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Elastic Scattering of Polarized Protons on Helium Three at 800 MeV

Alireza Azizi*

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ELASTIC SCATTERING OF POLARIZED PROTONS ON HELIUM THREE AT 800 MeV

by

Alireza Azizi

ABSTRACT

A set of spin dependent parameters and cross sections has been measured for $\vec{p}$ - ${}^3\text{He}$ elastic scattering over the range of q .7 - 4.2 fm^{-1} . The experiment was done at the Los Alamos Meson Physics Facility (LAMPF) using the High Resolution Spectrometer (HRS) with a polarized proton beam at .8 GeV. The focal plane polarimeter of the HRS was used to determine the spin direction of the scattered proton. Since ${}^3\text{He}$ is one of the simplest nuclei, $\vec{p}$ - ${}^3\text{He}$ scattering provides a very sensitive test of multiple scattering theories.

The theoretical analysis was done by using two different wave functions for ${}^3\text{He}$ as input to the multiple scattering theory. The theoretical calculations and experimental data together will give us useful information about nucleon-nucleon amplitudes and also help us to obtain a better understanding of the scattering process.

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Introduction

In order to understand both the strong interaction and nuclear structure, many different experimental and theoretical studies have been done in the last fifty years. By using different probes we are able to get different information about the strong interaction and nuclear structure.

Electrons as a probe are particularly suitable for accurate determination of charge and magnetic distributions. The interaction is described by quantum electrodynamics, and the strength of interaction is relatively weak. This means that they intrinsically see the charge distributions in nuclei, and not the matter distributions¹⁾.

Protons are used to extract information about spin dependence of nuclear forces, meson exchange currents, and testing the validity of different theoretical models for nuclear reactions^{2,3)}. High energy protons are believed to be useful for measuring the neutron densities of light nuclei⁴⁾.

The interest in the three-body nuclear systems is due to the fact that they represent the simplest nuclei that can be investigated by using the existing theoretical methods. Such methods are based either upon a Faddeev^{5,6)} or variational^{7,8)} calculations using

nucleon-nucleon forces as the basic input. The ability to predict the three-body properties is one of the strongest tests of the validity of the assumed nuclear force and the method of calculation.

Many different attempts had been made to measure and calculate ^3He charge and magnetic form factors. In Orsay Bernheim et. al.⁹⁾ measured the ^3He charge and magnetic form factors around $q \sim 4 \text{ fm}^{-1}$, then McCarthy et.al.¹⁰⁾ did a similar measurement for q up to 4 fm^{-1} . In 1978 Arnold et. al.¹¹⁾ measured the charge form factor for ^3He up to the second minimum ($q \sim 9 \text{ fm}^{-1}$), and the magnetic form factor up to $q \sim 6 \text{ fm}^{-1}$. Several attempts were made to explain the data using different methods of calculations: a Faddeev calculation in momentum space⁵⁾; a Faddeev calculation in configuration space⁶⁾; and a variational calculation⁷⁾. All three calculations used the Reid soft core NN interaction, and included meson-exchange currents as well as relativistic effects^{*12)}. The largest discrepancy between the data and calculations for the charge form factor is most prominent around the first minimum ($q \sim 3 \text{ fm}^{-1}$). The magnetic form factor has attracted less attention due to the less extensive and less precise experimental data¹⁰⁾. The discrepancy between the data and calculations is most prominent around the first minimum ($q \sim 4 \text{ fm}^{-1}$).

* Correction to the cross section of the order of $1/m^2$ (m is a nucleon mass).

In order to understand the origin of discrepancy between experimental and theoretical results, one can determine the density of point like nucleons. If we assume that charge form factor $F(q)$ is a product of the nucleon charge form factor $F_N(q)$, and a body form factor $F_p(q)$, then the resulting point density leads to a major surprise. The point density shows a pronounced central depression that is absent in all theoretical calculations. The extraction procedure for density of point like nucleons is not unique, because one has to include corrections due to meson exchange currents. In the method described above¹³⁾ a central depression in the point density is found. On the other hand another extraction procedures do not lead to a central depression¹⁴⁾. In the former analysis the central depression in the ^3He point density is due to the large value of $F(q)$ in the region of q between $3-5 \text{ fm}^{-1}$. One explanation is that there is a three-body force which is quite repulsive at short distances¹³⁾.

Many different types of three-body forces have been developed over the past years¹⁵⁾. Forces like two-pion-exchange three-nucleon ($2\pi, 3N$)¹⁶⁻¹⁸⁾, three-pion-exchange three-nucleon¹⁹⁾, have been studied extensively. In one of the more complete calculations, Hajduk et. al.¹⁸⁾ recently studied a ($2\pi, 3N$) force by including the Δ -isobar as an explicit component of the wave function (fig.1). The result of this calculation plus the effect of meson exchange currents for ^3He the charge and magnetic form factor are shown in figs.2,3.

Fig 1: Two-pion-exchange three-nucleon forces.

solve the discrepancy completely. In fig.4 we have the point charge density which shows a central depression that is absent in the Hajduk et. al. calculations. Several different new experiments on ^3H have been planned because, first one can do a comparable study between ^3He , ^3H , second one can study the effect of meson exchange current more extensively⁹⁾.

There have been many different measurements of the cross section and analyzing power for $\text{P}-^3\text{He}$ scattering from 100 MeV to 1700 MeV²⁰⁻²⁷⁾(fig.5). For analyzing these data different theoretical approaches have been taken. One approach is the Glauber multiple scattering theory with a simple spin independent form for the N-N

Fig 2: Charge form factor with the Hajduk's calculations¹⁸⁾.

Fig 3: Magnetic form factor with the Hajduk's calculations¹⁸⁾.

Fig 4: Experimental point density compared to the calculation¹⁵⁾.

scattering amplitudes²⁷⁾, another with a complete spin dependent form for the N-N amplitude²⁷⁾. Fain²⁵⁾ used the Glauber formalism with a Gaussian form for the amplitudes and the wave function. In another approach, a momentum space optical potential incorporating the full spin dependence of the N-N amplitudes was developed²⁸⁾. Frascaris²⁹⁾ used the Glauber formalism including spin-isospin dependence for the N-N amplitudes. The experiment reported in this thesis is the first measurement of the cross section and analyzing power at 800 MeV. It also represents the first measurement of all five Wolfenstein parameters for $\vec{p}$ - ^3He reaction. Since Wolfenstein parameters are sensitive to the spin dependent part of the interaction, they may be helpful in better understanding of spin

dependence of nuclear forces. We have begun a complete calculation of multiple scattering with full spin-isospin dependence. This calculation eventually should test multiple scattering theory.

The contents of this thesis are as follow: in the first chapter we discuss the basics of multiple scattering theory , the collision matrix for identical and non-identical spin 1/2-1/2 objects, and the calculation of amplitudes for $\vec{p}$ - ^3He elastic scattering. In the second chapter we discuss the experimental setup and the results of

Fig 5: Differential cross section with prediction based upon a simple Glauber formalism²⁷⁾.

the experiment. In the last chapter we present the results of calculations, and compare them with data.

CHAPTER 1

Theory

In this chapter we will review the basic concepts of multiple scattering theory, the scattering matrix, and structure of ^3He wave function. We also derive formulas for the relevant ^3He formfactors and relations between these formfactors and the experimental observables.

1.1 Multiple Scattering Theory

In multiple scattering theory we intend to derive an expression for the collision matrix for scattering of the projectile on the nucleus in terms of the known amplitudes for scattering on a nucleon. Here we deal with a rather simplified version of multiple scattering theory and we will not consider spin in our derivation. Later we will include spin in our calculation, but the basic formalism of the multiple scattering theory doesn't change. The two major modifications which we should consider are, inclusion of the spin dependence in the target wave function, and the projectile-target interaction. The theory is limited to be a non-relativistic potential theory¹⁾. In actual calculation relativity will be taken in to account.

For scattering of a nucleon from a system of A nucleons we can write the following Hamiltonian

$$H = H_0 + V = H_i + H_N + \sum_{i=1}^A v_i \quad (1-1)$$

where $H_i = k^2/2\mu$ is the kinetic energy of the projectile, v_i is the interaction potential between the projectile and the i^{th} nucleon, and

$$H_N = \sum_{i=1}^A \left(\frac{p_i^2}{2m} + \sum_{j>i}^A v_{ij} \right),$$

where $p_i^2/2m$ is the kinetic energy of the i^{th} nucleon in the target, and v_{ij} is the potential between i^{th} and j^{th} nucleons. The T matrix for the scattering process satisfies the Lippmann-Schwinger equation

$$T = V + VGT, \quad (1-2)$$

where the propagator G is

$$G = \frac{1}{E - H_0 + i\epsilon}. \quad (1-3)$$

If we write the Lippmann-Schwinger equation for the scattering process from an individual nucleon we have

$$t_i = v_i + v_i G_0 t_i, \quad (1-4)$$

where,

$$G_0 = \frac{1}{E - H_1 + i\epsilon}. \quad (1-5)$$

At a sufficiently high energy that the excitation energy of the target $\Delta\epsilon$ is much smaller than the kinetic energy of the projectile i.e. $\Delta\epsilon \ll k^2/2\mu$, the contribution of H_N can be ignored, and we can replace G with G_0 ¹⁾. This assumption is known as the "fixed scatterer" or "frozen nucleus" approximation. With this approximation the solution for equation (1-2) from the fixed nucleons at $r_1, r_2, \dots, r_A$ is in the following form

$$T = \sum_{i=1}^A t_i + \sum_{i=1}^A \sum_{j \neq i} t_i G_0 t_j + \sum_{i=1}^A \sum_{j \neq i} \sum_{k \neq j} t_i G_0 t_j G_0 t_k + \dots \quad (1-6)$$

where T is a function of $\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A$, initial and final momenta of the projectile, and $t_i = t_i(\vec{r}_i)$. In eq.(1-6) the first term shows the sum of all single scatterings, the second term represents the sum of all double scatterings, and the n th term represents the n -tuple scatterings. Then the scattering amplitude will be the average of the T over the momentum of projectile between successive scatterings and, the target wave function. It should be noted that we omitted the successive scatterings on the same nucleon from the series, because we have already summed over all these cases to produce the set of t_i 's.

Since in our case we are dealing with an $A=3$ nucleus, and a medium energy projectile, we can not use the Glauber model²⁾ which is based on high energy and small angle approximation, or the KMT method³⁾ which is based on large A approximation. In the KMT method we have an infinite series and we have to terminate it, also in this method there is a inconsistency in counting the number of multiple scattering terms. Some previous calculation showed that the Glauber Eikonal method is not a good approximation for this range of energy⁴⁾, it also does not give the possibility of treating spin relativistically. Therefore in our case we have to carry out the more complete calculation.

The objective of the multiple scattering theory is to construct the hadron-nucleus scattering amplitudes in terms of $N-N$ amplitudes, and the target wave function. Therefore in the following sections we will discuss the ingredients of multiple scattering theory.

1.2 Collision Matrix for Elastic Scattering of Two Nucleons

Since one of the ingredient in the multiple scattering theory $t_i(r_i)$, is related to $N-N$ amplitudes, in this section we will review the scattering matrix for nucleon-nucleon case.

The wave function which describes the elastic scattering of two particles with spin may be written as

$$\psi = \chi e^{i\vec{k} \cdot \vec{r}} + \tilde{M} \chi \frac{e^{ikr}}{r}, \quad (1-7)$$

where χ is product of spinors representing the incident and target particles, k is the momentum of the incident particle, $r = |\vec{r}_1 - \vec{r}_2|$, and $\tilde{M}$ is the scattering matrix which depends on spins of two particles and initial and final momentum of incident and scattered particles in center of mass frame. The matrix $\tilde{M}$ is related to operator t_i by the following relation

$$\langle \vec{P}' | t_i(\vec{r}_i) | \vec{P} \rangle = e^{i(\vec{P} - \vec{P}') \cdot \vec{r}_i} \tilde{M}(q), \quad (1-8)$$

where $\vec{P}$, and $\vec{P}'$ are the momentum of the projectile before and after scattering, and q is the momentum transfer between the projectile and target.

The general form of the scattering matrix follows from the requirements of invariance under time reversal, rotation, parity, and particle identity. The scattering matrix for nucleon-nucleon scattering has the following general form⁵⁾

$$\begin{aligned} \tilde{M} = & \alpha + i\gamma(\vec{\sigma}_p \cdot \vec{\sigma}_t) \cdot \hat{n} + \beta(\vec{\sigma}_p \cdot \hat{n})(\vec{\sigma}_t \cdot \hat{n}) + \delta(\vec{\sigma}_p \cdot \hat{q})(\vec{\sigma}_t \cdot \hat{q}) \\ & + \epsilon(\vec{\sigma}_p \cdot \hat{k})(\vec{\sigma}_t \cdot \hat{k}), \quad (1-9a) \end{aligned}$$

$\vec{\sigma}_p$, $\vec{\sigma}_t$, are the Pauli spin operators for the projectile and target and the amplitudes α , γ , β , δ , ϵ , are functions of the momentum transfer q . α is the spin-independent part of amplitude, γ is the spin-orbit amplitude and β , δ , ϵ , are the double spin flip amplitudes. These amplitudes can be further decomposed in terms of the isospins of the projectile τ_p , and target τ_t ,

$$\begin{aligned}\alpha &= \alpha_0 + \alpha_1 \tau_p \cdot \tau_t, \\ \beta &= \beta_0 + \beta_1 \tau_p \cdot \tau_t, \\ \gamma &= \gamma_0 + \gamma_1 \tau_p \cdot \tau_t, \\ \delta &= \delta_0 + \delta_1 \tau_p \cdot \tau_t, \\ \epsilon &= \epsilon_0 + \epsilon_1 \tau_p \cdot \tau_t,\end{aligned}\tag{1-9b}$$

The coordinate system is defined in the following way,

$$\begin{aligned}(\hat{y}) \quad \hat{n} &= \frac{\vec{k}_i \times \vec{k}_f}{|\vec{k}_i \times \vec{k}_f|}, \\ (\hat{x}) \quad \hat{q} &= \frac{\vec{k}_f - \vec{k}_i}{|\vec{k}_f - \vec{k}_i|}, \\ (\hat{z}) \quad \hat{k} &= \frac{\vec{k}_i + \vec{k}_f}{|\vec{k}_i + \vec{k}_f|}.\end{aligned}\tag{1-10}$$

Fig 1: The orthogonal coordinate system in center of mass frame.

It can be checked that matrix $\bar{M}$ in eq.(1-9a) is invariant under the time reversal, space reflection (parity), and rotational transformation of the reference frame.

$$\text{under parity: } \begin{cases} \vec{k}_i \rightarrow -\vec{k}_i \\ \vec{k}_f \rightarrow -\vec{k}_f \end{cases} \quad \begin{cases} \hat{x} \rightarrow -\hat{x} \\ \hat{y} \rightarrow \hat{y} \\ \hat{z} \rightarrow -\hat{z} \end{cases}$$

$$\vec{\sigma} \rightarrow \vec{\sigma}$$

(1-11)

$$\text{under time reversal: } \begin{cases} \vec{k}_i \rightarrow -\vec{k}_f \\ \vec{k}_f \rightarrow -\vec{k}_i \end{cases} \quad \begin{cases} \hat{x} \rightarrow \hat{x} \\ \hat{y} \rightarrow -\hat{y} \\ \hat{z} \rightarrow -\hat{z} \end{cases}$$

$$\vec{\sigma} \rightarrow -\vec{\sigma}$$

As an example we could not have a term proportional to $\vec{\sigma} \cdot \vec{q}$ in $\bar{M}$, because $\vec{\sigma} \cdot \vec{q}$ changes sign under the parity operation.

1.3 Collision Matrix for Scattering of Two Spin 1/2 Objects

By using similar arguments to those that lead us to eq.(1-9a), the scattering matrix for two non-identical spin 1/2 particles can be written in the following form

$$\bar{F} = \sum_{\xi=0,x,y,z} F_{\xi} \sigma_{p\xi}, \quad (1-12a)$$

where $\sigma_{p\xi}$ are the Pauli spin operators for the projectile and the F_{ξ} are defined in the following form

$$F_0 = F_0^0 + F_0^n (\vec{\sigma}_t \cdot \hat{n}),$$

$$F_x = F_q^q (\vec{\sigma}_t \cdot \hat{q}), \quad (1-12b)$$

$$F_y = F_n^0 + F_n^n (\vec{\sigma}_t \cdot \hat{n}),$$

$$F_z = F_k^k (\vec{\sigma}_t \cdot \hat{k}),$$

and $\vec{\sigma}_t$, is the Pauli spin operator for the target. If we put eqs.(1-12a,b) together we have

$$\begin{aligned} \vec{F} = F_0^0 + F_0^n (\vec{\sigma}_t \cdot \hat{n}) + F_n^0 (\vec{\sigma}_p \cdot \hat{n}) + F_n^n (\vec{\sigma}_t \cdot \hat{n})(\vec{\sigma}_p \cdot \hat{n}) \\ + F_q^q (\vec{\sigma}_t \cdot \hat{q})(\vec{\sigma}_p \cdot \hat{q}) + F_k^k (\vec{\sigma}_t \cdot \hat{k})(\vec{\sigma}_p \cdot \hat{k}), \end{aligned} \quad (1-12c)$$

We note that for two identical particles case we had $F_0^n = F_n^0$ (eq.1-9a). As we can see from eqs.(1-12) $\vec{F}$ is made up of six independent complex amplitudes. In order to completely determine the $\vec{p}$ - ^3He scattering amplitudes, eleven independent measurements are required. In our experiment we only measured five independent quantities, and therefore we will not be able to determine the $\vec{p}$ - ^3He scattering amplitudes completely, but the combinations of the amplitudes can be found (eqs.1-77).

1.4 Relation of Scattering Matrix to Experiment

Expressions for the cross section, analyzing power, and polarization for scattering of non-identical spin 1/2-1/2 objects can be written in terms of a density matrix formalism. For the case which the target is unpolarized, the density matrix for the system is the same as the projectile spin 1/2 density matrix and can be written in the following form

$$\rho^i = 1/2 [1 + \sum_{\xi=x,y,z} P_{\xi}^i \sigma_{p\xi}] \times 1, \quad (1-13)$$

where P_{ξ}^i is the polarization of the projectile, and $\sigma_{p\xi}$ is the spin component of the projectile along the ξ direction. For a pure spin state $\sum_{\xi} P_{\xi}^i{}^2 = 1$ and for a general ensemble $\sum_{\xi} P_{\xi}^i{}^2 \leq 1$ (6).

Using the initial density matrix, the density matrix after scattering can be written in the following form :

$$\rho^f = \bar{F} \rho^i \bar{F}^{\dagger}.$$

The expression for cross section is given by :

$$\frac{d\sigma}{d\Omega} = \text{Tr}(\rho^f), \quad (1-14)$$

where the trace is taken over the spin projections of the target and projectile. In terms of scattering matrix elements :

$$\frac{d\sigma}{d\Omega} = 1/2 \text{Tr}(\tilde{F}\tilde{F}^\dagger) + 1/2 \sum_{\xi}^i p_{\xi}^i \text{Tr}(\tilde{F} a_{\xi} \tilde{F}^\dagger),$$

or,

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol.}} [1 + P_y A_y], \quad (1-15)$$

where,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol.}} = \frac{1}{2} \text{Tr}(\tilde{F}\tilde{F}^\dagger),$$

and the analyzing power A_y (left-right asymmetry for a polarized beam) is,

$$A_y = \frac{\text{Tr}(\tilde{F} a_y \tilde{F}^\dagger)}{\text{Tr}(\tilde{F} \tilde{F}^\dagger)}. \quad (1-16)$$

By using time reversal invariance we can show that analyzing power and polarization (polarization of the scattered particle when the incoming particle is unpolarized) are equal, i.e.

$$A_y = \frac{\text{Tr}(\tilde{F} \tilde{F}^\dagger a_y)}{\text{Tr}(\tilde{F} \tilde{F}^\dagger)} = P. \quad (1-17)$$

In terms of the collision matrix components we have,

$$\frac{d\sigma}{d\Omega})_{\text{unpol.}} = \frac{1}{2} \{ |F_0^0|^2 + |F_0^n|^2 + |F_Q^0|^2 + |F_n^0|^2 + |F_n^n|^2 + |F_k^k|^2 \}, \quad (1-18)$$

and,

$$A_y = 2 \operatorname{Real}(F_0^n F_0^{*0} + F_n^n F_n^{*k}). \quad (1-19)$$

We can also obtain expressions for the spin pojection P_μ^f of the scattered beam,

$$P_\mu^f = \frac{\operatorname{Tr}(\rho^f \sigma_\mu)}{\operatorname{Tr}(\rho^f)},$$

$$= \frac{1/2 \operatorname{Tr}(\tilde{F}\tilde{F}^\dagger \sigma_\mu) + 1/2 \sum_{\xi} p_{\xi}^i \operatorname{Tr}(\tilde{F} \sigma_{\xi} \tilde{F}^\dagger \sigma_\mu)}{\operatorname{Tr}(\rho^f)},$$

$$P_\mu^f = \frac{1/2 \operatorname{Tr}(\tilde{F}\tilde{F}^\dagger \sigma_\mu)}{\operatorname{Tr}(\rho^f)} + \sum_{\xi} p_{\xi}^i D_{\xi\mu}, \quad (1-20)$$

where Wolfenstein parameters $D_{\xi\mu}^{7,8)}$ are defined in the following form

$$D_{\xi\mu} = \frac{\operatorname{Tr}(\tilde{F} \sigma_{\xi} \tilde{F}^\dagger \sigma_\mu)}{\operatorname{Tr}(\rho^f)}, \quad (1-21)$$

or in terms of the collision matrix components, we have :

$$I_0 D_{SS} = [|F_0^0|^2 - |F_0^{\Pi}|^2 + |F_{\Pi}^0|^2 - |F_{\Pi}^{\Pi}|^2] \cos(\chi - \chi') \\ + [|F_Q^0|^2 - |F_K^{\Pi}|^2] \cos(\chi + \chi') + 2 \text{Im}(F_0^{\Pi} F_0^{*0} + F_{\Pi}^{\Pi} F_{\Pi}^{*0}) \sin(\chi - \chi') \quad (1-22)$$

$$I_0 D_{S1} = [|F_0^0|^2 - |F_0^{\Pi}|^2 + |F_{\Pi}^0|^2 - |F_{\Pi}^{\Pi}|^2] \sin(\chi - \chi') \\ - [|F_Q^0|^2 - |F_K^{\Pi}|^2] \sin(\chi + \chi') + 2 \text{Im}(F_0^{\Pi} F_0^{*0} + F_{\Pi}^{\Pi} F_{\Pi}^{*0}) \sin(\chi - \chi') \quad (1-23)$$

$$I_0 D_{11} = [|F_0^0|^2 - |F_0^{\Pi}|^2 + |F_{\Pi}^0|^2 - |F_{\Pi}^{\Pi}|^2] \cos(\chi - \chi') \\ - [|F_Q^0|^2 - |F_K^{\Pi}|^2] \cos(\chi + \chi') - 2 \text{Im}(F_0^{\Pi} F_0^{*0} + F_{\Pi}^{\Pi} F_{\Pi}^{*0}) \sin(\chi - \chi') \quad (1-24)$$

$$I_0 D_{1S} = [-|F_0^0|^2 + |F_0^{\Pi}|^2 - |F_{\Pi}^0|^2 + |F_{\Pi}^{\Pi}|^2] \sin(\chi - \chi') \\ - [|F_Q^0|^2 - |F_K^{\Pi}|^2] \sin(\chi + \chi') - 2 \text{Im}(F_0^{\Pi} F_0^{*0} + F_{\Pi}^{\Pi} F_{\Pi}^{*0}) \sin(\chi - \chi') \quad (1-25)$$

$$I_0 D_{nn} = [|F_0^0|^2 + |F_0^{\Pi}|^2 - |F_Q^0|^2 + |F_{\Pi}^0|^2 + |F_{\Pi}^{\Pi}|^2 - |F_K^{\Pi}|^2] \quad (1-26)$$

and,

$$I_0 = [|F_0^0|^2 + |F_0^{\Pi}|^2 + |F_Q^0|^2 + |F_{\Pi}^0|^2 + |F_{\Pi}^{\Pi}|^2 + |F_K^{\Pi}|^2]$$

Where $\chi = \theta_B/2 + \alpha$, $\chi' = \bar{\theta}_{lab} - \theta_B/2 - \alpha$, and θ_B is the scattering angle in the Breit frame, $\bar{\theta}_{lab}$ is the scattering angle in antilab frame, and α is the angle between the Breit and Lab frames from antilab frame (Appendix C).

We can define D_{ξ} in terms of the experimentally measured parameters⁹⁾,

$$D_0 = 1/4 [1 + D_{nn} + (D_{ss} + D_{11}) \cos(\chi - \chi') + (D_{S1} - D_{1S}) \sin(\chi - \chi')], \quad (1-27)$$

$$D_x = 1/4[1 - D_{nn} + (D_{ss} - D_{ll})\cos(\chi + \chi') - (D_{sl} + D_{ls})\sin(\chi + \chi')], \quad (1-28)$$

$$D_y = 1/4[1 + D_{nn} - (D_{ss} + D_{ll})\cos(\chi - \chi') - (D_{sl} - D_{ls})\sin(\chi - \chi')], \quad (1-29)$$

$$D_z = 1/4[1 - D_{nn} - (D_{ss} - D_{ll})\cos(\chi + \chi') + (D_{sl} + D_{ls})\sin(\chi + \chi')], \quad (1-30)$$

The quantities D_ξ can be written in terms of components of scattering matrix $\tilde{F}$,

$$D_\xi = \frac{\text{Tr}(\tilde{F}_\xi \tilde{F}_\xi^\dagger)}{\text{Tr}(\tilde{F} \tilde{F}^\dagger)} \quad \xi = 0, x, y, z,$$

where traces are taken with respect to the target spin projection. In the expanded form we have,

$$I_0 D_0 = [|F_0^0|^2 + |F_0^N|^2], \quad (1-31)$$

$$I_0 D_x = |F_0^Q|^2, \quad (1-32)$$

$$I_0 D_y = [|F_N^0|^2 + |F_N^N|^2], \quad (1-33)$$

$$I_0 D_z = |F_K^K|^2, \quad (1-34)$$

The advantage of these definitions are the selective dependence of the D_ξ on components of scattering matrix F_ξ , which will be helpful in theoretical analysis.

1.5 Construction of the ^3He Wave Function

In this section we discuss the construction of ^3He wave function¹⁰⁾.

We start with

$$\begin{array}{ccc} \vec{r}_1 & \vec{r}_2 & \vec{r}_3 \\ \vec{\sigma}_1 & \vec{\sigma}_2 & \vec{\sigma}_3 \\ \vec{\tau}_1 & \vec{\tau}_2 & \vec{\tau}_3 \end{array}$$

and we intend to construct a complete antisymmetric wave function for ^3He with $J=1/2$ by combining these variables. We start by writing down a function for the $^2S_{1/2}$ state in spin space. Multiplication of this function by any scalar operator will provide another function with $J=1/2$. The desired values of the orbital angular momentum are obtained by choosing an operator with the transformation properties of an S, P, or D function under space rotations.

There are several ways of constructing an antisymmetric wave function. If we assume that the radial part of wave function is symmetric, then we only need to know all the possible combinations of spin and isospin part of wave function. We shall use the method of Young diagrams for construction of spin wave functions. We will adopt the following convention¹¹⁾:

With identical boxes we represent identical particles, and with nonidentical objects in the boxes we represent nonidentical states. By using this convention we can construct all the possible combinations of three spin 1/2 particles. The symmetric state in terms of Young diagram is

$$\psi_S = \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline \end{array} = S_{123} abc,$$

where,

$$S_{123} = 1 + P_{12} + P_{13} + P_{23} + P_{13}P_{12} + P_{12}P_{13}^*,$$

and,

$$\psi_S = abc + bac + cba + acb + cab + bca, \quad (1-35)$$

where a,b,c are three different states, and P_{ij} is the transposition operator[†]. Our convention in eq.(1-35) and thereafter is that we write first the state of particle 1, second the state of particle 2, and last the state of particle 3.

* S_{123} can be found by writing $S_{123} = S_{12}S_{23}S_{12}$ where $S_{ij} = 1 + P_{ij}$, and P_{ij} are the transposition operators.

[†] $P_{ij} |1,2\rangle = |2,1\rangle.$

For antisymmetric state of wave function we have

$$\psi_a = \begin{array}{|c|} \hline 1 \\ \hline 2 \\ \hline 3 \\ \hline \end{array} = A_{123} abc,$$

where,

$$A_{123} = 1 - P_{12} - P_{13} - P_{23} + P_{13}P_{12} + P_{12}P_{13}^{**},$$

and,

$$\psi_a = abc - bac - cba - acb + cab + bca, \quad (1-36)$$

For the mixed symmetry states of the wave function we get,

$$\psi_1 = \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & \\ \hline \end{array} = A_{13}S_{12} abc,$$

or,

$$\psi_1 = abc + bac - cba - cab, \quad (1-37)$$

and,

$$\psi_2 = \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & \\ \hline \end{array} = A_{23}S_{12} abc,$$

or,

$$\psi_2 = abc + bac - acb - bca, \quad (1-38)$$

and,

** $A_{123} = A_{12}A_{23}A_{13}$ where $A_{ij} = 1 - P_{ij}$.

$$\psi_3 = \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & \\ \hline \end{array} = A_{12}S_{13} \text{ abc,}$$

or,

$$\psi_3 = abc + cba - bac - bca, \quad (1-39)$$

and,

$$\psi_4 = \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & \\ \hline \end{array} = A_{23}S_{13} \text{ abc,}$$

or,

$$\psi_4 = abc + cba - acb - cab. \quad (1-40)$$

We could obtain two additional states by using the symmetrizer S_{23} , but these would not be independent of the states (1-37)-(1-38). Alternatively, the states $\psi = S_{ij}A_{ik} \text{ abc}$ are not independent of the four states (1-39)-(1-40). Note that a state formed by first symmetrizing with respect to i, j , and then antisymmetrizing with respect to j, k , loses its symmetry under the interchange of i and j . In other words the last performed symmetrizer or antisymmetrizer is controlling the symmetry of the state.

The functions $\psi_1, \psi_2, \psi_3, \psi_4$ are not orthogonal to one another. But we can construct linear combinations of them which are orthogonal to one another. For spin $1/2$ particle we have only two possible states, $a = |1/2, +1/2\rangle$, $b = |1/2, -1/2\rangle$, therefore we put $c=a$.

$$\chi_1 = aab + aba - 2baa,$$

$$\chi_2 = aab - aba.$$

The new forms of χ_1 , χ_2 , normalized to unity are

$$\chi_1 = (6)^{-1/2} [(+-) + (-+) - 2(++)],$$

$$\chi_2 = (2)^{-1/2} [(+-) - (-+)]. \quad (1-41)$$

The spin wave functions χ_1 , χ_2 , are related by $\chi_1 = (12)^{-1/2} (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \chi_2$, and under interchange of particles, they transform in the following way :

$$\begin{cases} P_{23} \chi_1 = \chi_1, \\ P_{23} \chi_2 = -\chi_2, \end{cases} \quad (1-42a)$$

$$\begin{cases} P_{12} \chi_1 = \frac{1}{2} (\sqrt{3} \chi_2 - \chi_1), \\ P_{12} \chi_2 = \frac{1}{2} (\chi_2 + \sqrt{3} \chi_1), \end{cases} \quad (1-42b)$$

$$\begin{cases} P_{13} \chi_1 = -\frac{1}{2} (\sqrt{3} \chi_2 + \chi_1), \\ P_{13} \chi_2 = \frac{1}{2} (\chi_2 - \sqrt{3} \chi_1), \end{cases} \quad (1-42c)$$

The isospin functions n_1, n_2 have the same form as χ_1, χ_2 except now +, - means proton and neutron.

In literature different admixtures of $S(L=0)$, $P(L=1)$, and $D(L=2)$ states for the ${}^3\text{He}$ wave function are suggested¹²⁻¹⁴). Most of them give about 90% S, 9% D and 1% for P state. The P part of the wave function is small because the P part can only contribute through coupling with the D part of the wave function^{10,12}). In our calculation we are going to ignore the P part of the wave function and use only the S, and D parts. Therefore, we discuss only the structure of these two parts of the wave function.

The ${}^3\text{He}$ wave function can be written in the following form

$$\Psi = \alpha_S \psi_S + \alpha_D \psi_D, \quad (1-43)$$

where ψ_S, ψ_D , represent the S and D parts of wave function, and α_S, α_D , are the weight factors for S and D components of the wave function. The D part of the wave function can be decomposed in the following form,

$$\psi_D = N_6 \psi_6 + N_7 \psi_7 + N_8 \psi_8, \quad (1-44)$$

where ψ_6, ψ_7, ψ_8 , are three components of the D part of the wave function which we define later, and N_6, N_7, N_8 , are the normalization factors.

For the S wave, if the spatial part f_1 is symmetric, then the fully antisymmetric combination of spin and isospin is required,

$$\psi_S = \psi_1 = (\chi_2 \eta_1 - \chi_1 \eta_2) f_1. \quad (1-45)$$

For construction of D wave we need to use $\vec{R}_1, \vec{R}_2$ (fig.2) and three spin operators $\vec{\sigma}_1, \vec{\sigma}_2, \vec{\sigma}_3$. The variables $\vec{R}_1, \vec{R}_2$ are the Jacobian coordinates and defined in the following form,

$$\begin{aligned} \vec{R}_1 &= \sqrt{3} \vec{r}_1, \\ \vec{R}_2 &= \vec{r}_2 - \vec{r}_3, \end{aligned}$$

where $\vec{r}_1, \vec{r}_2, \vec{r}_3$ are coordinates of nucleons relative to center of mass. Later for simplicity in our calculation we will rename $\vec{R}_1, \vec{R}_2$, as $\vec{x}, \vec{y}$ respectively. We should note that the new coordinates $\vec{R}_1, \vec{R}_2$, are following the same transformation as eqs.(1-42) under interchange of particles. By using the new variables $\vec{R}_1, \vec{R}_2$, the space-spin functions can be written in the following combinations :

$$\begin{aligned} \phi_1 &= [(\vec{\sigma}_1 \cdot \vec{R}_2)(\vec{\sigma}_2 \cdot \vec{R}_2) - (\vec{\sigma}_1 \cdot \vec{R}_1)(\vec{\sigma}_2 \cdot \vec{R}_1)] \chi_2, \\ \phi_2 &= [(\vec{\sigma}_1 \cdot \vec{R}_2)(\vec{\sigma}_2 \cdot \vec{R}_1) + (\vec{\sigma}_1 \cdot \vec{R}_1)(\vec{\sigma}_2 \cdot \vec{R}_2)] \chi_2, \\ \phi_a &= [(\vec{\sigma}_1 \cdot \vec{R}_2)(\vec{\sigma}_2 \cdot \vec{R}_1) - (\vec{\sigma}_1 \cdot \vec{R}_1)(\vec{\sigma}_2 \cdot \vec{R}_2)] \chi_2, \\ \phi_S &= [(\vec{\sigma}_1 \cdot \vec{R}_2)(\vec{\sigma}_2 \cdot \vec{R}_2) + (\vec{\sigma}_1 \cdot \vec{R}_1)(\vec{\sigma}_2 \cdot \vec{R}_1)] \chi_2, \end{aligned} \quad (1-46)$$

where ϕ_1, ϕ_2 , have mixed symmetry, ϕ_S, ϕ_a , are fully symmetric and antisymmetric respectively, and $\vec{\sigma}_{23} = \vec{\sigma}_2 - \vec{\sigma}_3$. We note that since $\chi_1 = (12)^{-1/2}(\vec{\sigma}_1 \cdot \vec{\sigma}_{23})\chi_2$, the space-spin function is expressed totally in terms of χ_2 , and since $(\vec{\sigma}_2 + \vec{\sigma}_3)\chi_2 = 0$, the spin of the target is $\vec{J} = \vec{\sigma}_1$. We have similar relation for isospin, i.e. $\tau_t = \tau_1$.

The antisymmetric part ϕ_a , contributes only in the P state, the other ϕ_S, ϕ_1, ϕ_2 , contribute to the S and D states. By combining these ϕ 's and subtracting their S parts we can construct the pure D state.

$$D_1 = [(\vec{\sigma}_1 \cdot \vec{R}_2)(\vec{\sigma}_2 \cdot \vec{R}_2) - (\vec{\sigma}_1 \cdot \vec{R}_1)(\vec{\sigma}_2 \cdot \vec{R}_1)] - 1/3(\vec{\sigma}_1 \cdot \vec{\sigma}_{22})(R_2^2 - R_1^2)]\chi_2,$$

$$D_2 = [(\vec{\sigma}_1 \cdot \vec{R}_2)(\vec{\sigma}_2 \cdot \vec{R}_1) + (\vec{\sigma}_1 \cdot \vec{R}_1)(\vec{\sigma}_2 \cdot \vec{R}_2)] - 2/3(\vec{\sigma}_1 \cdot \vec{\sigma}_{22})(R_1^2 + R_2^2)]\chi_2,$$

$$D_3 = [(\vec{\sigma}_1 \cdot \vec{R}_2)(\vec{\sigma}_2 \cdot \vec{R}_2) + (\vec{\sigma}_1 \cdot \vec{R}_1)(\vec{\sigma}_2 \cdot \vec{R}_1)] - 1/3(\vec{\sigma}_1 \cdot \vec{\sigma}_{22})(R_2^2 + R_1^2)]\chi_2,$$

(1-47)

Fig 2: Jacobi coordinates for the ^3He .

where D_3 is fully symmetric, and D_1, D_2 , have mixed symmetry, i.e. under interchange of particles, they are following the same transformation as eqs. (1-42). The D_1, D_2, D_3 , are the proper operators for generation of the D state from the $^2S_{1/2}$ state.

By using D_1, D_2, D_3 , we can construct the fully antisymmetric wave function for the D state.

$$\psi_6 = D_3(S_2 n_1 - S_1 n_2) f_6(S_3), \quad (1-48a)$$

$$\psi_7 = (D_2 n_1 - D_1 n_2) f_7(S_3), \quad (1-48b)$$

$$\psi_8 = (D_2 S_1 + D_1 S_2) n_1 - (D_2 S_2 + D_1 S_1) n_2 f_8(S_3), \quad (1-48c)$$

where $S_1 = R_2^2 - R_1^2$, $S_2 = 2 \vec{R}_1 \cdot \vec{R}_2$, $S_3 = R_2^2 + R_1^2$. It should be emphasised that these wave functions are not unique and we can use different combinations of D's, n 's, and f 's to make fully antisymmetric wave functions.

1.6 Calculation of Single Scattering Term

Since ^3He is composed of three nucleons, we can have single, double and triple scatterings, which are shown as the first three terms in eq.(1-6). By using fixed scatterer approximation, the single scattering amplitude (first term) in eq.(1-6), has the following form,

$$F = \langle \psi(1,2,3) | \tau_1(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{k}_i, \vec{k}_f) | \psi(1,2,3) \rangle,$$

$$= \langle \psi(1,2,3) | \sum_{j=1}^3 t_j | \psi(1,2,3) \rangle,$$

or,

$$F = \langle \psi(1,2,3) | \sum_{j=1}^3 e^{i\vec{q} \cdot \vec{r}_j} \bar{M}_j(q) | \psi(1,2,3) \rangle, \quad (1-49)$$

where $\vec{r}_j$, $j=1,2,3$ are the coordinates of three nuclei, $\psi(1,2,3)$ is the ^3He wave function, and $\bar{M}_j(q)$ is the collision matrix for nucleon-nucleon scattering which defined in eqs.(1-9) We can write the relation (1-49) in the following form

$$F = \langle \psi(1,2,3) | N_1(q) | \psi(1,2,3) \rangle + \langle \psi(1,2,3) | N_2(q) | \psi(1,2,3) \rangle$$

$$+ \langle \psi(1,2,3) | N_3(q) | \psi(1,2,3) \rangle, \quad (1-50)$$

where $N_j(q) = e^{i\vec{q} \cdot \vec{r}_j} \bar{M}_j(q)$. By exchanging 1,2 in the first term of (1-50) and using the symmetry property of $\psi(1,2,3)$ we get

$$\langle \psi(1,2,3) | N_1(q) | \psi(1,2,3) \rangle = \langle \psi(2,1,3) | N_2(q) | \psi(2,1,3) \rangle$$

$$= \langle \psi(1,2,3) | N_2(q) | \psi(1,2,3) \rangle. \quad (1-51)$$

Similarly we can show that the first term in (1-50) is equal to the third term, and we find

$$F = 3 \langle \psi(1,2,3) | \bar{M}_1 e^{i\vec{q} \cdot \vec{r}_1} | \psi(1,2,3) \rangle. \quad (1-52)$$

By combining eqs.(1-45),(1-48) we can write the ^3He wave function in the following form,

$$\psi(1,2,3) = \psi = u_2 n_1 - u_1 n_2, \quad (1-53)$$

where,

$$\begin{aligned} u_2 &= f_1 \chi_2 + D_3 S_2 f_6 + D_2 S_1 f_7 + D_2 S_1 f_8 + D_1 S_2 f_9, \\ u_1 &= -f_1 \chi_1 - D_3 S_1 f_6 - D_1 f_7 - D_2 S_2 f_8 + D_1 S_1 f_9. \end{aligned} \quad (1-54)$$

If we decompose M_1 in eq.(1-52) in terms of projectile isospin τ_p , and target nucleon isospin τ_1 , we have

$$M_1 = m_0 + m_1 (\tau_p \cdot \tau_1), \quad (1-55a)$$

where,

$$\begin{aligned} m_j &= \alpha_j + i\gamma_j (\vec{\sigma}_p + \vec{\sigma}_1) \cdot \hat{n} + \beta_j (\vec{\sigma}_p \cdot \hat{n}) (\vec{\sigma}_1 \cdot \hat{n}) + \delta_j (\vec{\sigma}_p \cdot \hat{q}) (\vec{\sigma}_1 \cdot \hat{q}) \\ &\quad + \epsilon_j (\vec{\sigma}_p \cdot \hat{k}) (\vec{\sigma}_1 \cdot \hat{k}) \quad j=0,1 \end{aligned} \quad (1-55b)$$

and $\vec{\sigma}_p, \vec{\sigma}_1$, are spins of projectile and target nucleon. Equation (1-52) can be written in the following form

$$F = \langle u_2 n_1 - u_1 n_2 | [m_0 + m_1 (\tau_p \cdot \tau_1)] e^{i\vec{q} \cdot \vec{r}_1} | u_2 n_1 - u_1 n_2 \rangle.$$

By using appendix B for isospin algebra we get,

$$F = \langle u_2 | m_0 e^{i\vec{q} \cdot \vec{r}_1} | u_2 \rangle + \langle u_1 | m_0 e^{i\vec{q} \cdot \vec{r}_1} | u_1 \rangle - \frac{1}{3} \langle u_2 | m_1 e^{i\vec{q} \cdot \vec{r}_1} | u_2 \rangle \\ + \langle u_1 | m_1 e^{i\vec{q} \cdot \vec{r}_1} | u_1 \rangle. \quad (1-56)$$

Let us consider only the first term in eq.(1-56) and substitute for m_0 from eq.(1-55b), then we get

$$\langle u_2 | m_0 e^{i\vec{q} \cdot \vec{r}_1} | u_2 \rangle = \langle u_2 | [\alpha_0 + i\gamma_0(\vec{\sigma}_p \cdot \vec{\sigma}_1) \cdot \hat{n} + \beta_0(\vec{\sigma}_p \cdot \hat{n})(\vec{\sigma}_1 \cdot \hat{n}) \\ + \delta_0(\vec{\sigma}_p \cdot \hat{q})(\vec{\sigma}_1 \cdot \hat{q}) + \epsilon_0(\vec{\sigma}_p \cdot \hat{k})(\vec{\sigma}_1 \cdot \hat{k})] e^{i\vec{q} \cdot \vec{r}_1} | u_2 \rangle,$$

or,

$$= [\alpha_0 + i\gamma_0(\vec{\sigma}_p \cdot \hat{n})] \langle u_2 | e^{i\vec{q} \cdot \vec{r}_1} | u_2 \rangle + i\gamma_0 \langle u_2 | (\vec{\sigma}_1 \cdot \hat{n}) e^{i\vec{q} \cdot \vec{r}_1} | u_2 \rangle \\ + \beta_0(\vec{\sigma}_p \cdot \hat{n}) \langle u_2 | (\vec{\sigma}_1 \cdot \hat{n}) e^{i\vec{q} \cdot \vec{r}_1} | u_2 \rangle + \delta_0(\vec{\sigma}_p \cdot \hat{q}) \langle u_2 | (\vec{\sigma}_1 \cdot \hat{q}) e^{i\vec{q} \cdot \vec{r}_1} | u_2 \rangle \\ + \epsilon_0(\vec{\sigma}_p \cdot \hat{k}) \langle u_2 | (\vec{\sigma}_1 \cdot \hat{k}) e^{i\vec{q} \cdot \vec{r}_1} | u_2 \rangle.$$

Therefore we need to calculate two terms $\langle u_2 | e^{i\vec{q} \cdot \vec{r}_1} | u_2 \rangle$, and $\langle u_2 | (\vec{\sigma}_t \cdot \hat{A}) e^{i\vec{q} \cdot \vec{r}_1} | u_2 \rangle$, where $\hat{A}$ can be in n, k , or q direction. By considering all four terms in eq.(1-56) and using similar arguments, we can show that for calculating F we need four following quantities,

$$G_1 = \langle u_1 | e^{i\vec{q} \cdot \vec{r}_1} | u_1 \rangle, \quad G_2 = \langle u_2 | e^{i\vec{q} \cdot \vec{r}_1} | u_2 \rangle, \\ G_3 = \langle u_1 | e^{i\vec{q} \cdot \vec{r}_1} (\vec{J} \cdot \hat{A}) | u_1 \rangle, \quad G_4 = \langle u_2 | e^{i\vec{q} \cdot \vec{r}_1} (\vec{J} \cdot \hat{A}) | u_2 \rangle, \quad (1-57)$$

where $\vec{J} = \vec{\sigma}_1$ is the spin of the target, and $\hat{A}$ is a unit vector which can be in n, k, or q direction.

As an example let us find an expression for F_0^0 , and F_0^n (eqs.1-18). Since now we are only interested in F_0^0 , F_0^n we substitute only the first two terms of m_j from eq.(1-55b) in (1-56)

$$\begin{aligned} F = & \langle u_2 | [\alpha^0 + \gamma_1^0(\vec{\sigma}_p \cdot \hat{n})] e^{i\vec{q} \cdot \vec{r}_1} | u_2 \rangle + \langle u_1 | [\alpha^0 + \gamma_1^0(\vec{\sigma}_p \cdot \hat{n})] e^{i\vec{q} \cdot \vec{r}_1} | u_1 \rangle \\ & - \frac{1}{3} \langle u_2 | [\alpha^1 + \gamma_1^1(\vec{\sigma}_p \cdot \hat{n})] e^{i\vec{q} \cdot \vec{r}_1} | u_2 \rangle \\ & + \langle u_1 | [\alpha^1 + \gamma_1^1(\vec{\sigma}_p \cdot \hat{n})] e^{i\vec{q} \cdot \vec{r}_1} | u_1 \rangle, \end{aligned}$$

or,

$$\begin{aligned} F = & \alpha^0 G_2 + \gamma_1^0(\vec{\sigma}_p \cdot \hat{n}) G_2 + \alpha^0 G_1 + \gamma_1^0(\vec{\sigma}_p \cdot \hat{n}) G_1 - \frac{1}{3} \alpha^1 G_2 \\ & - \frac{1}{3} \gamma_1^1(\vec{\sigma}_p \cdot \hat{n}) G_2 + \alpha^1 G_1 + \gamma_1^1(\vec{\sigma}_p \cdot \hat{n}) G_1, \end{aligned}$$

comparing with eq.(1-12c) we get

$$F_0^0 = (\alpha^0 - \frac{1}{3} \alpha^1) G_2 + (\alpha^0 + \alpha^1) G_1,$$

$$F_0^n = (\gamma_1^0 - \frac{1}{3} \gamma_1^1) G_2 + (\gamma_1^0 + \gamma_1^1) G_1.$$

Similarly we can calculate the other amplitudes. The complete expression for all six amplitudes are given in eq.(1-75).

1.6.1 Calculation of $\langle u_2 | e^{i\vec{q} \cdot \vec{r}_1} | u_2 \rangle$

For simplicity we substitute $\vec{R}_1, \vec{R}_2$, by $\vec{x}, \vec{y}$, then $\vec{r}_1 = (3)^{1/2} \vec{x}$, and G_2 is

$$G_2(q) = \int u_2^\dagger e^{i(3)^{1/2} \vec{q} \cdot \vec{x}} u_2 d^3r_1 d^3r_2 d^3r_3. \quad (1-58)$$

If we take $\vec{m} = (3)^{1/2} \vec{q}$ and write $d^3r_1 d^3r_2 d^3r_3 = (3)^{3/2} x^2 dx d\Omega_x y^2 dy d\Omega_y$ then

$$\begin{aligned} G_2 &= \frac{(3)^{3/2}}{8} \int x^2 dx d\Omega_x y^2 dy d\Omega_y e^{i\vec{m} \cdot \vec{x}} u_2^\dagger u_2, \\ &= \frac{(3)^{3/2}}{8} 16\pi^2 \int x^2 y^2 dx dy \frac{1}{4\pi} \int e^{i\vec{m} \cdot \vec{x}} d\Omega_x \frac{1}{4\pi} \int u_2^\dagger u_2 d\Omega_y, \end{aligned}$$

and,

$$\begin{aligned} u_2^\dagger u_2 &= f_1^2 + 1/2 f_6^2 S_2^2 C_{33} + 1/2 C_{22} (f_7 + S_1 f_8)^2 + 1/2 f_8^2 S_2^2 C_{11} \\ &\quad + C_{23} S_2 (f_7 + S_1 f_8) f_6 + C_{13} S_2^2 f_6 f_8 + C_{12} S_2 (f_7 + S_1 f_8) f_8, \end{aligned} \quad (1-59)$$

which $C_{ik} = D_i^\dagger D_k + D_k^\dagger D_i$. By using Appendix B for the average of C_{ik} over the solid angle Ω_y we have

$$\begin{aligned}
I_y = \frac{1}{4\pi} \int u_2^\dagger u_2 d\Omega_y = & f_1^2 + \frac{28}{45} (S_3^2 - S_1^2) S_3^2 f_6^2 + \frac{4}{15} S_1^2 (S_3^2 - S_1^2) f_6^2 \\
& + \frac{20}{9} (S_3^2 - S_1^2) (f_7^2 + S_1^2 f_8^2 + 2S_1 f_7 f_8) + \frac{28}{45} S_1^2 (S_3^2 - S_1^2) f_8^2 \\
& + \frac{4}{15} S_3^2 (S_3^2 - S_1^2) f_6^2 + \frac{16}{9} S_3 (S_3^2 - S_1^2) f_6 f_7 \\
& + \frac{16}{9} S_1 S_3 (S_3^2 - S_1^2) f_6 f_8 + \frac{16}{9} S_1 S_3 (S_3^2 - S_1^2) f_7 f_8 \\
& + \frac{16}{9} S_1 (S_3^2 - S_1^2) f_7 f_8 + \frac{16}{9} S_1^2 (S_3^2 - S_1^2) f_8^2.
\end{aligned}$$

Next by choosing the following substitution

$$\begin{cases}
h_1 = f_1, \\
h_6 = R^2 f_6, \\
h_7 = R^2 f_7, \\
h_8 = R^2 f_8,
\end{cases}
\quad
\begin{cases}
x = R \cos \theta, \\
y = R \sin \theta, \\
C = \cos(2\theta),
\end{cases}$$

Fig 3: The relation between R, θ , and x, y.

we have $S_3 = R^2$, $S_1 = -R^2C$ and I_y can be written in the following form

$$\begin{aligned}
 I_y = \frac{1}{4\pi} \int d\Omega_y u_2^\dagger u_2 = & h_1^2 + \frac{28}{45} h_6^2 - \frac{28}{45} h_6^2 C^2 + \frac{4}{15} h_6^2 C^2 - \frac{4}{15} h_6^2 C^4 \\
 & + \frac{20}{9} h_7^2 + \frac{20}{9} h_7^2 C^2 + \frac{20}{9} h_8^2 C^2 - \frac{20}{9} h_8^2 C^4 \\
 & - \frac{40}{7} h_7 h_8 C + \frac{40}{9} h_7 h_8 C^3 + \frac{28}{45} h_8^2 C^2 - \frac{28}{45} h_8^2 C^4 \\
 & + \frac{4}{15} h_8^2 - \frac{4}{15} h_8^2 C^2 + \frac{16}{9} h_6 h_7 - \frac{16}{9} h_6 h_7 C^2 \\
 & - \frac{32}{9} h_6 h_8 C + \frac{32}{9} h_6 h_8 C^3 - \frac{16}{9} h_6 h_8 C \\
 & + \frac{16}{9} h_7 h_8 C^3 + \frac{16}{9} h_8^2 C^2 - \frac{16}{9} h_8^2 C^4,
 \end{aligned}$$

or,

$$I_y = b_0 + b_1 C + b_2 C^2 + b_3 C^3 + b_4 C^4, \quad (1-60a)$$

where,

$$b_0 = h_1^2 + \frac{28}{45} h_6^2 + \frac{20}{9} h_7^2 + \frac{4}{15} h_8^2 + \frac{16}{9} h_6 h_7,$$

$$b_1 = -\frac{32}{9} h_6 h_8 - \frac{56}{9} h_7 h_8,$$

$$b_2 = -\frac{16}{45} h_6^2 + \frac{20}{9} h_7^2 + \frac{196}{45} h_8^2 - \frac{16}{9} h_6 h_7, \quad (1-60b)$$

$$b_3 = \frac{32}{9} h_6 h_8 + \frac{56}{9} h_7 h_8,$$

$$b_4 = -\frac{4}{15} h_6^2 - \frac{208}{45} h_8^2,$$

or in terms of $\sin\theta$

$$I_y = B_0 + B_1 \sin^2 \theta + B_2 \sin^4 \theta + B_3 \sin^6 \theta + B_4 \sin^8 \theta, \quad (1-61a)$$

where,

$$B_0 = b_0 + b_1 + b_2 + b_3 + b_4,$$

$$B_1 = -2(b_1 + 2b_2 + 3b_3 + 4b_4),$$

$$B_2 = 4(b_2 + 3b_3 + 6b_4), \quad (1-61b)$$

$$B_3 = -8(b_3 + 4b_4),$$

$$B_4 = 16b_4.$$

In order to integrate over the solid angle Ω_x we use formula (A-58) from appendix B,

$$I_x = \frac{1}{4\pi} \int d\Omega_x e^{i\vec{m} \cdot \vec{x}} I_y = j_0(mx) I_y. \quad (1-62)$$

Therefore,

$$\begin{aligned}
 G_2 &= 2 (3)^{3/2} \pi^2 \int R^5 \cos^2 \theta \sin^2 \theta j_0(mR \cos \theta) dR d\theta I_y, \\
 &= 4(3)^{3/2} \pi^3 \int R^5 g_2 dR,
 \end{aligned} \tag{1-63}$$

where,

$$g_2 = \frac{1}{2\pi} \int \cos^2 \theta j_0(mR \cos \theta) I_y d\theta.$$

By substituting for I_y from eqs.(1-61) and using Appendix B we get

$$g_2 = \frac{1}{4} \{ 4 B_0 \phi + 3 B_1 \phi + 15 B_2 \phi + 105 B_3 \phi + 945 B_4 \phi \},$$

or,

$$G_2(q) = 3 \sqrt{3} \pi^3 \int dR R^5 \sum_{n=2}^6 A_n \phi_n, \tag{1-64}$$

where,

$$\phi_n = \frac{J_n}{z^n},$$

with J_n is the Bessel function, and

$$z = mR = (3)^{-1/2} qR,$$

and the coefficients A_n are,

$$A_2 = h_1^2 + \frac{40}{9} h_7^2,$$

$$A_3 = \frac{32}{3} h_6^2 - \frac{80}{3} h_7^2 + \frac{815}{6} h_8^2 + \frac{64}{3} h_6 h_7 - \frac{128}{3} h_6 h_8 - \frac{244}{3} h_7 h_8,$$

$$A_4 = -\frac{160}{3} h_6^2 + \frac{400}{3} h_7^2 - \frac{4208}{3} h_8^2 - \frac{320}{3} h_6 h_7 + 640 h_6 h_8,$$

$$A_5 = 896 h_6^2 + \frac{46592}{3} h_8^2 - \frac{8960}{3} h_6 h_8 - \frac{15680}{3} h_7 h_8,$$

$$A_6 = -4032 h_6^2 + 69888 h_8^2.$$

1.6.2 Evaluation of $\langle u_2 | e^{i\vec{q} \cdot \vec{r}_1} (\vec{J} \cdot \hat{A}) | u_2 \rangle$

If we use the same conventions which we used for G_2 we will have

$$\begin{aligned} G_3 &= \langle u_1 | e^{i\vec{m} \cdot \vec{x}} (\vec{J} \cdot \hat{A}) | u_1 \rangle = \frac{3\sqrt{3}}{8} \int d^3x d^3y u_1^\dagger (\vec{J} \cdot \hat{A}) e^{i\vec{m} \cdot \vec{x}} u_1, \quad (1-65) \\ &= \frac{3\sqrt{3}}{8} 16\pi^2 \int x^2 y^2 dx dy \frac{1}{4\pi} \int e^{i\vec{m} \cdot \vec{x}} d\Omega_x \frac{1}{4\pi} \int u_1^\dagger (\vec{J} \cdot \vec{A}) u_1 d\Omega_y. \end{aligned}$$

Using $u_1 = f_1 x_1 + D_3 S_1 f_6 + D_1 f_7 + D_2 S_2 f_8 + D_1 S_1 f_8$ we get

$$\begin{aligned} u_1^\dagger (\vec{J} \cdot \vec{A}) u_1 &= f_1^2 (\vec{J} \cdot \vec{A}) + 1/2 E_{33} S_1^2 f_6^2 + 1/2 E_{11} f_7^2 + 1/2 E_{22} S_2^2 f_8^2 \\ &+ 1/2 E_{11} S_1^2 f_8^2 + E_{31} S_1 f_6 f_7 - E_{31} S_1^2 f_6 f_8 \\ &+ E_{23} S_1 S_2 f_6 f_8 + E_{12} S_2 f_7 f_8 + E_{11} S_1 f_7 f_8 + E_{12} S_2 S_1 f_8^2 \\ &+ \frac{8}{\sqrt{3}} (W_{yy} + W_{xx} - 1/3 S_3 (\vec{J} \cdot \hat{A})) S_1 f_1 f_6 \end{aligned}$$

$$\begin{aligned}
& + \frac{8}{\sqrt{3}} (W_{yy} - W_{xx} - 1/3 S_1 (\vec{J} \cdot \hat{A})) (f_7 f_1 - S_1 f_6 f_1) \\
& + \frac{8}{\sqrt{3}} (W_{xy} + W_{yx} - 1/3 S_2 (\vec{J} \cdot \hat{A})) S_1 f_1 f_8, \quad (1-66)
\end{aligned}$$

where,

$$E_{ik} = D_i^\dagger (\vec{J} \cdot \vec{A}) D_k + D_k^\dagger (\vec{J} \cdot \vec{A}) D_i,$$

$$W_{mn} = (\hat{A} \cdot \vec{m})(\vec{J} \cdot \vec{n}), \quad m, n = \vec{x}, \vec{y}.$$

By using Appendix B, the integration over solid angle Ω_y will give us

$$\frac{1}{4\pi} \int d\Omega_y u_1^\dagger (\vec{J} \cdot \hat{A}) u_1 = \phi_1 W_{xx} + \phi_2 (\vec{J} \cdot \vec{A}), \quad (1-67)$$

where,

$$\begin{aligned}
\phi_1 = & 4/3 (S_3 - S_1) S_1^2 f_6^2 + 4/3 (S_3 - S_1) (f_7^2 + S_1^2 f_8^2) \\
& + 4/3 (S_3 - S_1) (S_3 + S_1)^2 f_8^2 - 8/3 (S_3 - S_1) S_1 f_6 f_7 \\
& + 8/3 (S_3 - S_1) S_1^2 f_6 f_8 + 8/45 (S_3 + S_1) (17 S_3 - 3 S_1) S_1 f_6 f_8 \\
& + 8/45 (S_3 + S_1) (-3 S_3 + 17 S_1) f_7 f_8 - 8/3 (S_3 - S_1) S_1 f_7 f_8 \\
& - 8/45 (S_3 + S_1) (-3 S_3 + 17 S_1) S_1 f_8^2 + \frac{8}{\sqrt{3}} S_1 f_1 f_6 \\
& + \frac{8}{\sqrt{3}} (f_1 f_7 - S_1 f_1 f_8) + \frac{32}{\sqrt{3}} (S_3 + S_1) f_1 f_8,
\end{aligned}$$

and,

$$\begin{aligned}
\omega_2 &= f_1^2 + \frac{4}{9} S_1^3 S_3 f_6^2 - \frac{2}{3} S_1^2 f_6^2 + \frac{4}{3} S_1^2 S_3 f_6^2 + \frac{4}{9} S_1 S_3 f_7^2 \\
&\approx \frac{2}{3} (S_1^2 + S_3^2) f_7^2 + \frac{22}{45} (S_3^2 - S_1^2) f_8^2 + \frac{4}{9} S_1^3 S_3 f_8^2 - \frac{2}{3} S_1^2 f_8^2 \\
&+ \frac{4}{9} S_1 (S_1^2 + S_3^2) f_6 f_7 + \frac{8}{3} S_1^2 S_3 f_6 f_7 + \frac{4}{9} S_1^2 (S_1^2 + S_3^2) f_6 f_8 \\
&+ \frac{8}{3} S_1^3 S_3 f_6 f_8 - \frac{148}{135} (S_3^2 - S_1^2) S_1 S_3 f_6 f_8 + \frac{4}{45} (S_3^2 - S_1^2) S_1^2 f_6 f_8 \\
&- \frac{148}{135} (S_3^2 - S_1^2) S_1 f_7 f_8 + \frac{4}{45} (S_3^2 - S_1^2) S_3 f_7 f_8 - \frac{8}{9} S_1^2 S_3 f_7 f_8 \\
&+ \frac{4}{3} S_1^3 f_7 f_8 + \frac{4}{3} S_1 S_3^2 f_7 f_8 + \frac{148}{135} (S_3^2 - S_1^2) S_1^2 f_8^2 \\
&\approx \frac{4}{45} (S_3^2 - S_1^2) S_1 S_3 f_8^2 - \frac{4}{3\sqrt{3}} (S_3 - S_1) S_1 f_1 f_6 + \frac{4}{3\sqrt{3}} (S_3 - S_1) f_1 f_7 \\
&\approx \frac{4}{3\sqrt{3}} (S_3 - S_1) S_1 f_1 f_8 - \frac{8}{9\sqrt{3}} (S_3^2 - S_1^2) f_1 f_8.
\end{aligned}$$

As for the G_2 we choose the following substitution (fig.3) :

$$\begin{aligned}
h_1 &= f_1 & x &= R \cos \theta, \\
h_6 &= R^2 f_6 & y &= R \sin \theta, \\
h_7 &= R^2 f_7, & c &= \cos(2\theta),
\end{aligned}$$

$$h_8 = R^* f_8,$$

then we have $S_3 = R^2$, $S_1 = -R^2 C$, and ϕ_1 can be written in the following form

$$\phi_1 = a_0 + a_1 C + a_2 C^2 + a_3 C^3, \quad (1-68)$$

where,

$$a_0 = \frac{1}{R^2} \left[\frac{4}{3} h_7^2 + \frac{4}{3} h_8^2 - \frac{8}{15} h_7 h_8 + \frac{8}{\sqrt{3}} h_1 h_7 + \frac{32}{3\sqrt{3}} h_1 h_8 \right],$$

$$a_1 = \frac{1}{R^2} \left[\frac{4}{3} h_7^2 + \frac{28}{15} h_8^2 + \frac{8}{3} h_6 h_7 - \frac{136}{45} h_6 h_8 + \frac{8}{45} h_7 h_8 - \frac{8}{\sqrt{3}} h_1 h_6 + \frac{56}{3\sqrt{3}} h_1 h_8 \right],$$

$$a_2 = \frac{1}{R^2} \left[\frac{4}{3} h_6^2 - \frac{112}{45} h_8^2 + \frac{8}{3} h_6 h_7 + \frac{232}{45} h_6 h_8 + \frac{256}{45} h_7 h_8 \right],$$

$$a_3 = \frac{1}{R^2} \left[\frac{4}{3} h_6^2 + \frac{256}{45} h_8^2 + \frac{16}{5} h_6 h_8 \right].$$

and,

$$\phi_2 = b_0 + b_1 C + b_2 C^2 + b_3 C^3 + b_4 C^4, \quad (1-69)$$

where,

$$b_0 = h_1^2 - \frac{2}{3} h_7^2 + \frac{22}{45} h_8^2 + \frac{4}{45} h_7 h_8 + \frac{4}{3\sqrt{3}} h_1 h_7 + \frac{8}{9\sqrt{3}} h_1 h_8,$$

$$b_1 = -\frac{4}{9} h_7^2 + \frac{4}{45} h_8^2 + \frac{4}{3\sqrt{3}} h_1 h_6 + \frac{4}{3\sqrt{3}} h_1 h_7 + \frac{4}{3\sqrt{3}} h_1 h_8$$

$$= \frac{4}{9} h_6 h_7 + \frac{148}{135} h_6 h_8 + \frac{32}{135} h_7 h_8,$$

$$b_2 = -\frac{2}{3} h_6^2 + \frac{2}{3} h_7^2 + \frac{38}{27} h_8^2 + \frac{4}{3\sqrt{3}} h_1 h_6 + \frac{20}{3\sqrt{3}} h_1 h_8$$

$$= \frac{8}{3} h_6 h_7 - \frac{16}{45} h_6 h_8 - \frac{44}{45} h_7 h_8,$$

$$b_3 = -\frac{4}{9} h_6^2 - \frac{24}{45} h_8^2 - \frac{4}{9} h_6 h_7 + \frac{328}{135} h_7 h_8 + \frac{508}{135} h_6 h_8,$$

$$b_4 = -\frac{2}{3} h_6^2 + \frac{304}{135} h_8^2 + \frac{24}{45} h_6 h_8.$$

For integration over the solid angle Ω_x from Appendix B (A-58) we have

$$\frac{1}{4\pi} \int d\Omega_x e^{i\vec{m} \cdot \vec{x}} \phi_2(\vec{J} \cdot \hat{A}) = j_0(mx) \phi_2(\vec{J} \cdot \hat{A}), \quad (1-70)$$

and,

$$\begin{aligned} \frac{1}{4\pi} \int d\Omega_x e^{i\vec{m} \cdot \vec{x}} \phi_1 W_{xx} &= [-(\vec{J} \cdot \hat{m})(\hat{A} \cdot \hat{m}) + \frac{\vec{J} \cdot \hat{A}}{3}] x^2 j_2(mx) \phi_1 \\ &+ \frac{\vec{J} \cdot \hat{A}}{3} x^2 j_0(mx) \phi_1. \end{aligned} \quad (1-71)$$

Therefore,

$$G_3 = \frac{3\sqrt{3}}{8} \left\{ [-(\vec{J} \cdot \hat{q})(\hat{A} \cdot \hat{q}) + \frac{\vec{J} \cdot \hat{A}}{3}] L_1 + (\vec{J} \cdot \hat{A}) L_2 \right\},$$

where,

$$L_1 = 16 \pi^2 \int dx dy x^2 y^2 j_2(mx) \phi_1,$$

$$L_2 = 16 \pi^2 \int dx dy x^2 y^2 \left(\frac{x^2}{3} \phi_1 + \phi_2 \right) j_0(mx).$$

For integration over x , and y we substitute $x = R \cos \theta$, $y = R \sin \theta$, and $dx dy$ by $R dR d\theta$. By using Appendix B and integrating over θ we obtain,

$$G_3(q) = -(\vec{J} \cdot \hat{q})(\hat{A} \cdot \hat{q}) L_1(q) + \left[\frac{1}{3} L_1(q) + L_2(q) \right] (\vec{J} \cdot \hat{A}), \quad (1-72)$$

where,

$$L_1(q) = 3 \sqrt{3} \pi^3 \int dR R^5 \sum_{n=2}^6 C_n \phi_n,$$

$$L_2(q) = 3 \sqrt{3} \pi^3 \int dR R^5 \sum_{n=2}^6 D_n \phi_n,$$

with,

$$C_2 = \frac{8}{\sqrt{3}} h_1 h_6 + \frac{8}{\sqrt{3}} h_1 h_7 + \frac{8}{\sqrt{3}} h_1 h_8 + \frac{16}{3} h_6 h_7 + \frac{16}{3} h_6 h_8$$

$$+ \frac{16}{3} h_7 h_8 + \frac{8}{3} h_6^2 + \frac{8}{3} h_7^2 + \frac{8}{3} h_8^2,$$

$$C_3 = -\frac{96}{\sqrt{3}} h_1 h_6 - \frac{48}{3} h_1 h_8 + \frac{160}{\sqrt{3}} h_1 h_8 + 56 h_6^2 + 24 h_7^2$$

$$+ \frac{232}{5} h_6^2 + \frac{400}{3} h_6 h_8 + 80 h_6 h_7 + \frac{304}{3} h_7 h_8,$$

$$C_4 = \frac{384}{\sqrt{3}} h_1 h_6 + \frac{896}{\sqrt{3}} h_1 h_8 + 640 h_6^2 + 64 h_7^2 + \frac{4096}{3} h_8^2$$

$$+ 544 h_6 h_7 + 1696 h_6 h_8 + \frac{2688}{3} h_7 h_8,$$

$$C_5 = 4320 h_6^2 + \frac{34624}{3} h_6 h_8 + \frac{10240}{3} h_7 h_8 + 1600 h_6 h_7 + \frac{40576}{3} h_8^2,$$

$$C_6 = -13440 h_6^2 - 57344 h_8^2 + 32256 h_6 h_8,$$

and,

$$D_2 = h_1^2 + \frac{40}{9\sqrt{3}} h_1 h_8 + \frac{8}{9} h_6^2 + \frac{8}{9} h_7^2 + \frac{8}{9} h_8^2 + \frac{16}{9} h_6 h_7$$

$$+ \frac{16}{9} h_6 h_8 + \frac{16}{9} h_7 h_8,$$

$$D_3 = -\frac{128}{3\sqrt{3}} h_1 h_8 + 16 h_6^2 + \frac{16}{3} h_7^2 + \frac{208}{9} h_8^2 + \frac{64}{3} h_6 h_7$$

$$+ \frac{352}{9} h_6 h_8 + \frac{256}{9} h_7 h_8,$$

$$D_4 = \frac{640}{3\sqrt{3}} h_1 h_8 + \frac{560}{3} h_6^2 + \frac{80}{3} h_7^2 + \frac{3856}{9} h_8^2 + \frac{320}{3} h_6 h_7$$

$$+ \frac{1280}{3} h_6 h_8 + \frac{800}{3} h_7 h_8,$$

$$D_5 = \frac{4480}{3} h_6^2 + \frac{39424}{9} h_8^2 + \frac{16520}{9} h_6 h_8 + \frac{11200}{9} h_7 h_8,$$

$$D_6 = -6720 h_6^2 - 19712 h_8^2.$$

Using the same procedure which we used for calculating G_2 , G_3 , we can calculate G_1 , G_4 ; the results are

$$G_1(q) = 3 \sqrt{3} \pi^3 \int R^5 dR \sum_{n=2}^6 B_n \phi_n, \quad (1-73)$$

where,

$$B_2 = h_1^2 + \frac{8}{3} h_6^2 + \frac{8}{3} h_7^2 + \frac{8}{3} h_8^2 + \frac{16}{3} h_6 h_7 + \frac{16}{3} h_6 h_8 + \frac{16}{3} h_7 h_8,$$

$$B_3 = -48 h_6^2 - 16 h_7^2 - \frac{208}{3} h_8^2 - 64 h_6 h_7 - \frac{256}{3} h_7 h_8 - \frac{352}{3} h_6 h_8,$$

$$B_4 = 560 h_6^2 + 80 h_7^2 + \frac{3586}{3} h_8^2 + 320 h_6 h_7 + 1280 h_6 h_8 + 800 h_7 h_8,$$

$$B_5 = -4480 h_6^2 - \frac{39424}{3} h_7^2 - \frac{17920}{3} h_8^2 - \frac{17920}{3} h_6 h_8 - \frac{11200}{3} h_7 h_8,$$

$$B_6 = 20160 h_6^2 + 59136 h_8^2.$$

For G_4 we have

$$G_4(q) = -(\vec{J} \cdot \hat{q})(\hat{A} \cdot \hat{q}) K_1(q) + \left[\frac{1}{3} K_1(q) + K_2(q) \right] (\vec{J} \cdot \hat{A}), \quad (1-74)$$

where,

$$K_1(q) = \frac{16}{5} \sqrt{3} \pi^3 \int dR R^5 \sum_{n=3}^6 V_n \phi_n,$$

$$K_2(q) = 3 \sqrt{3} \pi^3 \int dR R^3 \sum_{n=2}^6 W_n \phi_n,$$

and

$$V_3 = -10 h_6^2 - \frac{35}{2} h_7^2 - \frac{95}{2} h_8^2 - 20 h_6 h_7 + 40 h_6 h_8 + 55 h_7 h_8,$$

$$V_4 = 140 h_6^2 + 140 h_7^2 + 1240 h_8^2 + 190 h_6 h_7 - 750 h_6 h_8 - 960 h_7 h_8,$$

$$V_5 = -810 h_6^2 - 14200 h_7^2 - 300 h_6 h_7 + 5140 h_6 h_8 + 5200 h_7 h_8,$$

$$V_6 = 2520 h_6^2 + 67200 h_7^2 - 10800 h_6 h_8,$$

$$W_2 = h_1^2,$$

$$W_3 = -\frac{32}{9} h_6^2 - \frac{80}{9} h_7^2 - \frac{176}{9} h_8^2 - \frac{64}{9} h_6 h_7 + \frac{160}{9} h_6 h_8 + \frac{224}{9} h_7 h_8,$$

$$W_4 = \frac{352}{9} h_6^2 + \frac{400}{9} h_7^2 + \frac{4208}{9} h_8^2 + \frac{320}{9} h_6 h_7 - \frac{800}{3} h_6 h_8 - \frac{1120}{3} h_7 h_8,$$

$$W_5 = -\frac{896}{3} h_6^2 - \frac{46592}{9} h_7^2 + \frac{12544}{9} h_6 h_8 + \frac{15680}{9} h_7 h_8,$$

$$W_6 = 1344 h_6^2 + 23296 h_8^2.$$

1.7 Results of Calculations

By collecting the results from the previous sections, we can now express the P-³He scattering amplitudes in terms of G_1 , G_2 , G_3 , G_4 , and N-N amplitudes.

$$F_0^0 = (\alpha^0 - \frac{1}{3} \alpha^1) G_2 + (\alpha^0 + \alpha^1) G_1,$$

$$F_0^1 = (\gamma_1^0 - \frac{1}{3} \gamma_1^1) G_2 + (\gamma_1^0 + \gamma_1^1) G_1,$$

$$F_n^0 = (\gamma_2^0 - \frac{1}{3} \gamma_2^1) (\frac{1}{3} L_1 + L_2) + (\gamma_2^0 + \gamma_2^1) (\frac{1}{3} K_1 + K_2), \quad (1-75)$$

$$F_n^1 = (\beta^0 - \frac{1}{3} \beta^1) (\frac{1}{3} L_1 + L_2) + (\beta^0 + \beta^1) (\frac{1}{3} K_1 + K_2),$$

$$F_q^0 = (\delta^0 - \frac{1}{3} \delta^1) (-\frac{2}{3} L_1 + L_2) + (\delta^0 + \delta^1) (-\frac{2}{3} K_1 + K_2),$$

$$F_k^1 = (\epsilon^0 - \frac{1}{3} \epsilon^1) (\frac{1}{3} L_1 + L_2) + (\epsilon^0 + \epsilon^1) (\frac{1}{3} K_1 + K_2),$$

where $\alpha^j, \beta^j, \gamma_1^j, \gamma_2^j, \delta^j, \epsilon^j, j=0,1$ are the isoscalar and isovectors of the N-N amplitudes for p-p, and p-n scattering in the Breit frame, e.g. $\alpha^0 = \alpha_{pp} + \alpha_{pn}, \alpha^1 = \alpha_{pp} - \alpha_{pn}$. It should be noticed that in center of mass frame γ_1^j, γ_2^j are equal. The formfactors are defined in the following form,

$$G_1(q) = 3 \sqrt{3} \pi^3 \int R^5 dR \sum_{n=2}^6 B_n \phi_n, \quad (1-76a)$$

$$G_2(q) = 3 \sqrt{3} \pi^3 \int dR R^5 \sum_{n=2}^6 A_n \phi_n, \quad (1-76b)$$

$$L_1(q) = 3 \sqrt{3} \pi^3 \int dR R^5 \sum_{n=2}^6 C_n \phi_n, \quad (1-76c)$$

$$L_2(q) = 3 \sqrt{3} \pi^3 \int dR R^5 \sum_{n=2}^6 D_n \phi_n, \quad (1-76d)$$

$$K_1(q) = \frac{16}{5} \sqrt{3} \pi^3 \int dR R^5 \sum_{n=3}^6 V_n \phi_n, \quad (1-76e)$$

$$K_2(q) = 3 \sqrt{3} \pi^3 \int dR R^5 \sum_{n=2}^6 W_n \phi_n, \quad (1-76f)$$

where the coefficients $A_n, B_n, C_n, D_n, V_n, W_n$ are defined in eqs. 1-64, 72, 73, 74.

As we can see from eq.(1-75), F_0^0 , and F_0^n are only functions of G_1, G_2 , and they don't depend on spin of the target. By substituting the amplitudes from eqs.(1-75) in to the following relations we can calculate all the observables.

$$I_0 D_{SS} = [|F_0^0|^2 - |F_0^n|^2 + |F_n^0|^2 - |F_n^n|^2] \cos(\chi - \chi') \\ + [|F_Q^0|^2 - |F_K^0|^2] \cos(\chi + \chi') + 2 \text{Im}(F_0^n F_0^0^* + F_n^n F_n^0^*) \sin(\chi - \chi'), \quad (1-77a)$$

$$I_0 D_{S1} = [|F_0^0|^2 - |F_0^n|^2 + |F_n^0|^2 - |F_n^n|^2] \sin(\chi - \chi') \\ - [|F_Q^0|^2 - |F_K^0|^2] \sin(\chi + \chi') + 2 \text{Im}(F_0^n F_0^0^* + F_n^n F_n^0^*) \sin(\chi - \chi'), \quad (1-77b)$$

$$I_0 D_{11} = [|F_0^0|^2 - |F_0^n|^2 + |F_n^0|^2 - |F_n^n|^2] \cos(\chi - \chi') \\ - [|F_Q^0|^2 - |F_K^0|^2] \cos(\chi + \chi') - 2 \text{Im}(F_0^n F_0^0^* + F_n^n F_n^0^*) \sin(\chi - \chi'), \quad (1-77c)$$

$$I_0 D_{1S} = [-|F_0^0|^2 + |F_0^n|^2 - |F_n^0|^2 + |F_n^n|^2] \sin(\chi - \chi') \\ - [|F_Q^0|^2 - |F_K^0|^2] \sin(\chi + \chi') - 2 \text{Im}(F_0^n F_0^0^* + F_n^n F_n^0^*) \sin(\chi - \chi'), \quad (1-77d)$$

$$I_0 D_{nn} = [|F_0^0|^2 + |F_0^n|^2 - |F_Q^Q|^2 + |F_n^0|^2 + |F_n^n|^2 - |F_K^K|^2], \quad (1-77e)$$

$$\frac{d\sigma}{d\Omega}_{\text{unpol.}} = \frac{1}{2} \{ |F_0^0|^2 + |F_0^n|^2 + |F_Q^Q|^2 + |F_n^0|^2 + |F_n^n|^2 + |F_K^K|^2 \}, \quad (1-77f)$$

$$A_y = 2 \operatorname{Real}(F_0^n F_0^0^* + F_n^n F_n^0^*), \quad (1-77g)$$

$$I_0 D_0 = [|F_0^0|^2 + |F_0^n|^2], \quad (1-77h)$$

$$I_0 D_x = |F_Q^Q|^2, \quad (1-77i)$$

$$I_0 D_y = [|F_n^0|^2 + |F_n^n|^2], \quad (1-77j)$$

$$I_0 D_z = |F_K^K|^2, \quad (1-77k)$$

and,

$$I_0 = [|F_0^0|^2 + |F_0^n|^2 + |F_Q^Q|^2 + |F_n^0|^2 + |F_n^n|^2 + |F_K^K|^2].$$

CHAPTER 2

Experimental Detail

The data for this experiment were taken at the High Resolution Spectrometer (HRS) at the Clinton P. Anderson Los Alamos Meson Physics Facility (LAMPF) with a polarized proton beam at 800 MeV. In the sections that follow we will give description of the beam line, the HRS facility, the experimental setup, and the results of the experiment.

2.1 Beam Line and Polarized Beam

The LAMPF linear accelerator is capable of accelerating simultaneously two beams : protons (H^+); and negative hydrogen atoms polarized (P^-) or unpolarized (H^-). The P^- beam is produced by a Lamb-shift ion source. The initial 750 KeV acceleration is provided by a Crockroft-Walton injector. In the next step the P^- ions are accelerated to an energy of 100 MeV in an Alvarez drift-tube linear accelerator which consists of four copper-lined tanks that operates at about 200 MHz frequency. In the last stage of acceleration the beam passes 44 sectors of side coupled cavities which operates at 805 MHz¹⁾. At completion of this stage the beam's energy has increased up to 800 MeV. Beam from this type of accelerator has a time structure. It has a microstructure consisting of a .25 nsec burst

every 5 nsec. The time sequence of the accelerator consists of 120 pulses per second, each one about 500 μ sec long. Therefore the duty factor is about 6%.

Once the final energy has been reached the P^- and H^+ beams are magnetically separated from each other and each one continues to different experimental areas. The H^+ enters area A while the P^- beams continue to line X where it passes through strippers and collimator jaws. At the stripper, electrons are removed from a fraction of the p^- beam which passes through it, and this portion of the beam, now H^+ , is magnetically bent into line C, and the unstripped p^- continues to other experimental areas (fig.1).

Line C consists of three sections, a separation section, a dispersion section, and a twisting and matching section (fig.2). In the separation section magnets LCBM1-3 bend and separate the H^+ beam from the H^- or p^- beam. Quadrupole magnets, LCQM1-2, provide focussing of the beam for input to the dispersion section. In the dispersion section LCQM3 and LCQM4-5 provide 9.03 cm dispersion per percent change in momentum. In the third section LCQM4-12 rotate the beam phase space by 90° and provide the requirements necessary to match the beam line optics to those of the HRS, operating in the momentum loss mode. Finally the beam reaches the HRS target¹⁾.

2.2 The High Resolution Spectrometer

HRS is a Quadrupole-Dipole-Dipole (QDD) system mounted in a vertical plane. If we assume Z is in the beam direction, X is perpendicular to Z in the momentum dispersion direction and Y is perpendicular to them both to make a right-handed system of coordinate. The quadrupole magnet provides focussing in the Y direction and defocussing in the X direction. The whole system provides parallel-to point focusing in the Y direction and point to point focussing in the X direction. This provides a precise angle and energy-loss measurement. A view of the HRS facility is shown in fig.3.

2.2.1 Focal Plane Polarimeter System

A diagram of the HRS focal plane system is shown in fig.4. The scintillators S_1 , S_2 , SP_1 , SP_2 provide the "event" trigger, and also give energy-loss and time of flight information for particle identification. Each scintillator is viewed from top and bottom, by two phototubes. Counters C_1 through C_8 are delay line readout multi-wire drift chambers. Each chamber consists of two orthogonal sets of four planes, a pair of planes for X information and another pair for Y information. The planes are of an alternating gradient design with 8mm wire spacing. The sequence planes are offset by 4mm.

Fig 1: Experimental Areas

Fig 3: High Resolution Spectrometer (HRS)

Cathode wires are bussed together, providing two cathode outputs per plane and the anode wires are coupled directly into a fast (2.5 ns/cm) delay line which provides two anode outputs per plane. Information about the dimensions of the detectors are given in Table 2.1.

The carbon analyzer has 10 slabs of $41 \times 41 \times 3$ cm³ graphite ($\rho = 1.78$ g/cm³). In the intermediate energy region carbon is a good analyzer because it is easy to handle and it has a large analyzing power and cross section²⁾.

The chambers are operated with a gas mixture of 65% argon, 35% isobutane, .10% freon and .50% isopropyl alcohol. More information about these chambers are given in References 3 and 4.

The chambers provide information about the trajectory of protons through the spectrometer which is used to determine the scattering angle and missing mass.

Table 2.1: Dimensions of detectors in focal plane

Detector	Dimension(cm ²)
S1,S2	10×56
SP1,SP2	38×76
C1,C2,C3,C4	10×60
C5,C6,C7,C8	60×60

2.3 Electronics

Signals from the scintillators first go to amplifiers and discriminators and convert to fast logic signals which are then transmitted to the counting house. The "event" signal is a coincidence $(S1 \cdot S2) \cdot (SP1 + SP2)$. The trigger pulse is the coincidence of $\text{event} \cdot \text{run} \cdot \text{beam} \cdot \overline{\text{busy}}$, i.e. event signal, run in progress (a signal which is triggered by the computer when we start collecting data), beam on and CAMAC not busy. The trigger pulse starts the TDC, ADC, scalars, and simultaneously triggers a 15 μ s busy signal which disables the generation of a new trigger pulse. During these 15 microseconds, the signals from wire chambers and scintillators are digitized and stored in a storage buffer at CAMAC units. The signals from CAMAC are read by a Microprogrammable Branch Driver (MBD). The

Fig 4: HRS Focal Plane Detectors

Fig. 5: HRS fast trigger electronics

MBD then applies various tests to determine whether the event is a "good event" and should be kept or rejected. Those events passing the MBD tests are written on tape. We also write a small fraction of events which didn't pass the MBD tests on tape for later check. Some fraction of these events are analyzed for on-line display and checking, this fraction depends on the rate of the events which are read and written on the tape. The diagrams of electronics system are shown in fig.5.

2.4 Target

The target was constructed in 1968-69 at High Energy Physics Laboratory at Stanford University.

The liquid ^3He target is a 6 feet long 2 feet diameter stainless steel cylinder. In this target ^3He is cooled and liquified in three steps. First, in contact with a 25 liter liquid nitrogen bath the temperature is brought down to 30°K . Then in contact with a 30 liter liquid ^4He bath the temperature reaches 4.2°K . Lastly the temperature of the ^3He in contact with 15 liters of superfluid ^4He is brought down to 1.5°K .

The closed-circuit ^3He system includes a 28 liters storage tank containing 17 liters of ^3He in STP. The ^3He was transferred to the target through an activated charcoal trap at liquid nitrogen temperature where impurities in the ^3He are removed. The outer and inner heat shields are connected to the liquid nitrogen and liquid helium baths. At a number of points temperature and liquid level are monitored by measuring the resistance of 200 Ω carbon resistors using an AC resistance bridge. The target and set up of the target in HRS are shown in fig.6.

Since aluminum windows hold the liquid ^3He , there are some scattering from the aluminum. Because of the kinematic shift at even the smallest angle (5 deg), the peaks from ^3He and aluminum scattering are well separated and they do not cause any problem. It should be mentioned that the scattering chamber is open to air. The distance which the protons travel through air is about 50 cm, which does not have a significant effect on the projectiles.

For monitoring the target we used two three-scintillator telescopes looking at the target from left and right inside the scattering chamber. The setup and diagrams of electronic for the monitoring system are shown in figs.7, and 8.

Fig 6a: Liquid Helium System-HRS

Fig. 6b: Liquid He Target System - IIRS

Fig 60: Cryogenic target Dewar (2).

Fig /: A Schematic layout of the Monitor Sc

Fig 8: Electronics for Target Monitor

2.5 Data Acquisition System

The data acquisition system "Q" has been explained in detail in references 5 and 6. This system consists of three major components: ANALYZER, ALLTEST, and DISPLAY PACKAGE. The analyzer indicates what data should be acquired from CAMAC in response to a particular trigger and what analysis is required on these data. Histograms are usually made by an analyzer. The analyzer runs simultaneously with a number of other subroutines, for example, a kinematics calculation. Two types of events were used during the experiment by analyzer, data events and scalar events. Usually in on-line data acquisition the "may process" mode is used for data events, i.e. the events will be analyzed if time permits and the "must process" for scalar events, i.e. the events will process.

The test package determines the condition which the data should satisfy⁷⁾. There are two different types of tests in a test file; the "Micro-Test" which contains a series of tests, boxes, gates and cuts on raw data or computed quantities, and the "Macro-Test" which is a series of logical tests on combinations of the micro-test.

The display package allows us to define as many as 40 histograms from raw and calculated data or display a two dimensional dot plot⁸⁾.

2.6 Analysis of the Data

First we need to calibrate the chambers. The objective of calibration is to convert time differences from the chamber's TDC into real positions. There are two major parts in the calibration. In the first part, time differences along a delay line are converted into a wire number. In the second part, drift times are converted into a position to be added or subtracted from the wire position to find the final position^{3,4)}.

In the first part a polynomial is constructed which gives a crude position from the time differences. This crude position is then truncated to obtain a wire number. By knowing the wire spacing this allows us to obtain a truncated position relative to the chamber. The procedure is as follows: If $t_d = t_1 - t_2$ where t_1, t_2 are the TDC times from each end of the delay line, then the anode wire position which is closest to the particle position can be written in terms of t_d ,

$$X = a_0 + a_1 t_d + a_2 t_d^2,$$

where the t_d^2 term is included to account for dispersion from a long delay line³⁾. We can define the truncated anode position K in the following way:

$$K = \text{INT}(X/w)w,$$

where w is the anode wire spacing and INT is a function which truncates X/w to the nearest integer. Now by minimizing the chi-square function

$$\chi^2 = \sum_{i=1}^n (X_i - K_i)^2,$$

for number of particles r we can find a_0 , a_1 , a_2 . A typical histogram of the calibration is shown in fig.9.

In second part, drift time histograms are used as input to a program which converts drift times to drift position (see 2.6.1). At last an absolute calibration is done by aligning all chambers in one dimension along a straight line. This alignment is not crucial, since the anode position calibration effectively shifts the wire position. Results of this calibration will be recorded as the new values of variables "R" (R157-164 for X wires and R221-228 for Y wires) in the file ALCOM.RDC. It should be noted that for our analysis we used RSX11-D system.

Fig 9: a) Histogram of the anode position $X_1^{(2)}$
 b) Histogram of the truncated anode position $X_1^{(2)}$
 c) a) and b) together

2.6.1 Left-Right Ambiguity

Calculating the position from a multiwire chamber plane requires knowing whether to add or subtract the drift distance from the anode wire position, in other words one must determine which side of the anode wire the track is located. There are two ways of solving this problem. One solution requires the use of multiple chamber planes. The second uses information obtained from the induced cathode pulses. We used the first method.

In the chambers we have adjacent pairs of anode planes, where one plane is offset one-half an anode wire spacing, to determine whether to add or subtract the drift distance from the anode wire position. Two sets of two offset planes are needed to fully solve the right-left ambiguity, since angle information is needed for large phase-space beams. Therefore we required eight anode planes (four measuring vertical coordinates and four measuring horizontal coordinates) to completely specify a particle trajectory.

In our procedure the position for each plane in a set can be found by trying all combinations of adding and subtracting drift distances and choosing the one which gives the smallest error for the difference between the two positions when they are projected to a common plane.

Next we create a proper test file and histogram file for off-line analysis. An example of these files is given in the appendix. Two histograms of missing mass and scattering angle are shown in fig.10. The analysis of data consists of two parts, 1) spin observable measurement, and 2) analyzing power and cross section measurement.

2.7 Polarization Calculation

The data are analyzed in two ways; "weighted sum" (cone test) and "fourier analysis" ($\phi+\pi$) which are explained by Besset (references 8,9).

The azimuthal distribution of particles which are scattered in the carbon blocks is given by:

$$I(\phi_c, \theta_c) = I_0(\theta_c) (1 + \epsilon_n(\theta_c) \cos\phi_c + \epsilon_s(\theta_c) \sin\phi_c) A(\theta_c, \phi_c), \quad (2-1)$$

where $I_0(\theta_c)$ is the unpolarized intensity, θ_c, ϕ_c are the polar and azimuthal angles in the carbon, and $A(\theta_c, \phi_c)$ is the acceptance of the apparatus. The asymmetries ϵ_n, ϵ_s , are functions of the carbon analyzing power $P(\theta)$, and transverse polarization components of the incident particles (for sign convention see reference 10), i.e.

Fig 10: Typical missing mass and scattering angle histogram (24.5 degree).

$$\epsilon_n(\theta) = P(\theta)P_n, \quad (2-2a)$$

$$\epsilon_s(\theta) = -P(\theta)P_s. \quad (2-2b)$$

2.7.1 Weighted Sum (Cone Test)

In this test for a given scattered particle we make a cone with θ_c and azimuthal angle ϕ_c . If this cone is completely inside the region accepted (area of the back chamber), the scattered particle will be included, otherwise it will be rejected. Therefore we require $A(\theta_c, \phi_c) = A(\theta_c)$, and we must estimate the following integrals,

$$\int_0^{2\pi} I(\theta_c, \phi_c) d\phi_c = 2\pi N(\theta_c), \quad (2-3a)$$

$$\int_0^{2\pi} I(\theta_c, \phi_c) \cos \phi_c d\phi_c = 1/2 N(\theta_c) \epsilon_n(\theta_c) = 1/2 N(\theta_c) A_c(\theta_c) P_n, \quad (2-3b)$$

$$\int_0^{2\pi} I(\theta_c, \phi_c) \sin \phi_c d\phi_c = 1/2 N(\theta_c) \epsilon_s(\theta_c) = -1/2 N(\theta_c) A_c(\theta_c) P_s. \quad (2-3c)$$

In other words in this method we calculate $\epsilon_n(\theta_i)$, $\epsilon_s(\theta_i)$, by making up the three sums : $\sum_{ev} 1 = N(\theta_i)$, $\sum_{ev} \cos \phi_c$, $\sum_{ev} \sin \phi_c$, where $\epsilon_n(\theta_i)$, $\epsilon_s(\theta_i)$, are the asymmetry for the i^{th} θ_c bin in the n and s direction. The sum $\sum$ represent the events in the i^{th} θ_c bin, and $N(\theta_i)$ is the sum of the events in the θ_i bin.

$$\begin{pmatrix} \sum_{ev} \cos \phi_c \\ \sum_{ev} \sin \phi_c \end{pmatrix} = \frac{N(\theta_i)}{2} \begin{pmatrix} \epsilon_n(\theta_i) \\ \epsilon_s(\theta_i) \end{pmatrix} \quad (2-4)$$

Now if we use analyzing power as a weighting factor we can find an estimator for the polarization,

$$\begin{pmatrix} \sum_{ev} A_c \cos \phi_c \\ \sum_{ev} A_c \sin \phi_c \end{pmatrix} = 1/2 \sum_{ev} A_c^2(\theta_c) \begin{pmatrix} p_n \\ -p_s \end{pmatrix} \quad (2-5)$$

where now the sum are evaluated over all the events for all the θ bins.

2.7.2 Fourier Analysis ($\phi_c + \pi$) Test

In this test for a given θ_c we look at an event (ϕ_c) and its "anti-event" ($\phi_c + \pi$). If both are in the acceptance region it will be accepted; otherwise will be rejected. Therefore it requires $A(\theta_c, \phi_c) = A(\theta_c, \phi_c + \pi)$. As we can see, this is a less restricted test than the cone test. We can do a similar estimation as the cone test for this test

$$\int_0^{2\pi} I(\theta_c, \phi_c) d\phi_c = 2\pi N(\theta_c), \quad (2-6a)$$

$$\begin{aligned} \int_0^{2\pi} I(\theta_c, \phi_c) \cos \phi_c d\phi_c &= \epsilon_n(\theta_c) \int_0^{2\pi} I(\theta_c, \phi_c) \cos^2 \phi_c d\phi_c \\ &+ \epsilon_s(\theta_c) \int_0^{2\pi} I(\theta_c, \phi_c) \cos \phi_c \sin \phi_c d\phi_c, \end{aligned} \quad (2-6b)$$

$$\begin{aligned} \int_0^{2\pi} I(\theta_c, \phi_c) \sin \phi_c d\phi_c &= \epsilon_n(\theta_c) \int_0^{2\pi} I(\theta_c, \phi_c) \cos \phi_c \sin \phi_c d\phi_c \\ &+ \epsilon_s(\theta_c) \int_0^{2\pi} I(\theta_c, \phi_c) \sin^2 \phi_c d\phi_c. \end{aligned} \quad (2-6c)$$

In the matrix form,

$$\begin{pmatrix} \sum_{ev} \cos \phi_c \\ \sum_{ev} \sin \phi_c \end{pmatrix} = \begin{pmatrix} \sum_{ev} \cos^2 \phi_c & \sum_{ev} \cos \phi_c \sin \phi_c \\ \sum_{ev} \cos \phi_c \sin \phi_c & \sum_{ev} \sin^2 \phi_c \end{pmatrix} \begin{pmatrix} \epsilon_n(\theta_i) \\ \epsilon_s(\theta_i) \end{pmatrix} \quad (2-7)$$

By using $A_c(\theta_c)$ as weighting factor we get:

$$\begin{pmatrix} \sum_{ev} A_c(\theta_c) \cos \phi_c \\ \sum_{ev} A_c(\theta_c) \sin \phi_c \end{pmatrix} = \begin{pmatrix} \sum_{ev} A_c^2(\theta_c) \cos^2 \phi_c & \sum_{ev} A_c^2(\theta_c) \cos \phi_c \sin \phi_c \\ \sum_{ev} A_c^2(\theta_c) \cos \phi_c \sin \phi_c & \sum_{ev} A_c^2(\theta_c) \sin^2 \phi_c \end{pmatrix} \begin{pmatrix} P_n \\ P_s \end{pmatrix} \quad (2-8)$$

Therefore for the $(\phi+\pi)$ test we need five sums, $\sum_{ev} \cos \phi_c$, $\sum_{ev} \sin \phi_c$, $\sum_{ev} \cos^2 \phi_c$, $\sum_{ev} \sin^2 \phi_c$, $\sum_{ev} \cos \phi_c \sin \phi_c$.

2.8 Coordinate Systems

For a complete description of the scattering process we define five coordinate systems. The first system is $(\hat{S}_0, \hat{N}_0, \hat{L}_0)$ which defines the beam polarization. The second coordinate is the coordinate system which defines the incident and scattered particles $(\hat{S}, \hat{N}, \hat{L})$ where $\hat{L}$ is in the same direction as $\hat{L}_0$, $\hat{N} = \hat{K}_i \times \hat{K}_f / |\hat{K}_i \times \hat{K}_f|$ is normal to $\hat{L}$ and $\hat{S} = \hat{N} \times \hat{L}$. The third coordinate system is the scattered particle frame $(\hat{S}', \hat{N}', \hat{L}')$ where $\hat{N}' = \hat{N}$, $\hat{L}'$ along $\hat{K}_f$ and $\hat{S}' = \hat{N}' \times \hat{L}'$. The fourth coordinate frame is $(\hat{S}'', \hat{N}'', \hat{L}'')$ where $\hat{S}'$ is rotated into the direction of dipole magnetic field $\hat{S}''$, and $\hat{L}'$ intact. The last coordinate frame is defined by the focal plane coordinates $(\hat{S}_{fp}, \hat{N}_{fp}, \hat{L}_{fp})$. These coordinates are shown in fig.11.

2.9 Spin Observables

The quantity which we measure is the polarization at the focal plane. Therefore we need to find a relationship between the focal plane coordinates and the incident beam coordinates¹²⁾. If the polarization of incoming beam in $(\hat{S}_0, \hat{N}_0, \hat{L}_0)$ system is,

Fig 11: Coordinate system

$$\bar{P}_B^0 = \begin{pmatrix} P_S^0 \\ P_N^0 \\ P_1^0 \end{pmatrix} \quad (2-9)$$

Then the polarization in the incident particle coordinate system will be obtained by rotating by angle ϕ about $\hat{L}$,

$$\bar{P}_B = \begin{pmatrix} P_S \\ P_N \\ P_1 \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \bar{P}_B^0 \quad (2-10)$$

The polarization of the scattered particle in the scattered particle system is given by,

$$\bar{P}' = \begin{pmatrix} P_{S'} \\ P_{N'} \\ P_{1'} \end{pmatrix} = \left[\begin{pmatrix} D_{SS} & 0 & D_{1S} \\ 0 & D_{NN} & 0 \\ D_{S1} & 0 & D_{11} \end{pmatrix} \bar{P}_B + \begin{pmatrix} 0 \\ P(\theta) \\ 0 \end{pmatrix} \right] \frac{1}{1 + P_N A_y} \quad (2-11)$$

where D_{SS} , D_{1S} , D_{NN} , D_{S1} , D_{11} are Wolfenstein parameters, $P(\theta)$ is induced polarization and A_y is analyzing power.

Now by rotating $\hat{S}'$ into the direction of dipole magnetic field we get*

$$\vec{P}'' = \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \vec{P}' \quad (2-12)$$

For the last coordinate system if we assume that the precession in dipoles takes place only about the $\hat{S}''$ axis, we get,

$$\vec{P}_{fp} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \chi & \sin \chi \\ 0 & -\sin \chi & \cos \chi \end{pmatrix} \vec{P}'' \quad (2-13)$$

where χ is the precession angle of the proton polarization with respect to its momentum and is given by $\chi = \gamma(g/2-1) \alpha = 269\gamma$ in which γ is the Lorentz factor, $g/2$ is the proton magnetic moment and α is the bending angle in the HRS ($\sim 150^\circ$).

* Since the vertical acceptance of the HRS is about 2° , it can be shown that the rotation angle for $\hat{S}'$ is equal to rotation angle ϕ in eq.(2-10)¹¹).

By measuring the $\hat{N}$ and $\hat{S}$ components of polarization at the focal plane for both normal and reverse polarizations of the incident proton beam, and by generating the proper histograms, we are able to calculate all the spin observables.

2.9.1 Calculation of D_{nn}

As an example we look at the complete calculation of the D_{nn} and the polarization. We start with n-type beam and follow the development of section 2.9.

$$\vec{P}_B^0 = \begin{pmatrix} 0 \\ P_n \\ 0 \end{pmatrix} \quad (2-14)$$

$$\vec{P}_B = \begin{pmatrix} \sin \phi \\ \cos \phi \\ 0 \end{pmatrix} P_n \quad (2-15)$$

$$\vec{P}' = \begin{pmatrix} P_n D_{ss} \sin \phi \\ P_n D_{nn} \cos \phi + P(\theta) \\ P_n D_{sl} \sin \phi \end{pmatrix} \frac{1}{1 + P_n A_y \cos \phi} \quad (2-16)$$

$$\vec{P}'' = \begin{pmatrix} P_n D_{ss} \cos \phi \sin \phi + [P_n D_{nn} \cos \phi + P(\theta)] \sin \phi \\ P_n D_{ss} \sin^2 \phi + [P_n D_{nn} \cos \phi + P(\theta)] \cos \phi \\ P_n D_{sl} \sin \phi \end{pmatrix} \frac{1}{1 + P_n A_y \cos \phi} \quad (2-17)$$

$$\vec{P}_{fp} = \begin{pmatrix} P_n D_{ss} \cos \phi \sin \phi + [P_n D_{nn} \cos \phi + P(\theta)] \sin \phi \\ ([P_n D_{ss} \sin^2 \phi + [P_n D_{nn} \cos \phi + P(\theta)] \cos \phi] \cos \chi + P_n D_{sl} \sin \phi \sin \chi \\ - ([P_n D_{ss} \sin^2 \phi + [P_n D_{nn} \cos \phi + P(\theta)] \cos \phi] \sin \chi + P_n D_{sl} \sin \phi \cos \chi) \end{pmatrix} \cdot \frac{1}{1 + P_n A_y \cos \phi} \quad (2-18)$$

Data are taken for both normal and reverse beam polarizations P_n^+ and P_n^- , respectively. We assume $P_n^+ = -P_n^- = P_n$.

Since the asymmetry for the n-type beam is in the horizontal plane, the distribution of ϕ is the same for normal and reverse beam:

$$\langle \{ \cos \phi \} \rangle^- = \langle \{ \cos \phi \} \rangle^+$$

$$\langle \{ \sin \phi \} \rangle^- = \langle \{ \sin \phi \} \rangle^+$$

Therefore we have

$$\left\{ \begin{aligned} P_{nfp}^+ &= [P_n D_{ss} \sin^2 \phi \cos \chi + P_n D_{nn} \cos \phi \cos \chi + P(\theta) \cos \phi \cos \chi \\ &\quad + P_n D_{sl} \sin \phi \sin \chi] * \frac{1}{1 + P_n A_y \cos \phi} \\ P_{nfp}^- &= [-P_n D_{ss} \sin^2 \phi \cos \chi + P_n D_{nn} \cos \phi \cos \chi + P(\theta) \cos \phi \cos \chi \\ &\quad - P_n D_{sl} \sin \phi \sin \chi] * \frac{1}{1 - P_n A_y \cos \phi} \end{aligned} \right. \quad (2-19)$$

and:

$$(P_{nfp}^+ - P_{nfp}^-) + P_n A_y \cos \phi (P_{nfp}^+ + P_{nfp}^-) = 2 P_n D_{ss} \sin^2 \phi \cos \chi + 2 P_n D_{nn} \cos \phi \cos \chi + 2 P_n D_{sl} \sin \phi \sin \chi \quad (2-20)$$

thus:

$$2 P_n D_{nn} \cos \phi \cos \chi = (P_{nfp}^+ - P_{nfp}^-) + P_n A_y \cos \phi (P_{nfp}^+ + P_{nfp}^-) - 2 P_n D_{ss} \sin^2 \phi \cos \chi - 2 P_n D_{sl} \sin \phi \sin \chi \quad (2-21)$$

After averaging over angles, we get

$$D_{nn} = D_0 - \frac{1}{\langle \cos \phi \rangle} [D_{ss} \langle \sin^2 \phi \rangle + D_{sl} \langle \sin \phi \rangle \langle \tan \chi \rangle] \quad (2-22)$$

where,

$$D_0 = \frac{1}{P_n \langle \cos \phi \rangle \langle \cos \chi \rangle} \left\{ \frac{P_{nfp}^+ - P_{nfp}^-}{2} + P_n A_y \langle \cos \phi \rangle \frac{P_{nfp}^+ + P_{nfp}^-}{2} \right\}$$

If we add Eqs. (2-19) we get $P(\theta)$,

$$P(\theta) = \frac{1}{\langle \cos \phi \rangle \langle \cos \chi \rangle} \left\{ \frac{P_{nfp}^+ + P_{nfp}^-}{2} + P_n A_y \langle \cos \phi \rangle \frac{P_{nfp}^+ - P_{nfp}^-}{2} \right\} \quad (2-23)$$

In D_{nn} in eq.(2-22), the correction term due to $\langle \sin \phi \rangle$ is either vanish or small, due to the fact that these corrections are important at small angles, where D_{s1} is generally small. The $\langle \tan \chi \rangle$ term will become important when n at the target precesses into l at the focal plane, which will not happen in our case (800 MeV)¹²⁾. For $P(\theta)$ if there is a large instrumental asymmetry, that will effect in the first term in eq.(2-23), which does not appear in our data.

2.10 Differential Cross Section and Analyzing Power

By measuring yields for normal and reverse beam we will be able to calculate the differential cross section and the analyzing power.

The analyzing power is calculated using the following formula

$$A_y = \frac{1}{P_B} \left[\frac{Y_N - Y_R}{Y_N + Y_R} \right], \quad (2-24)$$

where Y_N , Y_R are corrected yields for normal and reverse beam and P_B is the incident beam polarization. This formula is true if we assume that the polarization for normal and reverse beams are equal. If the polarizations for normal and reverse are not equal then we have the following relation¹³⁾,

$$A_y = \frac{2\epsilon}{P_N - P_R - \epsilon(P_N + P_R)}, \quad (2-25)$$

where,

$$\epsilon = \frac{Y_N - Y_R}{Y_N + Y_R}.$$

The relative cross section is calculated by the following formula,

$$\frac{d\sigma}{d\Omega} = \frac{\frac{Y_N^* \times CFN}{(ERO4)_N} + \frac{Y_R^* \times CFR}{(ERO4)_R}}{2}, \quad (2-26)$$

where Y_N^* , Y_R^* are the yields for normal and reverse polarizations, $(ERO4)_N$, $(ERO4)_R$ are the relative incident beam currents for normal and reverse beams, and CFN, CFR are correction factors for normal and reverse polarizations,

$$CF = \frac{1}{\text{computer live time} \times \text{chambers eff.}}$$

Since we could only measure the relative incident beam currents, we use proton proton scattering with CH_2 target to normalize the cross section. If the relative cross section for $p\text{-}^3\text{He}$ is σ_{He}^r , and if the relative cross section for P-P is σ_{pp}^r and further, if the absolute cross section for P-P is σ_{pp}^a , then the absolute cross section for $p\text{-}^3\text{He}$ will be

$$\sigma_{\text{He}}^a = \sigma_{\text{He}}^r \times \frac{\sigma_{pp}^a}{\sigma_{pp}^r}. \quad (2-27)$$

In our cross section measurement for P-P scattering only normal beam was available (for the normalization run, the polarization of the beam did not switch from up to down polarization). Therefore we used the relation given below to calculate absolute cross section. If the corrected yield for normal beam in P-P scattering is Y_{pp}^+ , then

$$Y_{pp}^+ = N_{pp}^t \sigma_{pp} (1 + P_B A_y) N_{pB}^{PP} \Delta\Omega_{pp}, \quad (2-28)$$

where N_{pp}^t is the number of particles in the target, P_B is the beam polarization, A_y is the analyzing power, N_{pB}^{PP} is the number of particles in the beam, and $\Delta\Omega_{pp}$ is the solid angle.

We can write the same formula for P-³He scattering where Y_{He} is now the average of normal and reverse beam polarizations,

$$Y_{He} = N_{He}^t \sigma_{He}^r N_{PB}^{He} \Delta\Omega_{He}. \quad (2-29)$$

Now by using Eq. (2-27) we have

$$\sigma_{He}^a = \frac{\frac{Y_{He}}{N_{PB}^{He}}}{\frac{Y_{pp}^a}{N_{PB}^{pp}(1+P_B A_y)}} \times \frac{N_{pp}^t}{N_{He}^t} \times \sigma_{pp}^a. \quad (2-30)$$

The quantity,

$$\frac{N_{pp}^t}{N_{He}^t} = \frac{2 \times T_{CH_2} \times M_{He}}{T_{He} \times M_{CH_2}},$$

is the ratio of the number of proton nuclei per unit area in CH₂ to the number of the ³He nuclei per unit area of the ³He target. The quantity T is the target thickness in g/cm² and M is the atomic mass.

The thickness of the ³He target is 1.318 ± .015 cm (with 5% bulging)¹⁴⁾. Variation of the pressure during the experiment was between 152-156 mmHg which by using fig. 12 gives a density of 77.7 ± .07 mg/cm³. By using this value for density we get .102 ± .001 gr/cm² for the target thickness. The thickness of CH₂ target was

Fig 12: Density and pressure of the target versus T .

$.148 \pm .005$ gr/cm². For A_y and σ_{pp}^a we used experimental value taken from the Arndt phase shift program¹⁵⁾, $A_y = .247 \pm .015$, $\sigma_{pp}^a = 13.5 \pm .33$ mb/sr. The absolute normalization factor was found to be $2.74 \pm .19$.

All the quantities that was used in the calculation of cross section and analyzing power have uncertainties associated with them. The uncertainties in the beam polarizations are about 1% (except for the normalization run in which the uncertainty is about 2%). The relative uncertainties in incident beam currents are about 1%, and uncertainties for the yields are about $\pm 1\%$. In calculating the absolute cross section the errors are mostly from uncertainty in measuring the thickness of the targets (2%), errors in experimental A_y (6%), and absolute cross section (2%) for P-P scattering.

The experimental results are shown in Tables 2.2 - 2.9 and figs.

13 - 20 .

Table 2.2: Analyzing power for P-³He scattering at T=800 MeV

$-t(\text{GeV}/c)^2$	$\Theta_{\text{lab}}(\text{degree})$	A_y
<hr/>		
0.013	4.5	$0.278 \pm .008$
0.016	4.9	$0.305 \pm .009$
0.018	5.2	$0.321 \pm .009$
0.020	5.6	$0.323 \pm .010$
0.023	5.9	$0.343 \pm .012$
0.037	7.5	$0.386 \pm .008$
0.040	7.9	$0.415 \pm .009$
0.044	8.2	$0.407 \pm .009$
0.047	8.6	$0.417 \pm .010$
0.051	8.9	$0.426 \pm .010$
0.071	10.5	$0.437 \pm .008$
0.076	10.9	$0.429 \pm .008$
0.081	11.2	$0.452 \pm .009$
0.086	11.6	$0.454 \pm .010$
0.091	11.9	$0.437 \pm .010$
0.118	13.5	$0.412 \pm .008$
0.123	13.9	$0.417 \pm .008$
0.130	14.2	$0.406 \pm .009$
0.136	14.6	$0.387 \pm .010$
0.142	14.9	$0.371 \pm .011$
0.166	16.2	$0.317 \pm .007$

0.172	16.5	0.277 ± .008
0.179	16.9	0.260 ± .009
0.187	17.2	0.251 ± .010
0.194	17.6	0.186 ± .012
0.196	17.7	0.190 ± .008
0.204	18.0	0.134 ± .009
0.212	18.4	0.100 ± .010
0.219	18.7	0.050 ± .012
0.230	19.2	0.014 ± .011
0.238	19.5	0.009 ± .011
0.246	19.9	-0.023 ± .012
0.254	20.2	0.020 ± .013
0.262	20.6	0.047 ± .013
0.265	20.7	0.068 ± .011
0.274	21.0	0.146 ± .011
0.282	21.4	0.220 ± .011
0.291	21.7	0.301 ± .011
0.300	22.1	0.351 ± .011
0.309	22.4	0.406 ± .010
0.321	22.9	0.480 ± .010
0.330	23.2	0.477 ± .010
0.339	23.6	0.507 ± .010
0.349	23.9	0.510 ± .010
0.362	24.4	0.547 ± .010
0.371	24.7	0.556 ± .010

0.381	25.1	0.569 ± .009
0.391	25.4	0.549 ± .010
0.405	25.9	0.565 ± .013
0.415	26.2	0.553 ± .013
0.425	26.6	0.560 ± .013
0.435	26.9	0.568 ± .013
0.495	28.9	0.524 ± .014
0.506	29.2	0.529 ± .014
0.517	29.6	0.512 ± .014
0.528	29.9	0.481 ± .016
0.581	31.5	0.489 ± .016
0.592	31.9	0.511 ± .016
0.604	32.2	0.467 ± .016
0.615	32.6	0.456 ± .017
0.626	32.9	0.447 ± .018
0.682	34.5	0.450 ± .021
0.694	34.9	0.450 ± .021
0.706	35.2	0.424 ± .023
0.718	35.6	0.396 ± .024
0.730	35.9	0.378 ± .025

Table 2.3: Cross section for $P-^3\text{He}$ at $T=800$ MeV

$-t(\text{GeV}/c)^2$	$\theta_{\text{lab}}(\text{degree})$	$\frac{d\sigma}{dt} \text{ (mb/}(\text{GeV}/c)^2\text{)}$
0.013	4.5	373.36 ± 43.71
0.016	4.9	342.03 ± 40.19
0.018	5.2	313.74 ± 36.97
0.020	5.6	285.26 ± 33.66
0.023	5.9	260.49 ± 30.84
0.037	7.5	181.22 ± 18.01
0.040	7.9	161.85 ± 15.98
0.044	8.2	149.04 ± 14.86
0.047	8.6	132.05 ± 13.24
0.051	8.9	115.62 ± 11.58
0.071	10.5	54.015 ± 4.393
0.076	10.9	47.370 ± 3.897
0.081	11.2	41.674 ± 3.426
0.086	11.6	36.263 ± 2.973
0.091	11.9	31.232 ± 2.589
0.116	13.5	16.423 ± 1.314
0.123	13.9	13.480 ± 1.087
0.130	14.2	11.107 ± 0.904
0.136	14.6	8.7335 ± 0.721
0.142	14.9	6.5501 ± 0.547
0.166	16.2	3.1042 ± 0.237

0.172	16.5	2.6105 ± 0.200
0.179	16.9	2.0979 ± 0.161
0.187	17.2	1.7372 ± 0.134
0.194	17.6	1.3385 ± 0.104
0.196	17.7	1.0051 ± 0.092
0.204	18.0	1.0062 ± 0.077
0.212	18.4	0.7974 ± 0.061
0.219	18.7	0.6360 ± 0.049
0.230	19.2	0.3892 ± 0.030
0.238	19.5	0.3417 ± 0.026
0.246	19.9	0.2943 ± 0.023
0.254	20.2	0.2658 ± 0.021
0.262	20.6	0.2468 ± 0.019
0.265	20.7	0.2487 ± 0.019
0.274	21.0	0.2478 ± 0.019
0.282	21.4	0.2440 ± 0.019
0.291	21.7	0.2468 ± 0.019
0.300	22.1	0.2573 ± 0.020
0.309	22.4	0.2658 ± 0.021
0.321	22.9	0.2563 ± 0.020
0.330	23.2	0.2658 ± 0.020
0.339	23.6	0.2753 ± 0.021
0.349	23.9	0.2724 ± 0.021
0.362	24.4	0.2563 ± 0.020
0.371	24.7	0.2705 ± 0.021

0.381	25.1	0.2810 ± 0.022
0.391	25.4	0.2715 ± 0.021
0.405	25.9	0.2421 ± 0.019
0.415	26.2	0.2297 ± 0.018
0.425	26.6	0.2193 ± 0.017
0.435	26.9	0.2050 ± 0.016
0.495	28.9	0.1699 ± 0.013
0.506	29.2	0.1443 ± 0.011
0.517	29.6	0.1414 ± 0.011
0.528	29.9	0.1395 ± 0.011
0.581	31.5	0.1044 ± 0.008
0.592	31.9	0.0864 ± 0.007
0.604	32.2	0.0769 ± 0.006
0.615	32.6	0.0712 ± 0.006
0.626	32.9	0.0683 ± 0.006
0.682	34.5	0.0494 ± 0.004
0.694	34.9	0.0456 ± 0.004
0.706	35.2	0.0418 ± 0.003

Table 2.4: D_{ss} for $p\text{-}^3\text{He}$ scattering at $T=800$ MeV

$-t(\text{GeV}/c)^2$	$\theta_{lab}(\text{degree})$	D_{ss}
0.016	5.0	.972 $\pm$.025
0.041	8.0	.891 $\pm$.020
0.078	11.0	.859 $\pm$.022
0.125	14.0	.872 $\pm$.022
0.183	17.0	.843 $\pm$.026
0.250	20.0	.507 $\pm$.036
0.287	21.5	.262 $\pm$.038
0.326	23.0	.369 $\pm$.028
0.410	26.0	.494 $\pm$.027
0.501	29.0	.544 $\pm$.037
0.599	32.0	.593 $\pm$.028
0.702	35.0	.496 $\pm$.069

Table 2.5: D_{sl} for $p\text{-}^3\text{He}$ scattering at $T=800$ MeV

$-t(\text{GeV}/c)^2$	$\theta_{lab}(\text{degree})$	D_{sl}
0.016	5.0	$-.074 \pm .035$
0.041	8.0	$-.150 \pm .028$
0.078	11.0	$-.230 \pm .031$
0.125	14.0	$-.336 \pm .029$
0.183	17.0	$-.545 \pm .033$
0.250	20.0	$-.534 \pm .044$
0.287	21.5	$-.559 \pm .046$
0.326	23.0	$-.590 \pm .033$
0.410	26.0	$-.573 \pm .030$
0.501	29.0	$-.571 \pm .040$
0.599	32.0	$-.583 \pm .030$
0.702	35.0	$-.627 \pm .071$

Table 2.6: D_{11} for $p\text{-}^3\text{He}$ scattering at $T=800$ MeV

$-t(\text{GeV}/c)^2$	$\Theta_{\text{lab}}(\text{degree})$	D_{11}
0.016	5.0	$.897 \pm .033$
0.041	8.0	$.817 \pm .029$
0.078	11.0	$.780 \pm .029$
0.125	14.0	$.771 \pm .028$
0.183	17.0	$.742 \pm .028$
0.250	20.0	$.524 \pm .044$
0.287	21.5	$.228 \pm .044$
0.326	23.0	$.543 \pm .030$
0.599	32.0	$.527 \pm .029$
0.702	35.0	$.582 \pm .037$

Table 2.7: D_{1s} for $p\text{-}^3\text{He}$ scattering at $T=800$ MeV

$-t(\text{GeV}/c)^2$	$\theta_{\text{lab}}(\text{degree})$	D_{1s}
0.016	5.0	.098 $\pm$.023
0.041	8.0	.191 $\pm$.020
0.078	11.0	.271 $\pm$.029
0.125	14.0	.390 $\pm$.021
0.183	17.0	.485 $\pm$.021
0.250	20.0	.598 $\pm$.036
0.287	21.5	.586 $\pm$.036
0.326	23.0	.522 $\pm$.029
0.410	26.0	.545 $\pm$.028
0.501	29.0	.589 $\pm$.028
0.599	32.0	.640 $\pm$.028
0.702	35.0	.741 $\pm$.036

Table 2.8: D_{nn} for p - ^3He scattering at $T=800$ MeV

$-t(\text{GeV}/c)^2$	$\Theta_{\text{lab}}(\text{degree})$	D_{nn}
0.016	5.0	$.961 \pm .036$
0.041	8.0	$1.015 \pm .032$
0.078	11.0	$1.026 \pm .033$
0.125	14.0	$.987 \pm .033$
0.183	17.0	$.978 \pm .034$
0.250	20.0	$.965 \pm .041$
0.287	21.5	$.855 \pm .058$

Table 2.9: Polarization for P-³He scattering at T=800 MeV

$-t(\text{GeV}/c)^2$	$\theta_{\text{lab}}(\text{degree})$	P
<hr/>		
0.016	5.0	.353 $\pm$.045
0.041	8.0	.440 $\pm$.041
0.078	11.0	.501 $\pm$.043
0.125	14.0	.419 $\pm$.043
0.183	17.0	.328 $\pm$.053
0.250	20.0	.020 $\pm$.063
0.287	21.5	.309 $\pm$.095
0.326	23.0	.478 $\pm$.066
0.410	26.0	.538 $\pm$.073
0.501	29.0	.518 $\pm$.106
0.599	32.0	.434 $\pm$.112
0.702	35.0	.154 $\pm$.319

Fig 13: D_{SS} at 800 MeV

Fig 14: D_{SL} at 800 Mev

Fig 15: D_{LL} at 800 Mev

Fig 16: D_{LS} at 800 Mev

Fig 17: D_{NN} at 800 Mev

Fig. 18: Polarization at 800 MeV

Fig 19: Analyzing power at 800 Mev

Fig 20: Cross section at 800 Mev

CHAPTER 3

In this chapter we compare the form factors of p - ^3He scattering for two different wave functions, and the effect of S and D parts of the wave functions on the form factors. We also compare the observables for these two wave functions in single scattering approximation.

3.1 Comparison of Two Wave Functions used in our calculations

We are going to use two different wave functions in our calculation. One is the Gibson wave function¹⁾ which is defined in the following form,

$$S : \begin{cases} N_1 R e^{-\gamma R/2} & R \leq R_0 = 4.5, & \gamma = 1.86 \\ N_2 \frac{1}{R} e^{-\delta R/2} & R > R_0 = 4.5, & \delta = .97 \end{cases}$$

(3-1a)

$$D : \begin{cases} f_6 = \alpha f_8 = N_6 \frac{1}{R} e^{-cR/2} & \alpha = \sqrt{2/15} \\ f_7 = \beta f_8 = N_7 \frac{1}{R} e^{-cR/2} & \beta = \sqrt{7/15} \\ f_8 = N_8 \frac{1}{R} e^{-cR/2} & c=1.5, \end{cases} \quad (3-1b)$$

where coefficients γ , δ , c , α , β were chosen by fitting the existing electron scattering data at that time, and the normalization factors are,

$$N_1 = \left\{ \frac{3\sqrt{3}\pi^3}{4} \left[\int_0^{4.5} R^7 e^{-\gamma R} dR + R_0^4 e^{-4} \int_{4.5}^{\infty} R^3 e^{-\delta R} dR \right] \right\}^{-1/2}, \quad (3-2a)$$

$$N_2 = N_1 R_0^2 e^{-2}, \quad (3-2b)$$

$$N_6 = \left[\frac{105\sqrt{3}\pi^3}{16} \int R^{13} f_6^2 dR \right]^{-1/2}, \quad (3-2c)$$

$$N_7 = \left[\frac{15\sqrt{3}\pi^3}{8} \int R^{13} f_7^2 dR \right]^{-1/2}, \quad (3-2d)$$

$$N_8 = \left[\frac{7\sqrt{3}\pi^3}{8} \int R^{13} f_8^2 dR \right]^{-1/2}. \quad (3-2e)$$

The S, and D parts of Gibson wave function are shown in figure 1.

Fig 1a: The S part of the Gibson wave function.

Fig 1b: The D parts of the Gibson wave function.

Fig 2a: The S part of the Friar wave function.

Fig 2b: The D parts of the Friar wave function.

The second wave function that we are going to use is Friar et al wave function²⁾. This wave function is the solution of Faddeev eqs. in configuration space using Reid soft-core potential.(fig.2).

3.2 Results For Form Factors

In this section we compare the form factors corresponding to the two set of wave functions. We also investigate the contribution of the S, and D parts of the wave functions in the form factors. By looking at each form factor individually (figs.3,4), we notice the following points: in all the form factors there is significant contribution from the D part of the wave function for the q values of the order 1 fm^{-1} ; the contribution of the D part of the wave function in L_1, L_2 , continue up to $q = 2 \text{ fm}^{-1}$; the S part of the wave function does not have any contribution in the form factor K_1 ; there is a large interference between the S, and D parts of the wave function in the form factors K_2, L . The comparison between two of the six form factors for two sets of wave functions are shown in fig.5. As can be seen from fig.5, the Gibson wave function give deepest slope than the Friar wave function in form factor G_2 ; in the form factor K_2 the Friar wave function shows more structure than the Gibson, which comes from interference between the S and D parts of the wave function. For the form factor G_1 , the situation is similar to the form factor G_2 . For the form factor K_1 , using the Gibson wave

Fig 3a: Contribution of the S and total S and D parts of the Gibson wave function in form factor G_1 .

Fig 3b: Contribution of the S and total S and D parts of the Gibson wave function in form factor G_2 .

Fig 3c: Contribution of the D part of the Gibson wave function in form factor K_1 (there is no S contribution).

Fig 3d: Contribution of the S and total S and D parts of the Gibson wave function in form factor K_2 .

Fig 3e: Contribution of the S and total S and D parts of the Gibson wave function in form factor L_1 .

Fig 3f: Contribution of the S and total S and D parts of the Gibson wave function in form factor L_2 .

Fig 4a: Contribution of the S and total S and D parts of the Friar wave function in form factor G_1 .

Fig 4b: Contribution of the S and total S and D parts of the Friar wave function in form factor G_2 .

Fig 4c: Contribution of the D part of the Friar wave function in form factor K_1 (there is no S contribution).

Fig 4d: Contribution of the S and total S and D parts of the Friar wave function in form factor K_2 .

Fig 4e: Contribution of the S and total S and D parts of the Friar wave function in form factor L_1 .

Fig 4f: Contribution of the S and total S and D parts of the Friar wave function in form factor L_2 .

Fig 5a: The form factor G_2 for both wave functions.

Fig 5b: The form factor K_2 for both wave functions.

function gives smaller values at minimum than using the Friar wave function. In the form factor L_2 the Friar wave function gives larger values (about two times) than the Gibson wave function.

3.3 Results For Observables

By using the form factors for two sets of the wave functions, and eqs.(1-70) we can calculate the observables. The results of the calculations are shown in fig.6.

We should mention that since our data for D_{nn} goes only up to $-t=3 \text{ (GeV/c)}^2$, for calculating D_0, D_x, D_y, D_z , (eqs.1-27-30), at larger t we assumed the value of $.9 \pm .1$ for D_{nn} . In all our calculations we only used single scattering approximation. In order to complete the theoretical analysis we should include double and triple scattering terms in our calculations.

In all the above calculations we assumed mixture of 94% S, and 6% D. In fig.7 we show some of the results with mixture of 90% S, and 10% D. As can be seen in fig.7, at this stage of calculation the results are not very sensitive to the mixture of S, and D in above range.

Fig 6a: The solid line is using the Friar wave function, and the dashed line, is using the Gibson wave function.

Fig 6b: The solid line is using the Friar wave function, and the dashed line is using the Gibson wave function.

Fig 6c: The solid line is using the Friar wave function, and the dashed line is using the Gibson wave function.

Fig 6d: The solid line is using the Friar wave function, and the dashed line is using the Gibson wave function.

Fig 6e: The solid line is using the Friar wave function, and the dashed line is using the Gibson wave function.

Fig 6f: The solid line is using the Friar wave function, and
the dashed line is using the Gibson wave function.

Fig 6g: The solid line is using the Friar wave function, and the dashed line is using the Gibson wave function.

Fig 6h: The solid line is using the Friar wave function, and the dashed line is using the Gibson wave function.

Fig 7a: Using the Friar wave function with two
different mixtures of the S, and D.

Fig 7b: Using the Friar wave function with two different mixtures of the S, and D.

3.4 Conclusions and Outline for the Further Calculations

As we can see from the figures, for both sets of the wave functions the D part has significant contribution to form factors and observables. But since the mixture of S and D shows its signature at $-t$ larger than $.2(\text{GeV}/c)^2$, where double and triple scatterings are important³⁾, we can not make a definite conclusion about the percentage of S, and D in the wave function (fig.7).

In order to complete the theoretical analysis we should include double and triple scattering terms in our calculations. For double and triple scatterings from eq.(1-6) we have

$$T_2(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{k}_i, \vec{k}_f, \vec{p}) = \sum_{\substack{j=1 \\ k \neq j}}^3 t_j G_0 t_k, \quad (3-3)$$

where $\vec{r}_1, \vec{r}_2, \vec{r}_3$, are the spatial coordinates of the nucleons, $\vec{k}_i, \vec{k}_f$, are the initial and final momentums of the projectile, and $\vec{p}$ is the momentum of the projectile after the first scattering. For triple scattering we have,

$$T_3(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{k}_i, \vec{k}_f, \vec{p}_1, \vec{p}_2) = \sum_{\substack{j=1 \\ k \neq j \\ l \neq k \neq j}}^3 t_j G_0 t_k G_0 t_l, \quad (3-4)$$

where $\vec{p}_1$, and $\vec{p}_2$, are the momentums of projectile after first and second scatterings. From eq.(1-8) we substitute for t_j in terms of M_j , and integrate over momentum we get

$$\tau_2(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{k}_i, \vec{k}_f) = \int \sum_{\substack{j=1 \\ k \neq j}}^3 M_j e^{i\vec{q}_1 \cdot \vec{r}_j} G_0(p) M_k e^{i\vec{q}_2 \cdot \vec{r}_k} d^3p, \quad (3-5)$$

where ,

$$\begin{cases} \vec{q}_1 = \vec{k}_i - \vec{p}, \\ \vec{q}_2 = \vec{p} - \vec{k}_f. \end{cases}$$

$$\begin{aligned} \tau_3(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{k}_i, \vec{k}_f) = \int \sum_{\substack{j=1 \\ k \neq j \\ l \neq k=j}}^3 M_j e^{i\vec{q}_1 \cdot \vec{r}_j} G_0(p_1) M_k e^{i\vec{q}_2 \cdot \vec{r}_k} G_0(p_2) M_l \\ \times e^{i\vec{q}_3 \cdot \vec{r}_l} d^3p_1 d^3p_2, \end{aligned} \quad (3-6)$$

where,

$$\begin{cases} \vec{q}_1 = \vec{k}_i - \vec{p}_1, \\ \vec{q}_2 = \vec{p}_1 - \vec{p}_2, \\ \vec{q}_3 = \vec{p}_2 - \vec{k}_f. \end{cases}$$

or for scattering amplitudes we have

$$F_2 = \langle \psi(1,2,3) | \tau_2(\vec{r}_1, \vec{r}_2, \vec{r}_3) | \psi(1,2,3) \rangle, \quad (3-7)$$

and,

$$F_3 = \langle \psi(1,2,3) | \tau_3(\vec{r}_1, \vec{r}_2, \vec{r}_3) | \psi(1,2,3) \rangle, \quad (3-8)$$

The recent relativistic calculations have been very successful in fitting data for several cases^{4,5}). For double and triple scattering we use relativistic propagator*. The relativistic propagator G_0 , is defined in the following way

$$G_0(p) = \frac{\vec{p} + m}{k^2 - p^2 + i\epsilon} = \frac{P_0 \gamma_0 - \vec{p} \cdot \vec{\gamma} + m}{k^2 - p^2 + i\epsilon}, \quad (3-9)$$

where $\vec{\gamma}$, and γ_0 are defined in terms of Pauli spin operators, and unit matrix,

$$\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix},$$

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

* It should be mention that for single scattering, relativistic and nonrelativistic calculations give the same results, because the propagator G_0 is not involved in the calculation.

For defining N-N operator M_j we use two component reduction method. Starting with relativistic N-N operator H_{NN} , we have

$$H_{NN} = H_1 + H_2 \gamma_p^\mu \gamma_{t\mu} + H_3 \sigma_p^{\mu\nu} \sigma_{t\mu\nu} + H_4 \gamma_p^3 \gamma_t^\mu \gamma_p^5 \gamma_{t\mu} + H_5 \gamma_p^5 \gamma_t^5, \quad (3-10)$$

in the Breit frame we can write

$$\bar{U}(-\frac{\vec{\Delta}}{2}) H_{NN} U(\frac{\vec{\Delta}}{2}) = M_j = h_0(q) + \vec{h}(q) \cdot \vec{\sigma}_{tj} \quad (3-11)$$

where $\vec{\Delta}$ is the momentum of the target in the Breit frame, and U

the target nucleon spinor is define in the following way,

$$U = \left(\frac{m+E}{2m} \right)^{1/2} \begin{pmatrix} 1 \\ \frac{\vec{k} \cdot \vec{\sigma}_t}{m+E} \end{pmatrix} \chi,$$

and,

$$\bar{U} = U^\dagger \gamma^0,$$

The h_0 , h , are

$$h_0(q) = H_1 + H_2 \gamma_p^0 + H_3 (\vec{p} - \vec{k}_i) \cdot \vec{\alpha}_p,$$

$$\vec{h}(q) = \frac{i}{2m} H_2 (\vec{p} - \vec{k}_i) \times \vec{\gamma}_i + 2 H_3 \vec{\Sigma}_p + H_4 \gamma^5 \vec{\gamma}_p + H_5 \frac{(\vec{p} - \vec{k}_i)}{2m} \gamma^5$$

where $\vec{p}$ is the momentum of the projectile between successive scatterings, and $\sigma^{\mu\nu}$, $\vec{\alpha}$, $\vec{\Sigma}$, γ^5 are defined in terms of γ matrices*,

$$\vec{\alpha} = \gamma^0 \vec{\gamma},$$

$$\vec{\Sigma} = \vec{\alpha} \gamma^5,$$

$$\gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3,$$

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu].$$

We should mention that in this calculation we considered the target nucleons nonrelativistic, and the projectile nucleon relativistic.

Calculation for double and triple scattering using relativistic propagator is in progress.

* We are using the Bjorkin & Drell notation⁵⁾.

A-1: Sample Test File

```

701, 52, 62, 3,      , [166, 100]X643. TST 6 BINS FOR X-SECTION
BOX 1,                ; 001 PID1 (PROTON PART 1, FROM PHS1 VS. PHS2
BOX 2,                ; 002 PID2      "      "      2      "      "      "
GA 55, 50, 600,       ; 003 NORMAL
GA 56, 50, 600,       ; 004 REVERSE
GA 59, -25000, -12000, ; 0005 MBDTSTS SKIPPED (1 OF N EVNT, IFLAG=-
IDR 1, 2,             ; 006 PID = PID1 + PID2
AND 6, 5, 3,          ; 0007 "      "      "      * NOR
AND 6, 5, 4,          ; 008 "      "      "      * REV
END,                  ; END OF LOOP 1
GA 131, 10005, 10085, ; 009 X-PLANE DRIFT TIME CHAMBER 1
GA 132, 10005, 10085, ; 010 "      "      "      "      2
GA 133, 10005, 10085, ; 011 "      "      "      "      3
GA 134, 10005, 10085, ; 012 "      "      "      "      4
GA 135, 10005, 10085, ; 013 "      "      "      "      5
GA 136, 10005, 10085, ; 014 "      "      "      "      6
GA 137, 10005, 10085, ; 015 "      "      "      "      7
GA 138, 10005, 10085, ; 016 "      "      "      "      8
GA 171, 10005, 10085, ; 017 Y-PLANE DRIFT TIME CHAMBER 1
GA 172, 10005, 10085, ; 018 "      "      "      "      2
GA 173, 10005, 10085, ; 019 "      "      "      "      3
GA 174, 10005, 10085, ; 020 "      "      "      "      4
GA 175, 10005, 10085, ; 021 "      "      "      "      5
GA 176, 10005, 10085, ; 022 "      "      "      "      6
GA 177, 10005, 10085, ; 023 "      "      "      "      7
GA 178, 10005, 10085, ; 024 "      "      "      "      8
GA 241, 9980, 10020,  ; 025 DIFF CORR POSIT X12
GA 242, 9980, 10020,  ; 026 "      "      "      X34
GA 243, 9980, 10020,  ; 027 "      "      "      X56
GA 244, 9980, 10020,  ; 028 "      "      "      X78
GA 245, 9980, 10020,  ; 029 DIFF CORR POSIT Y12
GA 246, 9980, 10020,  ; 030 "      "      "      Y34
GA 247, 9980, 10020,  ; 031 "      "      "      Y56
GA 248, 9980, 10020,  ; 032 "      "      "      Y78
BOX 4,                ; 033 XANGL CUT (FROM THTCUT VS. XF)
BOX 5,                ; 034 CARBON BLOCK CUT (FROM DCLOSE VS. ZCLOSE
GA 233, 9850, 10150,  ; 035 ← .8 DEG THTSC
GA 251, 10785, 13578, ; 036 THETAC OK (4.5 DEG < THETAC <= 20.5 DEG)
GA 256, 10090, 10110, ; 037 CONE TEST PASSED
GA 257, 10090, 10110, ; 038 PHI+PI TEST PASSED
IGATE 1,              ; 039 INDIRECT GATE 1
IGATE 2,              ; 040 "      "      2
IGATE 3,              ; 041 INDIRECT GATE 3
IGATE 4,              ; 042 "      "      4
GA 251, 10785, 10960, ; 043 4 5-5 5 DEG THETAC OK

```

```

IGATE 13.          ; 044 GATE ON YTGT
GA 233.9950.9910.  ; 045 THTSC REGION 1
GA 233.9910.9970.  ; 046 THTSC REGION 2
GA 233.9970.10020. ; 047 " " 3
GA 233.10030.10090. ; 048 " " 4
GA 233.10090.10150. ; 049 " " 5
GA 233.9950.10150. ; 050 " " 6
AND 09.10.11.12.23.26. ; 051 CHMBRS 1-4 X OK
AND 13.14.15.16.27.29. ; 052 " 5-8 X OK
AND 17.18.19.20.29.30. ; 053 " 1-4 Y OK
AND 21.22.23.24.31.32. ; 054 " 5-8 Y OK
AND 51.53.          ; 055 CHMBRS 1-4 X;Y OK
AND 52.54.          ; 056 CHMBRS 5-8 X;Y OK
AND 55.56.          ; 057 ALL CHAMBERS OK
AND 57.6.           ; 058 " " " * PID
AND 58.5. -36.      ; 059 " " " *MBDTSTSSKPD*—THETA
AND 6.55.33.44.34. ; 060 STDGEV = PID * CHM1-4 * XANGL * YTGT * THTPH
AND 39.60.3.        ; 061 PEAK 1*STGDEV *NOR
AND 39.60.4.        ; 062 PEAK 1* STGDEV *REV
AND 60.45.3.        ; 063 THTSC *REGION 1*NOR
AND 60.45.4.        ; 064 " " "*REV
AND 60.46.3.        ; 065 " " 2*NOR
AND 60.46.4.        ; 066 " " "*REV
AND 60.47.3.        ; 067 " " 3*NOR
AND 60.47.4.        ; 068 " " "*REV
AND 60.48.3.        ; 069 " " 4*NOR
AND 60.48.4.        ; 070 " " "*REV
AND 60.49.3.        ; 071 " " 5*NOR
AND 60.49.4.        ; 072 " " "*REV
AND 60.50.3.        ; 073 " " 6*NOR
AND 60.50.4.        ; 074 " " "*REV
AND 7.55.           ; 075 PID * MBDTST SKIPPED * NOR * CHMBRS 1-4 X&Y
AND 8.55.           ; 076 PID * MBDTST SKIPPED * REV * CHMBRS 1-4 X&Y
END.                ; END OF LOOP 2
FINISH.             ; END OF TEST DESCRIPTOR FILE

```


A-2: Sample of the Histogram File.

```

(166,100)HRS663.HST
:
:OFF-LINE HISTOGRAMS FOR EXP 663 (CYCLE 35) (19-DEC-82 AA)
:MODIFIED AGAIN FOR N-TYPE! (FEB-27-83 AA)
:TO BE USED WITH TEST FILE (166,100)HRS663.TST
:
FILE=HRS663.DSP
FREE ALL
:
: HORIZ SCAT ANG IN CARBON. GATED BY PID * ALL CHMS OK * MBDTSTS SKIPD
H1=249.9000.11000.5
T1=63
:
: VERT SCAT ANG IN CARBON. GATED BY PID * ALL CHMS OK * MBDTSTS SKIPD
H2=250.9000.11000.5
T2=63
:
: DX56PRJ. GATED BY PID * ALL CHMS OK * MBDTSTS SKIPD
H3=237.9500.10500.10
T3=63
:
: DX78PRJ. GATED BY PID * ALL CHMS OK * MBDTSTS SKIPD
H4=238.9500.10500.10
T4=63
:
: DY56PRJ. GATED BY PID * ALL CHMS OK * MBDTSTS SKIPD
H5=239.9500.10500.10
T5=63
:
: DY78PRJ. GATED BY PID * ALL CHMS OK * MBDTSTS SKIPD
H6=240.9500.10500.10
T6=63
:
:
: MISSING MASS. GATED BY STD GOOD EVENT* NOR * -0.8 - 0 DEG THTSC
H7=234.9000.11000.5
T7=86
:
: MISSING MASS. GATED BY STD GOOD EVENT* NOR * 0 - 0.8 DEG THTSC
H8=234.9000.11000.5
T8=87
:
: MISSING MASS. GATED BY STD GOOD EVENT* REV * -0.8 - 0 DEG THTSC
H9=234.9000.11000.5
T9=88
:
: MISSING MASS. GATED BY STD GOOD EVENT* REV * 0 - 0.8 DEG THTSC
H10=234.9000.11000.5
T10=89
:
: MISSING MASS. GATED BY STAND GOOD EVENT * -.8 ,-.3 THTSCT * NOR*
H11=234.9000.11000.5
T11=90
:
: MISSING MASS. GATED BY STAND GOOD EVENT * -.8 ,-.3 THTSCT * REV
H12=234.9000.11000.5
T12=91
:
: MISSING MASS. GATED BY STAND GOOD EVENT * -.3 ,+.2 THTSCT * NOR*
H13=234.9000.11000.5
T13=92
:
: MISSING MASS. GATED BY STAND GOOD EVENT * -.3 ,+.2 THTSCT * REV

```

```

H14=234.9000.11000.5
T14=93
:
:MISSING MASS. GATED BY STAND GOOD EVENT * +.2 .+.8 THTSCT * NORM
H15=234.9000.11000.5
T15=94
:
:MISSING MASS. GATED BY STAND GOOD EVENT * +.2 .+.8 THTSCT * REV
H16=234.9000.11000.5
T16=93
:YTGT GATED GOOD FRONT CHAMBERS
H17=229.8000.12000.10
T17=59
:
:THETA SCAT GATED ON PEAK 1 * STND GOOD EVENT * NOR
H18=233.9500.10500.10
T18=71
:
:THETA SCAT GATED ON PEAK 1 * STND GOOD EVENT * REV
H19=233.9500.10500.10
T19=72
: PRECESSION ANGLE CHI FOR PEAK # 1 * +- .8 DEG THTSC * STGDEV
:
H20=266.11700.13000.10
T20=113
:
:MISSING MASS GATED BY STGDEV+- .8 DEG THTSC*NOR
H21=234.9000.11000.5
T21=112
:MISSING MASS GATED BY STGDEV +- .8 DEG THTSC *REV
:
H22=234.9000.11000.5
T22=113
:
:MISSING MASS GATED BG STGDEV
H23=234.9000.11000.5
T23=64
CLEAR ALL
:
:END OF HRS663 HISTOGRAM SPECIFICATIONS
:
@CNICAL.DAT
:
EXIT
:

```

APPENDIX B

In this appendix we list some useful formulas in calculation of single and double scattering.

B.1

If τ_1, τ_2 are isospins of particles 1, 2, and η_1, η_2 are isospin wave functions of ${}^3\text{He}$, we can have the following relations,

$$\langle \eta_1 | \tau_{1z} | \eta_1 \rangle = -\frac{1}{3}, \quad (\text{B-1})$$

$$\langle \eta_2 | \tau_{1z} | \eta_2 \rangle = 1, \quad (\text{B-2})$$

$$\langle \eta_1 | \tau_{1z} | \eta_2 \rangle = \langle \eta_2 | \tau_1 | \eta_1 \rangle = 0, \quad (\text{B-3})$$

$$\langle \eta_1 | \tau_{2z} | \eta_1 \rangle = \frac{2}{3}, \quad (\text{B-4})$$

$$\langle \eta_2 | \tau_{2z} | \eta_2 \rangle = 0, \quad (\text{B-5})$$

$$\langle \eta_1 | \tau_{2z} | \eta_2 \rangle = \langle \eta_2 | \tau_{2z} | \eta_1 \rangle = \frac{1}{\sqrt{3}}, \quad (\text{B-6})$$

$$\langle \eta_1 | \tau_1 \cdot \tau_2 | \eta_1 \rangle = -2, \quad (\text{B-7})$$

$$\langle \eta_2 | \tau_1 \cdot \tau_2 | \eta_2 \rangle = 0, \quad (\text{B-8})$$

$$\langle \eta_1 | \tau_1 \cdot \tau_2 | \eta_2 \rangle = \sqrt{3}, \quad (\text{B-9})$$

$$\langle \eta_1 | \vec{\tau}_1 \times \vec{\tau}_2 | \eta_1 \rangle = 0, \quad (\text{B-10})$$

$$\langle \eta_2 | \vec{\tau}_1 \times \vec{\tau}_2 | \eta_2 \rangle = 0, \quad (\text{B-11})$$

$$\langle n_1 | (\vec{\tau}_1 \times \vec{\tau}_2)_z | n_2 \rangle = -\frac{2i}{1\sqrt{3}}, \quad (\text{B-12})$$

$$\langle n_2 | (\vec{\tau}_1 \times \vec{\tau}_2)_z | n_1 \rangle = \langle n_1 | (\vec{\tau}_1 \times \vec{\tau}_2)_z | n_2 \rangle^* = \frac{2i}{\sqrt{3}}, \quad (\text{B-13})$$

$$\chi_2^\dagger D_K + D_K^\dagger \chi_2 = 0, \quad (\text{B-14})$$

$$D_i^\dagger D_K + D_K^\dagger D_i = C_{iK}, \quad (\text{B-15})$$

or

$$C_{11} = \frac{4}{3} S_1^2 + 4 (S_3^2 - S_2^2), \quad (\text{B-16})$$

$$C_{22} = \frac{4}{3} S_2^2 + 4 (S_3^2 - S_1^2), \quad (\text{B-17})$$

$$C_{33} = \frac{4}{3} S_3^2 + 4 (S_2^2 + S_1^2), \quad (\text{B-18})$$

$$C_{iK} = \frac{16}{3} S_i S_K, \quad i \neq K \quad (\text{B-19})$$

where

$$S_1 = y^2 - x^2, \quad S_2 = 2 \vec{y} \cdot \vec{x}, \quad \text{and} \quad S_3 = y^2 + x^2.$$

B.2 Angular Average Over Solid Angle Ω_y

For calculating the average over solid angle Ω_y the following two integrals has been used

$$\int d\Omega_y (\vec{a} \cdot \vec{y})(\vec{b} \cdot \vec{y})(\vec{c} \cdot \vec{y})(\vec{d} \cdot \vec{y}) = \frac{4\pi}{15} [(\vec{a} \cdot \vec{b})(\vec{c} \cdot \vec{d}) + (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) + (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})], \quad (\text{B-20})$$

$$\int d\Omega_y (\vec{a} \cdot \vec{y})(\vec{b} \cdot \vec{y}) = \frac{4\pi}{3} y^2 (\vec{a} \cdot \vec{b}), \quad (\text{B-21})$$

If $W_{ab} = (\hat{A} \cdot \vec{a})(\vec{J} \cdot \vec{b})$, where $\vec{a}$, $\vec{b}$ can be $\vec{x}$, or $\vec{y}$, $\hat{A}$ is a unit vector, and $\vec{J}$ is the spin of the target, then by using two integrals B-20, B-21 we get

$$\langle W_{xx} \rangle = W_{xx}, \quad (\text{B-22})$$

$$\langle W_{yy} \rangle = \frac{1}{6} (\vec{J} \cdot \hat{A}) (S_1 + S_3), \quad (\text{B-23})$$

$$\langle W_{xy} \rangle = 0, \quad (\text{B-24})$$

$$\langle W_{xx} S_2^2 \rangle = \frac{1}{3} W_{xx} (S_3^2 - S_1^2), \quad (\text{B-25})$$

$$\langle W_{yy} S_2 \rangle = 0, \quad (\text{B-26})$$

$$\langle W_{xy} S_2 \rangle = \frac{2}{3} W_{xx} (S_1 + S_3), \quad (\text{B-27})$$

$$\langle W_{yy} S_2^2 \rangle = \frac{1}{3} (\vec{J} \cdot \hat{A}) (S_3 - S_1) (S_3 + S_1)^2 + \frac{2}{15} (S_3 + S_1)^2 W_{xx}, \quad (B-28)$$

$$\langle W_{xy} S_2^3 \rangle = \frac{2}{5} (S_3 - S_1) (S_3 + S_1)^2 W_{xx}, \quad (B-29)$$

$$\langle W_{xx} S_2^2 \rangle = \frac{1}{3} (S_3^2 - S_1^2) W_{xx}, \quad (B-30)$$

$$\langle W_{yy} S_2^2 \rangle = \frac{2}{15} (S_1 + S_3)^2 (S_3 - S_1) (\vec{a} \cdot \hat{A}), \quad (B-31)$$

$$\langle W_{xy} S_2^3 \rangle = \frac{2}{5} (S_3 - S_1) (S_3 + S_1)^2 W_{xx}, \quad (B-32)$$

If $E_{ik} = D_i^\dagger (\vec{J} \cdot \hat{A}) D_k + D_k^\dagger (\vec{J} \cdot \hat{A}) D_i$, where $\vec{J}$ is spin of target, and $\hat{A}$ is a unit vector in x, y, or z direction, then we have

$$\langle E_{11} \rangle = \frac{8}{3} (S_3 - S_1) W_{xx} + \frac{8}{3} \left(\frac{1}{3} S_1 S_3 - \frac{1}{2} S_1^2 - \frac{1}{2} S_3^2 \right) (\vec{a} \cdot \hat{A}), \quad (B-33)$$

$$\langle E_{22} \rangle = \frac{56}{9} (S_3 + S_1) W_{xx} - \frac{68}{27} (S_3^2 - S_1^2) (\vec{a} \cdot \hat{A}), \quad (B-34)$$

$$\langle E_{33} \rangle = \langle E_{11} \rangle, \quad (B-35)$$

$$\langle E_{13} \rangle = -\frac{8}{3} (S_3 - S_1) W_{xx} + \left(\frac{4}{9} S_1^2 + \frac{4}{9} S_3^2 - \frac{8}{3} S_1 S_3 \right) (\vec{a} \cdot \hat{A}), \quad (B-36)$$

$$\langle E_{12} S_2 \rangle = \frac{8}{45} (S_3 + S_1) (-3 S_3 + 17 S_1) W_{xx}$$

$$+ \frac{8}{270} (S_3^2 - S_1^2)(-37 S_1 + 3 S_3)(\vec{a} \cdot \hat{A}), \quad (\text{B-37})$$

$$\begin{aligned} \langle E_{2,1} S_2 \rangle &= \frac{8}{45} (S_3 + S_1)(17 S_3 - 3 S_1) W_{xx} \\ &+ \frac{8}{270} (S_3^2 - S_1^2)(-37 S_3 + 3 S_1)(\vec{a} \cdot \hat{A}), \end{aligned} \quad (\text{B-38})$$

$$\begin{aligned} \langle E_{1,1} S_2^2 \rangle &= \frac{8}{15} (S_3 + S_1)(S_3^2 + \frac{11}{3} S_1^2 - 2 S_1 S_3) W_{xx} \\ &+ \frac{4}{3} (S_3^2 - S_1^2)(-\frac{5}{9} S_1^2 - \frac{1}{5} S_3^2 + \frac{2}{15} S_1 S_3)(\vec{a} \cdot \hat{A}), \end{aligned} \quad (\text{B-39})$$

$$\langle E_{2,2} S_2^2 \rangle = \frac{8}{3} (S_3^2 - S_1^2)(S_3 + S_1) W_{xx} - \frac{44}{45} (S_3^2 - S_1^2)^2 (\vec{a} \cdot \hat{A}), \quad (\text{B-40})$$

$$\begin{aligned} \langle E_{1,3} S_2^2 \rangle &= \frac{8}{15} (S_3 + S_1)(S_1^2 + \frac{11}{3} S_3^2 - 2 S_1 S_3) W_{xx} \\ &+ \frac{4}{3} (S_3^2 - S_1^2)(-\frac{5}{9} S_3^2 - \frac{1}{5} S_1^2 + \frac{2}{15} S_1 S_3)(\vec{a} \cdot \hat{A}), \end{aligned} \quad (\text{B-41})$$

$$\begin{aligned} \langle E_{1,1} S_2^2 \rangle &= \frac{8}{45} (S_3 + S_1)(-3 S_1^2 - 3 S_3^2 + 14 S_1 S_3) W_{xx} \\ &+ \frac{8}{45} (S_3^2 - S_1^2)(\frac{1}{2} S_1^2 + \frac{1}{2} S_3^2 - \frac{22}{3} S_1 S_3)(\vec{a} \cdot \hat{A}), \end{aligned} \quad (\text{B-42})$$

$$\langle S_2^2 \rangle = \frac{1}{3} (S_3^2 - S_1^2), \quad (\text{B-43})$$

$$\langle S_2^4 \rangle = \frac{1}{5} (S_3^2 - S_1^2)^2, \quad (\text{B-44})$$

$$\langle C_{11} \rangle = \langle C_{33} \rangle = \frac{8}{3} (S_3^2 + S_1^2), \quad (B-45)$$

$$\langle C_{22} \rangle = \frac{40}{9} (S_3^2 - S_1^2) = \frac{40}{3} \langle S_2^2 \rangle, \quad (B-46)$$

$$\langle C_{12} \rangle = \langle C_{23} \rangle = 0, \quad (B-47)$$

$$\langle C_{13} \rangle = \frac{16}{3} S_1 S_3, \quad (B-48)$$

$$\langle C_{11} S_2 \rangle = \langle C_{22} S_2 \rangle = \langle C_{33} S_2 \rangle = 0, \quad (B-49)$$

$$\langle C_{12} S_2 \rangle = \frac{16}{9} S_1 (S_3^2 - S_1^2), \quad (B-50)$$

$$\langle C_{13} S_2 \rangle = 0, \quad (B-51)$$

$$\langle C_{23} S_2 \rangle = \frac{16}{9} S_3 (S_3^2 - S_1^2), \quad (B-52)$$

$$\langle C_{11} S_2^2 \rangle = \frac{8}{45} (7 S_1^2 + 3 S_3^2) (S_3^2 - S_1^2), \quad (B-53)$$

$$\langle C_{22} S_2^2 \rangle = \frac{24}{15} (S_3^2 - S_1^2)^2, \quad (B-54)$$

$$\langle C_{33} S_2^2 \rangle = \frac{8}{45} (7 S_3^2 + 3 S_1^2) (S_3^2 - S_1^2), \quad (B-55)$$

$$\langle C_{12} S_2^2 \rangle = \langle C_{23} S_2^2 \rangle = 0, \quad (B-56)$$

$$\langle C_{13} S_2^2 \rangle = \frac{16}{9} S_1 S_3 (S_3^2 - S_1^2). \quad (B-57)$$

The other integrals which we need are:

$$\frac{1}{4\pi} \int f(r) e^{i\vec{q} \cdot \vec{r}} d\Omega = j_0(qr) f(r), \quad (B-58)$$

$$\begin{aligned} \frac{1}{4\pi} \int f(r) e^{i\vec{q} \cdot \vec{r}} (\vec{a} \cdot \vec{r})(\vec{b} \cdot \vec{r}) d^3r &= [-(\vec{a} \cdot \hat{q})(\vec{b} \cdot \hat{q}) + \frac{\vec{a} \cdot \vec{b}}{3}] F_2(q) \\ &+ \frac{\vec{a} \cdot \vec{b}}{3} F_0(q), \end{aligned} \quad (B-59)$$

where

$$F_2(q) = \int_0^\infty j_2(qr) f(r) r^2 dr,$$

$$F_0(q) = \int_0^\infty j_0(qr) f(r) r^2 dr,$$

and

$$\frac{1}{2\pi} \int_0^{\pi/2} \sin^2 \theta \cos^2 \theta \sin^{2n} \theta j_0(z \cos \theta) d\theta = \frac{1}{4} (2n+1)!! \phi_{n+2}(z), \quad (B-60)$$

$$\begin{aligned} \frac{1}{2\pi} \int_0^{\pi/2} \sin^2 \theta \cos^4 \theta \sin^{2n} \theta j_0(z \cos \theta) d\theta &= \frac{1}{4} (2n+1)!! \\ &\times \{ \phi_{n+2} - (2n+3)\phi_{n+3} \}, \end{aligned} \quad (B-61)$$

$$\begin{aligned} \frac{1}{2\pi} \int_0^{\pi/2} \sin^2 \theta \cos^4 \theta \sin^{2n} \theta j_2(z \cos \theta) d\theta &= \frac{1}{4} (2n+1)!! \\ &\times \{ 2(2n+3)\phi_{n+3} - \phi_{n+2} \}, \end{aligned} \quad (B-62)$$

where

$$\phi_n = \frac{J_n(z)}{z^n},$$

where J_n is the Bessel function.

APPENDIX C

In this appendix we will give the relations between the Wolfenstein parameters in Laboratory and Breit frames.

We use the following definition for Pauli matrices in different coordinate systems,

in Breit or CM frames $(\sigma_x, \sigma_y, \sigma_z),$

in proton initial Lab frame $(\sigma_s, \sigma_n, \sigma_l),$

in proton final frame $(\sigma_s', \sigma_n', \sigma_l').$

The normal direction is the same for all three coordinate systems. Therefore we are interested in a rotation about the normal to the scattering plane. If χ is the rotation angle between the Breit and Lab frames, and χ' is the rotation angle between the proton final frame and the Breit frame, then we can write

$$\begin{pmatrix} \sigma_s \\ \sigma_l \end{pmatrix} = \begin{pmatrix} \cos\chi & \sin\chi \\ -\sin\chi & \cos\chi \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_z \end{pmatrix} \quad (C-1)$$

$$\begin{pmatrix} \sigma_S' \\ \sigma_L' \end{pmatrix} = \begin{pmatrix} \cos\chi' & \sin\chi' \\ -\sin\chi' & \cos\chi' \end{pmatrix} \begin{pmatrix} \sigma_X \\ \sigma_Z \end{pmatrix} \quad (C-2)$$

As an example, for calculating D_{SS}' we have

$$D_{SS}' = \text{Tr}(F\sigma_S F^\dagger \sigma_S')$$

$$\begin{aligned} D_{SS}' &= \text{Tr}\{F(\sigma_X \cos\chi + \sigma_Z \sin\chi)F^\dagger(\sigma_X \cos\chi' + \sigma_Z \sin\chi')\} \\ &= \text{Tr}\{F\sigma_X F^\dagger \sigma_X \cos\chi \cos\chi' + F\sigma_X F^\dagger \sigma_Z \cos\chi \sin\chi' \\ &\quad + F\sigma_Z F^\dagger \sigma_X \sin\chi \cos\chi' + F\sigma_Z F^\dagger \sigma_Z \sin\chi \sin\chi'\} \\ &= D_{XX} \cos\chi \cos\chi' + D_{XZ}(\cos\chi \sin\chi' - \sin\chi \cos\chi') + D_{ZZ} \sin\chi \sin\chi', \end{aligned}$$

or,

$$D_{SS}' = D_{XX} \cos\chi \cos\chi' + D_{XZ} \sin(\chi' - \chi) + D_{ZZ} \sin\chi \sin\chi'. \quad (C-3)$$

And in the same way we can find relations for other observables,

$$D_{LL}' = D_{XX} \sin\chi \sin\chi' - D_{XZ} \sin(\chi' - \chi) + D_{ZZ} \cos\chi \cos\chi' \quad (C-5)$$

$$D_{SL}' = -D_{XX} \cos\chi \sin\chi' + D_{XZ} \cos(\chi' - \chi) + D_{ZZ} \sin\chi \cos\chi' \quad (C-6)$$

$$D_{LS}' = -D_{XX} \sin\chi \cos\chi' - D_{XZ} \cos(\chi' - \chi) + D_{ZZ} \cos\chi \sin\chi' \quad (C-7)$$

In order to determine χ , χ' we can use Lorentz transformations or the method of relativistic velocity diagrams¹⁾.

In the framework of relativistic velocity diagrams the velocities of the particles in the reaction $a + b \rightarrow c + d$ are represented as points on the hyperboloid $U_\mu U^\mu = u_0^2 - \vec{u}^2 = 1$. where points on the diagram are defined in the following way (fig.1):

Fig 1: The velocity diagram with indicated scattering angles.

a = incident proton rest mass frame (antilab frame),
 b = incident ^3He rest mass frame (lab frame),
 c = scattered proton rest mass frame,
 d = scattered ^3He rest mass frame,
 B = Breit frame,
 o = center of mass frame.

In the method of velocity diagrams, the distances oa, ob, oc, od are related to the velocities of the particles in a center of mass frame. Notice that ad and bc are equal because this reaction is an elastic scattering¹⁾.

As we can see from figure 1 $\theta_{cm}, \theta_{lab}, \theta_B$ are the scattering angle of proton in the CM, Lab, and Breit frames respectively. The θ_R is the ^3He recoil angle in the lab frame and α is the angle between the Breit and lab frames as seen from antilab system.

We have three coordinate systems, CM, Lab, and Breit frames, which are shown in fig 2. Notice that $\hat{n}, \hat{n}'$, and y all are pointing into the paper.

Fig 2: The velocity diagram with indicated axes.

Since χ is the angle between l, z and χ' is the angle between l', z by using figures 1,2 we can calculate χ, χ' for any transformation between the two systems. For transformations between the Breit and lab frames we have

$$\chi = \frac{\pi}{2} - \theta_R - \delta(bBa), \quad (C-8)$$

$$\chi' = \frac{\pi}{2} - \theta_R - \theta_{lab} - \delta(bBc), \quad (C-9)$$

δ comes from hyperbolic geometry, that the sum of the angles in any triangle is generally smaller than $\pi^{1,2}$, and

$$\delta = \pi - \{ \text{sum of the angles in the triangle} \}.$$

Therefore

$$\delta(bBa) = \pi - \left(\theta_R + \frac{\pi}{2} + \frac{\theta_B}{2} + \alpha \right), \quad (C-10)$$

$$\begin{aligned} \delta(bBc) &= \pi - \left(\theta_{lab} + \theta_R + \frac{\pi}{2} - \frac{\theta_B}{2} + \bar{\theta}_{lab} - \alpha \right), \\ &= \frac{\pi}{2} - \theta_{lab} - \bar{\theta}_{lab} - \theta_R + \frac{\theta_B}{2} + \alpha, \end{aligned} \quad (C-11)$$

By combining eqs. (C-8)-(C-11) we get

$$\begin{cases} \chi = \frac{\theta_B}{2} + \alpha, \\ \chi' = \bar{\theta}_{lab} - \frac{\theta_B}{2} - \alpha. \end{cases} \quad (C-12)$$

For the determination of α , θ_B , $\bar{\theta}_{lab}$, we use the regular dot product of two vectors except that now we are on a hyperboloid surface so that all sin and cos change to sinh and cosh. The only

quantities that we need are invariant parameters s , t , u , and masses of four particles involved in the reaction¹⁾.

The dot product of two four momentum is

$$P_i^\mu P_{k\nu} = m_i m_k \cosh(ik), \quad (C-13)$$

or,

$$\cosh(ad) = \frac{-1}{2m_a m_d} (u - m_a^2 - m_d^2), \quad (C-14)$$

$$\cosh(ab) = \frac{1}{2m_a m_b} (s - m_a^2 - m_b^2), \quad (C-15)$$

$$\cosh(ac) = \frac{-1}{2m_a m_c} (t - m_a^2 - m_c^2), \quad (C-16)$$

$$\cosh(bd) = \frac{-1}{2m_b m_d} (t - m_b^2 - m_d^2), \quad (C-17)$$

$$\cos(bB) = \sqrt{1/2(\cosh(bd)+1)},$$

By using cosine and sine laws we have

$$\cos(\bar{\theta}_{lab}) = \frac{\cosh(ab)\cosh(ad) - \cosh(bd)}{\sinh(ab)\sinh(ad)}, \quad (C-18)$$

$$\sin(\theta_R) = \frac{\sinh(ad)}{\sinh(bd)} \sin(\bar{\theta}_{lab}), \quad (C-19)$$

$$\cos(aB) = \cosh(bB)\cosh(ab) - \sinh(bB)\sinh(ab)\cos(\theta_R),$$

$$\sin\alpha = \frac{\sinh(bB)}{\sinh(aB)} \sin(\theta_R), \quad (C-20)$$

$$\sin\left(\frac{\pi}{2} + \frac{\theta_B}{2}\right) = \frac{\sinh(ab)}{\sinh(aB)} \sin(\theta_R). \quad (C-21)$$

Appendix D

In this appendix we will give relations between the Gibson and Schiff³⁾ and the Friar et. al.⁴⁾ representations for the radial part of ^3He wave function.

In our calculation of $p\text{-}^3\text{He}$ scattering, we use the representation of Gibson and Schiff. The Gibson and Schiff wave functions f_1, f_s, f_v, f_a are functions of only one variable $R = \sqrt{x^2 + y^2}$. On the other hand the Friar et al wave functions are functions of $\vec{x}, \vec{y}$, and the angle between them. Therefore we have to average over the angle between $\vec{x}, \vec{y}$, and then change $\vec{x}, \vec{y}$ to two new variables R, θ , and finally average over the angle θ .

For better understanding of the convention of reference 4 we need the following definitions. If A, B are two irreducible representation of rank 1, we can have the following combinations

$$\{A \times B\}_{00} = -\frac{1}{\sqrt{3}} (\vec{A} \cdot \vec{B}) \quad \text{scalar (tensor of rank 0),} \quad (\text{D-1})$$

$$\{A \times B\}_1 = \frac{1}{\sqrt{2}} (\vec{A} \times \vec{B}) \quad \text{vector (tensor of rank 1),} \quad (\text{D-2})$$

$$\{A \times B\}_{1M} = \frac{1}{\sqrt{2}} [\vec{A} \times \vec{B}] = \sum_M C_{1\mu 10}^{1M} A_\mu B_\mu. \quad (\text{D-3})$$

$$\{A \times B\}_{2M} = \sum_{\mu\nu} C_{1\mu 1\nu}^{2M} A_{\mu} B_{\nu} = \left\{ \frac{(3|M|+2)}{(14|M|-12)} \right\}^{1/2} \sum_{\substack{\mu+\nu=M \\ \mu \geq \nu}} (A_{\mu} B_{\nu} + A_{\nu} B_{\mu}). \quad (D-4)$$

or (D-4) in the expanded form

$$\{A \times B\}_{2+2} = A_{+1} B_{+1},$$

$$\{A \times B\}_{2+1} = \frac{1}{\sqrt{2}} (A_{+1} B_0 + A_0 B_{+1}),$$

$$\{A \times B\}_{2+0} = \frac{1}{\sqrt{6}} (A_{+1} B_{-1} + 2 A_0 B_0 + A_{-1} B_{+1}), \quad (D-5)$$

$$\{A \times B\}_{2-1} = \frac{1}{\sqrt{2}} (A_{-1} B_0 + A_0 B_{-1}),$$

$$\{A \times B\}_{2-2} = A_{-1} B_{-1},$$

and

$$\{ (A \times B)_0 \times (C \times D)_0 \}_0 = \frac{1}{3} (\vec{A} \cdot \vec{B})(\vec{C} \cdot \vec{D}). \quad (D-6)$$

$$\begin{aligned} \{ (A \times B)_2 \times (C \times D)_2 \}_2 = \frac{1}{\sqrt{5}} \left[\frac{1}{2} (\vec{A} \cdot \vec{C})(\vec{B} \cdot \vec{D}) - \frac{1}{3} (\vec{A} \cdot \vec{B})(\vec{C} \cdot \vec{D}) \right. \\ \left. + \frac{1}{2} (\vec{A} \cdot \vec{D})(\vec{B} \cdot \vec{C}) \right]. \quad (D-7) \end{aligned}$$

If we assume $\sigma_1 = a$, $\sigma_2 = b$, $\sigma_3 = 2b$, $R_1 = x$, $R_2 = y$ and using relation (D-7) the eqs. (1-23) can be written in the following form

$$D_1 = 2\sqrt{5} \{ [(\vec{a} \times \vec{b})^2 \times (\vec{x} \times \vec{x})^2]^0 - [(\vec{a} \times \vec{b})^2 \times (\vec{y} \times \vec{y})^2] \} | \chi_2 \rangle,$$

$$D_2 = 4 \sqrt{5} \{ [(\vec{a} \times \vec{b})^2 \times (\vec{x} \times \vec{y})^2]^0 \} | \chi_2 \rangle, \quad (D-8)$$

$$D_3 = 2 \sqrt{5} \{ [(\vec{a} \times \vec{b})^2 \times (\vec{x} \times \vec{x})^2]^0 + [(\vec{a} \times \vec{b})^2 \times (\vec{y} \times \vec{y})^2]^0 \} | \chi_2 \rangle.$$

Now we need to write the relations in (D-8) in terms of spherical harmonics. We start with $[\vec{x} \times \vec{x}]^2$ using relations (D-5) we have

$$[\hat{x} \times \hat{x}]^2 = \begin{cases} x_{-1} x_{-1} \\ \frac{1}{\sqrt{2}} (x_{-1} x_0 + x_0 x_{-1}) \\ \frac{1}{\sqrt{6}} (x_{-1} x_{+1} + 2 x_0 x_0 + x_{-1} x_{+1}) \\ \frac{1}{\sqrt{2}} (x_{+1} x_0 + x_0 x_{+1}) \\ x_{+1} x_{+1} \end{cases} \quad (D-9)$$

or

$$= \begin{cases} x_{-} x_{-} \\ \sqrt{2} x_{-} x_0 \\ \sqrt{2/3} (x_{-} x_{+} + x_0 x_0) \\ \sqrt{2} x_{+} x_0 \\ x_{+} x_{+} \end{cases} \quad (D-10)$$

The relations in (D-10) are components of $Y_2(\hat{x})$. To see that let's check two of the components

$$\begin{aligned} x_{-x_{-}} &= \left[\frac{1}{2} \sqrt{3/2\pi} \right]^2 \sin^2 \theta e^{-2i\phi} = \frac{1}{4} \sqrt{3/10\pi} \sqrt{15/2\pi} \sin^2 \theta e^{-2i\phi}, \\ &= \sqrt{3/10\pi} Y_{2,-2}(\hat{x}), \end{aligned}$$

or,

$$\begin{aligned} \sqrt{2} x_{-x_0} &= \sqrt{2} \left(\frac{1}{2} \sqrt{3/2\pi} \right) \left(\frac{1}{2} \sqrt{3/\pi} \right) \sin \theta \cos \theta e^{-i\phi} \\ &= \sqrt{2/4} (\sqrt{3/5\pi}) (\sqrt{15/2\pi}) \sin \theta \cos \theta e^{-i\phi} = \sqrt{3/10\pi} Y_{2,-1}(\hat{x}). \end{aligned}$$

Therefore

$$[\hat{x} \times \hat{x}]^2 = \sqrt{3/10\pi} Y_2(\hat{x}). \quad (D-11)$$

or in terms of $I = \sqrt{4\pi} Y_2(\hat{x})$, we have

$$[\vec{x} \times \vec{x}]^2 = \frac{x^2}{2\pi} \sqrt{3/10} I. \quad (D-12)$$

With the same procedure we have

$$[\vec{y} \times \vec{y}]^2 = \frac{y^2}{2\pi} \sqrt{3/10} J, \quad (D-13)$$

where,

$$J = \sqrt{4\pi} Y_2(\hat{y}).$$

The other term which we should consider in (D-8) is $[\vec{\hat{x}} \times \vec{\hat{y}}]^2$

$$[\vec{\hat{x}} \times \vec{\hat{y}}]^2 = \begin{cases} x_{-1}y_{-1} \\ \frac{1}{\sqrt{2}} (x_{-1}y_0 + x_0y_{-1}) \\ \frac{1}{\sqrt{6}} (x_{+1}y_{-1} + 2x_0y_0 + x_{-1}y_{+1}) \\ \frac{1}{\sqrt{2}} (x_{+1}y_0 + x_0y_{+1}) \\ x_{+1}y_{+1} \end{cases} \quad (D-14)$$

Checking one of the components,

$$\begin{aligned} x_{+1}y_{+1} &= \left(\frac{1}{4} \frac{3}{2\pi}\right) \sin\theta_x \sin\theta_y e^{i\phi_x} e^{i\phi_y}, \\ &= Y_{1,1}(\hat{x}) Y_{1,1}(\hat{y}). \end{aligned}$$

Therefore,

$$[\vec{\hat{x}} \times \vec{\hat{y}}]^2 = [Y_1(\hat{x}) \times Y_1(\hat{y})]^2,$$

or,

$$[\vec{\hat{x}} \times \vec{\hat{y}}]^2 = \frac{xy}{4\pi} \sqrt{3/10} K, \quad (D-15),$$

where,

$$K = 4\pi [Y_1(\hat{x}) \times Y_1(\hat{y})] \left(\frac{10}{3}\right)^{1/2}.$$

Substituting (D-12,13,15) in (D-8) we will have all three D's in terms of Friar's notation:

$$\begin{aligned}
 D_1 &= \sqrt{3/2} \frac{1}{\pi} \{ x^2 I - y^2 J \}^2 \times (\vec{a} \times \vec{b})^2 | \chi_2 \rangle, \\
 D_2 &= \sqrt{3/2} \frac{1}{\pi} \{ K \}^2 \times (\vec{a} \times \vec{b})^2 | \chi_2 \rangle, \\
 D_3 &= \sqrt{3/2} \frac{1}{\pi} \{ x^2 I + y^2 J \}^2 \times (\vec{a} \times \vec{b}) | \chi_2 \rangle.
 \end{aligned}
 \tag{D-16}$$

If we assume $\phi_3 = \chi_3 n_1$, $\phi_4 = \chi_3 n_2$, where $\chi_3 = (+ + +)$ and also notice that $(\vec{a} \times \vec{b})^2 | \chi_2 \rangle \propto | \chi_3 \rangle$ we can substitute these D's in the eqs (1-28), (1-29) to get the wave function ψ ,

$$\begin{aligned}
 \psi &= \sqrt{3/2} \frac{1}{\pi} \{ (5 x^2 S_2 f_6 + x^2 S_2 f_8) I + (5 y^2 S_2 f_6 - y^2 S_2 f_8) J \\
 &\quad + (-2 S_3 f_6 + S_3 f_7 + S_1 f_8) K \} \times \phi_3 \\
 &\quad - \sqrt{3/2} \frac{1}{\pi} \{ (5 x^2 S_1 f_6 - 2 x^2 S_3 f_7 - x^2 S_1 f_8) I \\
 &\quad + (5 y^2 S_1 f_6 + 2 y^2 S_3 f_6 - y^2 S_3 f_7 + y^2 S_1 f_8) J \\
 &\quad + S_2 f_8 K \} \times \phi_4.
 \end{aligned}
 \tag{D-17}$$

In reference 4, ψ is defined in the following form

$$\psi = v_4 \times \hat{r}_3 - v_3 \times \phi_4.
 \tag{D-18}$$

We intend to define formulas for expressing the Gibson wave function in terms of the Friar wave function, or

$$(f_6, f_7, f_8) \rightarrow (v_3^I, v_3^J, v_3^K, v_4^I, v_4^J, v_4^K).$$

By comparing (D-17), (D-18) we get

$$\begin{cases} 5 x^2 S_2 f_6 + x^2 S_2 f_8 &= \sqrt{2/3} \pi v_4^I, \\ 5 y^2 S_2 f_6 - y^2 S_2 f_8 &= \sqrt{2/3} \pi v_4^J, \\ -2 S_3 f_6 + S_3 f_7 + S_1 f_8 &= \sqrt{2/3} \pi v_4^K, \end{cases}$$

(D-19)

$$\begin{cases} 5 x^2 S_1 f_6 - 2 x^2 S_3 f_6 + x^2 S_3 f_7 - x^2 S_1 f_8 &= \sqrt{2/3} \pi v_3^I, \\ 5 y^2 S_1 f_6 + 2 y^2 S_3 f_6 - y^2 S_3 f_7 + y^2 S_1 f_8 &= \sqrt{2/3} \pi v_3^J, \\ S_2 f_8 &= \sqrt{2/3} \pi v_3^K. \end{cases}$$

Since we should average over angle between $\vec{x}$, $\vec{y}$, and the average over S_2 is zero, we end up with only three equations

$$\begin{cases} -2 S_3 f_6 + S_3 f_7 + S_1 f_8 &= \sqrt{2/3} \pi v_4^K, \\ 5 x^2 S_1 f_6 - 2 x^2 S_3 f_6 + x^2 S_3 f_7 - x^2 S_1 f_8 &= \sqrt{2/3} \pi v_3^I, \\ 5 y^2 S_1 f_6 + 2 y^2 S_3 f_6 - y^2 S_3 f_7 + y^2 S_1 f_8 &= \sqrt{2/3} \pi v_3^K. \end{cases} \quad (D-20)$$

By solving these three equations in (D-20) we can find the formulas for expressing the Gibson wave functions f_6, f_7, f_8 , in terms of Friar wave functions $V_3^I, V_3^J, V_3^K, V_4^I, V_4^J, V_4^K$.

Results

The final results for relating these two wave functions are:

$$\begin{aligned}
 f_6 &= \frac{\sqrt{2/3} \pi (x^2 v_3^K + y^2 v_3^I)}{10 x^2 y^2 S_1}, \\
 f_8 &= \frac{\sqrt{2/3} \pi (x^2 v_3^K - y^2 v_3^I + x^2 y^2 v_4^K)}{4 x^2 y^2 S_1}, \\
 f_7 &= 2 f_6 - \frac{S_1}{S_3} f_8 + \sqrt{2/3} \frac{v_4^K}{S_3}.
 \end{aligned}
 \tag{D-21}$$

The S part (f_1) of Gibson & Schiff is equal to 'US' of the Friar's code.

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