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BEAM-BEAM EFFECTS IN CURVILINEAR INTERACTION REGIONS IN ISABELLE

M. CORNACCHIA AND G. PARZEN

MASTER

October 1979

ACCELERATOR DEPARTMENT

BROOKHAVEN NATIONAL LABORATORY
ASSOCIATED UNIVERSITIES, INC.

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ABSTRACT

The presence of experimental apparatus in the interaction regions of an intersecting beam accelerator changes the configuration of the crossing beams. This changes the space-charge forces with respect to the standard, magnet-free crossing. There is strong interest in instrumenting one of the intersections at ISABELLE with a major spectrometer facility, which involves a large aperture dipole magnet located at the intersection of the beams. Various configurations of the dipole spectrometer have been proposed, and they are analyzed in this paper from the point of view of the perturbation introduced to the beam-beam interaction.

I. INTRODUCTION

The presence of experimental apparatus in the interaction regions of an intersecting beam accelerator changes the configuration of the crossing beams. This changes the space-charge forces with respect to the standard, magnet-free crossing. In ISABELLE, the standard collision is for the two beams to cross at an angle of 11.188 mrad in the horizontal plane (Figure 1). There is strong interest in instrumenting one of the intersections at ISABELLE with a major spectrometer facility, which involves a large-aperture dipole magnet located at the intersection of the beams. Various configurations of the dipole spectrometer have been proposed,¹ and we have analyzed them from the point of view of the perturbation introduced to the beam-beam interaction. The "perturbation" is defined by the extent to which the linear beam-beam tune shift departs from the standard case and by the integrated strength of the higher-order derivatives of the field. This report is concerned with the variation of the beam-beam tune shift (at one point in the ring) with respect to the standard value, on the ground that such variation will reduce the periodicity of the space-charge field and thus decrease the spacing between the resonances that these space-charge fields will excite.

Figure 1

The result of the analysis given below is that the vertical bending configuration (Figure 2) is the most acceptable of all the configurations examined.

II. VERTICAL BENDING OF THE BEAM

The proposed layout¹ for this configuration is shown in Figure 2. The two beams are deflected vertically by the magnets A and brought to the same height in the interaction region by the magnets B. Figure 2a shows the vertical projection of the beam paths; in the horizontal plane (Figure 2b), the beams cross with the standard crossing angle of 11.188 mrad. The magnet C has a dipole field of effective length L_d and strength B_d . We are concerned here with the effect of the curved trajectories in C on the space-charge forces acting reciprocally on the beams. Two possibilities have been proposed¹ for the central magnet C: a system with a 30-kG field and 4.5-m effective magnetic length and a magnet with half the strength and twice the length.

Figure 2

A. Absence of Horizontal and Coupling Resonances

In Figure 3, the primed coordinates refer to the strong beam. In the small-angle approximation, the coordinates of the strong beam are related to those of the weak beam by the relationship

$$x' = x + s\alpha,$$

$$y' = y + \frac{s^2}{\rho},$$

$$s' = s + \frac{ys}{\rho} + x\alpha \sim s,$$

$$E_{x'} \sim E_x \quad (1)$$

where α is the horizontal crossing angle and ρ is the radius of curvature in the magnet C. $E_{x',x'}$ is the space-charge field in the ultrarelativistic approximation.

Figure 3

Let us consider, in Figure 3, two trajectories, a and b, having the same coordinate $y(s)$, but displaced by Δx in the coordinate system of the weak beam. If x_0 is the x-coordinate of trajectory a, we have

$$\int_{-\infty}^{\infty} E_x[x_0, y(s), s] ds = \int_{-\infty}^{\infty} E_x[x_0 + \Delta x, y(s), s] ds. \quad (2)$$

We now develop $E_x[x_0 + \Delta x, y(s), s]$ in a power series around x_0 :

$$E_x[x_0 + \Delta x, y(s), s] = E[x_0, y(s), s] + \left[\frac{\partial}{\partial x} E_x(x_0, y, s) \right]_{\substack{x = x_0 \\ y = y(s)}} \Delta x + \dots \quad (3)$$

Since Eq. (2) is true for any Δx , we have

$$\int_{-\infty}^{\infty} \frac{\partial}{\partial x} E_x(x, y, s) ds = 0. \quad (4)$$

Thus, no horizontal beam-beam tune shift is present in this geometry, but also for all the terms responsible for the higher-order resonances:

$$\int_{-\infty}^{\infty} \frac{\partial^n E_x}{\partial x^n} ds = 0, \text{ etc.} \quad (5)$$

The assumption has been made that the variation of the horizontal β -function in the interaction region can be neglected.

We can also consider the integral

$$\int_{-\infty}^{\infty} E_y ds$$

along the same trajectories a and b; clearly

$$\int_{-\infty}^{\infty} E_y[x_0, y(s), s] ds = \int_{-\infty}^{\infty} E_y[x_0 + \Delta x, y(s), s] ds, \quad (6)$$

from which, proceeding as above, it follows that

$$\int_{-\infty}^{\infty} \frac{\partial E_y}{\partial x} ds = \int_{-\infty}^{\infty} \frac{\partial E_x}{\partial y} ds = 0. \quad (7)$$

This is also true for the higher mixed derivatives:

$$\int_{-\infty}^{\infty} \frac{\partial^m E_x}{\partial x^m \partial y^{n-m}} ds = 0. \quad (8)$$

Thus, in the geometry under consideration, where we have horizontal crossing and vertical bending, the driving terms of the horizontal resonances and of the coupling resonances are zero (as in the standard case with horizontal crossing only).

All the vertical resonances $m_y = p$ are, however, driven, odd and even (unlike the standard case where only the even resonances are excited).

B. The Vertical Beam-Beam Tune Shift

The divergence theorem, for a Gaussian charge density distribution, gives the relationship between the electric field gradients:

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = \frac{2e\lambda}{\rho_o 2\pi\sigma_x\sigma_y} \exp\left(-\frac{(x+s\alpha)^2}{2\sigma_x^2} - \frac{(y+s^2/\rho)^2}{2\sigma_y^2}\right) \quad (9)$$

where λ = linear charge density, σ_x, σ_y = rms values of the horizontal and vertical densities of the particles, assumed the same in the two beams, and the other symbols have their usual meanings.

Integration along the reference orbit of the weak beam ($x = 0, y = 0$) yields

$$\int_{-\infty}^{\infty} \frac{\partial E_y}{\partial y} ds = \int_{-\infty}^{\infty} \frac{2e\lambda}{\rho_o 2\pi\sigma_x\sigma_y} \exp\left(-\frac{\alpha^2}{2\sigma_x^2} s^2 - \frac{s^4}{2\rho^2\sigma_y^2}\right) ds \quad (10)$$

since we proved in Section A that

$$\int_{-\infty}^{\infty} \frac{\partial E_x}{\partial x} ds = 0.$$

The vertical beam-beam tune shift is, in the extremely relativistic case $\beta \approx 1$,

$$\Delta\nu_y = \frac{r_p \beta^* i}{\pi\sigma_x\sigma_y \gamma ec} \int_{-\infty}^{\infty} \exp\left(-\frac{\alpha^2}{2\sigma_x^2} s^2 - \frac{s^4}{2\rho^2\sigma_y^2}\right) ds \quad (11)$$

where

r_p = classical proton radius,
 β_y^* = vertical β -function at the interaction point,
 γ = proton energy, normalized to the rest energy, and
 i = beam current.

If we put $t = 2\sigma_x^4 / (\alpha^4 \rho^2 \sigma_y^2)$, the integral can be expressed as a series expansion. As a fraction of the standard vertical tune shift $\Delta\nu_{yo}$, which, in the small crossing angle approximation, has the expression

$$\Delta\nu_{yo} = \frac{2\beta_y^* r_p i}{\sqrt{2\pi} e c \gamma \sigma_y \alpha_o} \quad (12)$$

with α_o = horizontal crossing angle = 11.188 mrad, we find

$$\frac{\Delta\nu_y}{\Delta\nu_{yo}} = 1 - 0.75 t + 3.3 t^2 - \dots \quad (13)$$

We have

$\rho = 445$ m,
 $\alpha = 11.188$ mrad,
 $\sigma_x = 1.79$ mm at 30 GeV,
 $\sigma_x = 0.49$ mm at 400 GeV,
 $\sigma_y = 0.77$ mm at 30 GeV,
 $\sigma_y = 0.21$ mm at 400 GeV.

Thus

$$\begin{aligned} t &= 1.11 \times 10^{-2} \text{ at } 30 \text{ GeV,} \\ t &= 8.3 \times 10^{-4} \text{ at } 400 \text{ GeV.} \end{aligned} \quad (14)$$

Retaining only the first term of the expansion, we get

$$\begin{aligned} \frac{\Delta\nu_y}{\Delta\nu_{yo}} &= 1 - 0.83 \times 10^{-2} \text{ at } 30 \text{ GeV,} \\ \frac{\Delta\nu_y}{\Delta\nu_{yo}} &= 1 - 6.2 \times 10^{-4} \text{ at } 400 \text{ GeV.} \end{aligned} \quad (15)$$

Noting that the sagitta of the vertical bend, y_s , over the interaction region length is

$$y_s = \frac{l_{\text{int}}^2}{4\rho} = \frac{4\sigma_x^2}{\alpha^2\rho}, \quad (16)$$

we find that the parameter t is twice the square of the sagitta divided by the vertical beam size:

$$t = 2 \left(\frac{y_s}{4\sigma_y^*} \right)^2. \quad (17)$$

Again, reducing the strength of the magnetic field in the dipole C quadratically decreases the perturbation of the linear beam-beam tune shift. We remark here, again, that our concern was for a variation of the space-charge force (as measured by the linear beam-beam tune shift) from the standard value at one of the six interaction regions. This reduces the periodicity of the perturbation and decreases the spacing between the resonance lines excited by the beam-beam interaction. As the calculations show, the effect is negligible for the case we have just considered.

C. Vertical Resonances

1. Odd-Order Vertical Resonances

The odd-order vertical resonances are excited because of the nonzero vertical displacement of the orbit in the crossing region. Odd-order vertical resonances are also driven by random orbit errors at the crossing point. It is of interest to compare the driving strength for these resonances due to the spectrometer with those due to the expected vertical orbit errors.

The vertical separation of the beams is given by s^2/ρ , at a distance s from the center of magnet C. A rough estimate of the average beam separation can be made by putting $s = \sigma_x/\alpha$, where σ_x is the standard deviation of the horizontal beam size at the crossing point ($\beta_x^* = 40$ m).

We have

$$\begin{aligned} \sigma_x &= 1.79 \text{ mm at } 30 \text{ GeV,} \\ \sigma_x &= 0.49 \text{ mm at } 400 \text{ GeV,} \\ \alpha &= 11.188 \text{ mrad.} \end{aligned} \quad (18)$$

We get for the average separation,

$$\begin{aligned} y_{\text{av}} &= \frac{\sigma_x^2}{\alpha^2\rho} = 0.056 \text{ mm at } 30 \text{ GeV,} \\ y_{\text{av}} &= 0.004 \text{ mm at } 400 \text{ GeV,} \end{aligned} \quad (19)$$

for a bending radius $\rho = 445$ m, corresponding to a field $B_d = 30$ kG.

The same driving strength for the odd-order resonances would be obtained with a random orbit error

$$\begin{aligned} y_{\text{rms}} &= \frac{y_{\text{av}}}{2\sqrt{6}} \text{ or} \\ y_{\text{rms}} &= 0.01 \text{ mm at } 30 \text{ GeV,} \\ y_{\text{rms}} &= 0.0008 \text{ mm at } 400 \text{ GeV.} \end{aligned} \quad (20)$$

It is not known with certainty, at present, how well the random orbit errors at the crossing points can be corrected. Something like $y_{\text{rms}} = 0.05$ mm may be a reasonable guess of what may be achieved. Thus, the driving terms for the odd-order vertical resonances due to the spectrometer magnet are probably smaller than those due to the remaining random orbit errors at the crossing points after these errors are corrected. The above results may be established more exactly. The half-width of the N^{th} -order resonance, N being odd, due to a vertical displacement of the beams of $y = y(s)$ at one crossing point is give by²

$$\begin{aligned} \Delta\nu_N &\approx \Delta\nu \frac{1}{2^{(N-1)/2}} \frac{1}{[(N-1)/2]!} \cdot A \\ A &= \alpha \int_{-\infty}^{\infty} \exp(-x^2/2\sigma_x^2) \cdot y/(\sqrt{2\pi} \sigma_x \sigma_y) \cdot ds \end{aligned} \quad (21)$$

where $\Delta\nu$ is the linear vertical beam-beam tune shift and x, y are evaluated along the test beam $x = \alpha s$, $y = y(s)$. This result is valid for beams which are Gaussian in the x and y directions with rms sizes σ_x and σ_y and for which the path of the test beam is $x = \alpha s$, $y = y(s)$.

For the spectrometer one can evaluate the integral A using $y = s^2/\rho$; one finds

$$A = \frac{\sigma_x^2}{\alpha^2 (\rho \sigma_y)} \quad (22)$$

For the orbit errors, y may be considered constant, and one finds for the rms value of A

$$A_{\text{rms}} = \sqrt{6} y_{\text{rms}} / \sigma_y. \quad (23)$$

These results give the same comparison as the previous qualitative estimate.

It is clear that, for a magnetic field having half the strength of the one we have considered, the resonance effects are correspondingly reduced.

2. Even-Order Vertical Resonances

Even-order vertical resonances will also be driven as the spectrometer reduces the periodicity of the beam-beam interaction from six to one. The half-width of the N^{th} -order resonance, N being even, due to a vertical displacement of beams $y(s)$ at one crossing point is given by 2

$$(\Delta v)_N = \Delta v \frac{1}{2^{(N-2)/2}} \cdot \frac{1}{[(N-2)/2]!} \cdot A$$

$$A = \alpha \int_{-\infty}^{\infty} ds \frac{e^{-x^2/2\sigma_x^2}}{(2\pi^{1/2})\sigma_x} \cdot \frac{y^2}{2\sigma_y^2} \quad (24)$$

For the spectrometer one can evaluate the integral using $y = s^2/\rho$; one finds

$$A = \frac{3y_{av}^2}{2\sigma_y}$$

$$y_{av} = \frac{\sigma_x^2}{\alpha^2 \rho} \quad (25)$$

For the random orbit error, one finds² the rms value of A

$$A_{rms} = \frac{\sqrt{6} \times 1.4 y_{rms}^2}{2\sigma_y^2} \quad (26)$$

Thus, the effect of the spectrometer in driving the even-order resonances is equivalent to the random orbit error with

$$y_{rms} = y_{av} \frac{(3/3.4)^{1/2}}{2},$$

$$y_{rms} = 0.021 \text{ mm at } 30 \text{ GeV},$$

$$y_{rms} = 0.0058 \text{ mm at } 400 \text{ GeV}. \quad (27)$$

The driving term for the even-order resonances due to the spectrometer is about the same as that due to the remaining random orbit errors at the crossing point after these orbit errors have been corrected.

The even-order resonances can also be driven by random errors in β_y at the crossing points. It has been found that this drives the even-order resonances with about the same strength as that due to random errors in the orbit. Thus, the spectrometer will excite the even-order resonances with about the same strength as that due to random errors in β_y .

III. HORIZONTAL BENDING

A. Effect of Horizontal Bending in the Crossing Region on the Beam-Beam Tune Shift.

Figures 4, 5, and 6 show other possible configurations of the dipole spectrometer.¹ All the bending occurs in the horizontal plane. We consider here the effect of curved trajectories in the interaction region, defined by magnet C in the figures. The ν -shift introduced in this region will turn out to be small compared with the ν -shift introduced in the drift space between magnets B and C, which is computed later in this section. The beam separation Δx is given by

$$\Delta x = \alpha s - \frac{s^2}{\rho} \quad (28)$$

where s , the longitudinal distance, is measured from the center of magnet C, and α is the crossing angle at the center of magnet C.

Figure 4

Figure 5

Figure 6

It is clear that, in case of curved trajectories, the assumption

$$\int_{-\infty}^{\infty} \frac{\partial E_x}{\partial x} ds = 0 \quad (29)$$

is no longer valid. Hence, a horizontal tune shift will result. Since we expect this to be small, we avoid solving for the field, which, for a beam that is not round, requires a rather cumbersome calculation,³ and are satisfied with establishing the order of magnitude of the perturbation.

The divergence theorem gives

$$\int_{-\infty}^{\infty} \frac{\partial E_x}{\partial x} ds + \int_{-\infty}^{\infty} \frac{\partial E_y}{\partial y} ds = \frac{2e\lambda}{\epsilon_0 2\pi \sigma_x \sigma_y} \int_{-\infty}^{\infty} \exp \left[-\frac{\left(\frac{s^2}{\rho} - s\alpha\right)^2}{2\sigma_x^2} \right] ds \quad (30)$$

where the integration is performed along the trajectory $x = x(s)$, $y = 0$. Since the beams are well separated at the entrance and exit of magnet C, we may assume that all the interaction takes place within the magnetic field region. After integration, we find

$$\Delta\nu_x \left(\frac{\beta_y^*}{\beta_x^*} \right) + \Delta\nu_y = \Delta\nu_{y0} \left[1 + 3t - \frac{15}{2} t^2 + \dots \right] \quad (31)$$

where $\Delta\nu_{y0}$ is the vertical beam-beam tune shift for standard crossing, β_x^* and β_y^* are the horizontal and vertical β -functions at the crossing point, and

$$t = \frac{2\sigma_x^2}{\alpha \rho^2} \quad (32)$$

is a measure of the perturbation introduced by the curved trajectories. Since $t \ll 1$, we may consider only the first term of the solution (31), which gives

$$\frac{\Delta\tilde{\nu}_{x,y}}{\Delta\nu_{y0}} = \frac{\Delta\nu_y - \Delta\nu_{y0}}{\Delta\nu_{y0}} + \frac{\Delta\nu_x (\beta_y^*/\beta_x^*)}{\Delta\nu_{y0}} \approx 6.0 \times 10^{-4} \quad (33)$$

for the layout of Figure 4, where

$$\alpha = 20 \text{ mrad}, \quad \rho = 445 \text{ m}. \quad (34)$$

For the layout of Figure 5, where $\alpha = 11.188 \text{ mrad}$, $\rho = 445 \text{ m}$,

$$\frac{\Delta\tilde{\nu}_{x,y}}{\Delta\nu_{y0}} \approx 6.2 \times 10^{-3}, \quad (35)$$

and, finally, for the layout of Figure 6, where $\alpha = 5 \text{ mrad}$, $\rho = 890 \text{ m}$,

$$\frac{\Delta\tilde{\nu}_{x,y}}{\Delta\nu_{y0}} \approx 3.9 \times 10^{-2}. \quad (36)$$

It is clear that in the cases shown in Figures 4 and 6 the effect of curved trajectories in the interaction region is negligible

compared with the variation of the crossing angle with respect to the standard value (11.188 mrad). In the case of Figure 5, the effect of curved trajectories is still small compared with the shift introduced by the interaction of the beams in the drift space between magnets C and B, as shown below.

B. Effect of "Parallel" Beams

In the layouts of Figures 5 and 6 the beam paths lie close to each other for a considerable distance. Since in both cases the beams are not exactly parallel, we write for the separation Δx in the field-free region

$$\Delta x = \Delta x_0 + K(s - s_0). \quad (37)$$

This beam separation in the drift space between magnets C and B (~ 5 m) varies, from right to left,

$$\begin{aligned} \Delta x_0 &= 13.8 \text{ mm} \quad \text{to} \quad \Delta x_1 = 19.2 \text{ mm, in Figure 5,} \\ \Delta x_0 &= 5.6 \text{ mm} \quad \text{to} \quad \Delta x_1 = 5.3 \text{ mm, in Figure 6} \end{aligned} \quad (38)$$

The longitudinal distance s is measured from the crossing point (center of magnet C); s_0 is the distance of the left edge of the magnet C from the crossing point.

Since the beam separation is many times its rms beam size, it is a reasonable approximation to assume round beams, which greatly simplifies the problem. We have

$$\left(\frac{\partial E_x}{\partial x} \right)_{\substack{x=\Delta x \\ y=0}} \approx - \left(\frac{\partial E_y}{\partial y} \right)_{\substack{x=\Delta x \\ y=0}} = \frac{\lambda e}{\pi \beta_0 \Delta x^2}. \quad (39)$$

The separation Δx is given by Eq. (37). In calculating the tune shift, we must also take into account the variation of the β -function along the trajectory:

$$\beta_{x,y}(s) = \beta_{x,y}^* + \frac{s^2}{\beta_{x,y}^*}. \quad (40)$$

We have, integrating over the field-free region $s_0 - s_1$

$$\Delta \nu_{x,y} = \frac{r_p i}{\pi \gamma c e} \int_{s_0}^{s_1} \frac{1}{[\Delta x_0 + K(s - s_0)]^2} \left(\beta_{x,y}^* + \frac{s^2}{\beta_{x,y}^*} \right) ds \quad (41)$$

After integration, we find for the ratios between the tune shift and the vertical tune shift for standard crossing $\Delta\nu_{yo}$,

$$\frac{\Delta\nu_x}{\Delta\nu_{yo}} = 0.35; \quad \frac{\Delta\nu_y}{\Delta\nu_{yo}} = 0.10 \text{ at } 30 \text{ GeV},$$

$$\frac{\Delta\nu_x}{\Delta\nu_{yo}} = 0.10; \quad \frac{\Delta\nu_y}{\Delta\nu_{yo}} = 0.027 \text{ at } 400 \text{ GeV}, \quad (42)$$

for the layout of Figure 5 and with the standard values $\beta_x^* = 40 \text{ m}$, $\beta_y^* = 7.5 \text{ m}$.

The layout of Figure 6 gives, with the same β_x^* , β_y^* ,

$$\frac{\Delta\nu_x}{\Delta\nu_{yo}} = 3.11; \quad \frac{\Delta\nu_y}{\Delta\nu_{yo}} = 0.90 \text{ at } 30 \text{ GeV},$$

$$\frac{\Delta\nu_x}{\Delta\nu_{yo}} = 0.85; \quad \frac{\Delta\nu_y}{\Delta\nu_{yo}} = 0.24 \text{ at } 400 \text{ GeV}. \quad (43)$$

Note the very large horizontal tune shift. Since $\Delta\nu_x = -0.0064$ at 30 GeV, $\Delta\nu_x = -0.02$ for the configuration of Figure 6.

Inside magnet B of length ℓ_B (Figures 5 and 6), the separation between the two beams is

$$\Delta x = \frac{(s - s_2)^2}{\rho} + \alpha(s - s_2) + \Delta x_0 \quad (44)$$

with

$\alpha = 1.08 \text{ mrad}$, $\rho = 740 \text{ m}$, $s_2 = 7.25 \text{ m}$, $\ell_B = 5 \text{ m}$, $\Delta x_0 = 19.2 \text{ mm}$ (Figure 5).

and

$\alpha = -0.06 \text{ mrad}$, $\rho = 2322 \text{ m}$, $s_2 = 7.25 \text{ m}$, $\ell_B = 10 \text{ m}$, $\Delta x_0 = 5.3 \text{ mm}$ (Figure 6).

The beam-beam tune shift is, in this case,

$$\Delta\nu_{x,y} = \frac{r_p i}{\pi \gamma c e} \int_0^{\ell_B} \frac{1}{\left(\Delta x_0 + \alpha s + \frac{s^2}{\rho}\right)^2} \left(\beta_{x,y}^* + \frac{(s + s_2)^2}{\beta_{x,y}^*} \right) ds. \quad (45)$$

After integration we find for the configuration of Figure 5:

$$\begin{aligned} \frac{\Delta\nu_x}{\Delta\nu_{y0}} &= 0.06; & \frac{\Delta\nu_y}{\Delta\nu_{y0}} &= 0.009 \text{ at } 30 \text{ GeV}, \\ \frac{\Delta\nu_x}{\Delta\nu_{y0}} &= 0.016; & \frac{\Delta\nu_y}{\Delta\nu_{y0}} &= 0.002 \text{ at } 400 \text{ GeV}; \end{aligned} \quad (46)$$

and for the configuration of Figure 6

$$\begin{aligned} \frac{\Delta\nu_x}{\Delta\nu_{y0}} &= 1.47; & \frac{\Delta\nu_y}{\Delta\nu_{y0}} &= 0.86 \text{ at } 30 \text{ GeV}, \\ \frac{\Delta\nu_x}{\Delta\nu_{y0}} &= 0.40; & \frac{\Delta\nu_y}{\Delta\nu_{y0}} &= 0.23 \text{ at } 400 \text{ GeV}. \end{aligned} \quad (47)$$

As far as the resonance excitation is concerned, the "quasi-parallel" beam layouts of Figures 5 and 6 will also excite the pure horizontal resonance, $n\nu_x = P$, as well as all the coupling resonances, $n\nu_x + m\nu_y = P$, for which x_m is even. These are the usual resonances present in a field with median plane symmetry. There is no special symmetry here that would cause the horizontal and coupling resonances to be absent, as in the case when the spectrometer field is removed.

IV. CONCLUSIONS

We have analyzed, from the point of view of the beam-beam interaction, the effect of magnetic fields of different geometry in the interaction region of the crossing beams in ISABELLE. The assumption was made that a linear beam-beam tune shift which, at only one of the six interaction points, is appreciably different from all the rest, will reduce from 6 to 1 the periodicity of all the resonances excited by the space-charge force and therefore is a potential source of trouble. Other effects of the presence of resonances excited by the beam-beam interaction which would not otherwise be present in standard crossing geometry have been considered. The result of the

analysis is that the vertical bending case (Figure 2) is the most acceptable of all the cases examined, at least from the point of view of the beam-beam interaction. The variation of the linear tune shift, as compared with the standard case, is negligible; no coupling resonances are driven by the space-charge field. The strength of the odd-order resonances (not present in a standard crossing geometry), after they are corrected, falls within the range of those excited by random vertical closed orbit errors.

The horizontal bending cases we have examined impose an obvious asymmetry on the regular distribution of the space-charge forces, and excite additional horizontal and coupling resonances. They also produce rather large linear tune shifts.

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