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DETERMINATION OF THE TIME EVOLUTION OF THE ELECTRON-TEMPERATURE
PROFILE OF REACTOR-LIKE PLASMAS FROM THE MEASUREMENT OF
BLACKBODY ELECTRON-CYCLOTRON EMISSION

by

P.C. Efthimion, V. Arunasalam, R.A. Bitzer, and J.C. Hosea

April 1982

PLASMA
PHYSICS
LABORATORY

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CNO-3073.

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Determination of the Time Evolution of the Electron-Temperature
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PPPL--1888
DEG2 012759

Abstract

Plasma characteristics (i.e., $n_e \gtrsim 1 \times 10^{13} \text{ cm}^{-3}$, $T_e \gtrsim 10^7 \text{ eV}$, $B_0 \gtrsim 20 \text{ kG}$) in present and future magnetically confined plasma devices, e.g., Princeton Large Torus (PLT) and Tokamak Fusion Test Reactor (TFTR), meet the conditions for blackbody emission near the electron cyclotron frequency and at few harmonics. These conditions, derived from the hot plasma dielectric tensor, have been verified by propagation experiments on PLT and the Princeton Model-C Stellarator.^{1,2} Blackbody emission near the fundamental electron cyclotron frequency and the second harmonic have been observed in PLT and is routinely measured to ascertain the time evolution of the electron temperature profile.^{2,3} These measurements are especially valuable in the study of auxiliary heating of tokamak plasma. Measurement and calibration techniques will also be discussed with special emphasis on our fast-scanning heterodyne receiver concept.⁴

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I. Introduction

In the study of astrophysics and gas-discharges, a plasma refers to an ionized gas. Over ninety-five percent of the matter in the Universe is in the plasma state including all of the stars and our sun. Controlled thermonuclear fusion research is directed toward producing a star-like plasma in a laboratory environment in order to generate limitless energy. There are a number of light element fusion reactions that generate energy. However, the fusion of deuterium (D) and tritium (T) ions has the largest cross section and the lowest temperature threshold to sustain a fusion burn. This fusion reaction is

where n is the neutron, and the energy of the particles is in parentheses. Even for the $D + T$ reaction plasma temperatures on the order of 10^8 K are required. In this paper we will discuss the determination of the plasma electron temperature by measuring the intensity of the electron cyclotron emission.

The study of fusion reaction in a controlled environment has developed plasma confinement devices based upon inertial confinement driven by potent laser beams⁵ and magnetic confinement in complex topologies.⁶ Presently, the magnetic confinement device, the "tokamak," has produced reactor-like temperatures.⁷ These conditions were obtained in the Princeton Large Torus (PLT) by heating the plasma with energetic neutral beams. Furthermore, the soon to be completed Princeton Tokamak Fusion Test Reactor (TFTR) is designed to achieve an energy yield in excess of the input energy to the tokamak plasma.

The tokamak⁸ is a donut-shaped device with a strong toroidal magnetic field ($B_\phi \sim 10 - 80$ kG) (Fig. 1). A current is induced in the toroidal direction by external coils in order to ohmically heat the plasma to 10^7 °K. This induced current gives rise to a weak poloidal magnetic field ($B \sim 1-10$ kG) that enhances particle confinement. A detailed description of the tokamak can be found in Ref. 8. The electrons follow a helical trajectory along the toroidal field that causes the electrons to radiate at the electron cyclotron frequency and at the harmonics. As will be discussed in Section II, the intensity of the electron cyclotron emission is a function of the electron velocity distribution $f(\underline{x}, \underline{v}, t)$, the electron density n_e , electron temperature T_e , and plasma geometry. Under appropriate plasma conditions the emission at the electron cyclotron frequency and the second harmonic is blackbody, and therefore the emission intensity is proportional only to the electron temperature. Section III is a description of the important cyclotron emission and absorption experiments along with examples of PLT temperature profiles obtained from such blackbody emission. Section IV will be a review of the diagnostic techniques with special emphasis to our fast-scanning heterodyne receiver concept.⁴

II. Theory

The principles of radiation transfer, Kirchhoff's radiation law, and waves in a magnetized plasma define the relationship between the plasma local electron temperature and the electron cyclotron emission.⁹ The equation of radiation transfer is a statement of electromagnetic energy conservation in a dielectric medium: The flow of spectral radiation

intensity I_ω along an incremental ray path length $d\ell$ is equal to the emission coefficient j_ω minus the absorption $\alpha_\omega I_\omega$,

$$n_r^2 \frac{d}{d\ell} \left(\frac{I_\omega}{n_r^2} \right) = j_\omega - \alpha_\omega I_\omega, \quad (2)$$

where α_ω is the absorption coefficient of radiation of frequency ω and n_r is the index of refraction of the magnetized plasma. (See Ref. 9 for a detailed description of n_r .) The solution of this equation of transfer may be written

$$I_\omega = I_\omega(\text{inc}) e^{-\tau_0} + \int_0^{\tau_0} S_\omega e^{-\tau} d\tau, \quad (3)$$

where $I_\omega(\text{inc})$ is the radiation intensity incident on the medium, $S_\omega = (1/n_r^2)(j_\omega/\alpha_\omega)$ is referred to as the source function, $\tau = -\int \alpha_\omega d\ell$ is the optical depth, τ_0 is the total optical depth of the medium. For the problems considered in this paper S_ω is at most a very weak function of I_ω and there is no incident radiation, then Eq. (3) becomes

$$I_\omega = S_\omega (1 - e^{-\tau_0}) \quad (4)$$

for situations where there are no reflecting boundaries (i.e., a single path absorption). However, if r is the reflection coefficient of the plasma containment vessel, then the multiple path solution of Eq. (3) may be written

$$I_{\omega} = S_{\omega} \frac{1 - e^{-\tau_0}}{1 - e^{-\tau_0}}. \quad (5)$$

The source function is derived from Planck's blackbody theory, Einstein's coefficient for emission and absorption, and Kirchhoff's law. Since photons obey Bose-Einstein statistics, the blackbody emission intensity $B_0(\omega, T)$ is

$$B_0(\omega, T) = S_{\omega} = \frac{\hbar \omega^3}{8\pi^3 c^2} \frac{1}{(e^{\hbar\omega/kT} - 1)}, \quad (6)$$

where $\hbar$ is Planck's constant, ω is the radian frequency, kT is the temperature in energy units, and c is the velocity of light. In the classical limit $kT \gg \hbar\omega$, Eq. (6) becomes the Rayleigh-Jeans Law

$$B_0(\omega, T) = \frac{\omega^2 kT}{8\pi^3 c^2}. \quad (7)$$

For a classical plasma,⁹ in the classical limit,

$$j_{\omega} = \int \eta_{\omega}(p) f(p) d^3p, \quad (8)$$

and

$$\alpha_{\omega} = \frac{8\pi^3 c^2}{n_r \omega^2} \int \eta_{\omega}(p) \frac{\partial f}{\partial \epsilon} d^3p, \quad (9)$$

where $\eta_{\omega}(p)$ is the Einstein's spontaneous emission coefficient for an electron of momentum p , and $\partial f / \partial \epsilon$ is the derivative of the electron velocity distribution with respect to energy. The appearance of $\partial f / \partial \epsilon$

in Eq. (9) is a reflection of the fact that α_ω is a consequence of the direct absorption minus the stimulated emission. Naturally, when the distribution function f is Maxwellian, the source function for a classical plasma may be written

$$S = j_\omega / n_r^2 \alpha_\omega = \frac{\omega^2 \kappa T_e}{8\pi^3 c^2} \quad (10)$$

Thus, the emission intensity from a Maxwellian plasma is determined by the electron temperature T_e when $\tau_0 \gg 1$. However, if the distribution function is non-Maxwellian, then the radiation temperature $T_r = S_\omega (8\pi^3 c^2 / \omega^2 \kappa)$ is in general not equal to the electron temperature T_e . Thus, for a non-Maxwellian plasma the temperature dependence of the emission is difficult to determine unless the details of the distribution function are exactly known. Substituting the expression for the source function Eq. (10) into Eq. (4) the intensity becomes

$$I_\omega = \frac{\kappa T_e \omega^2}{8\pi^3 c^2} (1 - e^{-\tau_0}) \quad (11)$$

Even if the distribution is Maxwellian the optical depth must be calculated to see if the plasma is a blackbody. When the plasma is highly absorptive, $\tau \gtrsim 2$, the plasma is considered black and

$$I_\omega = \frac{\kappa T_e \omega^2}{8\pi^3 c^2} \quad (12)$$

The calculation of τ_0 is not simple for a plasma in a strong magnetic field. In general, it is a complex function of electron temperature, electron density, plasma geometry, and the emission wave features of the magnetized plasma.

Plane polarized waves propagating perpendicular to the magnetic field (B_0) can have two possible orientations of the electric field E : the wave is referred to as "ordinary" when $E \parallel B_0$, and is referred to as "extraordinary" when $E \perp B_0$ (see Ref. 10 for a detailed discussion). But, for parallel propagation it is customary to talk of right-hand and left-hand circularly polarized waves. For the determination of T_e via blackbody electron cyclotron emission measurements from plasmas in tokamak type devices one is particularly interested in near-perpendicular propagating waves only. The index of refraction n_r can range from zero to infinity. When $n_r^2 = 0$, the wave enters a cutoff region of plasma where it is reflected. When $n_r^2 \rightarrow \infty$, the wave enters a resonance region where it may be absorbed, reflected, or converted to a different plasma wave. For the ordinary mode $n_r^2 = 0$ when $\omega = \omega_{pe}$ where $\omega_{pe} = (4\pi n_e e^2/m)^{1/2}$ is the electron plasma frequency. The extraordinary mode has two cutoff frequencies ω_{co1} and ω_{co2} when $\omega_{pe}^2/\omega^2 = 1 \pm \omega_{ce}/\omega_{co1,2}$ and has a resonance when $\omega = \omega_{UH}$ where ω_{UH} is the upper hybrid frequency defined as $\omega_{UH}^2 = (\omega_{pe}^2 + \omega_{ce}^2)^{1/2}$. In the preceding equation, the plus sign is associated with ω_{co1} and the minus sign with ω_{co2} . In Fig. 2 is a diagram of the characteristic frequencies of blackbody emission in the PLT plasma as a function of the plasma position. The curve of ω_{pe} has a conical shape because the density profile in PLT is parabolic. The ω_{ce} curve has the characteristic $1/R$ dependence for all tokamaks and is plotted here for $B = 32$ kG. As stated earlier, we are interested in waves of frequencies $\omega \approx \omega_{ce}$ and $\omega \approx 2\omega_{ce}$. There is a unique frequency for each harmonic at each radial position in the plasma and therefore the emission intensity as a function of position can be obtained by

measuring the emission over the indicated frequency range for a specific magnetic field. Now for $\omega < \omega_{pe}$, the ordinary mode wave will not propagate because of the cutoff condition. For the extraordinary mode, the wave will not propagate for $\omega < \omega_{co1}$, and from the right hand side of the diagram it can not propagate into the shaded region if $\omega < \omega_{co2}$. The right hand side of this diagram is the low magnetic field side of the plasma is the large major radius (R) side of the tokamak. In tokamaks $B_\phi \sim 1/R$. Thus for the extraordinary mode the cyclotron layer is not accessible from the low field side of the tokamak. It is accessible, however, from the high field side. On the other hand, the ordinary mode emission can propagate out of the plasma towards both the low and high field side of the tokamak as long as $\omega > \omega_{pe}$. Also, the extraordinary mode second harmonic electron cyclotron emission can propagate out from either side.

Many researchers (Schwinger, Landau, Oster, Schott, and Trubnikov) have considered the problem of the radiation spectrum due to the motion of electrons in a magnetic field.¹¹ The Schott-Trubnikov distribution of the emission intensity for a radiating electron is

$$\frac{dI}{d\omega d\Omega} = \frac{e^2 \omega^2}{2\pi c} \sum_{n=1}^{\infty} \left[\left(\frac{\cos\theta - \beta_{\parallel}}{\sin\theta} \right)^2 J_n^2 \left(\frac{\omega}{\omega_{ce}} \beta_{\perp} \sin\theta \right) + \beta_{\perp}^2 J_n'^2 \left(\frac{\omega}{\omega_{ce}} \beta_{\perp} \sin\theta \right) \right] \delta(n\omega_{ce} - \omega(1 - \beta_{\parallel} \cos\theta)) \quad (13)$$

where θ is the angle between the wavevector k and the magnetic field, n is the harmonic number, $\beta_{\parallel} = v_{\parallel}/c$, $\beta_{\perp} = v_{\perp}/c$, and v is the electron velocity. For perpendicular propagation ($\theta = \pi/2$) the emissivity η for linear polarized waves in the ordinary mode

$$\eta_{\omega}^{(o)} = \frac{e^2 \omega^2}{2\pi c} \sum_{n=1}^{\infty} \beta_{\perp}^2 J_n^2(n\beta_{\perp}) \delta(n\omega_{ce} - \omega), \quad (14)$$

and for the extraordinary mode

$$\eta_{\omega}^{(e)} = \frac{e^2 \omega^2}{2\pi c} \sum_{n=1}^{\infty} \beta_{\perp}^2 J_n^2(n\beta_{\perp}) \delta(n\omega_{ce} - \omega). \quad (15)$$

Using the expressions for η_{ω} in Eqs. (8) and (9), one can calculate the appropriate emission coefficient j_{ω} and the absorption coefficient α_{ω} for both the ordinary and the extraordinary modes of propagation. Knowing α_{ω} it is relatively easy to obtain closed form expressions for the optical depth $\tau_0 = -(\alpha_{\omega} d)$. If $\tau_0 \gg 2$, then the corresponding emission is a blackbody emission and is directly proportional to the electron temperature T_e .

Another group of researchers (Arunasalam et al.,^{1,12} Ridone et al.,¹³ Hosea et al.³) have used a somewhat different approach to the calculation of the optical thickness τ_0 . These authors make full use of the well known hot plasma dielectric tensor,^{9,10} $\underline{K}(\omega, k)$.

The waves in a plasma are complicated by the presence of a magnetic field that results in anisotropic wave propagation. After Fourier analysis in space and time, the Maxwell electromagnetic field equations yield, for plane waves of the form,

$$\begin{aligned} \vec{E} &= \vec{E}_0 \exp[i(\vec{k} \cdot \vec{r} - \omega t)] , \\ \vec{k} \times (\vec{k} \times \vec{E}) + \frac{\omega^2}{c^2} \vec{K} \cdot \vec{E} &= 0 . \end{aligned} \quad (16)$$

Here $\vec{K}(\omega, \vec{k})$ is the dielectric tensor of the magnetized plasma. The plasma dispersion relation is the condition for a nontrivial solution of Eq. (16) and is obtained by setting the determinant of the coefficients of E_x , E_y , and E_z equal to zero. Thus, knowing the dielectric tensor $\vec{K}(\omega, \vec{k})$ one can get a closed form expression for the allowed wave number k as a function of ω . The optical depth τ_0 is given by

$$\tau_0 = - \int \alpha_\omega d\ell = - \int (2 \text{Im } k) d\ell , \quad (17)$$

where Im stands for the imaginary part.

This dielectric tensor $\vec{K}$ for a hot plasma in a strong magnetic field predicts resonance phenomena for wave propagation at frequencies ω in the neighborhood of all the harmonics of the electron cyclotron frequency $\omega_{ce} = eB_0/mc$. Here m and e are the mass and charge of the electron. In the conventional hot-plasma theory the resonances at the second and higher harmonics arise from the effects of the finite Larmor

radii ρ of the electrons. Consequently, the relative strengths of these successive higher harmonic resonances are expected to fall off approximately as successive powers of $(\rho k_{\perp})^2 \sim (\kappa T_e / mc^2)$. Here, $k_{\perp}$ is the component of the wave vector in a direction perpendicular to the confining magnetic field B_0 . In present day fusion plasma devices the plasma conditions (e.g., n_e , T_e , and ω_{ce}) are such that only the resonances near the fundamental cyclotron frequency ($\omega \approx \omega_{ce}$) and the second harmonic ($\omega \approx 2\omega_{ce}$) are sufficiently strong to yield values of the optical depth $\tau_0 \gtrsim 2$. That is, only for $\omega \approx \omega_{ce}$ and $\omega \approx 2\omega_{ce}$, the plasma found in present day fusion devices emit like a blackbody. Hence, in this paper we will restrict our discussion to waves of frequencies $\omega \approx \omega_{ce}$ and $\omega \approx 2\omega_{ce}$ only.

For arbitrary angles of propagation vector $\underline{k}$ with respect to the magnetic field B_0 , all the nine elements K_{ij} ($i, j = x, y$, and z) of the dielectric tensor $\underline{K}$ are nonzero. Thus, it is extremely difficult if not impossible to obtain a closed form analytic expression for the plasma dispersion relation from Eq. (16) when the wave vector $\underline{k}$ makes an arbitrary angle with respect to the toroidal magnetic field B_0 . However, for near perpendicular (i.e., $k_{\parallel} \ll k_{\perp}$) propagation, one can obtain a meaningful closed form analytic expression for the plasma dispersion relation. Here $\perp$, and $\parallel$ refer to directions perpendicular and parallel, respectively, with respect to the toroidal magnetic field B_0 .

Arunasalam et al.¹ first applied this dispersion relation technique for near perpendicular propagation of the extraordinary mode in the

vicinity of the second harmonic of the electron cyclotron frequency ($\omega \approx 2\omega_{ce}$). Their result for propagation across a uniform cylindrical plasma column of diameter l whose axis is along B_ϕ is

$$\tau_o(\omega \approx 2\omega_{ce}) \approx A \exp \left[- \frac{mc^2}{2\kappa T_\perp \cos^2 \theta} \left(\frac{\omega - 2\omega_{ce}}{\omega} \right)^2 \right], \quad (18)$$

where

$$A \approx \frac{\pi^{1/2} \omega_{pe}^2 \kappa T_\perp \omega \tan \theta}{2mc^2 \omega_{ce}^2 (2\kappa T_\parallel / m)^{1/2}} \left(\frac{6\omega_{ce}^2 - \omega_{pe}^2}{6\omega_{ce}^2 - 2\omega_{pe}^2} \right)^2. \quad (19)$$

Equation (18) shows the familiar Doppler broadening of the second harmonic resonance. For propagation at exactly 90° (that is, in the limit $\cos^2 \theta \rightarrow 0$) it can be shown from Eqs. (18) and (19) that

$$\begin{aligned} \tau_o(\omega \approx 2\omega_{ce}) &= - \int_0^l (2\text{Im } k) dl = l(2\text{Im } k) \\ &= \frac{l \omega_{pe}^2 \kappa T_\perp}{mc^2} \left(\frac{6\omega_{ce}^2 - \omega_{pe}^2}{6\omega_{ce}^2 - 2\omega_{pe}^2} \right)^2 \delta \left(\frac{\omega - 2\omega_{ce}}{2\pi} \right). \end{aligned} \quad (20)$$

This result is for a uniform cylindrical plasma column of diameter l whose axis is along a uniform magnetic field B_ϕ and is applicable to the Model C stellarator plasma. However, for plasmas in tokamak geometry $\omega_{ce} \sim B_\phi \sim 1/R$, where R is the major radius of the torus. Then

$d\lambda = dR = - (R/\omega_{ce}) d\omega_{ce}$. Thus from Eq. (20) one sees readily that the optical depth for the second harmonic extraordinary mode near perpendicular propagation in a tokamak is given by

$$\tau_o(\omega = 2\omega_{ce}) = \frac{R\pi\omega_{pe}^2 \kappa T_{\perp}}{2\omega_{ce} mc^3} \left(\frac{\omega_{ce}^2 - \omega_{pe}^2}{\omega_{ce}^2 - 2\omega_{pe}^2} \right)^2. \quad (21)$$

Fidone et al.,¹³ Antonsen et al.,¹⁴ and Arunasalam et al.,¹² have also applied a similar dispersion relation technique to calculate the optical depth for near perpendicular propagation of the ordinary mode in the vicinity of ω_{ce} . Their results for the ordinary mode $\omega \approx \omega_{ce}$, $\theta \approx \pi/2$ propagation in tokamaks is

$$\tau_o(\omega \approx \omega_{ce}) = \frac{R\pi\omega_{pe}^2 \kappa T_{\parallel}}{2\omega_{ce} mc^3} \left(1 - \frac{\omega_{pe}^2}{\omega_{ce}^2} \right)^{1/2}. \quad (22)$$

Here we have generalized the results of Fidone et al. and Antonsen et al. to include the effects of pressure anisotropy (i.e., $T_{\perp} \neq T_{\parallel}$). It is interesting to note from Eqs. (21) and (22) that the second harmonic extraordinary mode absorption is due to the perpendicular electron temperature $T_{\perp}$, while the fundamental ordinary mode absorption is due to the parallel electron temperature $T_{\parallel}$ even though both these absorptions are a consequence of the finite size of the electron Larmor orbits. This conclusion is also apparent from the emissivity formulae of Eqs. (14) and (15).

In tokamak regimes for which the runaway population is small, the ordinary mode emission near the fundamental as well as both the ordinary and extraordinary mode emission near the second and higher harmonics can be well understood in terms of the conventional non-relativistic hot-plasma theory.⁹⁻¹⁰ However, Fidone et al.¹³ have shown that one must take account of the relativistic broadening of the resonance due to the variation of electron mass with energy in calculating the optical depth of the perpendicularly propagating extraordinary mode near ω_{ce} in a high density plasma $(\omega_{pe}^2/\omega_{ce}^2) \gg (\kappa T_e/mc^2)^{1/2}$. Their result for ω_{ce} - extraordinary mode is

$$\tau_o(\omega \approx \omega_{ce}) \approx \frac{3R\omega_{ce}^3}{2c\omega_{pe}^2} \left(2 - \frac{\omega_{pe}^2}{\omega_{ce}^2} \right)^{3/2} \frac{\kappa T_e^2}{mc^2} \quad (23)$$

Here Fidone makes use of the relativistic hot plasma dielectric tensor by Shkarofsky and obtains results similar to Dnestrovskii, et al.¹³ For current tokamak parameters, the extraordinary mode $\tau_o \ll 1$ and therefore does not predict blackbody cyclotron emission. However, the intensity of extraordinary mode emission in tokamaks near the fundamental electron cyclotron frequency is measured to be at the blackbody level, as will be discussed in Section III.

Mode conversion of blackbody electrostatic electron Bernstein waves into extraordinary mode electromagnetic waves is a possible physical mechanism to explain the observed blackbody emission. Hosea et al.³ calculate the optical thickness for the electron Bernstein mode to be

$$\tau_0 \approx a\pi^{1/2} (2\pi)^{4/3} R \frac{\omega_{ce}}{c} (\omega_{pe}^2 V_T / \omega_{ce}^2 c)^{1/3}, \quad (24)$$

where $a \approx 1$ for small $k_{\parallel}$ and $a = (8/\pi\epsilon)^{1/2}$ for the case of relativistic line broadening, and $V_T = (2kT_s/m)^{1/2}$. For the case of PLT tokamak, $\tau \sim 10^3 - 10^4$. This mode conversion at the upper hybrid frequency has been analyzed by Stix¹⁵ and calculated to be nearly 100% for PLT parameters by Schuss and Hosea.¹⁶

Thus the theory of blackbody emission from a plasma is well understood. Calculation of the optical thickness for the fundamental electron cyclotron frequency and the harmonics indicate the ordinary mode near $\omega = \omega_{ce}$ and the extraordinary mode near $\omega = 2\omega_{ce}$ are nearly completely absorbing, and thus by Kirchoff's law the emission is blackbody. The same conclusion is obtained for the extraordinary mode near $\omega = \omega_{ce}$ when mode conversion of the electrostatic Bernstein wave is invoked. Thus there are a number of modes and frequency ranges where the electron temperature can be ascertained from electron cyclotron emission.

III. Experiments and Temperature Measurements

Experiments verifying the theory presented in Section II have been two-fold: (a) wave propagation to measure the plasma absorptivity, and (b) measurements of the intensity of the plasma electron cyclotron emission. Arunasalam et al.¹ verified the hot plasma dispersion relation by verifying the expression Eqs. (18)-(20) for the optical thickness of a plasma for the extraordinary mode near the second harmonic of the electron cyclotron frequency. The experiments involved

propagating 70 GHz microwaves across the horizontal midplane of the homogeneous magnetic field straight-section of the Model C Stellerator. The absorptivity of the plasma as a function of $\omega/2\omega_{ce}$ was obtained by varying the magnetic field. The experiment was done at a few different angles (θ) of $\mathbf{k}$ with respect to the magnetic field. Thus the amount of Doppler broadening of the absorption as a function of $k_{\parallel}(2\kappa T_{\parallel}/m)^{1/2}$ could be changed. Figure 3a is a plot of the absorptivity $\ln(P/P_0) = \tau_0$ as a function of $\omega/2\omega_{ce}$ for $\theta = 90^\circ$. Here there is no Doppler broadening effect [see Eq. (20)] and the actual resonance width is due to magnetic field inhomogeneity. In Fig. 3b, $\theta = 103.7^\circ$ and the Doppler broadening makes the width significantly larger. The points represent actual measurements while the smooth continuous curve is the expected absorption from the theory with no temperature anisotropy [i.e., Eqs. (18) and (19) with $T_{\perp} = T_{\parallel}$]. However, the actual depth of the resonance is a function of $T_{\perp}$ while the width is a function of $T_{\parallel}$ where $T_{\parallel}$ and $T_{\perp}$ represent the electron temperatures in directions parallel and perpendicular to the magnetic field $\mathbf{B}_0$. There is a better fit of the data to the dashed curve in Fig. 3b which represents the theory Eqs. (18) and (19) with temperature anisotropy (i.e., $T_{\perp} \neq T_{\parallel}$). Although the plasma temperatures for this experiment were only in the range of 10-50 eV ($\sim 10^5$ °K), the hot-plasma theory still remains viable for 10^7 °K temperatures in present tokamaks. This experimental verification of hot-plasma theory became the basis for utilization of electron cyclotron emission to determine the electron temperature in present day tokamaks.

First measurements of blackbody electron cyclotron emission from the TFR tokamak were reported by Costley et al.¹⁷ These measurements were done with a Michelson interferometer followed by a cryogenically-cooled Putley indium antimonide detector. The interferometer swept out the emission intensity near the fundamental electron cyclotron frequency and the first few harmonics. Under certain plasma conditions the intensity of the emission in the extraordinary mode near $\omega = 2\omega_{ce}$ was blackbody. In addition the intensity of the emission for any frequency was the same for ordinary and extraordinary modes--a result contradictory to theory. This result is believed to be caused by the high reflectivity of the tokamak containment vessel depolarizing the emission. Later Cano et al.¹⁸ built a manually tunable millimeter wave heterodyne receiver and measured the electron temperature profile on the TFR tokamak from the extraordinary mode blackbody electron cyclotron emission near $\omega = 2\omega_{ce}$.

On PFM, Hosea et al.³ studied the electron cyclotron emission with a manually tunable heterodyne receiver. Measurements of the emission near $\omega = \omega_{ce}$ as well as $\omega = 2\omega_{ce}$ were completed. Figure 4 is a graph comparing the intensities of the extraordinary mode electron cyclotron emission at $\omega = 2\omega_{ce}$ measured from the low magnetic field side (open circles), with the extraordinary mode emission at $\omega = \omega_{ce}$ measured from the accessible high magnetic field side (solid circles), and with the electron temperature measured by ruby laser Thomson scattering (solid line) all as a function of plasma position. This graph shows the emission and temperature comparison only for half the plasma cross section located toward the low magnetic field side of the torus. The

agreement between the blackbody emission in the extraordinary mode near $\omega = \omega_{ce}$ and $\omega = 2\omega_{ce}$ and Thomson scattering is indeed remarkable. For $\omega = 2\omega_{ce}$ this result was expected because the optical thickness is greater than 2 from theory Eq. (21) and also from the earlier emission measurements of Costley from the TFR tokamak. However, the theory Eq. (23) for the extraordinary mode near $\omega = \omega_{ce}$ does not predict direct blackbody emission even if wall reflections are included in the calculations (since $\tau_0 < 0.01$). Hosea et al.³ suggested that the observed blackbody emission in the extraordinary mode near $\omega \approx \omega_{ce}$ is a result of efficient conversion of electrostatic electron cyclotron waves (i.e., Bernstein waves) into electromagnetic waves around the region of the plasma where $\omega = \omega_{UH}$. This mode conversion process has been analyzed by Stix,¹⁵ and later Schuss and Hosea¹⁶ calculated the efficiency of this particular process to be nearly 100%. Therefore the intensity of the electrostatic mode has to be at the blackbody level in order for the electromagnetic emission at $\omega = \omega_{ce}$ to be blackbody. Hosea et al. calculated the optical thickness of the electrostatic wave to be of the order of 10^3 - 10^4 for the PLT plasma [Eq. (24)]. Unfortunately, an experiment to verify this mode-conversion-emission process for the extraordinary mode near $\omega = \omega_{ce}$ has not been performed to date.

Recent interest in the ordinary mode near the fundamental electron cyclotron frequency was motivated by the desire to find a wave that is easily accessible, completely absorbing, and had a frequency as low as possible for electron-cyclotron resonance (ECR) heating. The accessible ordinary mode near the fundamental electron cyclotron

frequency was first studied by Fidone et al. who derived a closed form expression for the absorptivity of the mode Eq. (22). According to the optical thickness expression Eq. (22) the ordinary mode is nearly completely absorbed in a plasma with parameters on the order of $T_e \sim 10^3$ eV and density $\sim 3 \times 10^{13} \text{ cm}^{-3}$. These calculations motivated Efthimion et al.² to measure the emission intensity of the ordinary mode at $\omega = \omega_{ce}$ in PLT from the low magnetic field side of the plasma. Figure 5 is a comparison of the temperature profiles ascertained from the ordinary mode electron cyclotron emission and from ruby laser Thomson scattering. The temperature profile from the emission taken along the major radius in the horizontal midplane shows a 2-3 cm outward shift while the profile from the laser taken along a vertical plasma cord does not show this shift. The horizontal shift is attributable to either the outward shift of the magnetic field surfaces or the actual horizontal positioning of the plasma. Nevertheless the profiles indicate the emission intensity is at the blackbody level. For PLT the absorption theory Eq. (22) predicts that the inner 20 cm of the plasma is blackbody for single path absorption. The blackbody emission intensity observed outside of 20 cm can be explained by the reflections from the vacuum vessel allowing the cyclotron emission to build up to the blackbody level. However, the emission experiments could not accurately verify the ordinary mode theory because of wall reflections.

A verification of the theory involved measurements of the ordinary mode absorption in the PLT plasma.² Figure 6 is a block diagram of the experimental arrangement. The incident microwave power is supplied by

a klystron at a fixed frequency of 71 GHz. The antennas are standard 4 mm high gain horns that are oriented for the ordinary mode. The microwave power propagates through the plasma along the horizontal midplane, is collected by the receiving antenna, and detected by a heterodyne receiver that makes up the rest of the block diagram. A discussion of the swept heterodyne receiver will be presented in Section IV. The fractional microwave power (P_T/P_O) transmitted through the plasma may be written as a function of the optical thickness: (P_T/P_O) = $\exp(-\tau_O)$. The experiment consists of varying the magnetic field of the tokamak so the absorbing layer moves from the center of the plasma out to the edge. In P_O the refraction losses have been experimentally taken into account by measuring the power transmitted across the plasma with the cyclotron layer well outside the plasma. Measurements of the electron temperature and density in the absorbing layer were provided by ruby laser Thomson scattering. These local values of density and temperature enabled the calculation of the optical thickness from Eq. (22). The calculated values agreed well with the values obtained from the absorption experiment (see Ref. 2).

The collective knowledge of the experiments and theory is the basis for the determination of the electron temperature of magnetized plasmas from the measurement of the intensity of the emission near the electron cyclotron frequency and the harmonics. This technique is particularly useful in the study of auxiliary heating of plasma because it offers the time evolution of the temperature profile--information presently not available by any one technique. Here we will present some examples of the electron temperature measurements completed by this technique.

In Fig. 7 is a three dimensional computer plot of the time evolution of the electron temperature profile $[T_e(r,t)]$ measured for a typical ohmically heated PLT discharge. In order to visualize the contours, only seventeen radial positions are plotted. For ohmically heated plasmas the peak electron temperature is typically $1-2 \times 10^7$ °K. An example of auxiliary heating of the plasma electrons appears in Fig. 8. Here is a plot of the electron temperature profile before, during, and after injection of 2.1×10^6 watts of deuterium neutral beams at time $t = 450-600$ ms into a low density ($n_e = 1.8 \times 10^{13} \text{ cm}^{-3}$) hydrogen plasma. The observed peaking of the temperature after the termination of injection can be explained by the heating of the plasma by the still slowing-down beam ions and the removal of the heat loss due to the thermalization of cold electrons from the neutral beam particles. These examples of temperature determination from electron cyclotron emission illustrate the power and usefulness of this technique for tokamak fusion research.

A word of caution is necessary in utilizing electron cyclotron emission to ascertain the electron temperature. First, the proper plasma conditions must be present so that the plasma is highly absorbing and the emitting layer is optically thick ($\tau_o \gtrsim 2$). Second, the radiation should be in thermal equilibrium with the radiating electrons for the emission intensity to be at the blackbody level. Furthermore, if the plasma is a blackbody only with the help of multiple reflections from the vessel wall, energetic electrons (i.e., "runaways") can radiate with intensities which are orders of magnitude larger than that from the plasma electrons with a Maxwellian distribution. Nonthermal

emission near the electron cyclotron frequency and its harmonics is outside the subject addressed in this paper. All nonthermal processes can be held down to a minimum by properly controlling the plasma discharge conditions such as to keep the runaway population negligibly small.

IV. Measurement Techniques

Although the electron temperatures of tokamak plasmas is very large ($\sim 10^7 - 10^8$ °K), the actual broadband power measured in say a 250 MHz bandwidth is only of the order of 10^{-8} - 10^{-7} watts. In comparison to the radio astronomy emission measurements, this emission power level is quite large. Power detections in radio astronomy involves signal integration for many seconds. On the other hand tokamak plasmas are only ~ 1 second in duration and signal integration is not practical. Most tokamaks operate in the 10-60 kG range of magnetic fields. Therefore the spectral range of the emission near the fundamental electron cyclotron frequency and the next two harmonics is in the 10-0.5 millimeter wavelength region. Even if only one harmonic is to be measured, the radial variation in the magnetic field across the tokamak plasma dictates that the detection system should be at least 30% broadband in order to obtain the temperature profile.

There are two techniques for measuring the millimeter-submillimeter wavelength emission in tokamak plasmas: standard guided-wave heterodyne techniques, and quasi-optical interferometry employing cryogenically-cooled broadband bolometers and detectors. The interferometric devices could be a Michelson interferometer, a Fabry-Perot interferometer or a

grating spectrometer.¹⁹ As mentioned in Section III, both detection techniques have been successful in measuring plasma millimeter-submillimeter wave emission. However, the authors of this paper have made a large effort to refine the millimeter wave heterodyne techniques. Specifically, the fast-scanning millimeter wavelength heterodyne receiver has been developed by the authors at Princeton, and with this technique they have made emission measurements on PLT and expect to measure the time evolution of the electron temperature profile of the TFTR plasma.

Figure 9 is a block diagram of the millimeter wave heterodyne receiver on PLT. This receiver electronically scans the frequency range 60-90 GHz every 10 ms. Under typical PLT operating conditions ($B_0 \sim 32$ kG), this frequency range corresponds to the fundamental electron cyclotron emission between the center and the edge of the plasma on the low field side. There are two high-gain horns located on the equatorial plane along the major radius: (1) a horn located on the high field side of the torus oriented to receive the extraordinary mode, and (2) a horn located on the low field side of the torus to measure ordinary mode emission.

The millimeter wave mixer is single-ended with a flat response over the entire frequency range of the receiver and has a 16 dB conversion loss. The local oscillator is a Siemens Backward-Wave-Oscillator (BWO) that can be electronically swept over the 60-90 GHz range by sweeping the applied cathode voltage over a 1 kV range. In our system the 1 kV sweep is obtained with a low voltage signal generator and a high-

voltage amplifier. The internal capacitance of the BWO prevents sweep rates greater than 500 Hz. Because the BWO output power varies over a factor of ten, a commercially available feedback loop maintains a low but constant power level for the mixer. The feedback circuit has a 50 μ sec response time and levels the power with less than 1 dB variation.

The mixer output is amplified by a two stage 60 dB gain IF - amplifier with 3 dB points at 250 and 500 MHz. This frequency range is chosen to minimize the noise created by the mixing of the BWO carrier wings with the carrier peak and at the same time to minimize the frequency gap between the receiver sidebands. This selected IF bandwidth results in a 1 cm spatial resolution in the plasma. The amplified IF signal is detected by a Schottky barrier detector with a 100 KHz response. The detector output is amplified by a 20 dB gain line driver.

The receiver has a 23 dB noise figure while operating in the swept mode. This noise figure could be substantially reduced if either the IF range was selected to be in the 1-2 GHz range or the bandwidth of the receiver was restricted to allow the use of a balanced mixer to cancel the local oscillator noise. However, with typical electron temperatures of the order of 10^7 - 10^8 °K a noise figure of 23 dB ($\sim 6 \times 10^4$ °K) is acceptable. Essentially, the noise figure was sacrificed for bandwidth ($\sim 30\%$) and spatial resolution.

Calibration of the system is performed by two techniques. A calibration of the system can be made by normalizing the receiver

output to the electron temperature of a plasma measured by ruby laser Thomson scattering. This technique is quite simple and very effective but relies on the Thomson scattering measurements. An independent calibration is realized by measuring the emission of a well known blackbody object, and the calibration is obtained by measuring the voltage difference from the output of the receiver when the absorber is at room temperature (290°K) and at liquid nitrogen temperature (77°K). The absorber is maintained at 77°K by inserting it in a styrofoam dewar filled with liquid nitrogen. The styrofoam is transparent to the millimeter waves. Unfortunately, the receiver can only measure temperature above 6×10^4 °K. This problem is circumvented by switching the receiver input between the room temperature microwave absorber and the one cooled at liquid nitrogen temperatures. This technique to measure temperature lower than the noise temperature of the receiver was discovered by Dicke²⁰ for radio astronomy.

This switching technique in the millimeter wavelength region can be realized by a chopping wheel placed between the nitrogen filled dewar and the receiver antenna (Fig. 10). The chopping wheel blades can be covered with an absorber or merely be a metallic material to reflect the room temperature emission into the receiver antenna. A pulse train synchronized with the chopping wheel is generated. The receiver output and the synchronized pulse train are coupled to a lock-in amplifier to integrate the receiver signal and obtain the voltage difference of the receiver corresponding to the temperature difference between the blackbodies at two different temperatures.

V. Conclusions

In conclusion, we have attempted to give a balanced presentation of the theory, existing important cyclotron emission and absorption experiments, and a review of the diagnostic techniques which are involved in the determination of the time evolution of the electron temperature profile of reactor-like plasmas from the measurement of blackbody electron cyclotron emission. We sincerely hope that this paper illustrates the power and usefulness of the fast-scanning heterodyne technique for tokamak fusion research.

Acknowledgment

This work was supported by U.S. Department of Energy, Contract # DE-AC02-76-CHO-3073.

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Figure Captions

- Fig. 1. Schematic of the conventional tokamak configuration.
- Fig. 2. A plot of the characteristic electron cyclotron frequencies of the blackbody emission in the PLT plasma as a function of the plasma position. This plot illustrates the accessibility of polarized blackbody emission in a tokamak.
- Fig. 3a. Second cyclotron harmonic absorption resonance for $\theta = 90^\circ$. The resonance is narrower compared to the Fig. 3b case because of the absence of doppler broadening.
- Fig. 3b. Second cyclotron harmonic absorption resonance for $\theta = 103.7^\circ$ (solid curve) theoretical curve for $T_\perp = T_\parallel = T_e$; theory for $T_\perp = 15.5$ eV, $T_\parallel = 14.5$ eV (dashed curve).
- Fig. 4. (a) Cyclotron emission profiles for ω_{ce} (*) and $2\omega_{ce}$ (o) emission. The average Thomson scattering profile (-) is shown for comparison [$T_e(o) \sim 1$ keV, $n_e = 3 \times 10^{13} \text{ cm}^{-3}$]. (b) An oscilloscope trace of the emission as a function of time.
- Fig. 5. A comparison of the temperature profiles obtained from blackbody ordinary-mode fundamental cyclotron emission (i.e., dashed line) with the corresponding profile obtained from laser Thomson scattering, TS (i.e., solid line).

- Fig. 6. Block diagram of the experimental arrangement employed to verify the hot plasma ordinary mode theory.
- Fig. 7. A computer plot of the time evolution of the electron temperature profile of a typical ohmically heated PLT plasma discharge determined from the measurement of the ordinary-mode fundamental electron cyclotron emission by the fast-scanning heterodyne receiver. In order to easily visualize the contours of the three-dimensional graph only seventeen radial positions of every other temperature profile actually determined from the emission are plotted.
- Fig. 8. A computer plot of electron temperature (keV) ascertained from the cyclotron emission versus plasmas radius (cm) at four distinct times (ms) representing before (450 ms), during (575 ms), and after (630 and 660 ms) injection of 2.1 MW of deuterium neutral beams into a low-density ($\bar{n}_e = 1.8 \times 10^{13} \text{ cm}^{-3}$) hydrogen plasma.
- Fig. 9. Detailed block diagram of the 60-90 GHz fast scanning heterodyne receiver.
- Fig. 10. Calibration arrangement of the heterodyne receiver using the Dicke switching technique.

Fig. 1

BIX1087

Fig. 2

Fig. 3a

Fig. 3b

Fig. 5

Fig. 6

CYCLOTRON
TEMPERATURE

Fig. 7

Fig. 8

Fig. 9

CALIBRATION ARRANGEMENT

81X0589

Fig. 10