convection is much smaller than the ion heating by the cyclotron waves. This is examined later.

Let us first calculate the time required to heat the ions for $0 < x < l$ to the marginal stability point. To do so we estimate the time required to develop a plateau on the electrons. From a quasilinear theory (Drummond and Rosenbluth, 1962), the electron distribution may be determined from

$$\frac{\partial f^e(v_{\parallel})}{\partial t} = \frac{\partial}{\partial v_{\parallel}} \beta \epsilon \frac{\partial f^e(v_{\parallel})}{\partial v_{\parallel}}, \quad (8)$$

where

$$\beta = \frac{L}{7} \left(\frac{e}{m}\right)^2 \frac{v_{ti\parallel}^2}{v_{\parallel}} \quad (9)$$

and

$$\epsilon = \sum_{k_{\perp}} |E(k_{\perp}, v_{\parallel})|^2 \quad (10)$$

The diffusion time for plateau formation may be estimated from (Drummond and Rosenbluth, 1962)

$$\tau = \frac{(\Delta v_{\parallel})^2}{D} \approx \frac{v_{de}^2}{\beta \epsilon},$$

so that the distance λ in Fig. 2 may be given by

$$\lambda = \langle \tau_D v_{\parallel} \rangle = \frac{v_{de}^3}{\langle \beta \epsilon \rangle} \quad (11)$$

$$\begin{aligned} &= \frac{v_{de}^3}{2\pi(v_{de}/v_{te})^3 \Omega_i^2 (T_e/T_i) v_e^2} \\ &= \rho_e \frac{m_i}{m_e} \frac{T_i}{T_e} = \rho_i \left(\frac{m_i}{m_e} \right)^{1/2} \left(\frac{T_i}{T_e} \right)^{1/2}. \end{aligned} \quad (12)$$

It is interesting to observe that if we use $\tau_D^{-1} \approx \gamma \approx \Omega_i (T_e/T_i) (v_{de}/v_{te})$, then $\lambda = \langle \tau_D v_{de} \rangle$ will give the same result. The energy lost from the electrons by plateau formation during this time is calculated from

$$\Delta E^e = \frac{m_e n}{2} \int (f_0^e - f_{\infty}^e) v_{\parallel}^2 dv_{\parallel} dS, \quad (13)$$

where f_0^e is the initial drifting Maxwellian, and f_{∞}^e is the final electron distribution with a plateau, and S is the cross-sectional area of the electron beam. Equation (13) must be evaluated numerically for a given v_{de} and v_{te} in general. For a small drift, $v_{de}/v_{te} \ll 1$, we find

$$\Delta E^e \approx \frac{m_e n \lambda S v_{de}^5}{30 v_t^3},$$

and for a large drift, $v_{de}/v_{te} \gg 1$,

$$\Delta E^e \approx \frac{m_e n \lambda S}{3} v_{de}^2.$$

In the absence of energy loss, such as due to wave convection and ion heat conduction, most of the electron energy must go to the ion perpendicular energy so that the ion temperature increase during the diffusion time τ_D would be

$$\begin{aligned} \Delta T_{i \perp} &= \Delta E^e / n \lambda S \\ &= \frac{m_e}{30} \frac{v_{de}^5}{v_{te}^3} \quad \text{for } v_{de} \ll v_{te} \\ &\approx \frac{m_e}{3} v_{de}^2 \quad \text{for } v_{de} \gg v_{te}. \end{aligned} \quad (14)$$

The ion heating rate will then be given by

$$\begin{aligned} \frac{dT_{i \perp}}{dt} &\approx \frac{m_e}{30} \Omega_i \frac{T_e}{T_i} \frac{v_{de}^6}{v_{te}^4} \quad \text{for } v_{de} \ll v_{te} \\ &\approx \frac{m_e}{3} \Omega_i \frac{T_e}{T_i} \frac{v_{de}^3}{v_{te}} \quad \text{for } v_{de} \gg v_{te}. \end{aligned} \quad (15)$$

If the heating rate given by Eq. (15) is greater than the ion heat loss rate along and across auroral magnetic field, then the ion temperature rises to a point given in Fig. 1 where ion cyclotron waves are no longer unstable, so that the ionospheric electrons reach higher altitude to destabilize the ion cyclotron waves, and the whole process will repeat. If, on the other hand, ion heat conduction is greater than the heating from electrons given by Eq. (15),

then the ion temperature will not reach marginal stability point, and the ion cyclotron waves will be confined in a narrow region whose length is of the order of λ along auroral field lines.

Energy loss due to wave convection is much smaller than the ion heat loss. This is because both the group velocity of the cyclotron waves and the wave energy density are smaller than the ion thermal speed and ion energy density nT_i . The energy loss of ions associated with the streaming along auroral field lines during the time τ_η is given by $Q_{\parallel}^{il} = nT_{i\parallel}v_{i\parallel}\tau_\eta S$ so that for $v_{de} \geq v_{te}$ the heating will exceed the loss along field lines since $\frac{v_{de}}{v_{te}} \geq \frac{Q_{\perp}^{il}}{Q_{\parallel}^{il}}$ is satisfied for such a drift speed. Under such conditions, saturation of the instability occurs when the ion temperature anisotropy, $T_{i\perp}/T_{i\parallel}$, reaches the marginal stability given by Fig. 1. The time for saturation, τ_s , is estimated from

$$\tau_s = \tau_D T_{i\parallel}^f / \Delta T_{i\parallel} \quad (16)$$

where $T_{i\parallel}^f$ is the marginal perpendicular ion temperature determined from Fig. 1 for a given electron drift speed. The speed at which the ion heating propagates along the auroral field lines may be given by

$$\frac{v_{de}}{\tau_s} = v_{de} \frac{\Delta T_{i\parallel}^f}{T_{i\parallel}^f} \quad (17)$$

This speed must also be equal to the wave propagation speed along field lines since the unstable waves can propagate at higher altitudes only after the heating at low altitudes leading to the saturation of the ion cyclotron instability. If one considers the finite ion heat loss, then the speed of propagation must be less than that given by Eq. (17) because the ion

temperature increase $\Delta T_{i\perp}$ would be smaller.

So long as the above one-dimensional model is valid, ion cyclotron turbulence and ion heating move up at higher altitudes without limit along auroral field lines at the speed given by Eq. (17). One cannot, however, neglect the ion perpendicular heat loss at an altitude where the ion heat loss across magnetic field dominates the ion heating due to ion cyclotron turbulence. This is because the perpendicular heat loss is proportional to the length of the auroral field lines, while the heating as well as the parallel heat loss are independent of the length of the auroral field lines, l_t , on which ion cyclotron turbulence exists. l_t may be estimated by balancing the perpendicular ion heat loss with the heating from the electrons. Ion perpendicular heat loss during time τ_D is given by

$$Q_{\perp}^{i\perp} = \frac{1}{2} a \kappa_{\perp} \frac{\partial T_{i\perp}}{\partial r} \tau_D$$

$$= \frac{1}{2} a \kappa_{\perp} T_{i\perp} \tau_D, \quad (18)$$

where a is the size of the electron beam and $\kappa_{\perp}$ is the ion perpendicular heat conductivity. Balancing the heat loss, Eq. (18), with the heating from the electrons, Eq. (13), we find the length l_t will be given for $v_{de} \geq v_{te}$,

$$l_t = \frac{f E^c}{T_{i\perp} \tau_D}$$

$$= \left(\frac{a^2}{D_{i\perp} \tau_D} \right) \left(\frac{m_e v_{de}^2}{T_{i\perp}} \right) l. \quad (19)$$

Note that $\kappa_{\perp} = n D_{i\perp}$ where $D_{i\perp}$ is the cross-field particle diffusion coefficient and $S = a^2$. $\tau_D \approx (T_i/T_e)(v_{te}/v_{di})\Omega_i^{-1}$ so that Eq. (19) is reduced to

$$\frac{\lambda_t}{a} = \frac{a^2}{c_i^2} \frac{D_B}{D_\perp} \frac{v_{de}}{v_{te}} \frac{T_e}{T_i} \frac{m_e v_{de}^2}{T_i} \quad , \quad (20)$$

where $D_B \equiv \rho_i^2 \Omega_i$ is the Bohm diffusion coefficient. For most of the auroral plasma conditions at low altitudes, we assume $v_{de}/v_{te} \approx T_e/T_i \approx m_e v_{de}^2/T_i \approx 1$ so that the length λ_t over which the ion cyclotron turbulence exists will be

$$\frac{\lambda_t}{a} \approx \frac{a^2}{c_i^2} \frac{D_B}{D_\perp} \quad (21)$$

or

$$\lambda_t \approx \rho_i \left(\frac{m_i}{m_e} \right)^{1/2} \left(\frac{a^2}{c_i^2} \right) \left(\frac{D_B}{D_\perp} \right) \quad (22)$$

In the presence of ion cyclotron turbulence, $D_\perp$ may be smaller or comparable to D_B for $v_{de} \approx v_{te}$ (Drummond and Rosenbluth, 1962; Okuda et al., 1986). If we assume the transverse size of the auroral field line, a , is a few times of the ion gyroradius and that $D_B/D_\perp \approx 10$, we find $\lambda_t \approx 10^3 \rho_i \approx (500-1000)$ km at low altitudes. In the absence of spacecraft data on the ion cyclotron turbulence at low altitudes, one cannot compare this number with the observations. However, our theory predicts that the region in which the ion cyclotron turbulence exists should be localized at a narrow region along auroral field lines at low altitudes. The reason why the ion turbulence should be confined over the length of λ_t is the following. Suppose the turbulent regions exceeds λ_t . Then the ion temperature drops, thereby ion waves are destabilized, heating the ions back again to the stationary level,

and the system will maintain a steady state by repeating the stable and unstable cycles (Okuda and Ashour-Abdalla, 1982). Ionospheric electrons will develop a plateau when passing through such a turbulence region before reaching the distance of λ_t so that no turbulence develops beyond that point.

When a drift speed v_{de} is less than the thermal speed, heating of the ions associated with the cyclotron turbulence becomes much smaller as shown in Eq. (14). In such a case ion heat loss along field lines may dominate the heating from the electrons. Under such conditions ion temperature will rise to a point where the heat loss balances with the heating given by Eq. (14). The distance over which the ion cyclotron turbulence exists, λ_t , will be of the order of λ in this case, which is much shorter than the case where the drift speeds are comparable to or exceed the thermal speed of the electrons.

III. Simulation Models

In order to study the heating of hydrogen ions as well as electrostatic turbulence near the ion gyrofrequency on the auroral field lines at low altitudes, several different simulation models have been developed. It is clear that the conventional initial value simulations (Okuda et al., 1980, 1981; Pritchett et al., 1981) in which a particle distribution evolves in time starting from an initially unstable distribution is not applicable here. This is because for the case of auroral field lines, the presence of a plasma source at the ionosphere plays a decisive role in determining the evolution of the instability and the heating of ions. Plateau formation in the electron distribution due to quasilinear diffusion is constantly compensated by the fresh incoming electrons from the ionosphere. Under such conditions, it is clear that the plateau formation cannot stabilize the ion cyclotron instability.

In order to model the presence of fresh electrons, we have earlier developed a model in which velocities of small fraction of electrons, δ , are constantly replaced at each time step of simulations by the initial drifting Maxwellian (Okuda and Ashour-Abdalla, 1981; Ashour-Abdalla et al., 1981). The replacement or recycling was done uniformly across the plasma so that it may be considered as modeling the ionization or diffusion process which balances with the loss of plasmas to maintain a steady state. As pointed out earlier (Okuda and Ashour-Abdalla, 1981) cross-field diffusion of ions may be the dominant loss mechanism on auroral field lines. This model, which we will refer to as the uniform recycling model, gives rise to the results consistent with spacecraft observations, such as large amplitude, $e\phi/T_e \approx 1$, coherent, $\omega \approx \Omega_i$ ion cyclotron waves, and the generation of conics for hydrogen and oxygen ions by perpendicular heating of the order of $T_{i\perp}/T_{i\parallel} \approx (10-20)$.

While the recycling model has been successful in producing results similar to spacecraft observations, it assumed the presence of plasma source uniformly distributed on the return field lines, thereby neglecting the wave convection and heat loss. Clearly the model is valid at regions of auroral field lines near the ionosphere where fresh electrons stream into the plasma. When considering the global nature of auroral field lines, the localized source at the ionosphere must be taken into account. We have argued that (Okuda and Ashour-Abdalla, 1981) the electron beam from the ionosphere will reach higher and higher altitude on auroral field lines as the ion cyclotron instability saturates by producing the anisotropic temperature, $T_{i\perp} \gg T_{i\parallel}$, and therefore plateau formation due to quasilinear diffusion is not effective. This is equivalent to having a plasma source at higher altitudes.

In order to study the above assumption, we have developed a new model

(constant flux model) in which the plasma source at the ionosphere is treated as a boundary condition for the simulation plasma giving rise to a more or less constant electron flux determined from the initial drifting Maxwellian as sketched in Fig. 2. Electron distribution is the prescribed drifting Maxwellian at $x = 0$, and the distribution develops a plateau when it leaves the system at $x = L$ as a result of self-consistent wave-particle interactions. Particles leaving the end of the simulation system $x = L$ are reinserted at $x = 0$ with their flux distribution, $\Gamma(v)$, determined from the prescribed drifting Maxwellian for $v > 0$.

$$\Gamma(v) = \frac{1}{\sqrt{\pi} v_{te}} \exp\left[-\frac{(v - v_{de})^2}{v_{te}^2}\right] ,$$

modeling the flow of fresh electrons from the ionosphere. A similar procedure can be used for particles entering at $x = L$ and leaving at $x = 0$. These particles represent a tail of the Maxwellian distribution for $v < 0$ and constitute a small fraction of the total particles for the present simulations.

In addition to the presence of a plasma source at $x = 0$, we shall assume a small dc electric field E_0 on the auroral field lines (return field lines) so that the average drift speed or flux of the electrons is more or less maintained throughout the simulations (constant flux model). The electric field is quite small, such that $eE_0L/T_e < 1$, since the anomalous resistivity associated with the ion cyclotron wave is small. The role of the electric field is simply to maintain the electron flux at $x = 0$ close to its initial prescribed value, and its effect on particle distribution remains very small as shown in Sec. V. Such an electric field may be present on field lines or may also be considered a result of current driven ion acoustic or Buneman

instabilities. If the difference of electron flux at $x = 0$ and $x = L$ is large, charge neutrality of the plasma is violated and the total number of simulation electrons must be varied according to the continuity equation

$$\frac{dN_e}{dt} = \Gamma(x = L) - \Gamma(x = 0) .$$

In the presence of a small external field E_0 , we find in our simulations that the particle fluxes at $x = 0$ and $x = L$ are approximately equal so that it is not necessary to change the total number of simulation particles. Furthermore, exact flux conservation is obtained by changing the drift speed of the input flux slightly so that the fluxes at $x = 0$ and $x = L$ are equal. Note a similar process must take place on auroral field lines in which the drift speed of the upward flowing ionospheric electrons fluctuate according to the resistivity of the field line on auroral zone. Finally, we should point out that the recycling model and the constant flux model should correspond with each other in the following manner. In the recycling model, all the electrons will have new velocities at $T = \Delta t / \delta$, which corresponds to $T = L_{\parallel} / v_{de}$, for the constant flux model to refill the simulation system. Here $L_{\parallel}$ is the system length along the magnetic field lines which is related to the simulation length L by $L_{\parallel} = L(k_{\perp} / k_{\parallel})$. One can compare results from both models at the same time by using the above conversion.

IV. Results of Simulations from the Uniform Recycling Model

Using the two models described in the previous section, several runs have been made, both in one and two dimensions. We shall first describe the results obtained from the uniform recycling model in which the fraction of electron distribution is replaced by the initial drifting Maxwellian at a constant rate

in time and uniformly in space. These results will then be compared with the simulation data obtained from the constant flux model described in V.

Three different cases (case 1,2,3) have been run in one dimension using the one-coordinate space and three-velocity space code (Okuda and Ashour-Abdalla, 1981). These results are then compared with a two-dimensional run (case 4) (Lee and Okuda, 1978).

The simulation parameters for case 1 are the following: simulation system length $L = 1024 \Delta$, where Δ is the grid space which is equal to the initial electron Debye length, $v_{de}/v_{te} = 1.4$, $\Omega_e/\omega_{pe} = 5$, $m_i/m_e = 1837$, $\Omega_i \Delta t = 0.05$, $k_y/k = 0.1$ and $T_e = T_i$ initially. The recycling rate δ was chosen 10^{-2} .

Figures 3(a), (b), and (c) show the potential, $e\phi(x)/T_e$, electron density, $n_e(x)/n_0$, and ion density, $n_i(x)/n_0$, at $\Omega_i t = 50$ near saturation where n_0 is the average particle density. Note the presence of the large amplitude waves $e\phi/T_e \approx 2$ and $\delta n/n_0 \approx 0.5$. There are several large amplitude Fourier modes whose wave number is $k\rho_i = 0.58$ initially, and mode 11 is the fastest growing mode in this simulation. In the absence of plateau formation, the fastest growing mode quickly dominates other modes. This is because the difference in amplitude of various modes is amplified exponentially in time at the rate given approximately by the initial growth rates.

Figure 4a shows the time history of mode 11 and its frequency spectrum sampled for the entire length of time in Fig. 4b. Mode 11 reaches a peak amplitude of $e\phi_k/T_e \approx 1$ at $\Omega_i t = 40$ and settles down to a level slightly higher than the thermal noise, and it does not seem to be destabilized in spite of the presence of the recycling of electrons. The frequency spectrum shows a coherent peak at a frequency very close to the ion cyclotron frequency, Ω_i , as predicted from the linear theory given by Eq. (4) for

$$T_{i\perp} \gg T_{i\parallel}.$$

Ion and electron distributions are shown in Fig. 5 at the time of saturation, $Q_i t = 50$, and at a later time, $Q_i t = 145$. Strong heating of bulk ions is clearly seen in addition to the gain of ion parallel drift due to anomalous resistivity associated with the ion cyclotron instability. Note the perpendicular energy gain for high energy particles exceeds a factor of 100 of the thermal energy. Ion heating along the magnetic field is much smaller so that the temperature anisotropy at the time of saturation is large enough to move the system toward the marginal stability as discussed in Sec. II.

When the instability saturates at $Q_i t = 40$ anomalous diffusion diminishes, and the electron distribution quickly restores its initial drifting Maxwellian. It is interesting to observe that the restored distribution is capable of triggering the second phase of the instability for long wavelengths, modes 1-5. These modes, which are initially near marginal stability because their perpendicular wavelengths are too long ($k_{\perp} \rho_i \ll 1$), become unstable as a result of ion heating, resulting in the shift of $k_{\perp} \rho_i$ toward 1 for these modes.

This second phase of the instability is clearly seen in Fig. 6, where the total electric field energy and the average electron drift speed are shown in time. The electron drift shown in (a) rapidly decays initially until $Q_i t \approx 40$ as the development of the instability as evidenced by the growth of the field energy shown in (b). When the instability saturates, anomalous resistivity also disappears so that the drift speed is restored to its original value at $Q_i t \approx 80$. Then the process repeats again, however at a slower rate since the anisotropic ion distribution ($T_{i\perp} \gg T_{i\parallel}$) reduces the growth rate of the instability, causing the system to move toward marginal stability. If the simulation system is longer, it is possible that the system

repeats the stable-unstable oscillations several times before the marginal stability condition is met for a given electron drift speed. At that point, the system remains completely stable, and there would be no ion cyclotron fluctuations as long as $T_{i\perp}/T_{i\parallel}$ retains the value for marginal stability. Such a scenario holds only for a closed system without any ion heat loss. In the presence of cross-field heat loss as well as ion heat loss along field lines, ion temperature decays with a time constant determined from the heat conductivity, which then would cause the system to move from marginal stability to an unstable state. It is reasonable to assume that the system reaches a steady state, repeating stable and unstable cycles such as seen also in the simulation of ion acoustic instabilities when a plasma source exists (Okuda and Ashour-Abdalla, 1982).

Figure 7 shows the average temperature of ions along ($T_{i\parallel}$) and across ($T_{i\perp}$) magnetic field. The initial rapid rise of $T_{i\perp}$ stops at $\Omega_i t = 60$ followed by a slower increase. The rapid rise is associated with the first stage of the instability due to the shorter wavelength mode, $k_{\perp} \rho_i \approx 1$ initially, and the slower increase in $T_{i\perp}$ is accompanied by the second stage of the instability. Note the temperature has not saturated at $\Omega_i t = 220$, but the system is close to marginal stability as evidenced saturation of the field energy near the end of the run shown in Fig. 6.

Anomalous resistivity, η , between electrons and ions may be found from the gain of the ion momentum from the electrons rather than the electron drift speed itself. In the presence of recycling, electron drift speed shows a periodic motion in time due to the balance of anomalous resistance and recycling so that the slope of $v_{de}, dv_{de}/dt$ does not give the correct value of the resistivity. As shown in Fig. 8, ion momentum increases monotonically in time, and one can determine the anomalous resistivity from the slope giving

$\eta/\eta_0 = 6 \times 10^{-4}$ where $\eta_0 = 4\pi/\omega_{pe}$. Note $\eta/\eta_0 = v_{ei}/\omega_{pe}$ holds where v_{ei} is the electron-ion collision frequency.

It is interesting to compare the previous results with the second case in which all the simulation parameters are kept the same, except now the recycling rate δ is reduced to 10^{-3} (case 2). The replenishment of the electron distribution is now much slower, and it takes more than two ion cyclotron periods to recover the initial distribution. The recycling time in this case is approximately equal to the transit time of the electrons to cross the system length along the magnetic field. Since the recycling is much slower, ion cyclotron waves grow much more slowly due to the plateau formation on the electrons. It is, however, important to notice that the total ion heating is independent of the recycling rate, δ for a closed system. This can be proved in the following manner. Let us suppose the instability saturates for a smaller value of $T_{i\perp}/T_{i\parallel}$. Then, with the saturation of the instability, quasilinear diffusion disappears and the electron distribution tends to restore the original drifting Maxwellian, albeit slowly. Then the system must become unstable again for the same drift speed v_{de} until $T_{i\perp}/T_{i\parallel}$ reaches the same value for the previous case of $\delta = 10^{-2}$.

Figure 9 indicates the potential (top), electron density (middle), and ion density (bottom), at $Q_i t \approx 130$ near saturation. Note that the structure of the ion cyclotron waves is similar to that for the previous case where $\delta = 10^{-2}$ but the amplitude is smaller. Since the growth rate is reduced due to slower recycling, it takes a longer time to reach saturation.

The heating of ions is shown in Fig. 10. The initial increase of $T_{i\perp}$ is followed by a slower growth at $Q_i t \approx 300$ when $T_{i\perp}(t)/T_{i\perp}(0)$ reached about 10 as before. At a much later time, it was again observed that the dominant unstable modes shifted to longer wavelengths as a result of ion heating.

These observations confirm that the total heating of ions remains the same regardless of the values of the recycling rate, δ for a closed system.

The next case considered is a smaller electron drift $v_{de}/v_{te} = 0.85$ with $\delta = 10^{-2}$, and all the other parameters remain the same (case 3). Since the critical drift speed for marginally stable hydrogen cyclotron waves is about $v_{de}/v_{te} = 0.3$, $v_{de}/v_{te} = 0.85$ is still well above the marginal stability threshold. According to our theoretical predictions, ion heating should be less than in the previous case of $v_{de} = 1.4$ and $\delta = 10^{-2}$. Figure 11 shows the perpendicular and parallel temperature normalized to their initial values as a function of time. The maximum perpendicular temperature achieved is about eight times the original temperature, while the parallel temperature has increased to twice the original value. The electron velocity was maintained close to the original throughout the run, since the drift was so small and the recycling rate was fast as shown in Fig. 12. The waves grew to a small amplitude only, $e\phi/T_e = 0.2$. The saturation was due to the development of a temperature anisotropy which made the present drift subcritical. Figure 12 shows the heating of the distributions.

In order to supplement the one-dimensional results, simulations are extended to two dimensions in which the external magnetic field $B = (B_x, 0, B_z)$ is taken nearly perpendicular to the plane of simulations (Okuda et al., 1980). Earlier initial value simulations in two dimensions for the ion cyclotron instabilities gave nearly the same results obtained using a one-dimensional model (Okuda et al., 1980; Pritchett et al., 1981). This observation was attributed to the fact that the quasilinear diffusion in velocity space was the dominant nonlinear saturation mechanism, and other nonlinear effects such as mode-coupling were small since the ion cyclotron waves are the non-decay type. In the presence of electron recycling, we again

expect two-dimensional simulations for the ion cyclotron waves will give rise to results similar to those in one dimension.

Figure 13 shows the ion heating using a two-dimensional model (case 4) for a hydrogen plasma. Simulation parameters are the same as in one dimension except for $\Omega_e/\omega_{pe} = 10$, $v_{de}/v_{te} = 1.4$, and a 64×64 grid is used which is elongated by a factor of $2(\Delta y = 2\Delta x)$ in the y direction so that the longest wavelength mode in that direction satisfies $k_y \rho_i = 0.21$. As seen in Fig. 13, ion heating saturates at $T_{i\perp}(t)/T_{i\perp}(0) \approx 11$ and does not indicate the presence of a second stage of slower growth rate as seen in the one-dimensional simulations. This is due to the fact that, for two-dimensional simulations, system size in the y direction is 8 times shorter than that in one dimension so that no long wavelength modes are available to cause the second stage of the instability. The fact that the ion heating stopped at $T_{i\perp}(t)/T_{i\perp}(0) \approx 10$ for one and two dimensions for $v_{de}/v_{te} = 1.4$ suggests that the nonlinear saturation for both cases is the result of temperature anisotropy as discussed in Sec. II.

Figure 14 indicates the instantaneous electrostatic potential contours at an early time, $\Omega_i t = 55$ in (a), and after saturation, $\Omega_i t = 200$ in (b). Noting that the initial ion thermal gyroradius is 4.3 in the scale used in the figure, it is clearly seen in (a) that several structures corresponding to ion cyclotron waves are excited in the range of $k_{\perp} \rho_i = \approx 0.5 - 1$. At a much later time shown in (b), we observe a development of a new pattern in the x direction with $k_y = 0$ ($k_{\parallel} = 0$) in addition to the ion cyclotron structure ($k_y \neq 0$). Such two-dimensional structures ($k_{\parallel} = 0$) were seen to develop in previous initial value simulations (Okuda et al., 1981) as a nonlinear effect of the ion cyclotron waves.

Frequency analysis of one such mode, mode (1,0) whose $k_y = 0$ and $k_x = 0.1$

is shown in Fig. 15. Three peaks are seen at ω_{pi} , ω_i , and 0 corresponding to the lower hybrid, ion cyclotron and convective cell modes respectively. Since these modes are linearly stable for $k_y = 0$, nonlinear modes coupling is believed to be responsible for the excitation of such modes (Okuda et al., 1981). Once excited, convective cell modes are expected to lead to anomalous particle diffusion across the magnetic field. While convective cell modes are the largest amplitude in Fig. 15, it should be mentioned that the electrostatic ion cyclotron waves for $k_y \neq 0$ are much larger in their amplitude than the convective cells.

V. Results of Simulations from the Constant Flux Model

We have seen in the previous section that the electrostatic waves grow to a large amplitude $e\phi/T_e \approx 1$, resulting in the formation of an anisotropic ion velocity distribution with $T_{i\perp}/T_{i\parallel} > 10$ for $v_{de} \approx v_{te}$. Ion heating and the generation of a temperature anisotropy shuts off the instability by bringing down the wave frequency very close to ω_i as predicted by linear theory. However, that model assumed the presence of a uniform source of drifting electrons along the field line, which is different from the auroral field lines where the ionospheric electrons only enter at one end of the field line. In this section we consider the results from the constant flux model, described in Sec. III, in which a source of electrons is located at one end of the simulation system modeling the ionospheric source. We would expect that as the drifting electrons enter the low altitude auroral field lines, ion cyclotron waves would be generated near the entry point which, in turn, would cause plateau formation on the electron distribution as well as heating of the ions. We would also expect that as the ions are heated and develop a large temperature anisotropy, ionospheric electrons could reach higher altitudes

without suffering a plateau formation, thereby exciting the cyclotron waves at a higher altitude. This self-induced propagation of ionospheric electrons is a process similar to injecting beams into a target plasma. The speed of penetration of the beam or beam convection has been estimated in Sec. II. This convective nature of the instability can be studied with the present model, a capability not found in the uniform recycling model where the instability was treated as absolute. We expect, however, based on the above discussion, that the nonlinear saturation for both models to be due to the ion perpendicular heating, and it follows that the saturation value for $T_{i\perp}/T_{i\parallel}$ should stay about the same provided that the ion heat loss is much less than the heating from electrons. If, on the other hand, the energy loss due to wave convection or ion heat conduction becomes comparable to the heating from electrons, then the ion heating will be smaller. This is expected to happen for a small electron drift speed since the growth rate as well as the ion heating rate will be smaller. In the constant flux model, the source is located at the end of the field line, and therefore the plasma is inhomogeneous along the magnetic field so we need local diagnostics measured at different locations. We do this by dividing up the system length into five equal bins, each bin about 200 units long. The first bin is closest to the ionosphere, while the fifth bin is furthest from the ionosphere. Note the initial gyroradius of the thermal ion is 8.6 units so that the bin width is many ion gyroradii across. Density fluctuations, velocity distributions for both ions and electrons, parallel and perpendicular ion temperatures, as well as the local wave frequency are monitored for diagnostic purposes.

Figure 16 shows the profiles of ion density normalized by the average density for the entire system length plotted at three different times (case 5). The parameters of the simulation for this case is the same as case 1

where $v_{de}/v_{te} = 1.4$. The dashed vertical lines denote the boundary lines for different bins. We see that at time $\Omega_i t = 110$ (upper panel), the larger density perturbations, $\delta n/n = 0.25$, are mostly confined to the first two bins. At later times shown in the middle and lower panels, the ion density perturbations are seen to propagate and extend further right, confirming the physical picture presented in Sec. II in which the ω_i cyclotron waves are destabilized toward higher altitude as the electron beam from the ionosphere reaches higher altitudes without plateau formation. From plots such as Fig. 16, we can estimate the propagation speed to be $0.007 v_{de}$. This speed is much lower than the electron thermal speed and is in reasonable agreement with the self-induced propagation speed calculated to be $0.01 v_{de}$ from Sec. II. Note the speed of propagation measured in Fig. 16 is almost across the magnetic field, while the theoretical speed obtained in Sec. II is along the field lines so that a factor $k_{\parallel}/k$ must be multiplied to the theoretical value in order to compare with the simulation results. Note the propagation speed of the ion cyclotron turbulence, $0.01 v_{de}$, is larger than the ion thermal speed along the magnetic field projected onto the simulation plane, which is equal to $0.0017 v_{de}$. Therefore ion heat loss associated with the free streaming along field lines is smaller than the heating from electrons in this case. The fact that this ion density modulation is associated with the ion cyclotron waves is confirmed by measuring the frequency at several different locations along field lines. This can be seen in Fig. 17 where we show in the upper panel (a) the time history of electron density fluctuation normalized to its initial value, while the lower panel (b) shows its frequency spectrum. The left column of the plot is taken at $x = 320\Delta$, while the right column is at $x = 416\Delta$. From the upper panel we can see that large ion density perturbations arriving at $\Omega_i t = 100$ for $x = 320\Delta$ and at $\Omega_i t = 150$ for $x = 416\Delta$. The frequency

analysis confirms the presence of coherent peaks above Ω_i . In addition, there are much smaller, but clearly coherent, peaks near $\omega = k_y c_s$ indicating oblique ion acoustic waves propagating nearly perpendicularly to the magnetic field.

The effect of these large amplitude ion cyclotron waves on the particle distribution is shown in Fig. 18 at $\Omega_i t = 175$. Here we plot in the upper panel the profile of the ion density perturbation. In the lower panel we plot the perpendicular ion distribution in three different bins or, equivalently, at different altitudes along the field lines. It is clearly seen that the large ion density perturbations have penetrated up to the third bin by this time. We would expect that for the lower altitude where the ion cyclotron waves are excited, the ion distribution would be heated up to the point where the cyclotron waves are marginally stable as estimated in Sec. II. Note that the ion heat loss along the magnetic field is small in this simulation since the average ion displacement during this time is given by $v_{\text{diff}}(k_\parallel/k_\perp) \approx 140\Delta$, which is much less than the distance that the beam propagated. In fact, we note that the ion bulk heating in the first bin is more than ten times the original value. We also note the formation of a high energy tail which was not seen in the uniform recycling model. At higher altitudes the ion distributions are also heated, but to a lesser degree. This is simply because the heating has not saturated yet.

The high energy tails seen in Fig. 18 extend more than ten times of the initial thermal speed, and the total distribution is separated into bulk and tail parts. This can be seen more clearly in Fig. 19 where the logarithm of the distribution in bin 1 at $\Omega_i t = 175$ is plotted against particle energy normalized by the initial thermal energy v^2/v_{ti}^2 . The initial Maxwellian distribution is also shown for comparison in the figure. Clearly the heated distribution has a high energy tail extending almost as much as 100 times of

the initial thermal energy. The temperature of the tail as well as the bulk distribution may be estimated by fitting a straight line to the measured distribution function as shown in the figure. Comparing the temperatures of the tail and bulk parts reveals that the tail temperature is about 50-100 times as much as the initial temperature while the bulk temperature is about 5-10 times at the saturation of the ion cyclotron waves.

It is interesting to observe that the formation of a high energy tail is much more prominent in the constant flux model compared with the simulations using the uniformly recycling model for the same simulation parameters (case 1). This difference can be explained by the fact that the saturation level of the ion cyclotron waves for the recycling model is much higher than that from the constant flux model, so that the large amplitude waves in the recycling model can interact with particles in bulk distribution pulling the bulk particles toward the tail. Note that the maximum particle energy for both models is about the same, about 100 times that of the initial thermal energy. Therefore, it can be said that there are so many particles in the tail for the uniform recycling model that therefore particles in the high energy tail are not clearly distinguishable from the bulk distribution.

Figure 20 shows similar plots for the electron distributions at $\Omega_i t = 175$ for three different locations. It is seen that the electron distribution generally has a plateau at the higher altitudes such as in bin 4, however the distribution in general has positive slopes for $v < v_{de}$ in bins 1 and 2 since the saturated waves do not generate a plateau on the ionospheric electrons discussed in Sec. 11. One should note here that the plateau formation on the electron distribution is a very subtle process which does not require much energy at all. Therefore a small reduction in ion temperature due to ion heat loss or wave energy convection is able to shift the system into an unstable

cycle from marginal stability, thereby flattening of the ionospheric electron distribution can take place before the ionospheric electrons can reach the higher altitude. It appears that at low altitude the instability oscillates stable and unstable cycles around the marginal stability as seen in the uniform recycling model, thereby causing a partial flattening of the electron distribution before reaching the higher altitude. Electron distributions shown in Fig. 20 indicate partial flattening at lower altitudes, while they are more or less completely flattened at higher altitudes. These physical processes are consistent with the theoretical model discussed in Sec. II.

At a later time $Q_i t = 220$, using the same format, we plot in Fig. 21 the behavior of the ion distribution as well as the ion density fluctuation which has now propagated to the fourth bin by this time. There is considerable heating in bins 1 through 3. It should be noted that the heating in bin 1 and bin 2 are now comparable. Thus it seems that there is a maximum temperature that can be achieved similar to the case of uniform recycling discussed in Sec. IV, and this temperature is again determined from the maximum anisotropy shown in Fig. 1.

Let us now study the time development of the perpendicular ion temperature at several different locations on the auroral field lines. Figure 22 is the time history of the average perpendicular ion temperature normalized to the initial temperature for each bin. In Fig. 22, we have also marked the time of the onset of ion cyclotron waves at each bin. We note that the temperature in bin 1 increases rapidly, reaching a maximum value of roughly 10 times the initial value at $Q_i t = 150$, then stays around that fixed value. Note the maximum temperature attained here is only a factor of 10, whereas we had obtained a factor of 20 using the same plasma parameters for uniform recycling case 1 as shown in Fig. 7. The difference is mostly associated with

the absence of the second phase in the instability involving long wavelength modes seen in the recycling model. Note in Fig. 7, the fast rise in ion temperature stops at $\Omega_1 t = 60$, giving rise to a temperature increase of about 10, followed by a slower rise associated with the second phase of the instability. This initial fast rise in ion temperature is much the same seen in bin 1 of Fig. 22, confirming that the uniform recycling model and the constant flux model behave similarly when a loss is small. The second instability seen in the uniform recycling model has not been seen to develop in the constant flux model since the large amplitude ion cyclotron waves are mostly localized as seen in Fig. 16.

There is evidence for the ion heat conduction seen in bins 2, 3, and 4, where we have a small increase in the temperature at a time prior to the onset of the instability. The heating is due to the fact that high energy tail ions with large parallel speed along the magnetic field are moving from bin 1 to bin 2 to bin 3 and so forth, thereby increasing the average temperature in bins at higher altitude. As discussed previously, heat loss along field lines in this simulation is small compared with the heating due to cyclotron instability. After the onset of the instability in bin 2 at $\Omega_1 t \approx 100$, for example, we note that the temperature increases faster. It is clear that the heating in that latter phase, $\Omega_1 t > 100$, is due to the turbulent heating by the ion cyclotron waves. Heating at higher altitudes is delayed in time as expected; however, the saturated temperature is about the same. From Fig. 22 we can calculate a heating rate due to ion cyclotron waves. For bin 1, the heating rate is found to be $d(T_1/T_{10})/d(\Omega_1 t) \approx 0.12$, which gives good agreement with the theoretical estimate of 0.11 from Eq. (15).

The macroscopic ion perpendicular profile along auroral field lines is shown in Fig. 23 at different time steps. The different curves represent

different times. We note that at an early time the temperature gradient is confined to low altitudes, while at higher altitudes the temperature gradient is flat. At later times the gradient steepens and extends to higher altitudes as the ion cyclotron waves propagate along auroral field lines. The fact that the heating of ions propagate along auroral field lines is a consequence of energy conservation in a one-dimensional model in which the energy dissipated from electrons by the formation of a plateau must be absorbed by the ions. In the absence of energy loss across one-dimensional field lines, ion heating will continue moving to higher and higher altitudes. There are, however, cross-field heat losses across the magnetic field along auroral field lines. Perpendicular heat losses are proportional to the length of the field line on which the ion cyclotron waves are unstable so that the loss becomes larger as the unstable region extends to higher altitudes along auroral field lines. The heating zone can be determined by balancing the heat loss with the energy input from the electrons as estimated in Sec. II.

We have used the same model, the constant flux model, for a smaller electron drift speed at $v_{de}/v_{te} = 0.707$ (case 6). The rest of the simulation parameters remain the same as in case 5. Since the electron drift speed is smaller for this case, the ion heating rate from the electrons given by Eq. (14) is comparable to the ion heat loss along the magnetic field. With the onset of the instability at bin 1, the ion perpendicular temperature begins to rise. The saturated temperature will be somewhat lower than the theoretical predictions given in Fig. 1 since the ion heat loss carries the energy away towards higher altitudes. The ion cyclotron turbulence is expected to be confined mostly to lower altitudes as discussed in Sec. II. The majority of ion heating at higher altitudes is therefore associated with the thermal motion of ions along the magnetic field. Heating associated with wave

convection remains small in this simulation. Figure 24 indicates the ion temperature increase with time for bins 1, 2, and 3. In this case, due to the low value of the initial drift and the expected low temperature increase, we have divided the system into ten bins. Each bin is thus approximately one hundred units long. The temperature in the first bin increases to eight times the original value and fluctuate about its mean.

The ion perpendicular velocity distribution at bin 1 is shown in Fig. 25 at $\Omega_i t = 200$. Bulk heating, as well as the formation of a high energy tail, is clearly seen as before. The energy of the tail particles extend as much as 100 times the original thermal energy; however, their population is less than that found in case 5 where the drift speed $v_{de} = 1.4 v_{te}$ was used. Again the temperature of the tail particles determined from the tangent to the distribution in Fig. 25 is (50 ~ 100) times the initial temperature.

Ion cyclotron turbulence is more or less localized in this case where the ion heat loss is comparable to the ion heating from the electrons. Figure 26 shows the ion density plot $n_i(x)/n_0$ at three different times, $\Omega_i t = 150, 260$, and 340, respectively. Notice that the density modulations associated with the ion cyclotron waves are more or less confined in the first few bins, on the left in this case, again consistent with the theoretical predictions made in Sec. II. Certainly there are indications of wave convection at higher altitude; however, the amplitudes of ion cyclotron turbulence is much smaller as the waves are cyclotron damped as they propagate toward higher altitude.

VI. Discussion and Conclusions

We have investigated the excitation of ion cyclotron waves and the resultant acceleration of ions by a theoretical approach (Sec. II) as well as by simulation techniques, using two different models (Sec. IV and V). In

general, theoretical predictions and simulation results are in good agreement.

Let us summarize our findings.

Conclusions from Theory

- (1) The critical electron drift needed to excite ion cyclotron waves increases as the temperature anisotropy of the ion distribution $T_{i\perp}/T_{i\parallel}$ increases.
- (2) As the wave amplitude grows, the ions are heated primarily in the perpendicular direction as a result of cyclotron damping. The instability saturates and the ion heating ceases when the ion temperature anisotropy is large enough to bring the system toward marginal stability for a given electron drift v_{de} .
- (3) For a given electron drift v_{de} one can calculate the maximum ion temperature anisotropy from marginal stability conditions. For $T_{i\parallel}/T_e \approx 1$, the maximum temperature anisotropy, $T_{i\perp}/T_{i\parallel}$, is about 10 for $v_{de} \approx v_{te}$. For larger drift, or $T_e/T_{i\parallel} > 1$, this ratio can be larger.
- (4) When the ion heating due to cyclotron instability exceeds ion heat loss along auroral field lines, ion cyclotron turbulence extends towards higher altitudes at a speed given by Eq. (17). In this case, the ion cyclotron turbulence may be localized in a region determined from the perpendicular ion heat conductivity.
- (5) When the ion heating is less than the heat loss along and across the magnetic field, ion cyclotron turbulence is localized at a short distance on the order of λ given by Eq. (12).

Summary From the Uniforming Recycling Model

- (1) Large amplitude, $e\phi/T_e \sim 1$, coherent, $\omega = \Omega_i$, ion cyclotron waves are observed for $v_{de} \approx v_{te}$.
- (2) The ion distribution develops a temperature anisotropy $T_{i\perp}/T_{i\parallel} \approx 10 \sim 20$.
- (3) The maximum temperature anisotropy is a function of the electron drift, increasing as the electron drift increases, in agreement with theoretical predictions.
- (4) The maximum ion heating is independent of the recycling rate, while the saturation level of the ion cyclotron waves increases with the drift speed.
- (5) The anomalous resistivity η is small, $\eta/\eta_0 \lesssim 6 \times 10^{-4}$ for $v_{de} \approx v_{te}$ where $\eta_0 = 4\pi/\omega_{pe}$.
- (6) Two-dimensional simulations give rise to results similar to the one-dimensional simulations.
- (7) Saturation of the ion cyclotron waves is due to the development of a temperature anisotropy making the initial electron drift subcritical as predicted from theory.
- (8) Bulk heating of the ion distribution takes place, and the high energy particles are heated as much as 100 times the initial energy. There is no formation of a high energy tail.

Summary for the Constant Flux Model

- (1) Large amplitude ion cyclotron waves are observed to propagate towards higher altitudes along auroral field lines for $v_{de}/v_{te} = 1.4$. The propagation speed agrees with the theoretical prediction given by Eq. (17).

- (2) The ion distribution develops a temperature anisotropy $T_{i\perp}/T_{i\parallel} \approx 10$ as the ion cyclotron waves grow to large amplitudes. For $v_{de}/v_{te} = 1.4$, more and more ions are heated at higher altitudes as the ion cyclotron turbulence propagates along field lines.
- (3) The maximum temperature achieved for $v_{de}/v_{te} = 1.4$ is in agreement with the theoretical predictions where ion heat loss was neglected in comparison with ion heating.
- (4) Ion perpendicular velocity distribution develops a high energy tail whose temperature is (50 ~ 100) times the initial thermal energy.
- (5) When the heat loss is comparable to ion heating due to ion cyclotron waves, the region on which the ion wave turbulence develops is limited to a narrow region at low altitudes. Heating of ions at higher altitudes is mostly associated with the heat conduction along field lines.

Let us now consider a plasma at low altitude auroral field lines where strong field aligned current of a few $\mu A/m^2$ has been detected. Assuming the ionospheric plasma temperature is 1 eV or less, such a current, $v_{de} \approx v_{te}$, is strong enough to produce ion cyclotron turbulence on low altitude auroral field lines. The heating rate of ions from the electrons via ion cyclotron turbulence, Eq. (14), is estimated to be a few times larger than the ion heat loss along field lines. We therefore expect that the ions are heated up at low altitudes to a point where marginal stability is achieved. For $v_{de}/v_{te} \approx 1$, the maximum bulk heating is about a factor 10, and a higher energy tail temperature is as much as 100 times the initial temperature. The region on which the cyclotron turbulence develops is limited from parallel as well as perpendicular ion heat conduction. Assuming the perpendicular ion heat loss is dominant in this case, the length of ion turbulence, Eq. (22), is estimated

to be $l_t = (500 \sim 1000)$ km for $a/\rho_i = 5$ and $D_B/D_\perp = 5$, where a is the transverse dimension of the return flow of electrons on auroral field lines. Here we assumed that $B = 0.2$ gauss and $T_i \approx 1$ eV so that $\rho_i = 0.5$ km at low altitude.

We have assumed in our theory and numerical simulations that the magnetic field is uniform throughout the system, while the magnetic field strength on the auroral field lines varies as much as 20 ~ 40% at low altitude for the length $l_t = (500 \sim 1000)$ km. We do not believe this variation of magnetic field strength is important since the group velocity of the ion cyclotron waves is so much smaller than the speed at which the electron beams propagate through the field lines given by Eq. (17). Therefore as the electron beam penetrates through the auroral field lines, ion cyclotron waves are excited with their frequency determined from the local magnetic field strength.

While the present numerical simulations are only in one-dimension with three velocity space coordinates, it is straightforward to extend it to two-dimensions where two-dimensional effects, such as ion heat loss associated with the anomalous cross-field diffusion, may be incorporated.

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Figure Captions

- Fig. 1. Maximum attainable ion temperature anisotropy as a function of electron drift speed. Note for $v_{de} \approx v_{te}$, ion perpendicular temperature can be 10 times the parallel temperature for $T_{i\parallel} \approx T_e$.
- Fig. 2. Theoretical model for auroral field lines in which the ionosphere is the source of cold drifting Maxwellian electrons streaming upwards. As the electrons move up, ion cyclotron turbulence develops, causing the plateau on the electron velocity distribution and ion perpendicular heating.
- Fig. 3. Spatial structure of electrostatic potential $e\phi(x)/T_e$, (a), electron density $n_e(x)/n_0$, (b), and ion density $n_i(x)/n_0$, (c), at $\Omega_i t = 50$ for $v_{de}/v_{te} = 1.4$. Case 1.
- Fig. 4. Temporal development of mode 11, (a), and its frequency spectrum (b). Note the frequency is almost the ion cyclotron frequency as predicted for anisotropic ions in the text. Case 1.
- Fig. 5. Temporal development of electron parallel velocity distribution at $\Omega_i t = 0$ and 50, (a), ion perpendicular distribution at $\Omega_i t = 50$, (b), and 145, (c), and ion parallel distribution at $\Omega_i t = 145$, (d). Case 1.

- Fig. 6. Temporal development of the average electron drift, (a), and the total electric field energy (b). Note the oscillatory nature of the instability. Case 1.
- Fig. 7. Bulk ion perpendicular temperature and parallel temperature. Note the heating rate is reduced at $\Omega_i t = 60$ due to the saturation of the ion cyclotron instability for short wavelengths. Case 1.
- Fig. 8. Gain in ion drift along the magnetic field associated with the anomalous resistance with the electrons. Note the anomalous resistivity remains constant during the entire run. Case 1.
- Fig. 9. Electrostatic potential at the top, electron density in the middle, and ion density for uniform recycling model with $\delta = 10^{-3}$ at $\Omega_i t = 130$. Case 2.
- Fig. 10. Temporal development of the ion perpendicular and parallel temperatures for $\delta = 10^{-3}$. Case 2.
- Fig. 11. Perpendicular and parallel ion heating for $v_{de}/v_{te} = 0.85$ and $\delta = 10^{-2}$. Note the maximum heating is reduced from the previous case ($v_{de}/v_{te} = 1.4$) in agreement with Fig. 1. Case 3.
- Fig. 12. Temporal development of electron and ion velocity distributions at $t = 0$ and $\Omega_i t = 150$ for case 3. Note the electron distribution remains almost unchanged. Bulk heating of ion perpendicular temperature is observed. Case 3.

Fig. 13. Perpendicular and parallel ion heating using a two-dimensional model. Case 4.

Fig. 14. Snapshots of the constant potential contours for two-dimensional simulations at two different times. Note the potential structure whose size is of the order of ion gyroradius. At late times, $\Omega_1 t = 200$, a two-dimensional flute mode ($k_{\parallel} = 0$) develops to large amplitudes. Case 4.

Fig. 15. Frequency spectrum of the flute mode. Note the presence of large amplitude zero-frequency convective cells in addition to lower hybrid and ion cyclotron waves. Case 4.

Fig. 16. Ion density profile using the constant flux model in which the ionospheric electron source is located at the left boundary. Note the large density perturbation associated with the ion cyclotron waves propagates towards higher altitudes (right) with the speed predicted by Eq. (17). Case 5.

Fig. 17. Temporal behavior of the electron density perturbation at $x = 320\Delta$ and $x = 416\Delta$ and its frequency spectrum. Case 5.

Fig. 18. Ion density profile and the ion perpendicular velocity distribution at three different bins at $\Omega_1 t = 175$. Note the presence of high energy tails in the ion distribution. Case 5.

Fig. 19. Ion velocity distribution at bin 1 as a function of energy at $\Omega_1 t = 175$. Note the distribution may be divided into bulk and tail distributions in which the tail temperature may be as large as 100 times the initial temperature. Case 5.

Fig. 20. Electron density profile and the electron parallel velocity distribution at bins 1, 3, and 4. Note that for bins 1 and 3 where ion cyclotron waves have more or less saturated, distributions maintain a positive slope while for bin 4, it is flat. Case 5.

Fig. 21. Ion density profile and the ion perpendicular velocity distribution at four bins at $\Omega_1 t = 220$. Case 5.

Fig. 22. Temporal development of ion perpendicular temperature for bins 1, 2, 3, and 4. The heating rate of bin 1 agrees well with the theoretical estimate given in Sec. II of the text. Case 5.

Fig. 23. Ion perpendicular temperature profile at different times. Note at $\Omega_1 t = 250$, the temperature at low altitude has saturated. Case 5.

Fig. 24. Temporal development of ion perpendicular temperatures for bins 1, 2 and 3 for $v_{de}/v_{te} = 0.85$. Note the bin size here is one-half that of case 5. Heating at low altitudes (bins 1-3) is associated with the ion cyclotron turbulence while at higher altitudes, it is mostly due to ion thermal conduction. Case 6.

Fig. 25. Perpendicular ion velocity distribution for bin 1 at $\Omega_1 t = 200$. Note the presence of a high energy tail, albeit its population is smaller than that in case 5. Case 6.

Fig. 26. Ion density profile at $\Omega_1 t = 150, 260$ and 340 . Note in this case the turbulence is more or less confined to the lower field lines (left) and no clear sign is visible at higher altitudes. Case 6.

Fig. 1

81700.9

Fig. 3

81T0018

Fig. 4

Fig. 6

FIG. 2

Fig. 8

Fig. 9

Perpendicular and Parallel Temperature versus Time

Perpendicular and Parallel Temperature versus Time

Fig. 11

Fig. 12

Fig. 13

Electrostatic Potential

$\Omega_i t = 55$

$\Omega_i t = 200$

Fig. 14

Fig. 15

Fig. 16

Fig. 17

Fig. 18

Fig. 1a

Fig. 21

Spatial Distribution of Ion Heating

Temporal Evolution of Ion Heating

Fig. 2

Fig. 24

Fig. 25

Fig. 26