Inelastic Strong Interactions at High Energies

MASTER

Topical Report
October 20, 1977

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Prepared for

THE U.S. ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION

UNDER CONTRACT NUMBER EY-76-S-02-2978.*000

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Abstract

A method permitting the calculation of inclusive and semi-inclusive cross sections at high energies starting from an arbitrary field theory has been worked out. The method relies on a functional integral representation of the generating functional of inclusive cross sections and on the use of semiclassical methods. The inclusive cross section is proportional to the absolute square of the Fourier transform of a special soliton solution of the classical field equations in zeroth order semiclassical approximation. A conserved topological quantum number that has a nonzero value for the soliton solution was found. Future research to be performed in this field under the contract is described including applications to various field theories and the exploration of connections with Reggeon Field Theory.

1. Introduction

This report is being submitted because a major result that has a profound bearing on future investigations under this contract has been achieved.

The result we are going to describe concerns the calculation of cross sections in strong interactions. Until recently no systematic approximation scheme had been devised which would allow the calculation of strong interaction cross sections from a field theory without model assumptions (except in the deep inelastic region in non-abelian gauge theories). Many people even believe that such a calculation is impossible due to the large value of the coupling constant and the divergence of the perturbation series.¹

In a work recently submitted for publication we have shown that the functional integral approach, successfully applied to scores of problems not soluble by the methods of perturbation theory, can be applied to the evaluation of cross sections at high energies as well.²

We have shown - as a first step towards achieving our goal - that the generating functional of cross sections defined by³

$$\sigma[g] = \int \prod_{n=2}^{\infty} d^3 p_1 \dots d^3 p_n \sigma_n(p_a, p_b; p_1, \dots, p_n) g(p_1) \dots g(p_n) , \quad (1)$$

where $\sigma_n(p_a, p_b; p_1, \dots, p_n)$ is the n particle production differential cross section can be brought to the following compact functional integral form (for the sake of simplicity written for a single Hermitean scalar field)

$$\begin{aligned} \sigma[g] = & m^2 [(p_a p_b)^2 - m^4]^{-\frac{1}{2}} 64 \int dz dv dv' \exp\{i[z(p_a + p_b) + (v - v')(p_a - p_b)]\} \\ & \int D\phi D\bar{\phi} K_v \phi_v K_v \phi_v K_v' \bar{\phi}_v K_v' \bar{\phi}_v \exp\{\int d^4 x d^4 x' D_+(g, z + x - x') K\phi K'\bar{\phi}' \\ & + i \int d^4 x [L(\phi) - L(\bar{\phi})]\} , \end{aligned} \quad (2)$$

where

$$D_+(g, x) = (2\pi)^{-3} \int d^4 p \delta(p^2 - m^2) \theta(p_0) e^{-ipx} g(p) \quad (3)$$

$K_V = \square_V + m^2$, $K'_V = \square'_V + m^2$, $\phi_V = \phi(v)$ and $\bar{\phi}'_V = \bar{\phi}(v')$. The functional derivatives of $\sigma[g]$ at $g=0$ and 1 give the exclusive and inclusive cross sections, respectively.

At large values of the variable $(p_a + p_b)^2 = s$ and $g(p) = g = \text{const.}$ the functional integral of Eq. (2) can be evaluated using semiclassical methods.

The dominant contribution to cross sections occur where the action (the negative of the exponent of the functional integral) is minimal. This requires that such functions satisfy the following equations

$$\psi(x) = g \int d^4 y D_+(x-y) j^\dagger(y) + \int d^4 y D_F(x-y) j(y) \quad (4)$$

and

$$\begin{aligned} p_{a\mu} + p_{b\mu} &= ig \int d^4 x d^4 y \partial_\mu D_+(x-y) j(x) j^\dagger(y) \\ &= \int d^3 p [2(2\pi)^3]^{-1} |\bar{j}(p)|^2 p_\mu, \end{aligned} \quad (5)$$

where $j(x) = \delta L_{\text{int.}}(\psi)/\delta\psi(x)$ and $\bar{j}(p)$ is the Fourier transform of $j(x)$. The function $\psi(x)$ is related to $\phi(x)$ and $\bar{\phi}(x)$ as follows: $\psi(x+z/2) = \phi(x)$ and $\psi^\dagger(x-z/2) = \bar{\phi}(x)$. The function $\psi(x)$ that satisfies Eq. (4) satisfies the classical field equation as well.

The inclusive cross section is given by taking the functional derivative of (2) with respect to $g(p)$ and setting $g=1$. In the semiclassical approximation

$$(1/\sigma) d\sigma/d^3 p = [2E(2\pi)^3]^{-1} |\bar{j}(p)|^2 \quad (6)$$

which shows immediately that Eq. (5) is the sum rule for inclusive cross sections.³

Since the trivial solution of Eq. (4) does not satisfy Eq. (5) and the non-trivial solutions are singular in the coupling constant we could conclude that the asymptotic behavior of the cross sections cannot be calculated perturbatively.

One can substitute the fields ϕ and $\bar{\phi}$ by x and $x^\dagger$ directly in Eq. (2).

One can also separate the field ψ into $\psi = \psi_1 + \psi_2$ by writing the action as

$$A[\psi_1, \psi_2] = -\int d^4x d^4y D_+(x-y) K\psi_1(x) K\psi_1^+(y) + i\int d^4x [\psi_2 K\psi_1 + \frac{1}{2}\psi_1 K\psi_1 - L_{\text{int.}}(\psi_1 + \psi_2) - \text{h.c.}] \quad (6)$$

The variation of the action A with respect to $\psi_2, \psi_2^+, K\psi_1, K\psi_1^+$ gives ψ_1 and ψ_2 as the first and second term of the right hand side of Eq. (4). ψ_2 and ψ_2^+ are free fields and define a conserved current

$$J_\mu(x) = i(\psi_2 \partial_\mu \psi_2^+ - \psi_2^+ \partial_\mu \psi_2) \quad (7)$$

$Q = \int d^3x J_0(x)$ is a conserved topological charge (no invariance of A is generated by Q). So we have an expansion around a classical solution with a non-zero topological charge. This classical solution is not, however, an ordinary soliton. One can easily show that the original action, $-i\int L(\psi) d^4x$ is optimized by $\psi = \psi_1 + \psi_2$ defined in a way similar to Eq. (6) only if $\psi_2 = 0$, i.e. the topological charge is zero. Since our classical solutions are not solitons we propose a new name for them; sectons, underlining their relation to the calculation of cross sections.

In an application to ϕ^4 field theory we could find only one secton solution, giving a Feynman scaling single particle distribution with a transverse momentum distribution that cuts off exponentially at large values of p_T . The total cross section is of the form

$$\sigma = (\text{const.}/s) |\bar{j}(p_a)|^2 |\bar{j}(p_b)|^2 e^{-A} \quad (8)$$

where A has a logarithmic dependence on s , giving a powerlike dependence of σ on s . The semi-inclusive cross section and the multiplicity distribution are also exactly like those predicted in a generalized multiperipheral model.⁴

The results briefly described above signal only the beginning of an extensive study of the properties of cross section generating classical solutions (sectons) and their quantization. In the next section we shall describe the various questions and directions we intend to investigate in this field.

2. Proposed future investigations of sectons.

It is fairly obvious that a single secton solution is related to a single Reggeon (or Pomeron) exchange. By finding multi-secton solutions we wish to investigate the relation of these solutions to multi-Reggeon exchange diagrams and eventually to Reggeon Field Theory. It is possible that as in Reggeon Field Theory the Froissart bound is not imposed on a single secton solution, but rather the sum of all solutions exhibit the singularity at the point where the renormalized behavior of the sum corresponds to a constant total cross section. Because of their different singularity structure in the coupling constant the multi-secton solutions cannot be generated by a perturbation expansion in the single secton sector, just as the single secton solution cannot be generated by a perturbation expansion in the vacuum sector.

Further study of the single secton sector should reveal the properties of the correlation function of the two particle production inclusive cross section. The correlation function is identically zero in the semiclassical approximation, so we have to quantize the small oscillations around the classical solution to obtain information about it. The contribution of quantum oscillations around the secton solutions will strongly effect the behavior of the inclusive cross sections in the fragmentation regions as well.

We would like to study secton solutions in various field theories, namely fermion field theories and quantum chromodynamics (QCD) as well. The investigation of the inclusive cross section in QCD at large transverse momentum, $p_T \gg 1 \text{ GeV}$ should reveal a powerlike behavior.

We intend to investigate exclusive cross sections as well. It is not clear at the moment how the shrinkage of the elastic cross section enters into the picture.

Finally it would be interesting to see whether lower lying Regge trajectories appear as sectons as well after introducing internal symmetries.

Summarizing our future plans in investigating section solutions in field theories it is fairly safe to state that the listed problems form only a small subset of the questions that will eventually emerge during the investigations themselves. We believe that the investigation of cross section generating classical solutions and their quantization will form an important subfield of the theory of strong interactions.

References

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