Progress Report for
Application of Modern Mathematical Concepts to Plasma Confinement

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Abstract

Progress in three research areas has been achieved. (1) A new expression for the trapping probability for transitioning particles in asymmetric toroidal plasma confinement devices (e.g., stellarators and rippled tokamaks) has been derived. Comparison with numerical integrations shows that this new expression is more accurate than those of previous workers. (2) Simulations show that the dynamical enhancement of the turbulent diffusion coefficient over quasilinear observed in test particle simulations occurs also in selfconsistent plasma turbulence. (3) The simple method for calculating magnetic island widths discussed in the previous report has been published.

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I. Report

In the past year, progress in three research areas has been achieved. (1) A new expression for the trapping probability for transitioning particles in asymmetric toroidal plasma confinement devices (e.g., stellarators and rippled tokamaks) has been derived. Comparison with numerical integrations shows that this new expression is more accurate than those of previous workers. (2) Simulations show that the dynamical enhancement of the turbulent diffusion coefficient over quasilinear observed in test particle simulations occurs also in selfconsistent plasma turbulence. (3) The simple method for calculating magnetic island widths discussed in the previous report has been published in *The Physics of Fluids*.

In asymmetric toroidal plasma devices (stellarators and rippled tokamaks) there exist *transitioning particles*, particles that change back and forth between being trapped in the local magnetic ripple wells, and being able to pass over the local maxima, yet trapped by the poloidal variation of the magnetic field due to toroidicity. These particles can be the dominant contributors to transport, because their orbits deviate considerable from the flux surfaces. A critical aspect of this motion is determined by the trapping probability, which gives the fraction of the time that an orbit makes the transition from the toroidally trapped state to the locally trapped at each toroidal bounce. This trapping probability, together with what is known about the trajectory, determines the average deviation from a flux surface and, ultimately, the transport induced by these particles.

At least four calculations¹⁻⁴ of the trapping probability have been presented in the literature. These calculations differ substantially in their methodology (some expand in the rotational transform per field period, others neglect the lengthening of field lines in regions of small magnetic magnitude) and their results, as shown by our evaluations of these formulas. Hence, we set to calculate the trapping probability with a minimum of

approximations. Our results were quite good. We found that for typical stellarator parameters, our formula is a substantial improvement over all previous results. This work is described in App. A.

Last year we described a new direction of research: the study of the basic processes that occur in Langmuir turbulence. In our previous year's work we had calculated the diffusion of test particles in a field of randomly phase waves and found an enhancement of a factor of two or so in the regime of moderate overlap. This enhancement extends far above (by a factor of 60) the threshold for chaos. This work was published in the last year of support as a *Physical Review Letter*.⁵ This is attached as App. B.

We had surmised that our results may be connected to those Pesme and others, who announced that⁶ in the self-consistent evolution of a plasma, non-Gaussian effects can arise that may cause the velocity diffusion to differ from the quasilinear value by a factor of perhaps two. This was supposed to occur when the growth rate of the instability is small compared with the resonance broadening time, $\gamma_L < (k^2 D_{QL})^{1/3}$.

To test these ideas we began a program of selfconsistent plasma simulation of the warm beam-plasma instability. We have been able to show unequivocally that this enhancement does occur if the growth rate is sufficiently small. However, we find that the the region of enhancement is $\gamma_L < (k^2 D_{QL})^{1/3} / \epsilon^{2/3}$, where $\epsilon \approx 20$, the effective overlap parameter, is placed at the value of maximum enhancement observed in the test-particle simulations. Numerically, this amounts to an order of magnitude change of the region.

But more importantly, it represents a radical departure from previous ideas of plasma turbulence. It seems that the plasma, for a given field energy, arranges the statistics of the turbulence in such a way as to find the configuration with the largest diffusion, but putting the effective overlap parameter at about 20. This is akin to an entropy maximization principle and may have significant implications in other fields.

Our research group continues to be one of the leaders in the design of helical magnetic confinement systems. In a previous year we developed a simple method for calculating island widths in field-line following codes. This work has now been published in *The Physics of Fluids*.⁷ We attach a copy of this publication as App. C.

Because nearly all of the progress made in the previous year has been written up, we do not add to length of this report by presenting another discussion. Instead we simply append reports on each of the areas discussed above.

II. References

- ¹ V. P. Nagorny and V. A. Yavorskji, Sov. J. Plasma Phys. **15**, 309 (1989).
- ² J. R. Cary, C. L. Hedrick, and J. S. Tolliver, Phys. Fluids **31**, 1586 (1988).
- ³ V. P. Nagorny and V. A. Yavorskji, Sov. J. Plasma Phys. **15**, 309 (1989).
- ⁴ L. M. Kovrizhnykh and S. G. Shasharina, Nucl. Fusion **30**, 453 (1990).
- ⁵ J. R. Cary, D. F. Escande, and A. D. Verga, Phys. Rev. Lett. **65**, 3132 (1990).
- ⁶ J. C. Adam, G. Laval, and D. Pesme, Phys. Rev. Lett. **43**, 1671 (1979); G. Laval and D. Pesme, Phys. Fluids **26**, 53 (1983); G. Laval and D. Pesme, Phys. Fluids **26**, 66 (1983); G. Laval and D. Pesme, Phys. Rev. Lett. **53**, 270 (1983); K. Theilhaber, G. Laval, and D. Pesme, Phys. Fluids **30**, 3129 (1987).
- ⁷ J.R. Cary and J.D. Hanson, Phys. Fluids **B3**, 1006 (1991).

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