ATOMIC SCALE RESOLUTION WITH CORRELATION HOLOGRAPHY

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ABSTRACT

For many atoms (including atoms of interest in biology) the elastic and inelastic photon scattering cross sections (denoted respectively by σ_{el} and σ_{inel}) for photons in the wavelength region of interest, satisfy $\sigma_{el} << \sigma_{inel}$. Therefore, the probability is high that when illuminated with photons, such an atom will decay before a holographic picture of it can be taken. On the other hand, if certain nonlinear phenomena: correlations between photons are taken into account, a hologram of such atoms can nevertheless be generated. Observation of small objects is compatible with the principles of quantum mechanics, even if the probability of disturbing the object as a result of observation is arbitrarily small.

I. INTRODUCTION

Suppose we wish to make an optical image of an atom. Suppose further that for imaging we use photons with wavelength λ, for which the elastic scattering cross section on the atom, σ_{el}, and the inelastic scattering cross section (which includes e.g. the photoelectric cross section), σ_{inel}, satisfy the inequality:

$$\sigma_{el} << \sigma_{inel}. \qquad (1)$$

For simplicity, illuminate the atom by an incoming plane wave whose amplitude is $\underline{A}_i$. The photons in this wave have momentum $\underline{k}_i$, and frequency $w = 2\pi c/\lambda$. The wave scattered elastically by the atom has amplitude A_s.

The number of incoming photons crossing a unit surface at normal incidence per unit time is:

$$I_i = \alpha|A_i|^2,$$

DISCLAIMER

where α is a normalization constant. Similarly, the photon flux density in the elastically scattered wave is:

$$I_s = \alpha |A_s|^2.$$

II. IMAGING BY ELASTICALLY SCATTERED PHOTONS ONLY

The average number, N_s, of scattered photons impinging normally on a surface area S during a time interval T (see Fig. 1) is:

$$N_s = S\ T\ \alpha |A_s|^2. \tag{2}$$

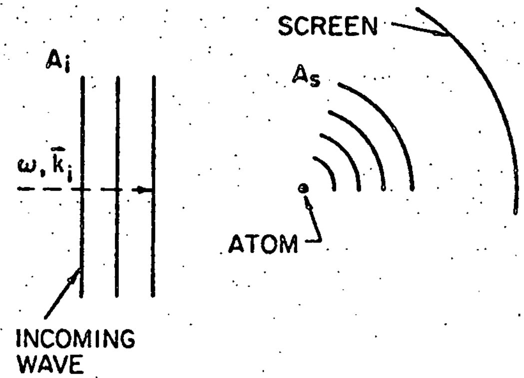


Figure 1

In order to indicate at least the presence of the atom in question, the number of scattered photons reaching a detector surface S must be equal to or larger than one:

$$N_s \geq 1. \tag{3a}$$

($N_s > 1$ is required if the image is to determine the position of the atom in three dimensional space). The total number of photons scattered by the atom , N_{st}, cannot be less than N_s, therefore:

$$N_{st} \geq 1. \tag{3}$$

What are the chances that at least one photon will be scattered elastically by the atom? If condition (1) holds, then the probability is $\sigma_{el}/\sigma_{inel} << 1$ that a photon will be scattered elastically before it will be scattered inelastically. Inelastic scattering processes, on the other hand, will result with a high probability in the destruction of the atom. Therefore, the probability is small, $\sim(\sigma_{el}/\sigma_{inel})$, that the image of the atom can be formed before the atom is destroyed by the photons which illuminate it.

III. HOLOGRAPHIC IMAGING

The final (i.e. the "total outgoing") amplitude of photons of wavelength λ, on the detector surface S, will be:

$$A_f = A_i + A_s.$$

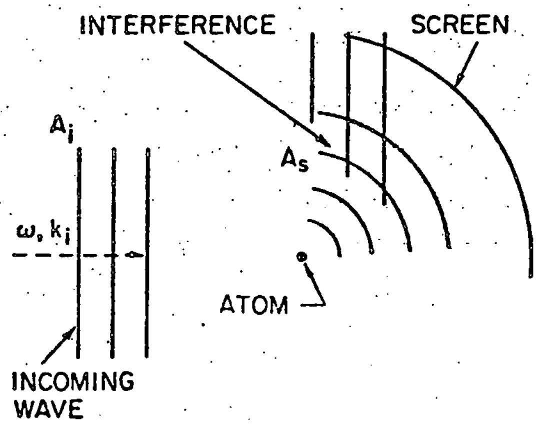
The final flux density of such photons on the surface S, is therefore,

$$I_t = \alpha|A_i + A_s|^2 = \alpha|A_i|^2 + \alpha 2\mathrm{Re}(A_i^* A_s) + \alpha|A_s|^2.$$

The first term is just the background intensity, I_i, a term which would be present, even if no scattering from the atom took place. When the last term is negligible, the I_t differs from the background term by the amount

$$\Delta I \equiv I_f - I_i = \alpha 2\mathrm{Re}(A_i^* A_s).$$

It is through this term that the prescence of the atom manifests itself in the final intensity distribution. This term contains all the information stored in the image about the atom. This is the term to be recorded on the surface S (See Fig. 2.).



HOLOGRAPHIC IMAGING

Figure 2

Note that ΔI is proportional to A_i^*, so that for large enough A_i^* it can be made as large as one wants to by increasing A_i.

However, in order to prove that ΔI can be observed, it is not enough to show that ΔI can be made arbitrarily large. In addition, one has to demonstrate that it can be distinguished from the "noise" generated by the background flux density I_i. This noise is the deviation of the number of photons actually impinging on the detector surface area from the expected number of these photons. The expected number is:

$$N_i = TS\alpha |A_i|^2, \tag{4}$$

and the expected deviation from it is, therefore, proportional to its square root, i.e. it is proportional to A_i. The signal ΔI is proportional to $A_i^* A_s$, but since A_s is itself proportional to A_i, the ΔI is proportional to A_i^2. The signal to noise ratio is, therefore, proportional to A_s (or A_i).

Next let us evaluate the critical value of A_s, i.e. that value which A_s must reach or exceed if ΔI is to be distinguished from the background noise.

For simplicity of discussion, assume that the detector consists of several identical detecting surface elements. The signal on the j^{th} surface will exceed the background fluctuation on that surface if:

$$ST\alpha 2|A_i|\ |A_s| \cos\theta_j \geq (ST\alpha |A_i|^2)^{\frac{1}{2}}.$$

In the above equation θ_j denotes the phase angle between A_i and A_s on that surface element. A summation over all surface elements shows that the signal will be distinguishable from the background, if:

$$ST\alpha |A_s|^2 \gtrsim 1 . \tag{5}$$

Comparison of expressions (2) and (5) shows that in order to distinguish ΔI from background noise, one needs:

$$N_s \gtrsim 1. \tag{6a}$$

Therefore:

$$N_{st} \gtrsim 1. \tag{6}$$

If condition (1) holds, the probability is $\sim(\sigma_{el}/\sigma_{inel}) \ll 1$ that a photon will be elastically scattered off the atom before the atom is destroyed by inelastic collisions with the photons which illuminate it.

To form an image of a molecule which contains n atoms for which condition (1) holds, one has to image n such atoms (plus possibly others). The probability that this can be done is $\sim (\sigma_{el}/\sigma_{inel})^n$, very small for large n.

So far our result is similar to what we found in section 1.

Note that it is not specific to photons. The same result holds for Boltzmann particles; if we record the impact of N^B of such particles against a random background of particles of the same kind whose average number is N_i, and if, furthermore N^B can be written as $N^B = N_{st}^{\frac{1}{2}} N_i^{\frac{1}{2}}$, then for N_{st} a result of type (6) holds.

The result just derived is the reason why it has been repeatedly stated in the literature that for wavelength λ for which inequality (1) holds, it will not be possible to produce holograms of atoms or molecules. Unfortunately, for most wavelengths of interest and many atoms which occur in biology, condition (1) is valid.

The purpose of my talk today is to point out that:

a) Holograms of atoms and molecules can nevertheless be made, even when $\sigma_{el} << \sigma_{inel}$, provided that correlations between photons are taken into account (Hanbury Brown & Twiss).

b) It is consistent with the principles of quantum mechanics to clearly observe a small object, even if the probability of disturbing the object as a result of observation (in the present instance the disturbance is inelastic photon-atom collisions) is arbitrarily small.

IV. CORRELATIONS

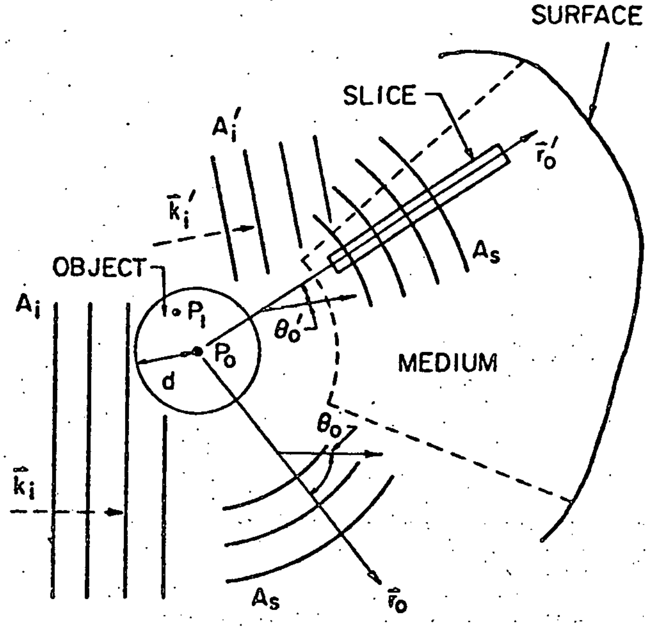
Photons are not Blotzmann particles. There are correlations between photons which can be detected. These non-linear phenomena may mainfest themselves as time correlations between photons as observed, e.g., by R. Hanbury Brown and R. Q. Twiss. They may also manifest themselves as spatial correlations. At the moment we are considering a steady state situaion (well determined $\bar{k}_i$). For these it is the spatial correlations which are of particular interest. They can be used to discriminate against background noise.

To avoid getting into unessential complications, we consider the particularly simple case of a small object located far away from the detector. The typical dimension of the object is d. (See Fig. 3.) We consider the interference pattern produced by incoming photons interfering with photons scattered by an atom located at point P_o inside the object (e.g., a molecule) whose image is to be produced. At distances large compated to d, along a radius vector $\bar{r}_o$ which originates at P_o, this pattern will be (almost) periodic, with a "beat wavelength":

$$\Lambda_o = \lambda(1-\cos\theta_o)^{-1}, \text{ for } |r_o| >> d.$$

Here θ_o is the angle between $\bar{r}_o$ and $\bar{k}_i$. Photons scattered by another atom inside the object, say one which is located at P_1, will produce a similar pattern along the radius vector $\vec{r}_l$, with periodicity:

$$\Lambda_\ell = \lambda(1-\cos\theta_\ell)^{-1}, \text{ for } |r_o| >> d,$$



CORRELATION HOLOGRAPHIC IMAGING

Figure 3

etc. All atoms will generate patterns, along radius vectors lying in the vicinity of $\vec{r}_o$, whose wavelengths will lie inside the range $(\Delta \pm \frac{1}{2}\Delta\Lambda_o)$. For small θ_o, $\Delta\Lambda_o \simeq 4d/r_o\theta_o$.

Dilute the recording medium: instead of recording only on the surface S, record inside the volume delineated by dashed line in Fig. 3. Investigate each slice in this volume. (One slice, near the radius vector $\vec{r}_o$, is shown in Fig. 3.) The background noise will generate a random intensity pattern. Only a fraction of that will lie in the wavelength region $(\Lambda_o \pm \frac{1}{2}\Delta\Lambda_o)$. The rest of it can be discarded, we will discuss briefly how.

If m is the number of distinquishable Δr intervals in the slice (assume $m < \Lambda_o/\Delta\Lambda_o$), then only $1/m$ fraction of the background noise will be in the wavelength region which contains the signal. Therefore, instead of satisfying conditions (6a), one only needs to satisfy:

$$N_s \gtrsim \frac{1}{m}. \tag{7a}$$

or, denoting by f the recording efficiency of scattered photons,

$$N_{st} \gtrsim \frac{1}{m} \; \frac{1}{f}. \tag{7}$$

When $m >> 1$, it can happen that $N_{st} << 1$. In this sense, disturbance of observed objects can be arbitrarily small.

Remarks for practical applications:

To record small Λ_o ($\lesssim 10 - 100$ Å) choose $\theta_o' \neq \theta_o$ so that $\theta_o' << 1$ and $\Lambda_o >> \lambda$ can be realized.

To Fourier analyze, one may for example count blackened silvergrains if the recording medium is a photographic emulsion. Simpler procedure consists of illuminating slice and observe light transmitted at particular directions.

V. CONCLUSIONS

If A_i is large enough, one may distinquish the hologram signal from the background, even if A_s is so small that $N_{st} << 1$.

It is possible to observe atoms (and molecules made up of such atoms), even when $\sigma_{el} << \sigma_{inel}$.

It is compatible with the principles of quantum mechanics to clearly observe an object, even if chances are arbitrarily small that the observation will disturb the object.