Gust Characteristics for WECS Design and Performance Analysis

**J. Christopher Doran
David C. Powell**

May 1980

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SUMMARY

This document provides a description of some gust characteristics which are useful in the study of wind turbine fatigue caused by a fluctuating wind environment. The particular gust form chosen can also be used in the analysis of the dynamic response of a turbine. The statistical behavior of such gust characteristics is not identical to that determined simply from the wind recorded by an anemometer. These modes of behavior may be related, however, by the application of appropriate digital filters to the anemometer data. This procedure has been carried out for a number of sample cases, and the variations of the resultant gust features are presented. A number of suggestions on specific applications and interpretations of the data are included.

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NOMENCLATURE

A	Amplitude of discrete gust
$\bar{A}$	Mean amplitude of gusts
$A_T(x)$	x percentile amplitude for a specified time T
C_A	Proportionality constant relating slope of $A_T(50)$ vs T to (σ_A/σ_T)
C_T	Proportionality constant relating slope of $T_A(50)$ vs T to (σ_T/σ_A)
$F(n)$	Filter function used to describe machine response to fluctuating winds
f	Dimensionless frequency = $nz/\bar{U}$
f_0	Peak value of neutral Kaimal spectrum, given value of 0.0144
K_1	Assumed proportionality constant relating σ_A to σ_F
K_2	Assumed proportionality constant relating σ_T to $1/N_0$
$N_{AT}(x,y)$	Frequency of occurrence of gusts with amplitudes $> x$ and times $> y$
$N_A(x)$	Frequency of occurrence of gusts with amplitudes $> x$
$N_T(y)$	Frequency of occurrence of gusts with times $> y$
N_0	Frequency of occurrence of all gusts
n	Frequency (Hz)
$P_{AT}(x, y)$	Joint probability of gust amplitude $> x$ and gust time $> y$
$P_A(x)$ $P(A > x)$	} Probability of gust amplitude $> x$
$P_T(x)$ $P(T > y)$	
	} Probability of gust time $> y$
S	Notation for general state of the atmosphere
$T_A(x)$	x percentile time for specified amplitude A

$\bar{T}$	Mean gust time
$\bar{U}$	Mean wind speed
z	Height of measurement
z_0	Roughness length
$\phi(n)$	Turbulence spectrum
ρ_{AT}	Correlation coefficient for gust amplitudes and times
σ	root-mean-square turbulent speed
σ_A	root-mean-square gust amplitude
σ_F	root-mean-square turbulent speed showing explicit dependence on filter $F(n)$
σ_T	root-mean-square gust time

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

The behavior of wind fluctuation, or gusts, is of fundamental importance in the design and performance of wind energy conversion systems (WECS). The mechanical and electrical responses of a wind turbine to varying winds, its susceptibility to fatigue failure and the likelihood of catastrophic failure from extreme wind events are all dependent upon the characteristics of gusts. These aspects of machine behavior depend upon a variety of wind features; consequently, it is unlikely that a single gust descriptor or model can be adequate for all purposes. For this document there were two points emphasized in the approach to the problem. The first was the identification of events which contribute to the fatigue of a WECS. Such events can be characterized by their amplitude and their duration. Hence, the second area, and the one with which this document is chiefly concerned, was the examination of the relationship between the magnitude and duration of gusts to which a machine may respond.

A number of previous studies have addressed one or more characteristics of gusts. From these, we have identified an event which is believed to be directly applicable to the fatigue problem mentioned above; in a previous document (Powell and Connell 1980) this event has been given the name $GUST_0$. It is simply defined as the sequence of wind speeds between two successive crossings of the mean speed. The time between these crossings is the duration or time of the gust, and the maximum deviation from the mean value is the amplitude of the gust (Figure 1.1).

There are a number of advantages to the selection of this event for study:

- It is roughly analogous to the 1-cosine gust form frequently used in the aircraft industry and proposed for design purposes for the Mod-2 wind turbine.
- It is conceptually simple.

FIGURE 1.1. GUST₀ Event. The horizontal line represents the mean wind speed, and the wavy line represents the fluctuating wind speed.

- There is evidence to suggest that such events may be directly related to the stress cycles experienced by a machine that contribute to fatigue failure (Crede 1963, Lockheed 1962). (There is also some evidence that a count of such events may underestimate the number of stress cycles, but alternative schemes, such as peak counts, provide their own difficulties of interpretation. This issue has not been clearly resolved for all circumstances, but the $GUST_0$ event appears to be an acceptable choice).
- It provides a reasonable framework in which to study machine response to gusts of varying amplitude and duration.

1.2 APPROACH

In principle we would like to determine the joint probability distribution of amplitude and frequency for the $GUST_0$ events, together with some basic frequency of occurrence of all gusts. In doing so we are interested in the gust environment as experienced by an actual wind turbine. This environment is by no means identical to that measured by an anemometer fixed at a single location. There are at least three significant factors responsible for the differences. The first is that the machine responds to some spatially averaged wind field whose extent is considerably larger than that to which an anemometer responds. The second is that a WECS may be capable of adjusting its operating characteristics to some degree, effectively "absorbing" some of the stresses to which it may otherwise have been exposed. An example of this is the use of blade pitch control to prevent excessive torque at the hub. Finally, the wind field experienced by a blade section of a WECS is not an Eulerian one but a "rotating" one. The first two factors are considered in this document, while the third will be treated in a subsequent report.

The spatial averaging of wind fluctuations is simulated by subjecting the data from an anemometer to a digital low pass filter. This filtering eliminates the higher frequency wind components which are relatively poorly correlated over any significant spatial extent.

The adjustment of a WECS to changing wind conditions may also be modeled by using a digital filter. Since the gusts of interest are described in terms

of the fluctuations about some mean wind speed, varying the time period over which this mean is determined simulates the adjustment of the operating characteristics. The procedure is as follows: take the original data string from an anemometer, subject it to a high pass filter, subtract the filtered data from the original string (thereby producing a running mean speed) and analyze the fluctuations about the resultant running mean.

The effects of spatial averaging and WECS adjustment may be combined by using a band-pass filter composed of the low and high pass filters. Results are presented using filtering times as a parameter.

1.3 STRUCTURE OF DOCUMENT

In Chapter 2 we give a brief description of the kinds of information provided in this report. Suggestions are given concerning the interpretation and application of these data. In some cases additional information must be provided by the design engineer, and these cases are noted. Finally, a brief discussion of the generality and limitations of the data are provided.

In Chapter 3 an extensive compilation of observed gust characteristics is given in graphical and matrix formats. In addition semi-empirical relationships for some of these characteristics are provided, along with comments on their use and anticipated errors.

In Chapter 4 a theoretical framework is given by which these gust characteristics may be understood. In cases where this is not yet possible, suggestions for possible future work are provided.

Finally, a set of Appendices are included which contain information of supplemental value or in greater detail than is required for a concise reading of the principal features of this document.

CHAPTER 2

APPLICATIONS OF GUST CHARACTERISTICS

2.1 TYPES OF INFORMATION PROVIDED

There are four basic types of information provided in this document. The first is a measure of the likelihood of occurrence of gusts of a given amplitude, regardless of the length of time such a gust lasts. An example of this is given in Figure 2.1, where the cumulative probability of occurrence of gusts is given as a function of amplitude. $P(A \geq x)$ is the probability that a given gust will have an amplitude A greater than or equal to some specified value x . Then if one has an estimate of the frequency of occurrence of all gusts, which we denote by N_0 , the number of gust events per unit time which have amplitude exceeding x is just $N_0 P(A \geq x)$.

It should be noted that both N_0 and $P(A \geq x)$ are functions of the filtering to which the data are subjected. Thus, one must determine each of these quantities for any particular application. In Chapter 3 some semi-empirical formulas are given which allow N_0 to be estimated from a knowledge of the turbulent spectrum. P is often approximated with a Gaussian curve, and examples of the quality of such approximation are also given.

One may also differentiate the curve in Figure 2.1 to yield the probability density of gust amplitudes. This, multiplied by N_0 , gives the number of gusts per unit amplitude interval occurring per unit time.

The second type of information provided is a description of the likelihood of occurrence of gusts of a given duration regardless of their amplitude. This is analogous to the amplitude distribution just discussed, and its properties are also clearly dependent upon the particular filters applied. These probabilities of amplitude or time exceeding specified values are examples of marginal probabilities.

The third kind of information is a description of the relationship between gusts of a given amplitude and duration. Since it has not been possible thus far to provide an analytical description of the joint frequency distribution

FIGURE 2.1. Probability Distribution for Amplitude

of these two quantities, we cannot provide a general procedure for determining this relationship. However, the following approach has been adopted and can be used to provide useful estimates of the behavior.

Given an amplitude A , what is the time such that $x\%$ of the gusts have durations less than or equal to T ? Let this time be denoted by $T_A(x)$. Alternatively given the length of time for the gust, what is the amplitude such that $x\%$ of the gusts have amplitudes less than or equal to A ? Let this amplitude be denoted by $A_T(x)$. We have determined the conditional probabilities $A_T(x)$ and $T_A(x)$ for values of x equal to 25, 50 and 75%, and for a number of different filtering combinations. An example of $T_A(25)$ and $T_A(75)$ is shown in Figure 2.2. Note that a ΔT and ΔA increment must be associated with such figures. These quantities denote the "box" size from which the curves were determined; e.g., if $\Delta A = 0.5$ m/s, the amplitude is assumed to be in the interval $(A, A + 0.5)$.

The fourth type of information given is a compilation of a number of parameters used in the current description of gusts or which may prove useful in future work (cf. Chapter 4). A list of all quantities and a definition of each is given in the Nomenclature.

2.2 USE OF INFORMATION

The above information can provide answers to the following questions:

- How often do gusts of a given amplitude range occur?
- How often do gusts of a given duration occur?
- Given the amplitude of a gust, what range of gust durations may be associated with it (25, 50, 75 percentile values)?
- Given the duration of a gust, what range of gust amplitudes may be associated with it (25, 50, 75 percentile values)?
- How do these various quantities differ with the size of the machine and its ability to respond to the changing ambient conditions?

The last question in particular requires careful consideration and judgment on the part of the design engineer. In the context of this document

FIGURE 2.2. 25 and 75 Percentile Times, $T_A(25)$ and $T_A(75)$

it implies the selection of a particular combination of high and low pass filters. Some suggestions on their selection follow.

As noted earlier, the high pass filter is a reflection of the machine's ability to react and adjust to variations in the mean wind. For example, if the time constant for this feedback loop is very short, the machine is able to compensate for all changes in wind speed, and no fatigue will result from excess torque. In this case, the running mean about which the gusts fluctuate will vary rapidly in time, and the high pass filtering time will be small. Conversely, if the time constant is large or if one wishes to estimate the number of times such compensation is required, the required averaging time may be much larger; one may wish to average for 10 minutes or more to achieve some measure of a "stationary" mean wind speed value. To accommodate these different requirements we have investigated the behavior of gusts for high pass filtering times of 25, 50, 100 and 2400 seconds. The design engineer must determine which of these values is most appropriate for his needs. We note here that the gust statistics are *relatively* insensitive to the high pass filter for filter times ≥ 100 seconds. This is probably a reflection of the existence of a "spectral gap" which implies relatively minor contributions to the fluctuating wind energy for time periods in the range of 10 minutes to 1 hour.

The size of the gust of interest provides the basis for a choice of low pass filters; the choice of filtering time, however, is not straight-forward. Roughly speaking, the higher the frequency of oscillation of a gust, the smaller the extent over which such an oscillation will occur. Hence, rapidly fluctuating gusts tend to encompass a small area while slowly fluctuating gusts are associated with eddies of large dimensions. What is required is a quantitative measure of this relationship. The coherence may be taken as such a measure. The coherence is defined by

$$\text{Coh}_{ij}(n) = \frac{\text{Co}_{ij}^2(n) + \text{Qu}_{ij}^2(n)}{F_i(n) F_j(n)} \quad (2.1)$$

where $\text{Co}_{ij}(n)$ is the cospectrum and $\text{Qu}_{ij}(n)$ is the quadrature spectrum of the two time series describing wind components i and j . The $F(n)$ are the normalized power spectra of the two series. An empirically determined approximation to Equation (2.1) is given by

$$\text{Coh}(n) = \exp \left(- \frac{\alpha n \Delta}{\bar{U}} \right) \quad (2.2)$$

for $i = j$ (the case of most interest for turbine fatigue analyses), where n is the frequency, Δ the lateral or vertical separation of the two points at which the wind is measured and $\bar{U}$ is the mean wind speed. The decay constant α is approximately equal to 16 (Pielke and Panofsky 1970). Some authors (e.g. Frost et al. 1978) define coherence as the square root of the quantity defined in Equation (2.1); in that case the value of α is about 8.

The coherence may be regarded as a frequency dependent correlation, ranging in value from 0 to 1. A low pass filter removes the higher frequency components which are less coherent for a given value of Δ . The difficulty lies in the selection of the value of coherence required over some specified distance Δ . If this is known, one may then determine the value of n for which this occurs and choose a low pass filter corresponding to the value of n . At the present time, however, the authors are unaware of any detailed simulation of WECS blade response which has addressed the question of what coherence is required over a given distance for a gust to influence a machine's performance or affect its fatigue. This is a prime responsibility of the design engineer but it does not seem to have been fulfilled. Similarly, the design engineer should determine which definition of coherence is appropriate, i.e., which value of the decay constant, α , should be selected. (Kirchoff [1979] has carried out some work on the electrical response of a small WECS to gusts, but the question of fatigue was not addressed.)

Table 2.1 gives some sample values of the distances for which the coherence has values of 0.05, 0.10 and 0.50. Frequencies of 0.05, 0.10 and 0.5 Hz are used, and $\bar{U}$ is assumed to be 10 m/s.

We do not wish to suggest that the use of coherence is necessarily the only means of selecting appropriate low pass filtering times. The information presented here can be adapted to any method a designer chooses to use, provided that the gusts of interest can be related to some suitable temporal average of the wind behavior measured at a "point."

Throughout this document results are presented using filtering time as a parameter. A filter time of t seconds corresponds to the use of a filter whose

TABLE 2.1. Values of Separation Δ (m) for which Coherence Assumes Indicated Values

		<u>n = 0.05 Hz</u>	<u>n = 0.10 Hz</u>	<u>n = 0.50 Hz</u>
Coh = 0.05	($\alpha = 8$)	75	37	7.5
	($\alpha = 16$)	37	19	3.7
Coh = 0.10	($\alpha = 8$)	58	29	5.8
	($\alpha = 16$)	29	14	2.9
Coh = 0.50	($\alpha = 8$)	17	8.7	1.7
	($\alpha = 16$)	8.7	4.3	0.87

power transfer function in the frequency domain has a half power point at a frequency $n = 1/t$. For some analyses, the mean wind speed was determined from a simple block average of 30 minutes duration rather than from the running mean procedure described previously. This corresponds to a high-pass filter with a characteristic time of approximately 2400 seconds. The basic sampling frequency of the anemometers was 10 Hz. Data were averaged in blocks of two, corresponding to a low pass filter of approximately 0.5 seconds for the "raw" data.

When the filter times have been selected, one may then use the various graphs, matrices and formulas to determine the quantities of interest. One also determines the value of N_0 for the particular filter used. This, coupled with the probability distributions for time and amplitude, will give the number of gusts per unit time exceeding a given time or amplitude. The conditional distributions $T_A(x)$ and $A_T(x)$ then can be used to relate gusts of a given amplitude to those of a given duration, and vice versa.

2.3 CONDITIONS AND LIMITATIONS

The gust descriptions proposed in this document are based upon analyses of data taken from a vertical array of anemometers 49 m in diameter and centered at a height of 36.6 m. The surrounding terrain is relatively flat and homogeneous, with a roughness length of 0.05 m. A further description of the experimental arrangement is given in Appendix A.

The largest part of the information presented in this document is derived from an analysis of 5 hours of data taken during the night of May 8 - May 9, 1979. During this period the wind speed at a height of 36.6 m averaged 11.6 m/s. The analyses were made on data collected at this same height. UNLESS OTHERWISE EXPLICITLY INDICATED, THE EXPERIMENTAL DATA EXHIBITED HERE ARE BASED ON THIS ONE PERIOD.

As will be evident in Chapter 3, an analysis of this type for a wide variety of wind speeds, heights, and stabilities etc. would be prohibitively expensive and time consuming. Thus, while greater generality would be preferable we have decided to present this information for a number of reasons.

- Additional analyses, not included in this report, suggest that at least some of the descriptors of amplitude and time distributions can be scaled with appropriate parameters to produce "universally" applicable results. Details will be provided in another report.
- The average wind speed for this data period has a value well within the operating range of many WECS. (For example the rated speed for the Mod 1 and Mod 2 wind turbines are approximately 15 m/s and 12 m/s, respectively, at hub height.
- At "typical" WECS sites with an average wind speed of 7 m/s, the wind speed may be expected to lie in the range 9.5 - 13.5 m/s over 18 percent of the time, based on an assumed Rayleigh distribution. For a machine with a cut-in velocity of 6 m/s and a cut-out velocity of 20 m/s, this represents over 30 percent of its operating time.
- The height of 36.6 m is a reasonable compromise between the hub heights of the larger machines (~60 m) and those of more modest dimension (~20 m).
- The upwind fetch during this data period was reasonably homogeneous and contained no significant roughness elements for a distance of 5 kilometers or more. The conditions are thus representative of those assumed in the formulation of many of the standard micrometeorological relationships found in the literature. Comparison of the results with various theoretical analyses should not be adversely affected by severe terrain

irregularities. The conditions at actual WECS sites may be far more complicated and best treated on a case-by-case basis; for a more general approach with a limited data set the conditions actually encountered for this analysis are more suitable.

- A number of semi-empirical relationships are proposed which relate the observed gust statistics to turbulence parameters which may be calculated from meteorological theory. The success of these approximations in describing the actual observations is assessed and provides a basis for estimating the utility of this approach in other conditions not directly represented by this data base. In this way gust behavior under more general circumstances may be determined.

Since the gust descriptions are primarily empirical, there are certain restrictions in the use of the data which must be observed if potentially serious errors are to be avoided.

- In complex terrain the gust structure may be seriously altered in comparison with that presented here. The same conclusion holds if the terrain is relatively homogeneous, but the roughness length z_0 is large and z/z_0 is significantly smaller than the present case ($z/z_0 \sim 700$). Here, z is the height of measurement.
- In light wind conditions, meandering of the wind may result in significantly different behavior. Similarly, rapidly changing mean wind speeds or directions or other obviously nonstationary situations are not covered by this analysis. In particular, gusts associated with severe weather conditions such as thunderstorms or frontal passages are excluded here.
- The application of these gust descriptions to extreme events should also be avoided. While some aspects of gust behavior may be well modeled by a Gaussian curve out to 3σ in the independent variable, this should probably be considered the exception rather than the rule. A large degree of scatter was found for many of the experimental curves, and the curves are obviously not reliable if the scatter becomes too severe. In no way has this analysis attempted to address questions such as what is the largest gust likely to occur in a period of N years.

- In this document positive and negative gusts have been treated as equivalent, i.e., we have been concerned only with the absolute value of the gust amplitude and not its sign. In fact, the distributions are always skewed, and the degree of skewness, although small, is probably statistically significant. In general, there seem to be more negative gusts with large amplitudes than positive ones. However, in view of the scatter in the data, the somewhat limited data base, the approximate nature of some of the semi-empirical relationships that were developed and the purposes of this study it was felt that little would be gained by attempting to discriminate between positive and negative gusts.

A final note on questions of accuracy in general is in order. As will be seen from the discussions of the semi-empirical relationships in Chapter 3, it is possible to err by more than a factor of two if the relationships are applied imprudently, even for the data presented here. Additional caution is advised, therefore, when applying these relationships to other situations. These results cannot be considered universal "handbook" values, nor are the authors aware of any gust descriptions of our type which qualify in this respect. Our results should be useful, however, in attaining some representative values which are applicable to design strategies and estimates of machine performance.

CHAPTER 3

GUST CHARACTERISTICS

3.1 INFORMATION FORMATS

The information provided in this section is of three types. The first is a series of graphs which describe the amplitude distribution of gusts, their time distribution, and the $T_A(x)$ and $A_T(x)$ conditional probabilities. The variation of each of these quantities with high and low pass filter combinations is also given. The notation for a filter combination is H/L, where H and L are the high- and low-pass filtering times in seconds, respectively.

The second type is that provided in matrix format. The matrix rows and columns are labeled by a particular high- and low-pass filter combination, and the matrix entry itself is a calculation of an observed gust characteristic for that particular filter combination. Then, for example, an entry might be $T_A(50)$ for a 100/10 filter, i.e., the 50 percentile conditional time determined for a 100 second high pass filter and a 10 second low pass filter.

We also introduce a number of semi-empirical relationships which characterize the gusts in terms of a small number of parameters. These parameters are, in turn, derived from basic turbulence properties such as the spectral energy density. The semi-empirical formulas may be considered as the third type of information provided; in practice it has been convenient to present much of this in matrix form as well.

In the last part of this section a discussion is given on the use of these relationships, and an assessment of some of the associated errors or uncertainties is also included.

3.2 OBSERVED GUST STATISTICS

3.2.1 Amplitudes

Figure 3.1 shows the amplitude distributions for gusts, using a filter combination of 2400/0.5. Here A is the gust amplitude and σ is the root-mean-square turbulent speed. Data from three runs taken under widely varying

FIGURE 3.1. Probability Distributions for Amplitude

wind speed conditions are shown. These include the 5-hr period described earlier, a 3-hr period when the winds averaged 14.5 m/s and a 2-hr period when the average wind speed was 18.3 m/s. While there is some scatter at the larger amplitudes, an exponential fit is quite good, at least out to values of A/σ a bit larger than 2.

The frequency of occurrence of gusts in a given amplitude range, $N_A(x \leq A \leq x + \Delta x)$, can be obtained from Figure 3.1 by differentiating the probability curves, multiplying by the appropriate N_0 (the frequency of occurrence of all gusts), scaling the amplitude value with the appropriate σ and multiplying the result by the amplitude range of interest:

$$N_A(x \leq A \leq x + \Delta x) = N_0 \frac{dP}{dA} \bigg|_{x + \frac{1}{2} \Delta x} \Delta x \quad (3.1)$$

An example is shown in Figure 3.2 for the 5-hr run. Here, the ordinate is in units of number of gusts per second per unit velocity interval (i.e., $N_A/\Delta x$ where $\Delta x = 1$ m/s), while the abscissa is the amplitude, in units of m/s. The exponential fit works well for values of amplitudes up to 3.5 m/s. A discussion of N_0 values is given later in this section.

Figures 3.3 and 3.4 show the effects of applying various filter combinations to the data. In each graph the high-pass filter time is fixed and the low pass filter time is allowed to vary. Lines have been drawn rather than individual points to simplify reading of the graphs.

The effects of various filters are perhaps seen more easily by looking at probability densities rather than the probability distribution. Figures 3.5 and 3.6 show the probability densities per unit amplitude for the same filter combination used in Figures 3.3 and 3.4. As the low-pass filtering time increases, the exponential form for the density gives a poorer fit. The density curve becomes flatter for some mid-range of amplitudes and then decreases rapidly for larger values.

3.2.2 Times

The probability distributions for gust times are shown in Figures 3.7 and 3.8 for two sets of filter combinations. For small times, a simple power

FIGURE 3.2. Frequency of Occurrence for Amplitudes

FIGURE 3.3. Probability Distributions for Amplitudes, 2400 s High-Pass Filter

FIGURE 3.4. Probability Distributions for Amplitudes, 50 s High-Pass Filter

FIGURE 3.5. Probability Densities for Amplitudes, 2400 s High-Pass Filter

FIGURE 3.6. Probability Densities for Amplitudes, 50 s High-Pass Filter

FIGURE 3.7. Probability Distributions for Gust Times, 2400 s High-Pass Filter

FIGURE 3.8. Probability Distributions for Gust Times, 50 s High-Pass Filter

law gives a rough fit to the time distributions. As the gust time approaches the value of the high pass filter time this relationship breaks down rapidly.

3.2.3 Tabulated Marginal Probabilities

To facilitate the construction of curves such as those shown in Figures 3.3 through 3.8, we have included a set of matrices of marginal probability levels which summarize the observed behavior. Each matrix represents the x percentile distribution value for either amplitude or time for 16 different filter combinations, where x takes on the values 25, 50, 75, and 95. Thus, for example, the 50 percentile value of amplitude for a 100/10 filter is 0.85 m/s, while the 95 percentile value is 2.3 m/s.

25 Percentile Amplitudes, m/s

$\begin{array}{c} H \\ \diagdown \\ L \end{array}$	2400	100	50	25
10	.21	.25	.25	.19
5	.21	.19	.21	.20
2.5	.26	.27	.27	.28
0.5	.25	.24	.23	.24

50 Percentile Amplitudes, m/s

$\begin{array}{c} H \\ \diagdown \\ L \end{array}$	2400	100	50	25
10	.87	.85	.76	.53
5	.65	.63	.63	.55
2.5	.64	.65	.64	.62
0.5	.63	.62	.61	.60

75 Percentile Amplitudes, m/s

$\begin{array}{c} H \\ \diagdown \\ L \end{array}$	2400	100	50	25
10	1.6	1.5	1.25	.99
5	1.5	1.4	1.3	1.0
2.5	1.4	1.3	1.2	1.1
0.5	1.3	1.3	1.2	1.1

95 Percentile Amplitudes, m/s

$\begin{array}{c} H \\ \diagdown \\ L \end{array}$	2400	100	50	25
10	2.6	2.3	2.0	1.4
5	2.7	2.4	2.1	1.7
2.5	2.7	2.4	2.2	1.8
0.5	2.6	2.4	2.2	2.0

25 Percentile Times, s

$\begin{smallmatrix} H \\ L \end{smallmatrix}$	2400	100	50	25
10	3.3	3.2	3.0	2.5
5	1.3	1.3	1.4	1.4
2.5	.74	.80	.80	.85
0.5	.42	.37	.32	.26

50 Percentile Times, s

$\begin{smallmatrix} H \\ L \end{smallmatrix}$	2400	100	50	25
10	8.4	8.6	7.9	6.0
5	3.6	3.7	3.9	3.4
2.5	1.7	1.6	1.6	1.6
0.5	0.84	.74	.77	.69

75 Percentile Times, s

$\begin{smallmatrix} H \\ L \end{smallmatrix}$	2400	100	50	25
10	18	16	13	8.5
5	10.5	9.9	9.3	9.1
2.5	4.5	4.2	3.8	3.4
0.5	1.9	1.9	1.7	1.6

95 Percentile Times, s

$\begin{smallmatrix} H \\ L \end{smallmatrix}$	2400	100	50	25
10	44	29	20	13
5	32	23	16.3	10
2.5	18.0	16	11.5	8.1
0.5	9.5	8.3	6.9	4.4

3.2.4 Miscellaneous Gust Parameters

Other possible quantities of interest for describing these distributions include the average gust amplitudes and times, the 1σ values of these quantities, where σ is measured about a zero value, and the kurtosises of the distributions. These quantities are given in the following sets of matrices.

Average Gust Amplitudes, m/s

$\begin{smallmatrix} H \\ L \end{smallmatrix}$	2400	100	50	25
10	1.03	.95	.82	.59
5	.94	.87	.80	.65
2.5	.91	.87	.82	.74
0.5	.89	.85	.81	.76

Average Gust Times, s

$\begin{smallmatrix} H \\ L \end{smallmatrix}$	2400	100	50	25
10	13.0	10.6	8.7	6.0
5	8.3	6.9	5.9	4.4
2.5	4.5	3.8	3.3	2.6
0.5	2.2	1.9	1.6	1.35

1 σ Amplitudes, m/s

$\begin{matrix} H \\ L \end{matrix}$	2400	100	50	25
10	1.35	1.21	1.03	.74
5	1.30	1.18	1.05	.84
2.5	1.24	1.16	1.07	.93
0.5	1.22	1.16	1.08	.99

1 σ Times, s

$\begin{matrix} H \\ L \end{matrix}$	2400	100	50	25
10	19.8	14.0	10.8	7.2
5	14.0	10.1	8.0	5.6
2.5	8.2	6.4	5.0	3.6
0.5	4.8	3.6	2.8	2.1

Kurtosises (Amplitude)

$\begin{matrix} H \\ L \end{matrix}$	2400	100	50	25
10	2.9	2.7	2.7	2.8
5	3.45	3.2	3.0	3.0
2.5	3.8	3.5	3.3	3.0
0.5	4.1	3.8	3.65	3.3

Kurtosises (Times)

$\begin{matrix} H \\ L \end{matrix}$	2400	100	50	25
10	8.9	3.2	2.6	2.25
5	11	4.7	3.4	2.8
2.5	16	8.1	5.7	4.4
0.5	34	16.4	12	7.8

The frequency of occurrence of all gusts, N_0 , is needed to convert probabilities into frequency of occurrence of gusts in a given amplitude or time range. The measured values of N_0 are shown in the next matrix; comparisons with calculated values are given later.

Observed N_0 (Frequency of Occurrence of All Gusts)(s⁻¹)

$\begin{matrix} H \\ L \end{matrix}$	2400	100	50	25
10	.077	.094	.115	.165
5	.120	.145	.168	.228
2.5	.225	.260	.305	.380
0.5	.460	.530	.616	.740

3.2.5 Conditional Probabilities

The relationships between the amplitude and duration of gusts would best be described by a joint frequency distribution of these two quantities. However, we have not succeeded in finding a general expression for this quantity and have chosen, therefore, to present the conditional probabilities of time and amplitude for selected percentile values, $T_A(x)$ and $A_T(x)$. The 25 and 75 percentile values are shown in Figures 3.9 [$T_A(x)$] and 3.10 [$A_T(x)$]. The discussion of the 50 percentile values appears later in this section. As noted in Chapter 2, associated with the $A_T(x)$ and $T_A(x)$ curves are "box" sizes for the amplitudes and times. These are given in the matrices below.

ΔA , m/s					ΔT , seconds				
$\begin{array}{c} H \\ \diagdown \\ L \end{array}$	2400	100	50	25	$\begin{array}{c} H \\ \diagdown \\ L \end{array}$	2400	100	50	25
10	.5	.4	.4	.4	10	8	1.6	2	1.6
5	.5	.4	.4	.4	5	2	1.2	1.2	.8
2.5	.4	.4	.4	.4	2.5	2.4	2.4	.8	.8
0.5	.5	.4	.5	.25	0.5	2	2	1.2	.4

The relationships between amplitude and time at larger values of these quantities differ significantly from those found at smaller values. There are at least two contributing causes. The first is that the number of larger amplitude and time gusts is relatively small, and the statistical accuracy of the curves is correspondingly less. The second is that the correlations between amplitude and time are rather poor for this region. The correlation for all values of these two parameters for all filter combinations is approximately 0.75; if the two lowest velocity and time classes for a given distribution are eliminated the correlation typically drops to 0.5. (The lowest velocity class is ~ 0.4 m/s; the smallest time class ~ 1 to 2 seconds. These numbers vary somewhat for analyses using different filter combinations.) The correlation for the largest events can be expected to be even worse. The result is that, for the $T_A(x)$ and $A_T(x)$ curves, little confidence can be given to those regions of the curves that show erratic behavior at large amplitudes

FIGURE 3.9. 25 and 75 Percentile Times
a. 2400 s High-Pass Filter

FIGURE 3.9. Continued
b. 100 s High-Pass Filter

FIGURE 3.9. Continued
c. 50 s High-Pass Filter

25 AND 75 PERCENTILE T(A)

FIGURE 3.9. Continued
d. 25 s High-Pass Filter

FIGURE 3.10. 25 and 75 Percentile Amplitudes
a. 2400 s High-Pass Filter

25 AND 75 PERCENTILE A(T)

FIGURE 3.10. Continued
b. 100 s High-Pass Filter

25 AND 75 PERCENTILE T(A)

FIGURE 3.10. Continued
c. 50 s High-Pass Filter

FIGURE 3.10. Continued
d. 25 s High-Pass Filter

and times. However, it is apparent from our data that the greatest amplitudes and longest times belong to different gusts.

Correlation Coefficients for Gust
Amplitudes and Times

$\begin{matrix} H \\ L \end{matrix}$	2400	100	50	25
10	.77	.77	.73	.68
5	.78	.80	.77	.71
2.5	.77	.77	.75	.69
0.5	.69	.71	.71	.70

3.3 SEMI-EMPIRICAL FORMULAS

For engineering approximations or in cases not covered by the information already provided, it would be useful to have a set of simple formulas that could be used to estimate the gust properties of interest from other, better known quantities. These latter quantities include the standard micrometeorological expressions for turbulence spectra, wind profiles, etc. We have attempted to give some working approximations based on our 5-hr test case. It should be emphasized that not only do these working approximations result in some errors when compared to our test case, but in general they have not been tested against other wind records. Thus, the user is advised to use discretion when applying them, remembering the spirit and intent for which they were provided. A discussion of some errors is given at the end of this section.

3.3.1 General Features

We begin by assuming that the gust properties of interest can be described in terms of one or more of the following three parameters, σ_A , σ_T , and N_0 (see Nomenclature). These quantities, in turn, will be related to the turbulent energy spectrum $\phi(n)$ and some appropriate filtering function $F(n)$. Here $\phi(n)$ is assumed to have the form recommended by Kaimal (1973) for the neutral limit of the stable case,

$$\frac{n\phi(n)}{\sigma^2} = \frac{0.164 (f/f_0)}{1 + 0.164 (f/f_0)^{5/3}} \quad (3.2)$$

where n is frequency, $f_0 = 0.0144$ (Frost 1978); $f = nz/\bar{U}$; z is the height of measurement and $\bar{U}$ is the mean wind speed. Strictly speaking, Equation 3.2 was proposed for the longitudinal wind component and not the speed, but the agreement with the speed spectrum can be expected to be quite good for moderate to strong winds. Figure 3.11 gives an example of the agreement between Equation 3.2 and the spectrum of a 28-min segment of our 5-hr test period.

The rms turbulent velocity is σ . If possible it should be measured, but if this is not practical it may be estimated from

$$\sigma \sim \frac{0.94 \bar{U}}{\ln(z/z_0)} \quad (3.3)$$

For the high- and low-pass filters used in this work, $F(n)$ is given by

$$F(n) = \left[\frac{\sin(S_1 \pi n \Delta t)}{S_1 \sin(\pi n \Delta t)} \right]^2 \left[1 - \frac{\sin(S_2 \pi n \Delta t)}{S_2 \sin(\pi n \Delta t)} \right]^2 \quad (3.4)$$

In this equation S_1 is the number of data points used in the low-pass filter, S_2 is the number used in the high pass filter and Δt is the time increment between successive data points. For this analysis, Δt was 0.2 s.

3.3.2 Frequency of Occurrence

We may then compute N_0 , the frequency of occurrence of all gusts, from the expression

$$N_0 = 2 \left[\frac{\int_0^\infty n^2 \phi(n) F(n) dn}{\int_0^\infty \phi(n) F(n) dn} \right]^{1/2} \quad (3.5)$$

The computed values of N_0 are given below, and the ratios of the measured to calculated values are also given. The agreement is quite good.

FIGURE 3.11. Comparison of Kaimal's Spectrum and a 28-Min Speed Spectrum

Calculated N_0					N_0 meas/ N_0 calc				
$\begin{matrix} H \\ L \end{matrix}$	2400	100	50	25	$\begin{matrix} H \\ L \end{matrix}$	2400	100	50	25
10	.081	.099	.120	.162	10	.95	.95	.96	1.02
5	.123	.145	.172	.221	5	.98	1.00	.98	1.02
2.5	.201	.235	.274	.341	2.5	1.12	1.11	1.11	1.12
0.5	.470	.544	.625	.759	0.5	.98	.97	.99	.97

3.3.3 Amplitude Distribution

While it is evident that a normal curve does not, in general, fit the data for the amplitude distribution, it is possible to approximate this distribution with a Gaussian form. Figure 3.12 gives two examples of the quality of such fits. Figure 3.13 gives the corresponding probability densities for the same cases. The quality of the fit is seen to depend upon the filtering combination, the amplitude range of interest, and the degree of error that is considered acceptable. The selection of a Gaussian curve requires an estimate of σ_A , the standard deviation of gust amplitudes. Figure 3.14 shows a plot of σ_A/σ vs band-pass filter ratio for all the filter combinations studied. σ is the turbulence rms speed; both σ_A and σ are derived from the filtered wind data. Note that to a high degree of accuracy σ_A/σ depends only on H divided by L and not the particular values of H or L . This must obviously break down as one moves to very high or very low frequency ranges. As $H/L \rightarrow 1$, however, the ratio should approach $\sqrt{2}$. A reasonable approximation for the band-pass filtered data here is $\sigma_A/\sigma = 1.3$.

3.3.4 Conditional Probabilities

Curves of $T_A(x)$ and $A_T(x)$ for the 25 and 75 percentiles were shown earlier. Efforts to represent these curves in terms of some simple relationships involving σ_A and σ_T , the standard deviations of gust amplitudes and times, respectively, were not very satisfactory. However, better success was realized for the 50 percentile curves. For these cases, the dependence of $T_A(x)$ on amplitude or $A_T(x)$ on time is essentially linear from zero up to some

FIGURE 3.12. Gaussian Probability Distribution for Amplitude
a. 50/5 Filter

FIGURE 3.12. Continued
b. 25/10 Filter

FIGURE 3.13. Gaussian Probability Density for Amplitudes
a. 50/5 Filter

FIGURE 3.13. Continued
b. 25/10 Filter

FIGURE 3.14. σ_A/σ as a Function of Band Pass Filter Ratio

"reasonable" value of amplitude or time. Figure 3.15 gives an example of this. For times up to approximately 10 seconds, $A_T(50)$ can be well represented by a straight line. In these linear regions, the slope of the $T_A(x)$ curve was found to be proportional to σ_T/σ_A , while the slope of the $A_T(x)$ curve depends upon σ_A/σ_T .

Assume that σ_A and σ_T may be approximated by

$$\sigma_A = K_1 \sigma_F \quad (3.6)$$

and

$$\sigma_T = K_2 / N_0 \quad (3.7)$$

Here, σ_F refers to the rms turbulent speed derived from the filtered data.

Then
$$\frac{\sigma_A}{\sigma_T} = \frac{K_1}{K_2} \sigma_F N_0 \quad (3.8)$$

or
$$\frac{\sigma_A}{\sigma_T} = 2\sigma_u \frac{K_1}{K_2} \left[\int_0^\infty \frac{n^2 \phi(n)}{\sigma_u^2} F(n) dn \right]^{1/2} \quad (3.9)$$

If we make the further assumption that $K_1 = K_2$, one can compute σ_A/σ_T and compare it with observations. This is shown in the next matrix.

(σ_A/σ_T) calculated/ (σ_A/σ_T) observed

$\begin{matrix} H \\ L \end{matrix}$	2400	100	50	25
10	1.35	1.07	.97	.87
5	1.57	1.26	1.11	.96
2.5	1.69	1.37	1.16	.96
0.5	2.37	1.86	1.57	1.24

FIGURE 3.15. Example of Straight Line Fit for 50 Percentile $A(T)$

If the longest high pass and shortest low-pass filter combinations are excluded the ratios of calculated to observed values are generally satisfactory. It is, therefore, possible to estimate σ_A/σ_T or σ_T/σ_A from a knowledge of the spectrum. The construction of the $A_T(50)$ or $T_A(50)$ curves then requires the proportionality constants between the slopes and σ_A/σ_T or σ_T/σ_A , respectively. Let

$$A_T(50) = C_A (\sigma_A/\sigma_T) T \quad (3.10)$$

and

$$T_A(50) = C_T (\sigma_T/\sigma_A) A \quad (3.11)$$

where T is the time and A the amplitude. Then the proportionality constants C_A and C_T are shown below.

C_A					C_T				
$\begin{array}{c} H \\ L \end{array}$	2400	100	50	25	$\begin{array}{c} H \\ L \end{array}$	2400	100	50	25
10	.76	1.16	1.03	.94	10	.71	.75	.85	.76
5	1.25	1.19	1.09	.97	5	.65	.73	.72	.77
2.5	1.06	1.05	1.28	.77	2.5	.62	.74	.73	.65
0.5	.96	.78	.71	.80	0.5	.65	.74	.72	.77

For high-pass filter times of 100 s or less and low-pass filter times of 2.5 s or more, the average value of C_A is 1.05 ± 0.15 and the average value of C_T is 0.74 ± 0.05 .

Hence to construct the 50 percentile $A_T(x)$ curve, one computes σ_A/σ_T from Equation 3.9, determines C_A from the matrix above, and multiplies the two quantities. The result is the slope of the amplitude versus time relationship. The procedure for the $T_A(x)$ curve is analagous.

3.4 ERROR ANALYSIS AND CONFIDENCE LIMITS

Since the determination of the various matrix elements has been based upon a single 5-hr data set, it is impossible to provide confidence limits in

the usual sense. However, it is possible to make some general observations about the internal consistency of some of the proposed relationships and the effects of relaxing or altering some of the basic assumptions.

The accuracy of the N_0 calculations has already been shown to be quite good for the test case. Additional comparisons were made for seven other runs using the 2400/0.5 filter. With this filter the calculated values of N_0 were generally higher than the measured ones; for eight runs the ratio of calculated to measured N_0 was 1.49 ± 0.49 . Equation 3.5 can then be used to give a conservative upper bound on the frequency of observed gusts for our data. The presence of the n^2 factor in the numerator of Equation 3.5 implies that it is the higher frequency portion of the spectrum that is more important. Hence, the variations commonly encountered in the lower frequency regions will not be as critical, unless a relatively long low-pass filtering time is used.

A procedure has also been given for determining the 50 percentile conditional probabilities for amplitude and time. One may write

$$A_T(50) = \bar{C}_A (\sigma_A/\sigma_T) T \quad (3.12)$$

and

$$T_A(50) = \bar{C}_T (\sigma_T/\sigma_A) A \quad (3.13)$$

where $\bar{C}_A = 1.05 \pm 0.15$ and $\bar{C}_T = 0.74 \pm 0.05$.

If (σ_A/σ_T) is computed from Equation 3.9, the computed slopes of $A_T(50)$ versus T and $T_A(50)$ versus A may be compared with the actual slopes. The ratios are shown below.

Ratio of Computed to Measured Slope of $A_T(50)$ versus T					Ratio of Computed to Measured Slope of $T_A(50)$ versus A				
$\begin{matrix} H \\ L \end{matrix}$	2400	100	50	25	$\begin{matrix} H \\ L \end{matrix}$	2400	100	50	25
10	1.87	.97	.99	1.00	10	.77	.92	.90	1.11
5	1.32	1.11	1.06	1.04	5	.72	.80	.92	1.01
2.5	1.07	1.37	.96	1.30	2.5	.57	.75	.88	1.19
0.5	2.01	2.50	2.32	1.65	0.5	.48	.54	.65	.97

For band pass filter combinations of 100/2.5 or narrower, the simple formulas of Equation 3.12 and 3.13 give generally satisfactory results.

The matrix entries were computed using the form of the spectrum proposed by Kaimal (1973), and assuming that the value of σ_u is known in Equation 3.9. For the case considered here, σ_u was found to be 1.32 m/s. If one uses the approximate relationship given in Equation 3.3, i.e., $\sigma_u \sim 0.94 \bar{U}/\ln(z/z_0)$ then $\sigma_u \sim 1.65$. The use of this value would raise the entries in the $A_T(50)$ slope ratio matrix by 25% and reduce the entries in the $T_A(50)$ slope ratio matrix by 20%. The conditions under which the data were taken were actually slightly stable, and σ_u can be expected to depart somewhat from the behavior given by Equation 3.3. Appendix C has a further discussion on the effects of stability on the computed slopes.

CHAPTER 4

GUST MODELS

4.1 GENERAL STRUCTURE OF GUST MODELS

In previous chapters we have given some specific descriptions of wind fluctuations of a particular kind, i.e., the $GUST_0$ events. In this chapter we propose a more general view of gust models and their relationship to turbulence quantities which does not depend on a particular choice of a discrete event. This view may provide a framework for the development of other gust descriptions. We begin with some general concepts, discuss some of the assumptions involved, define the moments of interest and finally relate the gust parameters to turbulence.

The basic structure of a discrete gust model can be expressed by a mathematical formalism developed from the following specifications:

- one basic state of the atmosphere S , which depends on mean wind speed $\bar{U}$, thermal stability, surface roughness z_0 , and height above the surface, z .
- one definition of discrete events.
- a subset of events that can be characterized by the magnitude classes x and y for gust amplitude and gust time (period), respectively.
- a turbulence spectral function $\phi(n)$, where n is cyclical frequency, (although $\phi(n)$ is a function of S , we suppress the S notation).
- one machine filter function $F(n)$.

We can now write the abstract equations of a gust model as follows:

$$N_{AT}(x,y;S,F) = N_0(S,F)P_{AT}(x,y|\sigma_A,\sigma_T,\rho_{AT},S,F) \quad (4.1)$$

$$\sigma_A = \sigma_A[\sigma(S,F),S,F] \quad (4.2)$$

$$\sigma_T = \sigma_T[N_0(S,F),S,F] \quad (4.3)$$

$$\rho_{AT} = \rho_{AT}(S, F) \quad (4.4)$$

$$\sigma^2(S) = \int_0^{\infty} \phi(n) dn \quad (4.5)$$

$$\sigma^2(S, F) = \int_0^{\infty} \phi(n) F(n) dn \quad (4.6)$$

In Equation 4.1 the term on the left is the frequency of occurrence of gusts in which A, the absolute value of gust amplitude, exceeds x while the gust time T, simultaneously exceeds y, and N_0 , the basic frequency factor, is the frequency of events. [For the GUST₀ model, N_0 is also twice the frequency of positive slope zero-crossings by the filtered turbulence time function, which we designate as Rice's frequency N_0 (Rice 1945).] P_{AT} is the probability that a GUST₀ event will simultaneously have amplitude A exceeding x and time T exceeding y. In the argument list both x and y are positive. We note that

$$P_{AT}(0,0) = 1 \text{ and} \quad (4.7)$$

$$N_{AT}(0,0) = N_0. \quad (4.8)$$

The S notation shows that all three factors are functions of the state of the atmosphere; the F notation shows that all three factors represent filtered turbulence, i.e., turbulence "as seen by the machine." The notation to the right of the vertical bar in Equation 4.1 shows that the probability is a function of σ_A (the gust amplitude rms), σ_T (the gust time rms), and ρ_{AT} (the correlation coefficient between gust amplitude [absolute value] and gust time). The S and F notation at the extreme right of Equation 4.1 shows that the relationship of the probability to the just-mentioned moment parameters is a function of both the state of the atmosphere and machine filtering.

4.2 ASSUMPTIONS AND LIMITATIONS

Certain assumptions are implicit in the treatment thus far. First, a more general approach would probably use the signed values of gust amplitude

rather than the absolute values. However, the notation above is consistent with the approach taken in this report. Also, the probability depends on all moments, not just second moments. Since a detailed approach based on higher moments may well be too complicated and since the accuracy of observed higher moments decreases rapidly with the order of the moment, we have chosen to treat the probability as a function of second moments only, noting that the relationship of the probability to the second moments depends on the state of the atmosphere and on filtering.

If the model of Equation 4.1 is to be practical, the probability structure must be amenable to description, and the dependencies on S and F as indicated at the extreme right of the equation must be known. Even if the dependencies on S and F can be treated separately, a practical model from Equation 4.1 is by no means certain, since we do not have a general method to derive the probability structure of filtered data from the probability structure of unfiltered data. This is true for both turbulence and gust data. As a result, it is probably necessary to make some reasonable assumptions about the structure and to test these assumptions for a variety of conditions. This was done in Chapter 3, where a Gaussian amplitude distribution for $GUST_0$ event was evaluated (see Figure 3.12).

Moreover, in the $GUST_0$ analysis in this report, the variation of some of the parameters with filtering was illustrated. In the limited analysis so far, it has been necessary to assume that such variation with filtering is independent of the state of the atmosphere. Many more tests would have to be analyzed to show how all these things vary as functions of S . Nevertheless, this is probably not a severe restriction for moderate to high wind speeds, where neutral conditions are often present.

The greatest aid to simplification of analysis of atmospheric turbulence is the use of dimensionless variables. This approach cannot be used in the present problem because the machine filter specifications are in terms of dimensional frequencies that are independent of such "atmospheric frequencies" as $\bar{U}/z$. As an example, consider the high- and low-pass filters used previously. These were determined by an assumed frequency range $[n_1, n_2]$ over which the

machine responds. The turbulent spectrum, however, was written in terms of a dimensionless frequency $f = nz/\bar{U}$. Integrals of the filtered turbulence, such as those entering into the calculation of N_0 or σ_A/σ_T , then correspond physically to the machine "seeing" the turbulence through a fixed spectral window. The quantity sampled by the window depends on $\bar{U}$ and z while the window does not. Hence, a dimensionless representation of gust characteristics is generally not possible. The presence of the (S,F) notation throughout the equation set 4.1 through 4.6 dictates that the entire problem is permeated with the same inapplicability of dimensionless analysis.

4.3 MOMENTS

Equation 4.2 through 4.4 deal with the individual and joint second moments of GUST₀ analysis. Since the maximum probability density of both gust amplitude and gust time is generally at the zero value, it is convenient to define the individual moments as non-central moments. Using E as the expectation operator, and remembering that A is the absolute value of gust amplitude, we define

$$\bar{A} = E[A] \quad (4.9)$$

$$\bar{T} = E[T] \quad (4.10)$$

$$\sigma_A^2 = E[A^2] \quad (4.11)$$

$$\sigma_T^2 = E[T^2] \quad (4.12)$$

$$\rho_{AT} = \frac{E[(A-\bar{A})(T-\bar{T})]}{[E[(A-\bar{A})^2]E[(T-\bar{T})^2]]^{1/2}} \quad (4.13)$$

4.4 RELATION OF GUST PARAMETERS TO TURBULENCE

Equations 4.2 and 4.3 indicate schematically how we expect σ_A and σ_T to be related to turbulence analysis. In Equation 4.3 we note that

$$N_0 = \bar{T}^{-1} \quad (4.14)$$

where $\bar{T}$ is the average duration of a gust. Again if the model is to be practical, expressions relating σ_A to σ and σ_T to $\bar{T}$ must be tractable. In Chapter 3, it was assumed that for a wide range of filter combinations

$$\sigma_A = K_1 \sigma \quad (4.15)$$

and

$$\sigma_T = K_2 \bar{T} \quad (4.16)$$

In practice, $K_1 \sim K_2 \sim 1.3$. Equation (4.15) owes its validity to the fact that as the filter bandwidth approaches zero, the ratio σ_A/σ increases asymptotically to $\sqrt{2}$. Equation (4.16) is valid if the probability distribution of the gust times is half-Gaussian, in which case $\sigma_T = \sqrt{\pi/2} \bar{T}$. This approximation is reasonable for band-pass filters of approximately one decade.

Since the correlation coefficient ρ_{AT} is entirely a function of the defined gusts' amplitudes and times, we have no reason to expect that we can formulate it as a function of turbulence variables. Indeed, we have not been able to exhibit it as a function of either F or S , but find that the correlation is a function of the magnitude of the variables being correlated. Thus when the entire sample is correlated the coefficient turns out to be

$$\rho_{AT} \sim 0.75 \quad (4.17)$$

As noted in Chapter 3, this value drops rapidly if low magnitudes of gust amplitude and gust time are arbitrarily removed from the sample population.

Equations 4.5 and 4.6 are included to emphasize that a gust frequency of occurrence model is in many respects a spectral model. Equation (4.6) emphasizes, additionally, that the effect of filtering is analytically linked to the turbulence spectrum.

Although Equation (4.1) exhibits only the joint probability of A and T , emphasis in the first three chapters has been on the more tractable probability

descriptors that may be derived from the joint probability. These are the conditional probabilities of A given T and of T given A, and the marginal probabilities of A and T. A set of equations for the marginal probabilities comparable to the set (4.1) through (4.6) is

$$N_A(x, S, F) = N_O(S, F) P_A(x | \sigma_A, S, F) \quad (4.18)$$

$$N_T(y, S, F) = N_O(S, F) P_T(y | \sigma_T, S, F) \quad (4.19)$$

plus Equations (4.2), (4.3), (4.5), and (4.6).

We do not recommend extrapolation of this approach to apply to machines of frequency sensitivity of less than one octave. As the bandwidth narrows, there comes a point when the maximum probability densities of amplitude and time are no longer at the zero value. In this case the applicable moments would be central moments rather than the non-central moments defined in this report.

It is apparent that a full description of the gust properties of interest is not possible at this time; however, the general formulas presented in this chapter should provide a useful framework within which further studies may be conducted.

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APPENDIX A

ANEMOMETER ARRAY

The array of Gill 3-component propeller anemometers used for the measurements described in this document consists of two concentric rings of instruments. Their center is at a height of 36.6 m; the outer ring is 49 m in diameter and the inner ring is 34.5 m in diameter. The outer ring consists of 8 anemometers spaced 45° apart and the inner one consists of 4 anemometers at 90° intervals. Figure A-1 gives a schematic diagram of the arrangement. Three other anemometers shown in the figure are currently in place but were not in service during the measurements described here, i.e., the center one and the ones at heights of 3 and 6.1 m.

For the present analysis any one of the anemometers at the 36.6-m level would have sufficed. In practice, the multiple instrumentation at that level provided a check on the data quality. In a subsequent report, a simulation of the gust environment experienced by a rotating blade will be obtained from an analysis of the data recorded by the outer ring of instruments.

Data from each instrument were recorded digitally on 9-track magnetic tape at a rate of 10 Hz using a Data General Nova 3 computer. The data were block averaged in groups of two, giving an equivalent recording rate of 5 Hz. A scheme developed by Horst (1972) was used to correct the wind records for the non-cosine response of the Gill anemometers.

The surrounding terrain was generally homogeneous semi-arid land with low grasses and scattered sagebrush giving a roughness length of ~ 0.05 m. About 100 m to the south was a group of low buildings; in general winds blowing from this direction are not analyzed for their gust characteristics.

FIGURE A-1. Schematic of Vertical Plane Array of Anemometers

APPENDIX B

ANALYTICAL AND DIGITAL FILTERS

B.1 ANALYTICAL FILTERS

Sharp cutoff filters are generally physically unrealistic since the change of machine responsiveness with frequency is a continuous function of frequency. An example of a low-pass filter function is that used by Cliff and Fichtl (1978) relating wind fluctuation frequency n to a frequency derived from the mean wind speed $\bar{U}$ and the WECS blade diameter D . Their formula is

$$F(n) = \frac{1}{1 + \left(\frac{nD}{\bar{U}}\right)^2} \quad (B.1)$$

which has a half-power point at $n = \bar{U}/D$.

B.2 DIGITAL FILTERS

The digital filters applied in the analysis of Chapter 3 are easy to specify and have frequency response functions $F(n)$ that are continuous functions of n . However, in the case of the high-pass filter employed, $F(n)$ oscillates to values greater than unity for some values of n . (See Figure B-1, where a 100/2.5 bandpass filter is plotted.)

The bandpass wind fluctuation filtering properties of a WECS machine were simulated by applying digital filters, which are moving averages, to the data. The formulation is as follows. Let the time series consist of N values at a sampling interval Δt :

$$u(0), u(\Delta t), u(2\Delta t), \dots, u[(N-1)\Delta t]$$

The filtering technique consists of using moving averages with the length adjusted so that the half-power point of the filter is at the required frequency. Let S_1 be the number of data required in a moving average that will filter the data such that the half-power point of a low-pass filter is at s_1 seconds

FIGURE B-1. 100/2.5 Band-Pass Filter

(reciprocal of a frequency n_1). Let S_2 be the number of data required in a moving average to be used to produce high-pass filtered data with half-power point at s_2 seconds. Then S_1 and S_2 must be optimal odd integers approximating

$$S_1 = 0.44 s_1 / \Delta t \quad (B.2)$$

$$S_2 = 0.76 s_2 / \Delta t \quad (B.3)$$

We define the moving average filter centered at time $m\Delta t$ by

$$\psi[u(m\Delta t), S_i] = \frac{1}{S_i} \sum_{j=m - \frac{S_i-1}{2}}^{m + \frac{S_i-1}{2}} [u(j\Delta t)], \quad \frac{S_i-1}{2} \leq m \leq (N-1) - \frac{S_i-1}{2} \quad (B.4)$$

$i = 1, 2$

depending on whether the moving average is to be used in a low-pass or high-pass filter. The data $u(m\Delta t)$ as seen through the low-pass filter or the high-pass filter are, respectively,

$$u_L(m\Delta t, S_1) = \psi[u(m\Delta t, S_1)] \quad (B.5)$$

$$u_H(m\Delta t, S_2) = u(m\Delta t) - \psi[u(m\Delta t, S_2)] \quad (B.6)$$

A bandpass filter using both characteristics is given by

$$u_B(m\Delta t, S_1, S_2) = \psi[u_H(m\Delta t, S_2), S_1] \quad (B.7)$$

In other words, the bandpassed data are run first through the high-pass filter and then through the low-pass filter.

The corresponding multipliers applied to the spectral estimates of the unfiltered data, which yield the spectra of the filtered data, are:

$$\phi_L(n, S_1) = \phi(n) \left[\frac{\sin(S_1 \pi n \Delta t)}{S_1 \sin(\pi n \Delta t)} \right]^2 \quad (B.8)$$

for the low-pass filter, and

$$\phi_H(n, S_2) = \phi(n) \left[1 - \frac{\sin(S_2 \pi n \Delta t)}{S_2 \sin(\pi n \Delta t)} \right]^2 \quad (\text{B.9})$$

for the high-pass filter. The expressions in the brackets are the spectral transfer functions. For the bandpass filter, $\phi(n)$ is multiplied by both bracketed squared functions.

APPENDIX C
THEORETICALLY PREDICTED ERRORS IN CALCULATION
METHOD FOR σ_A/σ_T ASSUMING NEUTRAL STABILITY

As shown in Chapter 3, the 50 percentile conditional probability for amplitudes is estimated by

$$A_T(50) = C_A \frac{\sigma_A}{\sigma_T} T \quad (C.1)$$

and

$$\frac{\sigma_A}{\sigma_T} = 2\sigma_u \left[\int_0^\infty \frac{n^2 \phi_u(n)}{\sigma_u^2} F(n) dn \right]^{\frac{1}{2}} \quad (C.2)$$

where the value of C_A was about 1.05 for certain decade bandpass filters, but varied significantly from 1.05 for some other filterings. All of this was developed assuming neutral conditions. Turbulence theory predicts that for non-neutral conditions there would be systematic errors in calculations based on the assumption of neutral stability. These errors arise because of changes in the spectral function in the integral in Equation C.2. Also, if σ_u is being estimated from $\bar{U}$, this estimate must vary with stability. Further uncertainty arises from the fact that different spectral functions are proposed for the same physical power spectrum by different investigators.

The u-component power spectrum that has been suggested for WECS research is taken from Kaimal (1973):

$$\frac{n\phi_u(n)}{\sigma_u^2} = \frac{0.164 (f/f_0)}{1 + 0.164 (f/f_0)^{5/3}} \quad (C.3)$$

where

$$f = nz/\bar{U}$$

This has been adapted to neutral conditions by Frost et al. (1978) by the specification

$$f_0 = 0.0144$$

Actually, f_0 is a function of stability. For stable conditions, Kaimal (1973) suggests

$$f_0 = 0.5 Ri \quad (C.4)$$

where Ri is the Richardson number:

$$Ri = \frac{g}{\bar{\theta}_0} \frac{d\bar{\theta}/dz}{(d\bar{U}/dz)^2} \quad (C.5)$$

where $\bar{\theta}$ is potential temperature and g is gravitational acceleration. For unstable conditions, Equation C.3 seriously underestimates the low-frequency portion of the power spectrum.

For stable conditions σ_u can be estimated from the mean wind speed $\bar{U}$ using the following

$$\frac{\sigma_u}{\bar{U}} = \frac{\sigma_u}{u_*} \frac{u_*}{\bar{U}} \quad (C.6)$$

where (see Dyer 1974)

$$\frac{u_*}{\bar{U}} = \frac{k}{\ln(z/z_0) + 5.0 z/L} \quad (C.7)$$

$$\frac{\sigma_u}{u_*} = 2.2 - 0.086 z/L \quad (C.8)$$

(see McBean 1971).

In these equations k is Von Karman's constant, taken to be 0.4, and L is the Monin-Obukhov stability length

$$L = \frac{-u_*^3}{k(g/\bar{\theta}) \overline{w'\theta'}} \quad (C.9)$$

where $\overline{w'\theta'}$ is the vertical temperature flux.

Using these equations we can test theoretically the errors involved by applying Equation C.2 to stable conditions. We form the ratio:

$$R = \frac{(\sigma_A/\sigma_T)_{\text{stable}}}{(\sigma_A/\sigma_T)_{\text{neutral}}} \quad (C.10)$$

where the turbulence is described by Equations C.2 through C.9, filter combinations of 2400/0.5, 50/0.5, 2400/5, or 50/5 are used, and various degrees of stability are defined by allowing L to equal ∞ , 300, 100, and 50 m in separate ratio calculations. In practice, the differences caused by using different filters are relatively unimportant.

A four dimensional matrix of ratios was produced by letting

$L = \infty, 300, 100, 50 \text{ m}$

$\bar{U} = 4, 8, 12, 16, \text{ and } 20 \text{ m sec}^{-1}$ at a reference level of 10 m

$z = 10, 20, 40, 80, 120 \text{ m}$

$z_0 = 0.10 \text{ m}, 1.0 \text{ m}.$

For these values, the ratio R varied from 0.85 to 1.9.

For the unstable case, the situation is more complicated since the ratio σ_u/u_* is a function of the height to the lowest temperature inversion aloft, z_i (Panofsky et al. 1977). A proposed formula for σ is

$$\frac{\sigma}{u_*} = [4 + 0.6(z_i/-L)^{2/3}]^{1/2} \quad (C.11)$$

where σ refers to either σ_u or to σ_v . Experimental data from Panofsky et al. (1977) show values of σ/u_* as great as 4.5, which is twice the expected

neutral value. Also, when using Paulson's formula (1970), $u_{*}/\bar{U}$ was found to be up to half again as great as for neutral conditions at WECS heights.

In evaluating the possible error as a result of modeling diabatic conditions with a formula for neutral conditions, one must remember that the extremes of diabatic conditions do not apply since wind-energy-producing conditions tend to favor neutrality. On the other hand, more variation in the estimates could be introduced by using an alternative form of the power spectrum.

Taking all this into account, it seems that an estimate of σ_A/σ_T using Equation C.2 is accurate within a factor of two, and further attempts at refinement are probably not warranted.

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