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COMPUTATIONAL FUSION MAGNETOHYDRODYNAMICS

By

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COMPUTATIONAL FUSION MAGNETOHYDRODYNAMICS*

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Simple magnetohydrodynamic models provide the framework for much of our understanding of the macroscopic behavior of magnetically confined laboratory plasmas. In even the simplest of models, however, the many different time and spatial scales, the multidimensionality, and the nonlinearity of the equations make finding solutions difficult. In realistic geometries obtaining quantitative results to aid our understanding, to interpret experiment, and to design new devices, involves the development of large scale numerical codes. During the past decade considerable effort has been extended in the fusion community to develop equilibrium, linear stability, and nonlinear time evolution codes in two and three dimensions, some of which have had a considerable impact on the fusion program. An overview of the various types of codes and numerical methods is given. Emphasis is on the spectrum of linear perturbations and ideal MHD stability, boundary layer methods and resistive MHD stability, and modeling of nonlinear, time evolution resistive MHD phenomena in tokamak configurations.

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§1. INTRODUCTION

In the magnetic confinement approach to controlled thermonuclear fusion, very high temperature hydrogen plasmas must be contained by a magnetic field for long enough times to allow fusion reactions to proceed at a rate large enough that net energy is produced. Over the past decade, significant advances have been made in fusion experiments, especially tokamaks, and the goal of achieving reactorlike conditions is not very distant. Because the size, cost, and construction times for presently conceived reactors are very large, it becomes increasingly important to develop a detailed theoretical understanding of the basic physical processes involved in reacting plasmas. The process by which this understanding is achieved relies heavily on studying the consequences of simple physical models, usually involving extensive computational methods. Our understanding is advanced by a comparison of the solutions of the model equations with experimental measurements. If this is successful, it is usually followed by extensive use of the tools developed to predict more optimal configurations or operating conditions.

In many ways the simplest physical model which has been shown to be capable of describing many of the macroscopic features of magnetically confined plasmas is the single fluid, magnetohydrodynamic (MHD) system of equations [1,2]. This system treats the plasma as a simple, electrically conducting fluid, which interacts self-consistently with the confining magnetic field. Its solutions can give useful information about many of the instabilities which arise in fusion experiments because of the free energy available either from the internal energy of the fluid or from the energy of the magnetic field. The model is studied extensively to find interesting steady-state (equilibrium) configurations, to determine the stability properties with respect to the many parameters which describe them, and to determine the consequences of the nonlinear evolution of these configurations.

One issue, which is central to the success of the magnetic confinement program and to which the MHD model has been extensively applied, involves an evaluation of the maximum plasma pressure which a specific magnetic field can contain. This is important because, when the temperature is sufficiently large that fusion reactions occur, the ability to sustain the reaction is determined by the product, $n\tau$, of the number of particles (n) in the device and their confinement time (τ). Interesting configurations then have a finite plasma pressure [$p \sim nT$] and a commonly used figure of merit is the quantity, $\beta \equiv 2\langle p \rangle / \langle B^2 \rangle$, the ratio of the average plasma pressure $\langle p \rangle$, to the pressure of the magnetic field $\langle B^2 \rangle / 2$. Instabilities can occur, even at very small values of β . There can be a variety of consequences, ranging from catastrophic large scale dynamical processes leading to loss of equilibrium, to more benign (but as potentially disastrous) small scale turbulence leading to enhanced plasma transport and a reduction in the confinement time. While it is possible to avoid (or control) many of these instabilities, the situation usually becomes worse as β is increased. Hence, it is obviously important to know if a β limit exists, and if it does, what the consequences of exceeding the limit will be.

Over the past decade extensive numerical solutions of the MHD equations have greatly increased our understanding of these processes, and the ability to describe the pressure and current-driven instabilities in present day experiments (often with remarkable agreement) has greatly increased our confidence in the model. The principal objective of this paper is to provide an overview of the various categories of numerical work which are involved in gaining this understanding. While the list of topics is by no means complete, consisting mainly of issues of importance to the tokamak approach, the mathematical and numerical complexities which underly the various methods are

often interesting in their own right or analogous to similar problems in other fields of large scale numerical simulation.

This review is arranged as follows. In the next section the MHD model and some of its general properties are described, including the concept of equilibrium configurations and the characteristic multiple time and spatial scales involved. Section 3 illustrates some of these aspects by considering the spectrum of linear perturbations of the simple one-dimensional cylindrical diffuse pinch configuration. The numerical techniques involved in generalizing this discussion to two-dimensional, axisymmetric, toroidal configurations follow in Sec. 4. Methods of extracting the long time, quasistatic, evolutionary MHD behavior appropriate for transport time scales are the subject of Sec. 5. Fully three-dimensional simulations, based on "reduced" MHD equations, are reviewed in Sec. 6. In the final section the value of coordinating the various computational activities is emphasized.

§2. THE MHD MODEL

The single fluid MHD model describes the macroscopic state of the system by equations for the conservation of mass, momentum, and energy of the fluid, coupled to Maxwell's equations for the electromagnetic field. We consider the simplest form, in which the equations are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \underline{v} = 0 \quad , \quad (1)$$

$$\rho \frac{d\underline{v}}{dt} = \underline{j} \times \underline{B} - \nabla p \quad , \quad (2)$$

$$\rho \gamma \frac{d}{dt} (p \rho^{-\gamma}) = 0 \quad , \quad (3)$$

together with,

$$\nabla \times \underline{B} = \underline{J} \quad , \quad \nabla \cdot \underline{B} = 0 \quad , \quad \frac{\partial \underline{B}}{\partial t} = -\nabla \times \underline{E} \quad . \quad (4)$$

The system is closed with a constitutive relation, the elementary Ohm's law,

$$\underline{E} + \underline{v} \times \underline{B} = \eta \underline{J} \quad , \quad (5)$$

in which the only form of dissipation considered here, plasma resistivity (η), enters the model. Equations (1) - (5) are a quasilinear, semi-hyperbolic system of partial differential equations which, for most fusion applications, are solved as mixed initial-value/boundary-value problems.

It is not always a simple matter to justify the use of these equations to describe magnetically confined plasmas [3]. More sophisticated fluid models can be developed from kinetic theories to include those additional effects which are needed for detailed comparison with experiment. However, many general qualitative features of such refined models can be well represented within this simplest MHD model, and it is natural that it should be extensively studied first. Its (sometimes deceptive) simplicity allows one to concentrate on basic physics issues such as the characteristic motions of the system, and to incorporate important effects related to the complicated geometry of fusion experiments in a realistic way.

Equation (1) through Eq. (5) are capable of describing a wide range of characteristic phenomena with time and spatial scales varying over many orders of magnitude. For confinement purposes we are particularly interested in configurations in which the fluid is contained by the magnetic field in a near steady-state (equilibrium) situation, where (hopefully) only small amplitude excursions can occur.

The most frequently studied equilibria are stationary ($\gamma = 0$) and are solutions of the equations,

$$\underline{J} \times \underline{B} = \nabla p, \quad \nabla \cdot \underline{B} = 0, \quad \nabla \times \underline{B} = \underline{J}. \quad (6)$$

Since $\underline{B} \cdot \nabla p = \underline{J} \cdot \nabla p = 0$, both the current and magnetic field lines lie on constant pressure surfaces. The most interesting configurations are topologically toroidal (Fig. 1), and the magnetic field lines map out nested toroidal surfaces, called magnetic surfaces [4]. The basic hope for confinement arises because charged particles follow these magnetic surfaces for sufficiently long times before collisions or other processes transport them outwards.

For tokamaks we consider axisymmetric toroidal equilibria ($\partial \phi \equiv 0$). We decompose $\underline{B}$ into poloidal and toroidal components, and introduce the poloidal magnetic flux function, $\Psi(X, Z)$. Then

$$\underline{B} = \nabla \phi \times \nabla \Psi + g(\Psi) \nabla \phi, \quad (7)$$

and Eq. (6) gives the Grad-Shafranov equilibrium equation [5,6],

$$\Delta^* \Psi \equiv X^2 \nabla^2 \Psi + \frac{1}{X^2} \nabla \Psi = -(X^2 p' + g g') \quad (8)$$

where ' denotes $d/d\Psi$. This is a two-dimensional, nonlinear, elliptic equation which can be solved numerically [7] after specifying $p(\Psi)$ and $g(\Psi)$. Interesting configurations involve the calculation of a free boundary separating the plasma from a surrounding vacuum region ($p' = g' = 0$). The shape of the plasma cross section is then determined by imposing appropriate

boundary conditions on the flux function ψ in the vacuum region to represent the externally applied vertical magnetic field which prevents the plasma ring from outwardly expanding.

Once equilibria have been determined, one of the first aspects to study is that of the linear stability of small perturbations about an appropriate equilibrium solution. We find three classes of wave motion. For typical parameters, one of these represents essentially ordinary sound waves which travel along the magnetic field lines with characteristic phase velocity, $v_s^2 \sim \gamma p / \rho$. The other two are Alfvén waves which have no counterpart in the absence of the magnetic field. One branch, consisting of discrete, high frequency modes, is compressional in character and propagates isotropically with phase velocity scaling as $v_A^2 \sim B^2 / \rho$. The other low frequency branch of shear Alfvén waves is essentially incompressible. It is this branch of modes which leads to the most dangerous instabilities. Since the instability time scale (typically measured in microseconds) is very small compared to the time scale associated with effects due to the finite resistivity of Eq. (5), it is usual to study this phenomena first in an infinitely conducting fluid ($\eta = 0$). The system of equations is then called ideal MHD. It possesses several exact nonlinear constants of the motion, including the total energy (which underlies the energy principle of the stability analysis) and the magnetic flux through a closed contour consisting of fluid elements (which conserves the topology of the magnetic field structure).

Equilibria which are stable on the ideal MHD time scale may be unstable in the presence of resistivity [8], to modes with characteristic time scales (typically measured in milliseconds) intermediate between those of the Alfvén waves and that of resistive diffusion. These instabilities can allow changes in the topology of the magnetic field which permit the system to find a lower

energy state. In many situations these new magnetic configurations may have large regions of stochastic field lines leading to anomalously large fluid transport and even to loss of confinement as the fluid elements conduct and convect internal energy and particles to the walls of the containment device.

Configurations which are stable to these ideal and resistive instabilities evolve quasistatically on the much longer transport time scales (seconds/minutes) associated with dissipative processes in the plasma or with slow changes in the external conditions (such as preprogrammed external magnetic fields).

Because of the wide range of scales involved, it is not realistic to attempt to solve the full, time dependent, three-dimensional model numerically in order to understand all the MHD phenomena. Even on the fastest available computers, a straightforward solution with crude resolution would take thousands of hours of computing time to reach the transport time scale. Demonstrating the numerical accuracy of such a calculation would probably be impossible. Because of this a variety of codes are needed; to compute interesting equilibria, to understand the spectrum of small perturbations including instabilities, to understand the most important nonlinear effects on these instabilities, and to model the essential features of the long time behavior.

§3. MHD SPECTRUM IN THE DIFFUSE LINEAR PINCH

Many of the issues important in more general configurations can be seen clearly in a study of a cylindrical plasma column, of length $2\pi R$, carrying a J_z current, which is surrounded by a vacuum region in which there is an applied external, constant B_z field. Cylindrically symmetric equilibria satisfy the pressure balance relation, Eq. (8),

$$\frac{d}{dr} \left(p + \frac{1}{2} B^2 \right) + \frac{B^2}{r} = 0 \quad , \quad (9)$$

with $d(rB_\theta)/dr = rJ_z$. The magnetic surfaces are nested cylinders on which the magnetic field lines spiral helically. The pitch of these field lines is measured by the safety factor, $q(r) \equiv r B_z(r)/R B_\theta(r)$. This equilibrium parameter plays an important role in the stability analysis.

The stability analysis proceeds by considering small perturbations δp , δB , δv , and linearizing about the equilibrium state [9]. Introducing the displacement, $\underline{\xi}$, of a fluid element through $\dot{\underline{\xi}} \equiv d\underline{\xi}/dt = \delta \underline{v}$, Eq. (2) gives

$$\rho \ddot{\underline{\xi}} = \delta \underline{J} \times \underline{B} + \underline{J} \times \delta \underline{B} + \nabla(\delta p) \quad . \quad (10)$$

In the ideal MHD limit, Eq. (1) through Eq. (5) can be used to express $\delta \underline{J}$, $\delta \underline{B}$, and δp in terms of $\underline{\xi}$ and, taking $\underline{\xi}(x,t) = \underline{\xi}(x) \exp(i\omega t)$, Eq. (10) can be written,

$$-\rho \omega^2 \underline{\xi} = F(\underline{\xi}) \quad (11)$$

where F is a self-adjoint, linear, partial differential operator [10]. This equation is an eigenvalue problem for the normal modes of the system. Since F is self-adjoint, the eigenvalues, ω^2 , are real. Positive values of ω^2 correspond to stable oscillatory waves, while negative values give exponentially growing instabilities.

Specializing to the cylindrical model, we assume the perturbations vary as $\underline{\xi} = \underline{\xi}(r) \exp(ik_z z) = \underline{\xi}(r) \exp i(m\theta - nz/R)$, and find that Eq. (11) can be reduced to a pair of coupled first order equations [11]

$$D(r\xi_r)' = C_1 r\xi_r - rC_2 P$$

and

$$D P' = C_3 \xi_r - C_1 P, \quad (12)$$

where $P \equiv \delta p + B \cdot \delta B$, and C_1 , C_2 , and C_3 depend on the equilibrium quantities $p(r)$, $B_\theta(r)$, $B_z(r)$, and ω^2 . The coefficient, $D(r)$, can be written

$$D(r) = (\omega^2 - F^2/\rho)[\omega^2 - (\gamma p/\rho) F^2/(\gamma p + B^2)] \quad (13)$$

with $F(r) \equiv B_\theta[m - nq(r)]/r = k \cdot B$.

The eigenvectors of Eq. (12) will be singular where D vanishes, and Eq. (13) then implies the existence of two continuous spectra, the shear Alfvén or sound continuum, depending on whether the first or second term on the right-hand side is zero. In addition, there are discrete normal modes that may include unstable solutions.

While Eq. (12) is useful for analyzing the properties of the normal modes, a direct numerical solution of Eq. (11) (to be described in the next section) gives a good overview. Figure 2 shows the results of such a calculation [12] for a case in which the equilibrium current, $J_z(r)$, is constant, and $p(r)$ is parabolic in r . We have plotted the eigenvalue spectrum for each of a set of equilibria in which J_z (i.e., q) was varied. In the stable half of the diagram we can easily identify the largest frequency, discrete compressible Alfvén branch, and below it the shear Alfvén and sound continua, both of which reach down to $\omega^2 = 0$ when nq is an integer. From the

unstable part of the diagram we can see that instabilities are associated with the shear Alfvén branch, and a further analysis shows that when q is rational the unstable modes accumulate at $\omega^2 = 0$.

By varying the equilibrium $p(r)$ and $q(r)$ profiles, it is possible to remove the unstable solutions, but, in general, the shear Alfvén continuum will still reach down to $\omega^2 = 0$, because $F(r)$ usually vanishes at some point in the plasma column. The corresponding marginally stable eigenfunctions are singular at the point where $F(r_s) = 0$. An analysis of the singular behavior [9] shows that

$$\xi(r) \sim A|r - r_s|^{v_+} + B|r - r_s|^{v_-} \quad (14)$$

where $v_{\pm} = (-1/2) \pm (-D_I)^{1/2}$, with $D_I = -1/4 - 2p'q^2/rB_z^2q'^2$. Since these solutions are not square integrable, additional physics must be included near such rational surfaces to give meaningful results. Allowing a finite resistivity in Eq. (5) is one choice; it leads to a higher order system of equations in the vicinity of the rational surfaces and resolves the singularities [8]. As often happens in such boundary layer problems, however, the spectrum is greatly modified and we find the possibility of new "resistive" instabilities, with growth rates which scale as fractional powers of the resistivity [8,13].

While a boundary layer analysis (see §4) is essential in two dimensions, in the cylindrical pinch progress can be made using standard numerical techniques to determine the resistive spectrum. In this case Eqs. (10), (5), and (4) can be reduced to a set of three, coupled, second order differential equations of the form

$$\frac{d}{dr} \left(\underline{A} \cdot \frac{d\underline{Z}}{dr} \right) + \underline{B} \cdot \frac{d\underline{Z}}{dr} + \underline{C} \cdot \underline{Z} = 0 \quad , \quad (15)$$

with $\underline{Z} = [r \xi_r, r (\delta B_r - F \xi_r), r \delta J_z]$ and $\underline{Z}(r=0) = \underline{Z}(r=1) = 0$. $\underline{A}$, $\underline{B}$, and $\underline{C}$ are 3×3 complex matrices involving ω , $\underline{k}$, η , and the equilibrium quantities. The equations are not self-adjoint so that the eigenvalue ω is, in general, complex.

Numerical solutions can be obtained by finite differencing in r and solving the resulting block tridiagonal system of equations. An iterative technique, matrix shooting [14,15], is used to adjust ω until the boundary conditions are satisfied. Since the solutions are typically highly localized and oscillatory in r (see Fig. 4), many grid points ($\sim 1000-3000$) are needed, even at relatively large values of η .

An example of the way in which the ideal spectrum is changed is given in Fig. 3, which shows that with finite resistivity the shear Alfvén continuum is resolved into discrete, damped oscillatory modes, which lie on well-defined loci extending from the accumulation points of the continuum [15]. As the resistivity is reduced, modes move along the purely damped axis towards the point A, becoming oscillatory. They also move in the directions $A\vec{B}$, $B\vec{C}$, $B\vec{D}$, $D\vec{E}$, and $D\vec{O}$. The nature of a typical eigenfunction, as the resistivity is reduced, is shown in Fig. 4. It becomes highly localized and oscillatory around the surface which is singular for this frequency in the ideal MHD model. Because of the interest in heating plasmas by the absorption of energy from radio frequency waves, it is very important to understand the nature of the stable spectrum, including the damping rates and the regions in the plasma where the energy would be deposited.

§4. AXISYMMETRIC TOROIDAL CONFIGURATIONS

In the cylindrically symmetric pinch configuration, the linear normal mode analysis reduced to a one-dimensional problem because the equilibrium solutions depended on the coordinate, r , alone. In the axisymmetric torus, however, both the equilibrium and stability analysis are two-dimensional problems and numerical solution is essential. While many of the spectral properties are similar in the two systems, the toroidal geometry can substantially modify the stability properties (e.g., the β limits) and major computational efforts have been undertaken to study the effects in detail. The first attempts to do this involved finite difference numerical solutions [16,17] to the time dependent problem of Eq. (10). For an unstable equilibrium, such a method follows the fastest growing mode and, by integrating long enough in time, it is possible to project out the eigenfunction and linear growth rate. These methods were superseded by more accurate and efficient Galerkin procedures for the normal modes.

The efficiency of the Galerkin procedure derives partly because, since the operator, $\underline{F}$, of Eq. (11) is self-adjoint, the normal mode problem can be posed variationally. Taking the scalar product of Eq. (11) with $\underline{\xi}$ and extremizing over the space of square integrable functions, we have the Rayleigh quotient estimate, $\omega^2 = \text{extremum } (\underline{\xi}, \underline{F}(\underline{\xi})) / (\underline{\xi}, \rho \underline{\xi})$, for the normal mode frequencies. Since $(\underline{\xi}, \rho \underline{\xi})$ is positive definite, the occurrence of unstable modes ($\omega^2 < 0$) is associated with the existence of perturbations for which the change in potential energy, $\delta W(\underline{\xi}^*, \underline{\xi}) \equiv (\underline{\xi}, \underline{F}(\underline{\xi}))$, is negative. Moreover, it is possible to demonstrate an energy principle [10], which states: if, for any $\underline{\eta} \in L_2$ and satisfying the appropriate boundary conditions, $\delta W(\underline{\eta}^*, \underline{\eta}) < 0$, then there exists a solution of Eq. (11) with $\omega^2 < 0$. Thus, while an accurate calculation of the growth rates and eigenfunctions

requires a demonstration of numerical convergence, the question of stability does not. This feature can be extremely valuable when it is necessary to determine the equilibrium parameter ranges where important instabilities occur.

The Galerkin procedure is straightforward to implement [18,19,20,21]. We expand $\xi(\underline{x})$ in a convenient set of known functions, $\Phi_n(\underline{x})$

$$\xi = \sum_{n=1}^N a_n^{(N)} \Phi_n \quad , \quad (16)$$

substitute, and carry out the extremization with respect to the coefficients, $a_n^{(N)}$. This leads to the generalized matrix eigenvalue problem,

$$\sum_{n=1}^N [\omega_N^2(\Phi_m, \rho \Phi_n) - (\Phi_m, F(\Phi_n))] a_n^{(N)} = 0 \quad , \quad (17)$$

whose solutions give approximations to the normal modes.

To carry out this approach successfully, attention must be given to selecting an appropriate coordinate system, to decomposing the vector ξ into appropriate scalar variables, and to choosing convenient basis functions, $\Phi_n(\underline{x})$. These choices influence the ease and accuracy of computing the matrix elements, the rate of convergence of the expansion and, ultimately, the efficiency of solving the matrix eigenvalue problem.

It is essential to choose a coordinate system based on the equilibrium magnetic flux surfaces, since the physics and the mathematical structure is different in directions parallel and perpendicular to the magnetic field [18]. Both the sound and shear Alfvén waves propagate along magnetic surfaces, and both can possess eigenfunctions which are highly localized (even singular) around the special magnetic surfaces where the helicity of the perturbation

matches the pitch of the field lines. In addition, the boundary conditions for ξ , which must be imposed at the plasma edge (the last plasma flux surface), are much easier to implement if this is a coordinate line.

To incorporate these aspects a nonorthogonal, flux coordinate system (ψ, θ, ζ) is employed (see Fig. 1) [18,21]. An appropriate flux label, $\psi = \psi(\Psi)$, is chosen to represent the radial variation, and the poloidal (θ) and toroidal (ζ) angle coordinates are chosen so that the field lines are straight on each surface. The coordinate system can be constructed numerically from the equilibrium solution, $\Psi(X, Z)$, by integrating a pair of ordinary differential equations to obtain $X(\psi, \theta)$, $Z(\psi, \theta)$ once the Jacobian $J \equiv (\nabla\psi \times \nabla\theta \cdot \nabla\zeta)^{-1}$ has been specified [21]. An example of several different straight field line flux coordinate systems is given in Fig. 5.

The vector ξ is decomposed [18] into directions parallel and perpendicular to $\underline{B}$ to separate out the sound branch, ξ_b ,

$$\underline{\xi} = \xi_\psi \nabla\psi + \xi_s \underline{B} \times \nabla\psi + \xi_b \underline{B} \quad (18)$$

The motion perpendicular to $\underline{B}$ is often nearly incompressible and this can be approximately accounted for by introducing a stream function for ξ_ψ and ξ_s . The remaining dependent variable is related to the compressibility of the mode. Each of the three components is then expanded in a mixed, finite element, Fourier series [18,21], e.g.,

$$\xi_\psi(\psi, \theta, \zeta) = \sum_{l, m} \xi_{lmn} u_m(\psi) \exp i(l\theta - n\zeta) \quad (19)$$

The Fourier expansion is particularly convenient since the calculation of the variation of perturbed quantities along the field lines can be handled

algebraically; from Eq. (7),

$$(\underline{B} \cdot \underline{\nabla}) \exp i(l\theta - i\zeta) \sim i l^{-1} (l - nq(\psi)) \exp i(l\theta - n\zeta) .$$

Linear piecewise finite elements are chosen for the $u_m(\psi)$, leading to large, sparse, block tridiagonal matrices for the eigenvalue problem, Eq. (17).

With these choices the calculation is relatively straightforward, although tedious, since many of the matrix elements involve long expressions in this coordinate system. The elements involve two-dimensional integrals of equilibrium quantities over the ψ and θ coordinates. These are computed by quadratures in ψ and fast Fourier transforms in θ . Typical calculations use from 50 ~ 100 finite elements and 20 ~ 50 poloidal harmonics, leading to matrices of order 3000 ~ 10,000. The eigenvalues (most often the lowest) are found by inverse iteration [19]. A typical high β , pressure driven, unstable eigenfunction, computed from the PEST code [18], is illustrated in Fig. 6. These low n ballooning modes, which occur theoretically under a wide variety of conditions, will set a β -limit in tokamak devices and their effects are probably present even in the discharges of existing devices.

For toroidal configurations which are stable within the ideal MHD model, we again must consider the possibility of resistive instabilities [22]. This is a much more complicated problem than in a cylindrical system, because the poloidal variation of the solutions does not consist of a single Fourier mode and, in cases of interest, there will be multiple singular layers. These singularities occur on flux surfaces for which the ideal MHD operator, $F \equiv \underline{F} \cdot \underline{\nabla} \psi$, of Eq. (11), vanishes. In the (ψ, θ, ζ) coordinate system it can be shown that [21]

$$F = [\partial_\psi (\partial_\theta - inq) + Q^+] \mathcal{L} [(\partial_\theta - inq) \partial_\psi + Q] - R \quad , \quad (20)$$

where $q(\psi) = g(\psi)/\phi(xd\theta/|V\psi|)$ is the toroidal safety factor, $\mathcal{L}$ is an elliptic partial differential operator, and Q, R are operators involving derivatives only within the magnetic surfaces. Singularities occur, therefore, for values of ψ where $q(\psi) = k/n$. For low $n(\sim 1, 2)$ modes in typical tokamak configurations there will be typically 2 ~ 6 such rational surfaces of interest.

Because the resistivity is very small in high temperature plasmas, it is only important in narrow boundary layers near these rational surfaces, and it is possible to compute the resistive modes approximately by a multiple boundary layer matching method. This has recently been implemented in the resistive PEST code [23] which solves three separate problems (schematically shown in Fig. 7a). Since the growth rates of the resistive modes are small compared to ideal MHD time scales, first the solutions of the marginal ($\omega^2 = 0$), two-dimensional, ideal MHD equations $F\zeta = 0$, are determined in the regions between the singular surfaces. The solutions are described in terms of the ratios of the asymptotically singular solutions which are computed by a finite element method (similar to that employed for the ideal MHD normal analysis) which incorporates special singular finite elements with the correct asymptotic behavior near rational surfaces. An example of one such solution is shown in Fig. 7b. At each resistive boundary layer the resistive MHD equations are reduced to a set of three coupled second order equations [22]. The numerical solution of these constitutes the second problem of the resistive PEST code. Finally, asymptotically matching the singular ψ -behavior of these solutions to those of the outer ideal MHD solutions, gives a matrix dispersion relation, from which the growth rates may be determined

numerically. In general, the roots are complex and are determined by iteratively adjusting the growth rates, re-solving the sixth order system on each surface simultaneously at each step until the asymptotic behaviors match on all rational surfaces.

A relatively simple example of a finite pressure system, where only one rational surface exists, is illustrated in Fig. 8. This shows how, as the magnetic Reynolds number ($S \sim \eta^{-1}$) varies between 10^6 to 10^8 , two purely growing modes coalesce, become overstable, become damped oscillatory modes, and eventually approach the point of marginal stability as the resistivity is reduced to zero. Parameter studies of the effects of plasma cross-sectional shape, current, pressure profiles, etc. on these resistive modes are an area of current activity.

§5. TWO-DIMENSIONAL, QUASISTATIC, MHD EVOLUTION

Theoretical studies based on methods like the last section show that tokamaks are unstable to a number of long wavelength and many short wavelength modes. Experimental techniques currently employed show that it is possible to confine tokamak plasmas over time scales very long compared to those of ideal and resistive MHD. A common interpretation is that the modes can saturate nonlinearly. For large scale modes the process is often violent, sometimes terminating discharges as macroscopic regions of the plasma become turbulent. The short wavelength modes are usually highly localized and the saturation process can be thought of as leading to enhanced transport across the magnetic surfaces. It is then of interest to model the evolution of these states, quasistatically, as the plasma evolves through a sequence of axisymmetric equilibria [24].

While it is relatively simple to state the physical model, it is more complicated to formulate a computational model based on an appropriate limit of the MHD equations. Straightforward solution of the time dependent equations is not appropriate, since, unless the characteristic MHD wave motion is filtered out, the time step restrictions of the numerical method would not permit calculations on the transport time scales. Two approaches have been successful in modelling different aspects of tokamak experiments. The most aesthetically pleasing (partly because of the mathematical structure of the resulting equations) is a massless fluid limit, which reduces the characteristic times to zero [24,25]. Consequently, the hyperbolic system arising from Eq. (2) is replaced by the elliptic equilibrium condition of Eqs. (6) and (8), so that the plasma is always in MHD equilibrium. The diffusive flow across the surfaces is determined from Eq. (5), which when substituted into Eq. (1), (3), and (4), determines the evolution of the surface averages of $\rho(\psi,t)$, $p(\psi,t)$, and $g(\psi,t)$. Substitution of these into the right-hand side of Eq. (8) allows the flux surfaces to be advanced in time. There are at least two practical difficulties which arise. One involves the satisfaction of various nonlinear integral constraints which must be solved for simultaneously by iteration, and the second concerns the complications which arise when the topology of simply nested flux surfaces is broken by field line reconnection on the resistive skin time ($\sim \eta^{-1}$). Nevertheless, most tokamak laboratories have developed such codes which are used regularly to help interpret experimental results in terms of conventional measures of energy and particle confinement times.

A second, and cruder, approach is useful when we are more interested in the resistive evolution of the magnetic field structure than in the details of particle and energy transport. This situation arises, for example, in the

formation of interesting noncircular cross section configurations (e.g. spheromaks [26]) and in compression experiments. The method involves the adjustment of the natural time scales of the MHD system, by artificially increasing the mass density and adding viscosity terms to Eq. (21) to slow the ideal MHD motions, and increasing the resistivity to enhance field diffusion and reconnection. Assuming incompressible flow also helps by removing the fastest waves. For a zero pressure configuration, Eqs. (2), (4), and (5) can be written

$$\rho \frac{\partial \underline{v}}{\partial t} - \nu_1 \nabla^2 \underline{v} - \nu_2 \nabla^2 \underline{v} = (\nabla \times \underline{B}) \times \underline{B} \quad , \quad (21)$$

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{v} \times \underline{B}) - \nabla \times (\eta \nabla \times \underline{B}) \quad . \quad (22)$$

Equations (21) and (22) are then solved on an Eulerian (X,Y) mesh with implicit finite difference schemes. Boundary conditions may be chosen to represent preprogramming of the currents in external field coils, feedback systems which react to center the plasma column, and other external electromagnetic circuits.

A simple example [27], which illustrates this method, is shown in Fig. 9. Here a rectangular plasma column undergoes magnetic reconnection into two separated columns under the influence of two "pinching" coils placed above and below at $X = 0$. As the original flux surfaces are compressed (Figs. 8a-8c), the J_z current peaks at the origin and reconnection occurs. In these calculations, where only the term involving ν_1 was kept on the left-hand side of Eq. (21), the reconnection rate depends on the resistivity, the elongation of the column, and the strength of the external field. In other models this is not the case, and the reconnection is modelled by jump conditions [28] similar to those involved in the treatment of shocks.

§6. THREE-DIMENSIONAL, NONLINEAR, MHD CALCULATIONS

Detailed experimental measurements clearly identify magnetic fluctuations which occur intermittently during discharges with well-defined three-dimensional structures. Many of these can be correlated with the nonlinear development of resistive instabilities discussed in §4, and involve the growth of helical islands of magnetic flux as the field lines reconnect around rational surfaces. A simple example involving the $l = 1, n = 1$ mode is shown in Fig. 10 [29]. Here magnetic reconnection occurs to the left of the set of nested circular surfaces, leading to large lens-shaped islands on the right-hand side. As time passes, the original central core shrinks, Fig. 9(b), and the island evolves turbulently through the nonlinear mode coupling of modes with similar helicity (i.e., $l/n = 1$). Eventually many small scale structures develop, allowing the temperature to flatten out across the region and the system returns to simple nested surfaces. This relaxation process is repeated many times during tokamak discharges and provides a basic mechanism which limits plasma overheating.

The single helicity, two-dimensional calculation of Fig. 10 is symptomatic of far more complicated topological changes which occur when modes with different helicity interact. Then fully three-dimensional simulations are required. Since these phenomena occur on time scales intermediate between the ideal MHD waves and resistive diffusion, the full MHD equations must be simplified to permit numerical work. A systematic ordering procedure, which involves an expansion in powers of the inverse aspect ratio, $\epsilon \equiv a/R$, is invoked to develop the Reduced MHD Equations [30]. In the simplest version, the low β ordering, the assumptions

$$B_T \sim \partial_r \sim \partial_\theta \sim O(1) \quad , \quad B_P \sim \underline{B} \cdot \nabla \sim v_P \sim \partial_t \sim O(\epsilon) \quad ,$$

and $p \sim v_T \sim \nabla \cdot \underline{v}_P \sim O(\epsilon^2)$, with subscripts P, T referring to poloidal and toroidal components respectively, result in the simple model equations [32],

$$\rho(\partial_t + \underline{v}_P \cdot \nabla - \frac{2}{R} \partial_z U) \nabla^2 U = R \underline{B} \cdot \nabla (\Delta^* \psi) \quad , \quad (23)$$

$$\partial_t \psi = X^2 \underline{B} \cdot \nabla U + \eta \Delta^* \psi \quad . \quad (24)$$

Here $U = U(r, \theta, \phi)$ is a stream function for the poloidal velocity and ∇^2, Δ^* are two-dimensional elliptic operators [see Eq. (8)].

These equations are solved using finite differences in r and a Fourier series expansion in θ, ϕ [31]. From 100 - 200 radial points (with grid refinement near important rational surfaces) and up to 100 Fourier modes have been employed. Equation (23) is advanced with an explicit time stepping scheme while the resistive term in Eq. (24) is treated implicitly. As with other spectral methods, the convolutions associated with convective terms in Eq. (23) are expensive in computer time, and careful thought has been given to selection schemes so that only the most important mode numbers are included. Fast block tridiagonal inversion schemes for the elliptic operators are essential. The schemes are usually restricted by a Courant condition on the time step. Running times are measured in hours of CRAY machine time for typical applications.

In Fig. 11, "puncture" plots of the magnetic field lines are shown at one toroidal angle during the simulation of a plasma disruption [32]. In the early phases we can see that the field lines form reasonably good magnetic surfaces, with only small $l = 2, n = 1$ and $l = 3, n = 2$ islands localized

around the $q = 2$ and $q = 3/2$ surfaces of the initial equilibrium. At later times the islands grow and interact producing numerous secondary islands of different helicity. Eventually, there is an overlap of islands and stochasticity of the field lines occurs over large regions near the plasma edge. Magnetic field lines are then connected from the plasma interior directly to the vessel surrounding the plasma. The discharge is rapidly quenched as the particles move down the lines carrying the plasma internal energy with them.

Simulations such as these have greatly advanced our understanding of plasma confinement and can come very close to explaining a wide range of experimental measurements. Additional extensions of the model to include finite aspect ratio and pressure effects will hopefully bring theory and experiment even closer.

§7. CONCLUDING REMARKS

In this paper I have tried to review some of the fascinating problems which can occur in a computational study of the simple, single fluid, MHD equations. From the physics viewpoint, the fact that the solutions can help to understand and predict plasma behavior under a wide range of conditions makes the study rewarding. The complexity and wide range of different mathematical problems involved provides a constant challenge for the development of new solution methods, both analytical and numerical.

The wide range of interesting phenomena has its origin in the many different characteristic time and spatial scales present in the MHD model. There is no single technique available to obtain a general solution, and progress is made by isolating a sequence of special problems, which together enable us to understand the MHD behavior of a fusion plasma. While each

aspect may involve its own special subtle features, it has been important to appreciate that essentially all of the phenomena play a role in our understanding, and that it is necessary to organize these studies as part of an overall research program.

Such an integrated approach has been one of the objectives of the MHD research program of the Plasma Physics Laboratory's Theoretical Division at Princeton University, where many of the techniques described in this paper were developed over the past decade. Aside from the many physics contributions, one of the practical outcomes is a set of MHD codes which can easily interact with each other through shared disk files, which make the results of one calculation available for subsequent analysis by others. As illustrated schematically in Fig. 12, axisymmetric equilibria can be generated from experimental results, from quasistatic evolution codes or from engineering design studies. There are about ten major codes, each with special features, involved in this step. The resulting equilibria are available for study by at least three major activities. These include linear studies of localized stability properties (six codes) and codes for long wavelength ideal and resistive instabilities (five codes). Finally, multi-dimensional nonlinear codes can be initiated to study the development of those equilibria shown to be interesting by the other numerical studies. The advantage of the structural organization is, of course, not only the ability to examine systematically the same configuration to a wide variety of physics issues but also that improved codes and new physics efforts can be added with considerable computational flexibility.

While the simple MHD models will continue to inspire future computational efforts to unravel their properties, it must be remembered that they only just begin to describe real laboratory plasmas. They provide a framework for the

inclusion of important kinetic effects which clearly influence plasma heating and confinement. Nevertheless, with the cost of proposed major fusion experiments now reaching the billion dollar level, it can clearly be anticipated that these computational activities will continue to be pursued vigorously.

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Figure Captions

Fig. 1 Tokamak equilibrium geometry showing magnetic (pressure) surfaces and coordinate systems.

Fig. 2 The ideal MHD spectrum of the diffuse linear pinch (from [12]). On the left of the diagram the highest frequency (Ω^2) modes are discrete compressional Alfvén waves. Below these are the shear Alfvén ($\Omega^2 \sim 10^{-2}$) and sound ($\Omega^2 \sim 10^{-4}$) continua. The most unstable mode is a current-driven kink mode ($nq < 2$). It becomes the lowest fast mode for $nq > 2$. The other instabilities are pressure driven.

Fig. 3 The stable part of the resistive MHD spectrum of the diffuse linear pinch (from [15]).

Fig. 4 Effect of resistivity for a typical mode on B-C in Fig. 3. a) $\eta = 4 \times 10^{-3}$, b) 2×10^{-3} , c) 10^{-3} , and d) 5×10^{-4} .

Fig. 5 Three different straight field line, flux coordinate, computational grids for the spheromak configuration: (a) PEST coordinates, $J \propto x^2$, (b) equal arc length coordinates $J \propto x/|\nabla\psi|$, (c) $J \propto 1/B^2$ (from [21]).

Fig. 6 Poloidal projection of $\underline{E}$, showing the displacement (flow) vector of a typical pressure driven, ballooning mode in a high- β tokamak.

Fig. 7 a) Flux surfaces for a bean-shaped tokamak showing $n = 1$ rational surfaces ($q = 2, 3, 4$), across which ideal MHD solutions are matched to resistive solutions.

b) Radial variation of different poloidal harmonics (l) of an ideal MHD solution, with dominant singularity at the $q = 2$ surface.

Fig. 8 In a toroidal system with favorable average magnetic field curvature, resistive instabilities become stable as the magnetic Reynolds number ($S \sim 1/\eta$) is varied (from 10^6 - 10^8). Arrows follow the loci of the resistive mode frequencies (from [23]).

Fig. 9 Resistive reconnection in a straight system caused by the forced pinching effect of increasing currents in two external coils, above and below $X = 0$.

Fig. 10 Simulation of the tokamak sawtooth relaxation effect, in which magnetic reconnection of $m = 1$ resistive modes causes drastic changes to the magnetic topology near the plasma center (from [29]).

Fig. 11 Magnetic field pattern at four different times during tokamak evolutions showing the growth of the $2/1$ island ($t = 485$), overlap with the $3/2$ island (555) and destruction of magnetic surfaces (890) (from [32]).

Fig. 1

Fig. 2

Fig. 3

Fig. 4

81T0385

Fig. 5

83T0458

Fig. 6

Fig. 7

#82T0134

Fig. 8

#83T0459

Fig. 9

(1)

(2)

(3)

(4)

Fig. 10

* 62 P 0289

Time = 100

Time = 485

Time = 555

Time = 890

Fig. 11

#83T0455

Fig. 12

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