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ROCK EQUATION OF STATE: A CONSTITUTIVE COMPUTER MODEL

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August 24, 1977

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Prepared for U.S. Energy Research & Development Administration under contract No. W-7405-Eng-48.

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SYMBOLS

B_L	longitudinal bulk modulus
C_0	longitudinal sound velocity
E	Young's modulus
EL	elastic limit ($-\sigma_{EL}$)
E_v	internal energy at vaporization
G	shear modulus
g	acceleration due to gravity
ΔH	heat of vaporization
K	bulk modulus
P_S	peak pressure
V_{FS}	true free surface velocity without gravitational acceleration
X	ground depth
Y	yield
Z	% of water by weight
ν	poisson's ratio
ν_0	dry poisson's ratio
ϕ	input constant (e.g., b_0, b_1, b_2)
Γ	gamma law gas constant input
σ_x	stress
ρ	bulk density
ρ_g	grain density
τ	rise time of velocity
μ	volume compression $\left(\frac{\rho}{\rho_0} - 1\right)$
μ_S	compression at peak pressure

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ROCK EQUATION OF STATE: A CONSTITUTIVE COMPUTER MODEL

ABSTRACT

A constitutive model for a rock equation of state was developed in HEMP and CEL-HEMP computer codes. The model includes the mechanical and thermal properties of soil, such as elasticity, pore closure, compaction, and vaporization. Three test problems were computed using the equation of state: ground motion following the LATIR event, impact vaporization of preheated plates, and surface motion produced from exploding a 3.7-kg(10-lb) charge at a depth of 6.5 m. Agreement between our computations from the model and actual measurements is remarkably close.

INTRODUCTION

A constitutive model for a rock equation of state (EOS) was developed for use in HEMP¹ and CEL-HEMP² computer codes. This model includes an elastic property, pore closure, compacted states, and a vapor phase. Since the rock equation of state is often used to calculate phenomena deep in the ground, the equation is coupled to a gravity term to account for gravitational overloading.

In this report, we describe this model together with tables showing input data. We have used the EOS to compute three problems with practical applications.

CONSTITUTIVE MODEL

A graphic description of the model is shown in Fig. 1. The loading path consists of three regions: elastic region, pore closure, and compacted state. Unloading paths include two phases: solid and gas.

Fig. 1. Constitutive model of rock EOS: 1, elastic; 2 and 2a, pore closure; 3, compacted state; 4, solid phase; 5, gas phase.

Region I: Elastic Region

The bulk modulus K of soil increases monotonically with depth due to the gravity compaction in soil. Therefore, the longitudinal bulk modulus B_L is given as a function of depth. This term is obtained from measurements of the longitudinal sound wave and the bulk density in the ground. Knowing the longitudinal bulk modulus B_L , we can calculate the bulk modulus K and the shear modulus G at each depth from the following relationships³:

$$K = G \frac{2(1 + \nu)}{3(1 - 2\nu)} ,$$

$$B_L = K + \frac{4}{3} G ,$$

where ν = Poisson's ratio.

Poisson's ratio is determined from the water content in soil; Poisson's ratio of dry rock can be measured in the laboratory.^{4,5} If the soil is fully water-saturated, it is a liquid-like substance with a Poisson's ratio of 0.5. The ν of partially saturated wet soil is determined by an extrapolation:

$$\nu = \nu_0 + \frac{0.5 - \nu_0}{Z_{\max}} Z ,$$

where

ν_0 = dry Poisson's ratio,

Z = water (wt %),

and

$$Z_{\max} = \frac{\rho_g - \rho}{\rho_g \rho - \rho} = \text{maximum weight percent of water.}$$

Poisson's ratio of rocks increases asymptotically with confining pressure up to about 1 kbar.⁶ This is because the confining pressure tends to close the open pores and cracks, stiffening the rocks and resulting in an increased Young's modulus and Poisson's ratio. Natural pores and cracks strongly dominate the elastic properties of rocks at low pressures.⁷ In this model, however, Poisson's ratio is treated as a constant.

Since ν is related to both K and G , such pressure dependence of ν has important effects on low-stress seismic waves but is negligible on the intense shock waves near an underground nuclear explosion.

Region II: Pore closure

Region II is above the elastic limit (EL); it simulates pore closure by volume compression. The pore closure is assumed to be irreversible. When the pressure unloads from partially closed states in region II, it follows an elastic unloading path, producing a hysteresis.

Region III: Compacted states

The third region is the Hugoniot state, a fully compacted region. This state is described by a Tillotson-type EOS.⁸ The fully compacted states unload pressure along adiabatic release paths. If the residual internal energy at zero compression ($\mu = 0$) is equal to or greater than the vaporization energy of the soil, the material expands smoothly into a gas phase.

The algorithm for the constitutive model is as follows:

Loading path ($\dot{V}/V < 0$)

For $\mu \leq \mu_a$, EOS (1),

$$P = K\mu,$$

where

$$\mu = (\rho/\rho_0) - 1.$$

The bulk modulus K and shear modulus G are calculated from B_L and ν by:

$$B_L = b_0 + b_1 X + b_2 X^2 + b_3 X^3,$$

$$K = \frac{B_L(1 + \nu)}{3(1 - \nu)},$$

$$G = \frac{B_L(1 - 2\nu)}{2(1 - \nu)},$$

where b_0, b_1, b_2, b_3 are input constants, and X = ground depth.

For $\mu_a < \mu < \mu_b$, EOS (2),

$$P = K\mu_a + B(\mu - \mu_a) + \left[\gamma_0 + \frac{\gamma_1}{1 + \frac{E}{\gamma_2(1 + \mu)^2}} \right] (1 + \mu) E ,$$

or EOS (2a),

$$P = B_0 + B_1\mu + B_2\mu^2 + B_3\mu^3 + \left[\gamma_0 + \frac{\gamma_1}{1 + \frac{E}{\gamma_2(1 + \mu)^2}} \right] (1 + \mu) E ,$$

whichever is greater. Here, B , B_0 , B_1 , B_2 , B_3 , γ_0 , γ_1 and γ_2 are input constants.

For $\mu_b \leq \mu$, EOS (3),

$$P = C_1(\mu - \mu_c) + C_2(\mu - \mu_c)^2 + C_3(\mu - \mu_c)^3 + C_4(\mu - \mu_c)^4 + \left[\gamma_0 + \frac{\gamma_1}{1 + \frac{E}{\gamma_2(1 + \mu)^2}} \right] (1 + \mu) E ,$$

where C_1 , C_2 , C_3 and C_4 are input constants.

Unloading path ($\dot{V}/V > 0$)

When unloading starts, record the peak pressure P_S and the compression μ_S .

1. If $\mu_S \leq \mu_a$, unload along EOS (1).
2. If $\mu_a \leq \mu_S < \mu_b$, calculate

$$K_0 = K + (C_1 - K) \frac{\mu_S - \mu_a}{\mu_b - \mu_a} ,$$

and

$$\mu_0 = \mu_S - \frac{P_S}{K_0} .$$

Then unloading occurs along

$$P = K_0(\mu - \mu_0) .$$

3. If $\mu_b \leq \mu_s$, unloading occurs along EOS (3). If $E < E_v$ at $\mu = 0$, where E_v is internal energy at evaporation temperature, unloading occur along EOS (3). If $E \geq E_v$ at $\mu = 0$, calculate

$$F = \exp [-\phi (V/V_0 - 1)]$$

and EOS (4),

$$P = (\Gamma - 1) (\mu + 1) (E - \Delta H),$$

then unloading occurs along

$$P = \text{EOS}(4) + F \cdot [\text{EOS}(3) - \text{EOS}(4)],$$

where ϕ = input constant (default 0.768).

If the residual internal energy at zero compression is greater than the total energy required for vaporization, the solid phase transforms exponentially into an ideal gas phase. The default constant of 0.768 for the exponential function is chosen so that, at a four-fold volume expansion, 90% of the material is in the gas phase. This was chosen because the triple point in the phase diagram occurs near a four-fold expansion in many materials.

GRAVITATIONAL LOAD

The acceleration along the x axis is calculated by the equation of motion;

$$\ddot{x} = \frac{1}{\rho} \frac{\partial \sigma_x}{\partial x} + g.$$

The initial condition is that the acceleration is zero everywhere, i.e.,

$$\ddot{x} = 0 \text{ at } t = 0.$$

then

$$\sigma_x = -g \int_0^x \rho dx = -g \rho_0 \int_0^x (1 + \mu) dx,$$

where

$$\mu = \frac{\rho}{\rho_0} - 1.$$

In numerical calculations, the integral is replaced by a summation:

$$\sigma_x = -g\rho_0 \sum_{i=1}^n (1 + \mu_i) \Delta x . \quad (1)$$

The initial gravitational overburden σ_x and the corresponding volume compression μ must satisfy Eq. (1) for zero acceleration. Since the gravitational force is in the x direction, we have a uniaxial stress along the x axis. Then the strains in the y and z axes must vanish, i.e., $E_y = E_z = 0$,

where

$$E_y = \frac{1}{E} [\sigma_y - \nu(\sigma_x + \sigma_z)] ,$$

and

$$E_z = \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)] ,$$

where E is Young's modulus.

Then

$$\sigma_y = \sigma_z = \frac{\nu}{1 - \nu} \sigma_x .$$

Since the pressure is

$$P = -\frac{1}{3} (\sigma_x + \sigma_y + \sigma_z) = -\frac{\sigma_x}{3} \left(\frac{1 + \nu}{1 - \nu} \right) ,$$

$$\sigma_x = -3P \left(\frac{1 - \nu}{1 + \nu} \right) . \quad (2)$$

We set the gravitational pressure and compression in each zone as follows: referring to Fig. 2, the overburden from Eq. (1) in any two adjacent zones must satisfy the expression:

$$\sigma_x^n - \sigma_x^{n-1} = -\frac{1}{2} \rho_n^0 g(1 + \mu_n) \Delta x - \frac{1}{2} \rho_{n-1}^0 g(1 + \mu_{n-1}) \Delta x .$$

However, using Eq. (2), we can also write:

$$\sigma_x^n - \sigma_x^{n-1} = -3P_n \left(\frac{1 - \nu_n}{1 + \nu_n} \right) + 3P_{n-1} \left(\frac{1 - \nu_{n-1}}{1 + \nu_{n-1}} \right) .$$

Combining these two equations,

$$A^n (1 + \mu_n) + A^{n-1} (1 + \mu_{n-1}) = -\alpha^n P_n + \alpha^{n-1} P_{n-1} , \quad (3)$$

Fig. 2. Gravitational overburden.

where

$$\alpha^n \equiv 3 \left(\frac{1 - \nu_n}{1 + \nu_n} \right), \text{ and } \alpha^{n-1} \equiv 3 \left(\frac{1 - \nu_{n-1}}{1 + \nu_{n-1}} \right),$$

$$A^n \equiv -\frac{1}{2} \rho_n^0 g \Delta x \text{ and } A^{n-1} \equiv -\frac{1}{2} \rho_{n-1}^0 g \Delta x.$$

Since the overburden pressure is well within the elastic limit of rocks,* we assume that P_n and P_{n-1} are elastic equations of state, i.e.,

$$P_n = K_n \mu_n \text{ and } P_{n-1} = K_{n-1} \mu_{n-1}$$

where K_n and K_{n-1} are bulk moduli.

Substituting the equations of state into Eq. (3), the initial compression at the nth zone is:

$$\mu_n = \frac{1}{K_n \alpha^n - A^n} \left[\alpha^{n-1} P_{n-1} - A^n - A^{n-1} (1 + \mu_{n-1}) \right]. \quad (4)$$

In the top zone,

$$\mu_1 = \frac{-A^1}{K_1 \alpha^1 + A^1}. \quad (5)$$

* At 800 m depth, the gravity overburden yields

$$\sigma_x = -\rho g x = -15.7 \text{ MPa},$$

and

$$\sigma_y = \sigma_z = \frac{\nu}{1 - \nu} \sigma_x = -3.9 \text{ MPa},$$

then

$$P = -\frac{1}{3} (\sigma_x + \sigma_y + \sigma_z) = 7.8 \text{ MPa},$$

where

$$\rho = 2 \text{ Mg/m}^3$$

$$\nu = 0.2$$

$$g = 9.8 \text{ m/s}^2$$

For example, the elastic limit of Paintbrush Tuff⁹ is 13.3 MPa, and the overburden pressure is well below this limit.

Thus, we initialize by starting at the top zone, and calculate the overburden pressure and the corresponding compression at successive zones.

INPUT QUANTITIES

The constitutive model was implemented in HEMP and CEL-HEMP computer codes. The model is large enough to tie up three standard equations in HEMP. The rock EOS can be constructed using EOS (4, 5, 6), (14, 15, 16), (24, 25, 26), (34, 35, 36), or (44, 45, 46). The input quantities for EOS 4, 5, 6 as an example are listed in Table 1. In Table 1, the yield Y is a function of pressure,

$$Y = a + bP ,$$

with $Y_{\max}$ as an upper limit. The shear modulus G is also a linear function of pressure,

$$G = G_0 (\bar{a} + \bar{b}P) ,$$

where G_0 is calculated from the longitudinal bulk modulus and Poisson's ratio, as shown in the algorithm for the loading path. $G_{\max}$ gives the upper limit for the shear modulus.

Table 2 shows the input card for gravity loading. The gravity constant must be present to access the rock EOS. These input constants are punched on a single card in the order shown in the table.

The gravitational constant is $9.8 \times 10^{-10} \text{ cm}/\mu\text{s}^2$. However, in HEMP the dimension can be scaled down for problems of large dimensions. In this case, the gravitational constant must be scaled accordingly. For example, if 10 m is reduced by 10^3 down to 1 cm, the gravitational constant is 9.8×10^{-7} . In interpreting the computed results, the distance and the time must be multiplied by 10^3 .

OUTPUT EDIT

In addition to a standard edit of P , E , V , Q , etc., the output edit provides the various quantities calculated for the rock EOS shown in Table 3.

Table 1. Input for rock EOS.

Constant	Input	Constant	Input
b_0	C1(4)	a	C8(5)
b_1	C2(4)	b	C9(5)
b_2	C3(4)	$Y_{\max}$	C10(5)
b_3	C4(4)	$\bar{a}$	C11(5)
v	C5(4)	$\bar{b}$	C12(5)
B	C6(4)	$G_{\max}$	C1(6)
γ_0	C7(4)	Overburden flag	C2(6) = 0.
γ_1	C8(4)	ΔH	C3(6)
γ_2	C9(4)	C_1	C4(6)
B_0	C10(4)	C_2	C5(6)
B_1	C11(4)	C_3	C6(6)
B_2	C12(4)	C_4	C7(6)
B_3	C1(5)		
μ_a	C2(5)		
μ_b	C3(5)		
μ_c	C4(5)		
ϕ	C5(5)		
Γ	C6(5)		
E_V	C7(5)		

The edit also shows, by F-numbers, the EOS regions and that phase the material is in. The meanings of the F-number are shown in Table 4. For example, when the material is in gas phase, F is set to 3, and SHOCKMU is set to 9.99999E10. Therefore the F-number shows the vaporized zones as well as the various phases and regions of the EOS in each zone.

Table 2. Gravity input.

Input name	Meaning
Gravity constant	The acceleration is $9.8 \times 10^{-10} \text{ cm}/\mu\text{s}^2$
$\sin \theta$	The sin of the angle between the gravity in x direction and the ground surface (1.0 if the gravity is in the positive x direction and -1.0 if in the negative x direction).
$\cos \theta$	The cos of the above described angle ($\emptyset$.)
Bulk modulus	This quantity is not used ($\emptyset$.)
x1	x coordinate of one end of the ground surface
y1	y coordinate that goes with x1
x2	x coordinate of the other end of the ground surface.
y2	y coordinate that goes with x2
Δx	Uniform zone thickness

Table 3. Edit output.

Output	Meaning
KZERO	K_0 calculated for unloading
MUKZERO	μ_0 calculated for unloading
MUZERO	Initial compression due to the gravity overburden
BLKMOD	Calculated bulk modulus K
SHRMOD	Calculated shear modulus G
SHOCKMU	Compression μ_s corresponding to the peak pressure
POLD	Pressure from the previous cycle
MUOLD	Compression from the previous cycle

Table 4. F-Number.

F-Number	Meaning
11,12,13	Loading on EOS (1), EOS (2) (2a), EOS (3) respectively.
21	Unloading along EOS (1)
23	Unloading along EOS (3)
25	Unloading along $P_2 = K_0 (\mu - \mu_0)$
26	Unloading along $P_3 = K_0 (\mu - \mu_0)$
31	Reloading on EOS (1)
33	Reloading on EOS (3)
36	Reloading on $P_3 = K_0 (\mu - \mu_0)$
37	Reloading on $P_2 = K_0 (\mu - \mu_0)$
38	Reloading on EOS (2) or (2a)
3	Gas phase (SHOCKMU = 9.99999E10)

COMPUTATION AND DISCUSSION

We tested the rock EOS on three problems. Our first calculation was the ground motion of the LATIR detonation. The second problem was the impact vaporization of preheated plates made of a hypothetical material, such as the impact vaporization of a heated meteorite. Results of our third problem showed what effects the yield strength of rocks can have on seismic waves. This third study can be applied to the seismic study of a building foundation, i.e., the measurements of the elastic limit and the yield strength of the foundation soil.

LATIR Event

LATIR was detonated on February 27, 1974, 640 m deep within a well (Boring U4d) at the Nevada Test Site. The geologic structure of the site is shown in Fig. 3. Seven ground-surface stations were located at 15 m (SZ), 160 m (S2), 320 m (S3), 607 m (S4), 707 m (S5), 807 m (S6) and at 1173 m (S7) away from ground zero. Station SZ was oriented to respond only to vertical motion. All other stations were situated so they could respond to vertical, radial, and tangential motions. The measurements from all seven stations are described in Ref. 10.

Fig. 3. Approximate west-east geologic cross-section of Nevada LATIR detonation.

A simple geologic model was used for a HEMP computer simulation. The model is shown in Fig. 4. The energy source is a gas with a density of 1.93 Mg/m^3 , the normal soil density at that depth. The EOS we used is a γ -law gas equation.

The longitudinal sound velocity (Fig. 5) and the bulk density (Fig. 6) were measured at depths up to 800 m. We determined the longitudinal bulk modulus from these data (Fig. 7). A Poisson's ratio of 0.5 was given to both Rainier Mesa and Paintbrush Tuff. That is, the shear moduli and the yield strengths in these strata were zero. The Poisson's ratio of alluvium was estimated to be 0.48 while its yield strength was 10 MPa (100 bars). Paleozoic rock, the deepest rock layer, was treated as a rigid wall.

The first signal was a longitudinal wave that propagated with a pressure equal to the elastic limit of the soil EOS. However, our computations show that the longitudinal wave had a long pressure ramp rather than a sharp wavefront. The pressure gradually increased, leading to the zone of the wave front (maximum Q , an artificial viscosity). Illustrated in Fig. 8 is the wave front propagating directly upward, and measurement of sound propagation along the hole, Boring U4d. The directions of the sound wave are indicated by the arrows. Agreement between our computations and the acoustical measurement is remarkably good.

Ground motion was measured at seven stations at the surface. We computed the surface motion for stations, SZ, S2 and S3. The long pressure ramp preceding the wave front made it difficult for us to determine the signal arrival time. The arrival time was defined as the time at which the surface velocity is about 1% of the maximum velocity at a location.

In Fig. 9, we plotted both our actual measurements and the computations of the wave front arrival times at the three stations. Standard errors (the bars) represent one-half of the time-spread among all the detectors at each station. The error bars on the computations also represent the signal transit time through one zone near the ground surface. Considering that the geologic model used for the HEMP simulation is a greatly simplified model of the geologic structure of the Nevada Site, agreement is still quite close. The maximum surface velocities agree within 35% of our measurements. However, our computations were not run long enough for us to see a gravitational fallback of the ground surface.

Fig. 4. Geologic model for a HEMP simulation of the LATIR detonation.

Fig. 5. Longitudinal sound velocity as a function of depth for the LATIR detonation.

Fig. 6. Bulk density as a function of depth for the LATIR detonation.

Fig. 7. Longitudinal bulk modulus as a function of depth for the LATIR detonation.

Fig. 8. Propagation of wavefronts in the ground following the LATIR detonation.

Fig. 9. First wavefront signal arrival times at three stations: comparison of actual measurement and computer calculations.

A large shock wave that follows the first signal sets the ground in motion. The grid lines, which were printed out every 25 ms, show the ground motion by their displacement. We took successive plots at 275 and 300 ms, and superimposed them, as illustrated in Fig. 10. The shock front where there is a large displacement of the grids produces double lines from the two frames. At 220 m from the source, a material boundary forms where the wave front is greatly distorted, and a reflected wave from the boundary can be seen.

If there are any faults in the ground, they can be simulated by slide planes. For this computation, edits of various quantities were put out every 25 ms. The binaries were saved every 50 ms for the x and y velocity components, volume compression, pressure, stress deviator S_{xx} , shear stress and internal plastic distortion. Thus isocontours of these quantities can be plotted every 50 ms, if necessary. Computing for 500 ms took about 1.7 h on the CDC 7600.

Impact Vaporization

Our second study was that of the impact vaporization of preheated plates constructed of a hypothetical material (Fig. 11). This problem can be envisaged as the study of an impact vaporization of a heated meteorite upon the earth. The inset in Fig. 11 shows the initial conditions. A 4 mm flyer plate impacts on a 4 mm target at a velocity of 10 mm/ μ s. Both plates are preheated to 1630 J/g. Assuming the specific heat of 0.75 J/g-K, the preheat energy is equivalent to 2170 K. The vaporization temperature is set at 1685 J/g, or 2250 K. The heat of vaporization ΔH is assumed to be 562 J/g.

Figure 11 shows the assumed principal Hugoniot for this material. The Hugoniot at 2170 K is determined by editing the computed shock pressures and the corresponding compressions in the plates. The computed vaporization process is determined by the adiabatic release paths taken from three zones.

Yield Strength of Rocks

For the third problem, we examined the effects of the rock yields strength on seismic waves. Figure 12 compares two problems. A 10-lb charge is set off at a depth of 6.5 m. The ground structure is two layers of alluvium with a varied yield strength. The bottom layers have the strength of 10 MPa and lie on rigid rock. In problem 1, the top layer has a yield strength Y_0 of 1 MPa, whereas in problem 2, the top layer has no yield strength. This can be regarded as a comparison between layers of hard coral and montmorillonite, both

Fig. 10. HEMP simulation of LATIR explosion shock waves: superposition of two grid plots at 275 and 300 ms.

Fig. 11. Release paths and Hugoniot determined from preheated plate-flyer collision.

Fig. 12. HEMP calculations of seismic waves propagating through rocks with two yield strengths.

common foundation layers in the Hawaiian Islands. These layers are known to cause serious problems in the foundations of buildings in many parts of the Islands. This is an interesting problem that should be studied.

The vertical velocities of the ground surface are computed at two stations, S1 (0.5 m) and S2 (5.0 m). Figure 13 illustrates the stress contours at 4 ms for problem 2; Fig. 14 shows the vertical surface velocities calculated at the two stations.

The longitudinal sound velocity determines the arrival time of the first signal, and it is given by:

$$C_0 = \sqrt{\frac{B_L}{\rho}} ,$$

B_L and ρ being measured values. The peak vertical velocity of the first signal gives the elastic limit on the equation of state as follows: The Rankine-Hugoniot relation gives:

$$-\sigma_{EL} = \frac{1}{2} \rho C_0 V_{FS} .$$

The expression must be multiplied by 1/2 because the free surface velocity is twice the particle velocity.¹¹ The peak vertical velocity is related to the free surface velocity by:

$$V_{max} = V_{FS} - g \tau .$$

Then the elastic limit becomes:

$$-\sigma_{EL} = \frac{1}{2} \rho C_0 (V_{max} + g \tau) . \quad (6)$$

If Poisson's ratio is measured, the bulk and shear moduli are calculated by:

$$K = \frac{B_L (1 + \nu)}{3(1 - \nu)}$$

$$G = \frac{B_L (1 - 2\nu)}{2(1 - \nu)} .$$

Fig. 13. Stress contours at 4 ms from problem 2, Fig. 12.

Fig. 14. Ground surface vertical velocities at two stations from Fig. 12.

Assuming the von Mises yield condition,^{1,12,13} shown in Fig. 15, the yield strength is then:

$$Y_0 = \frac{3}{2} (-\sigma_{EL} - P_{EL}) \quad (7)$$

To determine the elastic limit and the yield strength, we must measure first, the longitudinal sound velocity as a function of depth, second, the bulk density and water content, third, the Poisson's ratio under carefully reproduced in-situ conditions, i.e., the water content and the equivalent gravity load at each depth, and last, the arrival time of the signal and the vertical surface velocities.

These problems are quite small, and the 30 ms computation takes about 5 minutes on CDC 7600 computer.

SUMMARY

Using the HEMP and CEL-HEMP codes, we have constructed a constitutive model for soil. The main features of the equation of state for this model are:

- The bulk modulus and the shear modulus are functions of depth.
- Poisson's ratio is assumed to be a constant.
- Pore closure is simulated by a model of volume compression.
- The compacted state is given by a Tillotson-type equation.
- The gas phase is included.
- The equation of state is coupled to the gravity term to account for gravitational overloading.

The three sample problems we have shown demonstrate that the model has the general thermal and mechanical properties possessed by soil. The range of application for the EOS, is quite broad. For example, it can be applied to simulate the cavity and crater formation by nuclear explosives, to study meteorite impact, or to determine the seismic properties of building foundations.

Computer models such as these should be effectively used to simulate actual field experiments. The weakest link between field experiments and these computer models are data on the *in-situ* properties of soil that are not readily available. The measurement of the in-situ properties is difficult and costly, but the development of such technology certainly deserves more consideration.

Fig. 15. The elastic limit on the equation of state based on the von Mises yield condition.

ACKNOWLEDGMENT

This work was supported by the Air Force Technology Applications Center. The author wishes to thank the H-Division management for their support. Discussions with Doug Norris, Bill Hoover, and Dave Young were very helpful. Conversations with Danny Chung of K-Division are also appreciated. The containment groups in L-Division provided the information on LATIR. Programming was done by Pat Weidhaas and John French. John Hobson is responsible for implementing in CEL-HEMP. Computer production was assisted by Nora Anson, Margaret Tracy, and Diane Giroux.

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