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QUASI-STEADY OPERATION OF REVERSED FIELD PINCHES

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QUASI-STEADY OPERATION OF REVERSED FIELD PINCHES

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A three fluid, Lagrangian mesh, transport and stability code (RFPBRN) has been developed and applied to the Reversed Field Pinch reactor concept. Using a circular cylinder, quasi-static approximation, RFPBRN follows the time evolution of the temperature, density, and magnetic field profiles for the RFP while simultaneously monitoring ideal MHD stability. Local stability is monitored for Suydam modes while global stability is monitored using a Rayleigh-Ritz expansion of the energy principle.

Initial studies were done to analyze the Los Alamos RFPR reactor which was designed using a global code. The global code and RFPBRN behave similarly although center-line ignition allows the plasma to be ignited with less current in the one-dimensional simulations. This led to new reactor parameters with a more favorable energy gain. Turbulent transport due to Suydam instabilities have also been included, but this has only a small effect on the reactor performance.

A smaller, Compact Reversed Field Pinch (CRFP) has also been designed and evaluated. Performance for the non-refueled small reactor is marginal, so refueling using gas puffing and pellets was simulated. This improved reactor performance near the level of the large reactor.

Work has also been done on simulating stable startup of an RFP. Using a convective startup technique, a scenario has been found which maintains ideal MHD stability to global and local modes during the

■

startup and burn. This requires initially peaking the toroidal current off-axis, which is a new stability constraint.

This work has led to new ideas for multipole stabilized startup and resistive MHD stable startup. These are discussed and a user guide for RFPBRN is also included.

■

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TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION	1
II. BACKGROUND	5
A. Experimental	5
B. Theoretical	11
III. THE MODEL	20
A. Transport Model	20
B. Transport Coefficients	23
C. Solution Technique	26
D. Stability Model	26
IV. BATCH BURN RFP REACTORS	30
A. Introduction	30
B. Discussion of Case A Results	31
C. Comparison of Constant Radius Cases With and Without Enhanced Transport, Cases B and C	39
D. Optimization of RFPR Design Point by One-Dimensional Model	44
E. Parametrically Varied Transport Simulations (Case E) ..	50
F. Compact Reversed Field Pinch Reactor (Case F)	52
G. Conclusions	55
V. STARTUP	56
A. Introduction	56
B. Modeling Technique	57
C. Results	59
D. Stability Considerations	63
E. Conclusions	70

Chapter	Page
VI. REFUELING	72
A. Introduction	72
B. Background	73
C. Gas Puffing Simulations	77
D. Pellet Refueling	79
E. Conclusions	92
VII. CONCLUSIONS AND PROPOSED EXTENSION	95
A. Introduction	95
B. Conclusions	95
C. Future Work	99
D. Summary	101
REFERENCES	103
APPENDIX	
A. TRANSPORT	109
A. Equations	109
B. Transport Boundary Conditions	112
C. Numerics	113
B. ALPHA PARTICLE PROMPT LOSS MODEL	116
C. MESH ADJUSTMENT	118
A. Pressure Balance	118
B. Boundary Conditions	121
D. PARTICLE BALANCE AND ENERGY BALANCE CHECKS	123
E. RFPBRN USER'S GUIDE	125
VITA	134

I. INTRODUCTION

The Reversed Field Pinch (RFP) is a magnetic confinement device similar to a tokamak in structure. It is toroidal and has both toroidal (B_ϕ) and poloidal (B_θ) magnetic field components (Figure 1).¹ The major difference is that $B_\theta \sim B_\phi$ and B_ϕ is reversed on the outside edge of the plasma for an RFP while $B_\theta \ll B_\phi$ for a tokamak (Figure 2).²

Reversing the toroidal field results in high magnetic shear which allows the RFP to remain stable at high β for pressure driven instabilities. A conducting wall is necessary to stabilize current driven modes. Unlike tokamaks which require $q > 1$ ($q = (r_p/R) B_\phi/B_\theta$) the RFP operates with $q < 1$ and hence has no aspect ratio constraint.

Since $B_\theta \sim B_\phi$, the RFP has large currents and the reactor can be ignited by ohmic heating alone. This feature, coupled with stability at high β and the lack of an aspect ratio constraint, potentially provide significant engineering advantages over tokamaks.^{1,2}

One problem the RFP has as a reactor is that the toroidal field B_ϕ resistively decays. Still, the decay is slow enough at thermonuclear temperatures to allow operation for tens of seconds. However, since the field, temperature and density profiles change in time, so do the stability characteristics of the plasma.

Ohmic heating can cause the central core of the plasma to become unstable^{3,4} to Suydam modes.⁵ However, it is believed these modes will saturate and result only in enhanced transport⁶ provided they occur only in weakly sheared plasma regions.⁷ Gross pressure driven^{7,8} and current driven modes can also exist. These modes have been extensively studied

Figure 1. Conceptual design of a 600 MWe RFP reactor.

Figure 2. Comparison of magnetic field and pressure profiles for a $dq/dr \neq 0$ stabilized RFP and a $q > 1$ stabilized tokamak.

for relaxed profiles^{9,10} and some analytic⁸ constraints have been derived. However, the more general problem of the time dependent evolution of stability is not well known.

This study considers transport and stability simultaneously. A one-dimensional Lagrangian mesh transport code solves the time dependent evolution of the field, density, and temperature profiles, while a separate algorithm monitors ideal MHD stability. The code, which was developed by the author and Dr. Ron Moses, is called RFPBRN. Details of the model are presented in Chapter 3.

Chapter 2 reviews previous RFP experiments and theoretical studies. It also describes the motivation and details the specific problems considered in this thesis. Unlike most previous one-dimensional RFP studies, the problems considered here are principally related to reactor regimes.

Results are divided into three studies (Chapters 4-6), each considering a particular RFP reactor related problem. Chapter 4 examines the RFPR batch burn scenario and compares the one-dimensional results with previous global studies.² The plasma's time dependent stability characteristics are included as well as Suydam-induced turbulent transport. The RFPR design is then optimized with the one-dimensional code and the design's performance is checked using different transport assumptions. A smaller, compact RFP (CRFP) is also optimized with the one-dimensional code.

Chapter 5 considers the RFP startup problem using a pitch programming technique.¹¹ New stability constraints are found and a scenario is described for maintaining ideal MHD stability to both local and global modes. Chapter 6 examines the refueled CRFP for both gas puffing and pellet refueling. Refueling by either method significantly improves the plasma performance. Chapter 7 is a summary.

II. BACKGROUND

A. Experimental

The RFP concept has evolved from some of the earliest toroidal confinement experiments. The most important of these experiments was Zeta (Culham).¹³⁻¹⁷ Two similar experiments, Alpha^{18,19} and SCEPTRE IV²⁰ exhibited behavior similar to Zeta but neither was as successful. These were slow, large, long pulse experiments (1-20 ms). They were operated between 1958 and 1968.

Of these experiments, Zeta was the only one that showed a period of greatly improved confinement, since labeled the "quiescent period." The quiescent period is important since the RFP reactor hopes to achieve the same behavior. A detailed discussion of quiescence and RFP's in general is presented in reference 13.

Quiescence was first noticed late in the life of the Zeta experiment.²¹ Shortly after the toroidal current peaked, a period of a few milliseconds occurred when current fluctuations (dI_ϕ/dt) decreased by an order of magnitude. Decreased fluctuations improved the energy confinement which caused an increase in temperature and β (see Figure 3).¹⁴ The maximum β achieved was about 10% and the maximum electron temperature was 100-200 eV.¹³ The effect was reproducible, but only after extensive discharge cleaning.

However, even during the quiescent period energy losses were strongly anomalous ($\tau_E \sim 3-10$ ms compared to 90 ms for classical).¹³ Magnetic field fluctuations were 10^{-3} at best¹⁴ compared to 10^{-4} for tokamaks. This is believed to be caused by MHD turbulence and should lead to Bohm-like energy confinement scaling.⁶ We have assumed this type of scaling for thermal

Gas current and rate of change of current showing the period of improved stability.

The electron temperature and density during the relatively stable period.

Figure 3. Example of quiescence in the Zeta discharge.

$$B_{\phi} = 9.0(10^{-2})T$$

$$P_o = 2 \text{ mtorr of } D_2$$

conduction in our RFP model.

Other observations during quiescence were a flattening of the density profile¹⁶ and reversal of both the toroidal field and the toroidal current. These effects are believed to be interrelated. Reversal of the toroidal current leads to strong Suydam stabilization in the outer plasma regions which in turn leads to much improved confinement. This, coupled with weak turbulence (due to small shear) in the central plasma region, causes the density to flatten. This result has been verified numerically by both Culham workers⁶ and by ourselves.⁴

The other experiments, Alpha and SCEPTRE IV, did not demonstrate quiescence. Both of these machines operated at lower current and temperature than Zeta (see Table 1).¹³ The reason they did not show quiescence is believed to be that the toroidal current did not reverse.¹³ Any negative toroidal current decayed quickly due to large resistivities caused by low edge temperatures. In SCEPTRE IV impurity induced radiative losses appear to be the major contributing factor. SCEPTRE IV did not have a liner and suffered from plasma arcing across the breaks in the copper chamber. This deposited large quantities of copper impurities in the plasma. Indeed, quiescence was achieved on Zeta only after extensive discharge cleaning to minimize impurities.

One of the major problems with all of these experiments is that they were concluded before many diagnostics were developed. Also, much of the theoretical understanding of the experiments was developed long after the experiments were dismantled. The first theoretical explanation of self-reversal as a natural relaxation phenomena was done in 1974.²² (It has since been extended to a more general turbulence theory.)²³ Consequently,

Table I. Slow Pulse Experiment Parameters

Machine	Minor Radius (m)	Shell Radius (m)	Major Radius (m)	Current (MA)	Current Rise (ms)	Pulse Length (ms)
Zeta	0.48	0.52	1.5	0.3-1.05	0.8-2.4	3-20
Alpha	0.45	0.50	1.6	0.3	0.5	2-3
SCEPTRE IV	0.15	0.15	0.95	0.5	0.5	1-2
$\eta\beta$ -II	0.125	0.29	0.65	0.2	.003-0.2	.2-1
ZT-40	0.20	0.22	1.14	0.6	.025-1	1-5
HBTX-IA	0.26	0.29	0.8	0.4	0.1-0.5	1-10

much of the RFP theoretical understanding is somewhat speculative since it is based on old, incomplete data.

In order to get better experimental justification for the more recent RFP theory, a new series of large, slow experiments are being built at Culham (HBTX-IA)²⁴, Padova (η - β II)²⁵ and Los Alamos (ZT-40)²⁶. η - β II is already operational and has recently reproduced the quiescence seen in Zeta. ZT-40 has recently become operational and HBTX-IA is due to be operational very soon.

Since the demise of Zeta, there have been a number of small, fast RFP experiments. These experiments include HBTX-I^{9,27-33} at Culham, η β -I^{34,35} at Padova, ZT-1 and ZT-S³⁶⁻³⁹ at Los Alamos, TPE-1⁴⁰⁻⁴² at Tanashi and STP⁴² at Nagoya University, Japan. These are characterized by small radii (4.8 cm-6.0 cm) and fast (1-10 μ s) field programming (See Table II).¹³

The purpose of fast-field programming is to achieve a reversed-field state on a time scale shorter than that required for an MHD instability to grow. This alleviates the impurity problem caused by the plasma striking the wall during startup that was experienced on the slow experiments. Unfortunately, this technique does not scale to large machines due to the large electric field required.

Since the small experiments have short lived, turbulent (non-quiescent) plasmas, their transport scaling is not applicable to the reactor study we have done. Their principle value is that they have verified many of the theoretical stability constraints for RFPs.

Ideal MHD yields analytic constraints on both the pinch ratio,

Table II. Parameters of Fast Pulse Experiments

Machine	Minor Radius (m)	Shell Radius (m)	Major Radius (m)	Current (KA)	Rise Time (μ s)
HBTX-I	.06	.075	1.0	30-300	3-15
ZT-1	.051	.057	.38	60-250	0.8-2.5
ZT-S	.075	.083	0.4	65	2.5
$\eta\beta$ -I	.048	.062	0.4	30-150	2
TPE-1	.05	.058	0.4	40-150	3.8
STP	.042	.048	0.12	60-110	3.6-5.3

θ , ($\theta \equiv \frac{\mu_0 I r_w}{2\Phi(r_w)}$ where I is the toroidal current, r_w is the wall radius and Φ is the toroidal flux) and poloidal beta ($\beta_\theta = \frac{4\mu_0 \int_0^{r_w} r p(r) dr}{r_w^2 B(r_w)^2}$). Here θ is limited to about 2.5 by current driven instabilities^{22,23} while β_θ is limited to about 0.5-1.0 by pressure driven modes.⁸ HBTX-I exhibited kink modes due to over-compression²⁷ ($\theta \sim 3$) with subsequent relaxation into the stable θ range.²⁸ Also, β_θ was found to peak at about .6,²⁹ consistent with theoretical predictions.⁸ Similar values of β_θ were achieved on ZT-1³⁶ and $\eta\beta$ -1³⁵.

B. Theoretical

Over the past several years, there have been two principle groups designing RFP conceptual reactors. One study is a combined effort by researchers from Culham^{1,43-47} and Padova⁴⁸ and the other is by CTR-12^{2,49-51} at Los Alamos. Although during the original design phases there was little interaction between the groups, their final designs are remarkably similar. A comparison of the two designs is in Table III.^{46,51}

The major differences are that the RFPR (LASL) is smaller with lower density and a shorter cycle time than the Culham reactor. RFPR has higher wall loading and higher energy multiplication, also. This results in a higher electric power output (750 MW_e vs. 600 MW_e).

Both of these reactors were designed principally using "global" or "point model" burn codes. For this kind of code, magnetic field, density and temperature profiles are assumed along with a formulation for the pressure balance. The spatially averaged density and temperature equations are then integrated forward in time to model the burn.

The Culham burn code was developed originally by Newton and Yeung.^{3,52}

Table III. RFP Reactor Parameters

	<u>Culham</u>	<u>LASL (RFPR)</u>
Minor radius, r_w (m)	1.75	1.5
Major radius, R (m)	15.5	12.7
Toroidal current, I_ϕ (MA)	19	20
Burn time, τ_B (s)	28	21.6
Cycle time, τ_C (s)	40	26.6
Average plasma density, $n(m^{-3})$	2×10^{20}	2×10^{20}
Average first wall neutron load, $P_w(MW/m^2)$	1.5	2.7
Recirculating power fraction, ϵ	.36	.17
Net electrical power, P_E (MWe)	600	750

Important features of this model include assumed limits in β_θ and a resistivity modification to account for helicity of field lines. Pressure balance is assumed to be the High Beta Model (HBM)⁵² which is a numerical modification of the Bessel function fields²² to allow finite pressure. Pressure and magnetic field are assumed to scale such that Suydam's criterion⁵ is marginally satisfied. A brief comparison of the Culham and LASL models appears in Table IV.

Yeung's reference reactor size⁵² is about twice the size of present designs (see Table V). The magnetic fields, like the power density and wall loading, are comparatively small. Yeung did numerous simulations with refueling which indicated that the RFP could burn for hundreds of seconds. However, workers doing later reactor designs^{1,45-47} opted for a batch burn as did the LASL researchers.

The LASL model⁵³ differs from the Culham model principally by the energy confinement scaling assumed and the pressure balance model. Energy confinement is assumed to be fractional Bohm ($1/200$) while resistivity is classical. We have incorporated this energy confinement model into our one-dimensional model. The plasma is assumed to end at the reversal point with the pressure varying as $J_0(\alpha r)^2$. This is consistent with Bessel function field profiles with unequal coefficients.

There have also been some global studies done at the University of Illinois^{12,54} which embodied our initial effort in RFP work. Our model used a combination of the Culham and LASL formulations. Energy confinement was assumed to be β limited; the limiting beta provided by the Suydam criterion in Robinson's stability diagram⁹ (see Figure 4). Hagenson's pressure balance model was used. Simulations principally considered

Table IV. Reactor Physics Models

	Yeung (Culham) High β model	Hagenson (LASL)
Pressure balance		$B_\theta = A_\theta J_1(\alpha r)$ $B_\phi = A_\phi J_0(\alpha r)$ $P \propto J_0^2(\alpha r)$
Number of species	3	3
Energy confinement	$\tau_E = \tau_{E_{class}} \exp(K(\beta_\theta - .5))$ (K is a parametric constant ~ 100)	$\tau_E = 200 \tau_{Bohm}$
Resistivity	Classical modified by β effect	Classical

Table V. Yeung and Newton's Reactor

Minor radius, r_w (m)	3
Major radius, R(m)	15
Toroidal current, I_ϕ (MA)	25
Pulse time, t(sec)	10-100
Average plasma density, $n(m^{-3})$	$7 \times 10^{19} m^{-3}$
Average first wall neutron load, $P_w(MW/m^2)$	1.6
Net electrical power, $P_E(MWe)$	~ 600

Figure 4. Regions of stability for an RFP.

$$\beta_0 \equiv \frac{\sum n_{j0} K_j T_{j0}}{B_{z0} / 2\mu_0}$$

$$\theta \equiv \frac{B_\theta(r_w)}{\langle B_\phi \rangle}$$

refueled reactor burns. The results of the global studies were the main impetus for this work.

The most significant result of the global studies was that a significant gain in energy multiplication was achieved by refueling the plasma (a result also found by Yeung⁵²). The reason is that the "batch burn" scenario requires one to repetitively ohmically heat and ignite the plasma while refueling allows the already burning plasma to ignite the cold refuel. In the batch burn scenario the plasma is extinguished by fuel depletion rather than ash buildup. Thus, batch burns extinguish quicker than refueled ones and have a smaller energy gain.

However, the net result was that more questions were introduced by the global studies than were answered. One problem was that the stability limits used were derived for specific analytic profiles. Whether these criteria are sufficient for predicting stability of general profiles was an open question.

Another open question concerned the β -limited transport. It was questionable whether turbulent transport would be sufficient to keep the plasma pressure beneath the gross instability β limit (as was assumed by the Culham and Illinois global models). This problem is further aggravated for a refueled reactor. Since energy transport is not well known in the RFP this question has not been completely answered, but it has been quantified parametrically in this study.

RFP startup was another problem that the global studies did not attempt to address. As pointed out in the previous section, present slow startup techniques require the plasma to pass through a turbulent,

unstable regime which dissipates and transports large amounts of energy to the wall resulting in a large influx of impurities. Previous analytic and numeric simulations¹² have suggested this problem can be resolved by properly programming the magnetic fields and fuel influx at the wall. Ideal MHD stability was claimed for these simulations since it was shown that the pitch function ($P = rD_\phi / D_\theta$) was a decreasing function with radius (a necessary criterion for stability). We have extended this work by including a full-blown energy principle stability check⁵⁵ in the one-dimensional model. Results indicate that a decreasing pitch profile is not sufficient to ensure stability during startup and we have derived some new stability constraints. Using these new constraints we have demonstrated how to program the fields for an RFP reactor so both global and local (Suydam modes)⁵ ideal MHD stability is maintained through the startup, heating and burn phases.

A final problem encountered by the global simulations is that of plasma penetration by the cold refuel molecules. Previous work¹² indicated that plasma penetration using pellet injection was a problem. However, one-dimensional simulations on tokamaks indicated that pellets do not need to penetrate very far to be effective.⁵⁶ On the other hand, unlike the tokamak, a RFP has large resistively decaying currents near the plasma edge so pellet injection not only affects density and temperature profiles but also affects the field profiles. How pellet refueling affects both the burn and stability is another question addressed in this thesis.

All of the aforementioned problems have one thing in common. They are all a result of spatial profile effects. In order to address these

problems a one-dimensional Lagrangian mesh transport code was developed for operating in reactor regimes. A stability check was included by using a Rayleigh-Ritz approximate expansion of the energy principle to answer the questions of the time evolution of the MHD stability. This code (RFPBRN) is unique for RFP codes in that it does both transport and stability self-consistently and simultaneously.

There have been some previous one-dimensional RFP studies, but most have been for non-fusing plasmas (i.e. simulations for current or proposed experiments). Two of these codes are currently being used at LASL. Christian⁵⁷ has considered heating and maintenance of Suydam stable profiles while Caramana⁵⁸ is principally concerned with simulations of ZT-40, LASL's new large RFP. Caramana's code includes line radiation losses which are important for low temperature operation and burning through impurities.

Fairly extensive studies have been done at Culham with the ATHENE⁵⁹ code, developed principally by Christiansen and Roberts. The numeric formulation of this code is very similar to our code, except it is written in the convective fluid frame rather than the Lagrangian frame. They divide the plasma into three regions and assume different physics in each region. The central part of the plasma is dominated by turbulent transport due to Suydam instabilities and is surrounded by a stable confining region. The third region is adjacent to the wall and has enhanced electron thermal conduction. Other coefficients are classical. This code has been used to simulate Zeta and RFX,⁶ a proposed experiment.

Yeung used the ATHENE code to do some reactor simulations,⁵² but did

not consider the physics questions we have raised here. He did not do extensive parametric studies, and results indicate he may have had some difficulties with boundary conditions (modeling the edge region can be difficult). The simulations showed the plasma overheating in the center and piling up against the wall which is an unphysical condition. This type of problem is common in one-dimensional simulations.⁶⁰

Thus, the study reported in this thesis is the first detailed one-dimensional study of the RFP reactor regime. We have not intended to do a reactor design here, but rather have attempted to consider some of the unresolved physics questions of RFP reactor operation. Our code is unique in the sense that it includes both transport and stability and the study is unique in that it addresses problems that have not been previously explored.

III. THE MODEL

A. Transport Model

As stated previously, the RFP is simulated using a combined transport and stability model. The model has three species, two temperatures and two magnetic field components (toroidal and poloidal). Since the RFP has a large aspect ratio, flux surfaces are assumed to be circular and concentric. The code uses the three fluid equations to describe the following balances:

continuity

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \vec{\Gamma}_e = S_i \quad (3.1)$$

$$\frac{\partial n_i}{\partial t} + \nabla \cdot \vec{\Gamma}_i = S_i - 2f(1-f) \langle \sigma v \rangle n_i^2 \quad (3.2)$$

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot \vec{\Gamma}_\alpha = f(1-f) \langle \sigma v \rangle n_i^2 f_\alpha \quad (3.3)$$

momentum conservation

$$m_e n_e \frac{d\vec{v}_e}{dt} + \nabla p_e + en_e (\vec{E} + \vec{v}_e \times \vec{B}) = \vec{R}_{ei} - \vec{R}_{\alpha e} \quad (3.4)$$

$$m_i n_i \frac{d\vec{v}_i}{dt} + \nabla p_i - en_i (\vec{E} + \vec{v}_i \times \vec{B}) = -\vec{R}_{ei} - \vec{R}_{\alpha i} \quad (3.5)$$

$$m_\alpha n_\alpha \frac{d\vec{v}_\alpha}{dt} + \nabla p_\alpha - 2en_\alpha (\vec{E} + \vec{v}_\alpha \times \vec{B}) = \vec{R}_{\alpha e} + \vec{R}_{\alpha i} \quad (3.6)$$

energy transport

$$\frac{3}{2} n_e \frac{dT_e}{dt} + p_e \nabla \cdot \vec{v}_e = -\nabla \cdot \vec{q}_e + Q_e \quad (3.7)$$

$$\frac{3}{2} n_i \frac{dT_i}{dt} + p_i \nabla \cdot \vec{v}_i = -\nabla \cdot \vec{q}_i + Q_i \quad (3.8)$$

$$\frac{3}{2} n_{\alpha} \frac{dT_{\alpha}}{dt} + p_i \nabla \cdot \vec{v}_{\alpha} = - \nabla \cdot \vec{q}_{\alpha} + Q_{\alpha} \quad (3.9)$$

along with Maxwell's equations,

$$\nabla \times \vec{B} = \mu_0 \vec{J} \quad (3.10)$$

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad (3.11)$$

$$\epsilon_0 \nabla \cdot \vec{E} = \sum_j Z_j n_j e, \quad j = i, e, \alpha \quad (3.12)$$

$$\nabla \cdot \vec{B} = 0 \quad (3.13)$$

and an equation of state (required for closure)

$$P = \sum_j n_j T_j, \quad j = i, e, \alpha \quad (3.14)$$

where

$$\vec{J} = -en_e \vec{v}_e + en_i \vec{v}_i + 2en_{\alpha} \vec{v}_{\alpha} \quad (3.15)$$

$$\vec{I}_j = n_j \vec{v}_j, \quad j = i, e, \alpha \quad (3.16)$$

$$\vec{q}_j = -K_j^{\perp} \nabla T_j, \quad j = i, e, \alpha. \quad (3.17)$$

The fluid velocity with respect to the Eulerian coordinates is $\vec{v}_j$, S_i is a refuel ion source, f is the tritium fraction, and $f_{\alpha} (\leq 1.0)$ is an alpha particle confinement fraction based on finite orbit effects. Q_j is a heat source/sink including alpha heating, ohmic heating, ion-electron equilibration, Bremsstrahlung and cyclotron radiation; and $\vec{R}_{ij}$ is the collisional force of the j^{th} specie on the i^{th} specie.

In order to solve the equations, the following assumptions are made:

1. The torus can be modeled as a finite cylinder
2. Charge neutrality
3. Ions and alphas transport at the same rate.
4. Alpha slowing down is immediate and occurs in-situ
5. Ions and thermalized alphas have the same temperature

Assumption 1 yields circularly concentric flux surfaces of constant temperature since equilibration along flux surfaces is much faster than cross field transport. Thus, transport is reduced to a one-dimensional radial problem.

Charge neutrality and equal specie velocities reduce the continuity equations from three equations to two equations with the same fluid velocity. Assumptions 4 and 5 reduce the energy transport equations from three equations to two equations with simplified source terms.

Fluid plasmas typically have two time scales of interest. The fast time scale (μsec) is due to bulk plasma motion and is governed by the inertial terms in the momentum equations (Eqs. 3.4-3.6). The slower time scale is a resistive time scale (typically msec. to sec) and is due to the collisional term ($\vec{R}$). Since we are interested in the plasma evolution on the resistive time scale, the plasma is assumed to converge to an equilibrium on the fast time scale (i.e. neglect inertial terms in Eqs. 3.4-3.6). Then, adding Equations 3.4-3.6 we obtain the pressure balance:

$$\nabla P = \vec{J} \times \vec{B} \quad (3.18)$$

while multiplying by $\frac{Z_j e}{m_j}$ and adding (neglecting small terms) we find a generalized Ohm's law:

$$\vec{E} + \vec{v} \times \vec{B} = \frac{1}{en_e} \vec{R}_{Th} + \eta_{||} J_{||} \hat{e}_{||} + \eta_{\perp} J_{\perp} \hat{e}_{\perp} \quad (3.19)$$

where $\vec{R}_{Th}$ is the thermal force and $\eta_{||}$, $\eta_{\perp}$ are the parallel and perpendicular resistivities respectively.

Thus, the transport equations reduce to:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot \vec{\Gamma}_i = S_i - 2f(1-f)\langle\sigma v\rangle n_i^2 \quad (3.20)$$

$$\frac{\partial n_{\alpha}}{\partial t} + \nabla \cdot \vec{\Gamma}_{\alpha} = f(1-f)\langle\sigma v\rangle n_i^2 f_{\alpha} \quad (3.21)$$

$$\frac{3}{2}n_j \frac{dT_j}{dT} + p_j \nabla \cdot \vec{\Gamma}_j = -\nabla \cdot \vec{q}_j + Q_j \quad j = e, i + \alpha \quad (3.22)$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left[-\vec{v} \times \vec{B} + \frac{1}{en_e} \vec{R}_{Th} + \eta_{||} j_{||} \hat{e}_{||} + \eta_{\perp} j_{\perp} \hat{e}_{\perp} \right] \quad (3.23)$$

along with charge neutrality and Eq. 3.18. Details of the transport equations, coefficients, boundary conditions, initial conditions and numerics are in Appendix A. The alpha particle confinement model is discussed in Appendix B.

Integrating the pressure balance (Eq. 3.18) yields:

$$p + B^2/2\mu_0 = (2/r^2) \int_0^r (p + B_{\phi}^2/2\mu_0) r' dr' , \quad (3.24)$$

This equation is treated as a constraint and is solved by a separate algorithm from the transport. Details of the pressure balance formulation and numerics are in Appendix C.

B. Transport Coefficients

Transport coefficients are assumed to be classical⁶¹ except for thermal conductivity and perpendicular resistivity which both have anomalous scaling. Due to the high fluctuation level in the plasma,

energy confinement is strongly anomalous.¹⁵ To account for this we have assumed Bohm-like scaling for the thermal conductivity. This scaling is conservative compared to classical and other anomalous scaling laws (see Table VI) in the reactor regime.

$$K_{\perp} = K_{\perp}^C + f_B n k_B T D_B \quad (3.25)$$

where $K_{\perp}^C$ is the classical conductivity,⁶¹ k_B is Boltzmann's constant, D_B is the Bohm diffusion coefficient and f_B is a fractional Bohm constant which is parametrically varied.

Another associated problem is that magnetic field shear is weak in the central part of the plasma which leads to violation of Suydam's criterion. The resulting instabilities are assumed to remain localized and to be manifested as enhanced transport.⁶ Specifically, the perpendicular resistivity, $\eta_{\perp}$ was modified relative to the classical value, $\eta_{\perp C}$, according to:

$$\eta_{\perp} = \eta_{\perp C} + f_T f_s C B^2 D_B / n_e, \quad (3.26)$$

where f_T is a triangular function that spreads the region of enhanced transport over several mesh points (~ 10 Larmor radii) adjacent to the location of Suydam instability, $C = 10^{-4}$, and f_s is given by

$$f_s = \begin{cases} 0 & \text{if } \Sigma < 1 \\ (\Sigma - 1)/\Sigma & \text{if } \Sigma > 1 \end{cases} \quad (3.27)$$

$$\Sigma = - \frac{dp/dr}{(B_{\theta}^2/8r_o)(dp/dr)^2} \quad (3.28)$$

and

$$P = r B_{\phi} / B_{\theta} \quad (3.29)$$

Bohm-like transport is used since this scaling is generally consistent with

Table VI. Classical and Anomalous Energy Confinement Times

Scaling	$\tau_E(\text{sec})$ (RFPR) ^a	$\tau_E(\text{sec})$ (Zeta) ^b
1. Classical ⁶¹	145.0	$9.0(10)^{-2}$
2. 1/200 Bohm	0.6	$9.0(10)^{-2}$
3. Hugill-Sheffield ⁶²		
a. Scaling 1	1.2	$3.9(10)^{-2}$
b. Scaling 2	12.5	$6.9(10)^{-2}$
c. Scaling 3	27.1	$6.7(10)^{-2}$
4. Pfeiffer-Waltz ⁶³		
a. Best scaling	16.6	$1.7(10)^{-1}$
b. Modified Alcator	125.6	$4.4(10)^{-1}$
5. Experimental ¹³	-	$\sim 3.0(10)^{-3} - 1.0(10)^{-2}$

^aSee Table III for RFPR parameters, $T = 15 \text{ KeV}$, $n = 2.0(10)^{20} \text{ m}^{-3}$

^bSee Table I for Zeta parameters, $T = 200 \text{ eV}$, $n = 6.0(10)^{19} \text{ m}^{-3}$

MHD turbulence.⁶ The triangular function, f_T , accounts for the width of a Suydam mode while f_S is a function varying from 0 to 1 which provides a measure of the degree of Suydam violation. The exact scaling of the perpendicular resistivity has been found to be relatively unimportant as long as it substantially exceeds (by 2 to 3 orders of magnitude) the classical values. The higher perpendicular resistivity increases radial transport of particles and energy, thereby reducing the pressure gradients that initiated the instability. Parallel resistivity is left classical since experimental ohmic heating rates are classical.¹³

C. Solution Technique

A solution of the transport equations is affected using a variable implicit/explicit Lagrangian meshing technique. First of all, the transport equations (Eqs. 3.19-3.23) are integrated forward in time on a fixed radial finite difference mesh. The mesh is then adjusted to satisfy Eq. 3.24 under the constraints of an ideal gas and flux conservation. Since the technique is partly implicit, a predictor-corrector forward differencing (in time) technique is used. Variable time steps are used to assure fast convergence, while energy balance and particle balance are used to check the accuracy of the model (See Appendix D).

D. Stability Model

Two methods are used to monitor the ideal MHD stability. Local modes are assumed to saturate and are monitored using Suydam's criterion.⁵ A Rayleigh-Ritz type energy principle is used to monitor global modes. Both methods for monitoring stability are derived from the ideal MHD equations using a cylindrical approximation.

In one-dimension the energy principle assumes the following form.⁶⁴

$$\delta W = (\pi/2) \int_0^a (f(d\xi/dr)^2 + g\xi^2) dr, \quad (3.30)$$

where

$$f = \frac{r(krB_\phi + mB_\theta)^2}{\mu_0((kr)^2 + m^2)} \quad (3.31a)$$

$$g = \frac{2(kr)^2}{(kr)^2 + m^2} \left(\frac{dp}{dr} \right) + \frac{(krB_\phi + mB_\theta)^2((kr)^2 + m^2 - 1)}{\mu_0 r((kr)^2 + m^2)} + \frac{2(kr)^2((kr)^2 B_\phi^2 - m^2 B_\theta^2)}{\mu_0 r((kr)^2 + m^2)^2}, \quad (3.31b)$$

The axial wave number is k , the mode number is m , and ξ is the radial displacement. The transform $\psi = \xi\sqrt{f}$ is employed in order to avoid the singularity in ξ when f approaches zero.⁸ Then Eq. (3.30) transforms to

$$\delta W = (\pi/2) \int_0^a \{ (d\psi/dr)^2 - (\psi/f)(df/dr)(d\psi/dr) + ((df/dr)^2/4f^2 + g/f) \psi^2 \} dr. \quad (3.32)$$

The parameter ψ is then expanded into a Fourier Series according to

$$\psi = \sum_{i=0}^n C_i \sin(\alpha_i r), \quad (3.33)$$

where $\alpha_i = (i+1)\pi/r_p$. The constant C_0 is normalized to unity and the remaining constants, C_i ($i = 1, \dots, n$), are computed to minimize the energy δW by requiring that $\partial \delta W / \partial C_i = 0$ for $i=1, \dots, n$. The resulting set of n inhomogeneous equations in n unknowns is inverted and solved for C_i . If

the resulting value of δW is negative, the associated plasma configuration is considered unstable. This check is applied to $m=0$, $k \rightarrow 0$ and $m=1$, $-30/R < k < 30/R$, since the long wavelength modes are typically the most unstable.⁸

In general, $\partial \delta W / \partial C_i = 0$ only provides a stationary solution rather than a minimum for the energy principle. A minimum requires that $\partial^2 \delta W / \partial C_i^2 > 0$ also. However, if $\partial^2 \delta W / \partial C_i^2 < 0$, it can be shown that the plasma is always unstable and the required answer is provided without inverting the matrix.

The Suydam criterion is included in the more general energy principle stated by Eq. (3.30). These Suydam modes, however, have eigenfunctions that are asymptotically singular at rational surfaces. Since the mesh size used by the one-dimensional computation is finite and determined by the previously described transport computation, eigenfunctions that are more localized than the mesh size will not be detected by the global energy principle analysis. Consequently, the global stability analysis can isolate gross modes that may also occur simultaneously with Suydam modes (which are assumed to saturate). The global energy principle analysis also quantifies the validity of the assumption that Suydam modes are localized in that the modes will be detected if the associated eigenfunctions extend over a "macroscopic" (i.e., mesh size) volume of the plasma.

Other expansion functions were also tried for the energy principle (Eq. 3.30) and gave similar results, both giving good agreement with analytic stability constraints.^{8,22} These functions are

$$\xi = \sqrt{f} \sin(\alpha_0 r) + \sum_{i=1}^n C_i \sin(\alpha_i r) \quad (3.34a)$$

for the $m = 0$ case and

$$\xi = \sqrt{f} \cos(\alpha_0 r/2) + \sum_{i=1}^n C_i \cos(\alpha_i r/2) \quad (3.34b)$$

for the $m = 1$ case where $\alpha_i = (i + 1)\pi/r_p$. These functions are finite everywhere, thus avoiding the singularity at the rational surfaces.

IV. BATCH BURN RFP REACTORS

A. Introduction

As mentioned earlier, previous RFP reactor studies have been principally based on "global" or "point" models which average plasma parameters over the plasma cross-section. One purpose of this research is to check the validity of one of the global models using a one-dimensional model that includes all profile effects. Specifically, the global model considered is that of R. Hagenson.⁵³ That comparison, as well as turbulent transport simulations, RFPR optimization studies, and compact RFP studies are all reported in this chapter.

Plasma performance was evaluated for six cases:

- A: The one-dimensional model was forced to follow exactly the radius trajectory predicted by the zero-dimensional model.
- B: The more desirable boundary condition of constant radius and moderately programmed toroidal current was examined.
- C: With the plasma radius fixed, regions of Suydam instability were subjected to enhanced transport in order to model a flattening of the pressure profile (e.g., Eqs. 3.26 to 3.29).
- D: Using Case B parameters, the magnetic field and density were varied to optimize Q_p while keeping the wall load constant.
- E: Keeping the wall load constant, transport was parametrically varied along with density and field to optimize Q_p .
- F: Same as Case D except reactor dimensions are cut in half.

For all cases except Case E the radial energy confinement time is set equal to 200 Bohm times and it is assumed that both the alpha-particle energy

and mass are deposited at the place of birth. Table VII summarizes the essential elements of the three simulation cases described above.

Table VII. Summary of Conditions Assumed for One-Dimensional Simulations

Case	Radius Trajectory	Energy Transport (f_B in Eq. 3.25)	Burn time ^b τ_B (s)
A ^a	moving	1/200	21.6
B	fixed	1/200	25
C	fixed	1/200 plus Suydam Turbulence	25
D	fixed	1/200	21.6
E	fixed	parameter	parameter
F	fixed	1/200	10.3

^aclosest simulation of zero-dimensional model

^bcycle time taken as burn time plus 5 s

B. Discussion of Case A Results

In order to match the point model as closely as possible, the plasma radius was confined to a trajectory $r_p(t)$ given by

$$r_p(t) = r_p(t=0)/\sqrt{1-\beta_\theta(t)}, \quad (4.1)$$

where the plasma radius r_p is defined by the toroidal field reversal point, and β_θ is the minimum poloidal beta. The initial temperature distribution was assumed uniform, and the initial density and toroidal field profiles within the plasma region were modeled by Bessel functions

(consistent with the point model)⁵³

$$n(r,t=0) = n(r=0,t=0) J_0^2(0.94\alpha r) \quad (4.2)$$

$$B_\phi(r,t=0) = B_\phi(r=0,t=0) J_0(\alpha r) \quad (4.3)$$

where $\alpha = 2.405/r_p(0)$. The initial poloidal field profile is computed to satisfy pressure balance.

Figure 5 compares the average ion and electron temperatures obtained with the one-dimensional model for Case A with the predictions of the zero-dimensional model. Shown also is the time dependence of poloidal beta, β_θ , and the fractional burnup. The radial dependences of temperatures and fields at $t = 12$ s into the burn are depicted in Figure 6. Figure 7 shows the density profile, the shear parameter (Eq. 3.29), P and the Suydam parameter, f_s (Eq. 3.27) at 12 seconds. For the parameters shown, $f_s > 0$, everywhere so the plasma is unstable to local Suydam modes over its entire volume. Figure 8 compares the neutron, transport, and radiation energy fluxes at the first wall.

The differences between the point and one-dimensional models illustrated in Figures 5-8, can be attributed to the self-consistent profiles contained in the one-dimensional code. The point model assumes a uniform plasma temperature, while the one-dimensional code has radially varying temperatures which are higher on-axis and cause enhanced fusion reactivity (and alpha-particle heating) in the central plasma region. This results in a higher average temperature shortly after ignition (Figure 5). The one-dimensional model burns out more rapidly, and has a lower value for f_B (Figure 5). This lower burnup is principally caused by small burnup

Figure 5. Comparison of averaged parameters versus time as predicted by the Case A one-dimensional simulation and the zero-dimensional model.

Figure 6. Temperature and field profiles for the Case A simulation at 12.0 s into the burn phase.

Figure 7. Profiles of density, pitch parameter P , and Suydam parameter, f_s , for the Case A simulation at 12.0 s into the burn phase.

Figure 8. Comparison of first-wall neutron, thermal conduction, and radiation fluxes for the Case A one-dimensional simulation and the zero-dimensional model.

near the plasma edge where the plasma is cooler.

The point burn model must specify a magnetic field profile to assure pressure balance and does not account for resistive decay of the toroidal field B_ϕ . In forcing a radius trajectory, $r_p(t)$, the one-dimensional model forces the toroidal current, I_ϕ , to decay, as resistive losses consume B_ϕ field. This results in somewhat less ohmic heating, although this effect is small.

The radial profiles shown in Figures 6 and 7 remain relatively unchanged throughout the burn period. Both the temperature and pressure show a maximum on the plasma center-line because of the assumed Bohm-like thermal conduction. However, since the bulk plasma is unstable to Suydam modes, enhanced transport occurs in this region. This is examined in Case C.

Table VIII summarizes performance parameters for both the zero-dimensional and Case A one-dimensional simulations. Of particular interest is the plasma energy gain factor, Q_p , defined as:

$$Q_p = 1.25 \cdot W_N / (W_{OHM} + W_{Bf}) \quad (4.4)$$

It is seen that for the conditions under which Case A was evaluated, a factor ~ 1.5 decrease in Q_p relative to the point model is predicted. This significant difference in Q_p illustrates the net influence of profile effects, with the reduction in Q_p a direct result of the lower fusion yield. However, as for any model, Q_p values are sensitive to modelistic assumptions. Some of the assumptions are examined further in the following cases.

Table VIII. Comparison of Energy Balance for Zero- and One-Dimensional Burn Models

<u>PARAMETER (MJ/m)</u>	<u>ZERO-D</u>	<u>ONE-D^a CASE A</u>
Burn time τ_B (s)	21.6	21.6
Cycle time τ_C (s)	26.6	26.6
Toroidal field on centerline, $B_{\phi 0}$ (t)	6.93	6.93
Toroidal current, I_ϕ (MA)	20.0	20.0
Average plasma density, $n(1/m^3)$	2.0 (10) ²⁰	2.0(10) ²⁰
Radiation energy, W_{RAD}	28.1	14.3
Ohmic heating energy, W_{OHM}	7.1	8.9
Plasma energy loss (conduction), W_{COND}	147.5	98.0
Plasma expansion energy, W_{PV}	0.7	1.1
Magnetic-field energy inside first wall at end of burn cycle, W_{Bf}	51.6	45.0
• Fusion neutron energy, W_N (14.1 MeV)	678.5	410.0
• $Q_p = (\text{fusion energy})/(\text{ohmic} + \text{field energy})$	14.4	9.5

^a radius forced to follow zero-dimensional case

C. Comparison of Constant Radius Cases With and Without Enhanced Transport, Cases B and C

The one-dimensional model was evaluated for a programmed rather than a constant toroidal current, using the formalism for Suydam turbulent transport represented by Eqs. 3.26-3.29. A comparison for the constant radius case with and without Suydam turbulence is given in Figure 9, where the time dependence of average temperature, β_0 , and the Suydam factor f_s are given. Typical radial profiles of temperatures and fields are depicted in Figure 10. Like the global parameters in Figure 9, Suydam turbulent transport has little effect on the temperature and field profiles. Figure 11 gives the radial dependence of densities and the Suydam parameter, f_s . Key plasma parameters for Case B and Case C are summarized in Table IX.

Even without Suydam turbulent transport a greater fraction of the plasma volume becomes Suydam stable (Figure 11) for the constant radius Case B. Suydam turbulence causes anomalous Bohm-like transport to occur in unstable regions (Eqs. 3.26 - 3.29), flattening the density profiles in the center of the plasma. There is a similar leveling of the pressure profile, pushing the plasma towards the marginal Suydam stability limit.

Comparison of the Suydam stability parameter, f_s , for Cases B and C shows that enhanced transport changes the plasma from one of uniform Suydam instability to a pointwise, stochastic violation of the Suydam criterion. As an unstable plasma region is stabilized by anomalous transport, pressure gradients in adjacent regions are increased, causing these latter regions to become unstable. The net result is a randomized or statistical violation of the Suydam criterion in regions of weak shear.

Figure 9. Comparison of Case B and Case C one-dimensional simulations showing the influence of Suydam turbulence on the average temperatures, poloidal beta, and burnup fraction. For f_B and β_θ , no difference was observed.

Figure 10. Typical temperature and field profiles at $t=10$ s for Cases B and C.

Figure 11. Comparison of Case B (no turbulence) and Case C (with turbulence) one-dimensional simulations in terms of density and f_s profiles that have evolved at ~ 10 s into the burn.

Table IX. Comparison of Energy Balance for
Nonturbulent and Turbulent Transport

PARAMETER (MJ/m)	ONE-D ^a CASE B	ONE-D ^b CASE C
Burn time τ_B (s)	24.6	25.0
Cycle time τ_C (s)	29.6	30.0
Toroidal field on centerline, $B_{\phi 0}$ (T)	6.93	6.93
Toroidal current, I_{ϕ} (MA)	15.2	15.2
Average plasma density, $n(1/m^3)$	$1.59(10^{20})$	$1.59(10^{20})$
Radiation energy, W_{RAD}	14.8	13.9
Ohmic heating energy, W_{OHM}	10.4	12.4
Plasma energy loss (conduction), W_{COND}	115.2	114.5
Plasma expansion energy, W_{PV}	0.0	0.0
Magnetic-field energy inside first wall at end of burn cycle, W_{BF}	45.1	45.5
Fusion neutron energy, W_N (14.1 MeV)	493.5	469.7
$Q_p = (\text{fusion energy})/(\text{ohmic} + \text{field energy})$	11.1	10.1

^afixed plasma radius, no Suydam turbulence

^bfixed plasma radius, Suydam turbulence

These results indicate that a major part of the plasma confinement is provided by the plasma edge regions.

The magnetic fields maintain a continuous topological structure despite the enhanced transport associated with the Case C model. Aberrations in both field and pitch parameter profiles are small compared to density perturbations (Figures 10, 11). This result is characteristic of anomalous transport scaling through only the perpendicular resistivity, $\eta_{\perp}$ (Eqs. 3.26-3.29).

Despite the significant differences in particle transport models used in Cases B and C, the plasma Q_p is increased only 10-20% from the Case A comparison case. The principle reason there is little change in Q_p is that the assumed Bohm-like thermal conductivity (Eq. 3.25) dominates the energy transport even in Suydam unstable regions. This also causes the differences in the trajectories of Figure 9 to be small. The only effect inclusion of Suydam turbulence has on plasma performance is that it levels the density profile which slightly reduces the fusion power by reducing n_i^2 . This, in turn, reduces Q_p .

D. Optimization of RFPR Design Point by One-Dimensional Model

The optimization of the RFPR design point by the one-dimensional model was a primary motivation for the "batch burn" studies. Overall reactor system considerations are not within the scope of this work, but a comparison with the point model is used to gauge if the predictions of the one-dimensional model are favorable. Establishing a 14.1-MeV neutron wall loading near the reference design⁵³ (2.7 MW/m²), while maintaining a similar power output, fixes the plasma line density

at $1.5(10)^{21} \text{ m}^{-3}$.

Using Case D (with fixed radius) listed in Table X, the magnetic field levels were reduced to increase Q_p (see Figure 12). If $B_{\phi 0}$ is reduced below $\sim 5.8 \text{ T}$, plasma ignition does not occur due to insufficient ohmic heating. As noted in Figure 12, the poloidal beta increases from 0.39 to 0.51 as the magnetic fields are reduced, still remaining within stability limits as predicted by ideal MHD theory.

The one-dimensional model requires less current for ignition than the global model because it allows both the current density and the temperature to peak on the plasma axis. This causes ignition on-axis initially with the burn spreading radially outward. The global model assumes a flat temperature and radially averages ohmic heating, which essentially requires the plasma to ignite everywhere at the same time.

The time dependence of the electron and ion temperatures, f_B and β_θ for the $Q_p = 20.4$ case are plotted in Figure 13. The plasma power fluxes are also given in Figure 13. The radial dependence of the magnetic fields is shown in Figure 14 for $t=0, 10$, and 20 s . At $t=0$ the plasma pressure is essentially zero, and the magnetic field profiles represent a pressureless plasma state. Expansion occurs as the plasma heats, causing the magnetic fields near the center to decrease in order to maintain pressure balance. This behavior is illustrated by the $t=10 \text{ s}$ case in Figure 14. Near the end of the burn ($t \approx 20 \text{ s}$) the plasma is again cool, and the fields relax toward the center. The effect of plasma expansion is also exhibited by the electron density profiles depicted in Figure 14.

The effects of fuel burnup are illustrated in Figure 14 by the decreasing ion density and corresponding increases in alpha-particle density

Table X. Comparison of Energy Balance for
Zero- and Optimized One-Dimensional
Burn Models

<u>PARAMETER (MJ/m)</u>	<u>ZERO-D</u>	<u>ONE-D^a CASE D (OPTIMIZED)</u>
Burn time τ_B (s)	21.6	22.0
Cycle time τ_C (s)	26.6	27.0
Toroidal field on centerline, $B_{\phi 0}$ (T)	6.93	5.80
Toroidal current, I_{ϕ} (MA)	20.0	12.9
Average plasma density, n (1/m ³)	2.0(10 ²⁰)	2.1(10 ²⁰)
Radiation energy, W_{RAD}	28.1	19.3
Ohmic heating energy, W_{OHM}	7.1	8.93
Plasma Energy loss (conduction), W_{COND}	147.5	146.7
Plasma expansion energy, W_{PV}	0.7	0.0
Magnetic-field energy inside first wall at end of burn cycle, W_{BF}	51.6	30.6
• Fusion Neutron Energy, W_N (14.1 MeV)	676.5	646.2
• $Q_p = (\text{fusion energy})/(\text{ohmic} + \text{field energy})$	14.4 ^b	20.4

^aFixed plasma radius

^bThe previous one-D simulations had $Q_p \sim 10$ (see Tables VIII and IX) so optimization approximately doubles Q_p .

Figure 12. Dependence of plasma Q-values and associated maximum poloidal beta on the initial toroidal bias field.

Figure 13. Time dependence of average plasma temperature, β_0 , and DT fuel burnup fraction for the optimum design point. Also shown are the neutron, radiation and conduction powers expressed per unit first-wall area.

Figure 14. Radial profiles of the magnetic fields, particle densities, ion and electron temperatures, magnetic field pitch and the Suydam stability parameter (f_s) for the optimal design point at various times.

as the burn proceeds. Regions of Suydam instability violation are shown in Figure 14 as f_s . Only a small region of the plasma is initially unstable, but the region grows as the temperature is increased. The region of Suydam instability is reduced as the plasma temperature decreases near the end of the burn ($t = 20$ s, Figure 14), although relaxation of the profiles has also contributed to the degradation of the stability.

The existence of a turbulent Suydam-unstable central region is common to axisymmetric configurations (both the tokamak and the RFP) and is not expected to limit severely the confinement properties because of the stable region between the turbulent plasma core and first wall.

E. Parametrically Varied Transport Simulations (Case E)

Since the exact energy confinement of an RFP is not known, the Bohm transport factor f_B in Eq. 3.25 was varied parametrically to determine its effect on plasma performance. Cases were run from $f_B = 1/100$ to $f_B = 1/300$. Results are shown in Figure 15.

Generally, better confinement allows longer burn times (τ_B) and lower currents for ignition. Both of these factors improve Q_p (since densities are maintained to keep the wall load approximately constant). It appears that the present RFPR design would be adequate for energy confinement times as poor as 100 Bohm times but not too much beyond this limit. If transport scaling is significantly worse, the reactor size will have to be increased to reduce thermal conduction.

MHD stability was maintained for $\tau_E \leq 200 \tau_{Bohm}$ but the pressure driven limit was exceeded for the $\tau_E = 300 \tau_{Bohm}$ case. However, this problem is easily rectified by increasing the current. Although this

Figure 15. Q_p , W_{B_0} (magnetic field energy) and τ_B versus transport Bohm factor (f_B).

lowers Q_p somewhat, it is already large enough that the reactor still performs well.

F. Compact Reversed Field Pinch Reactor (Case F)⁶⁵

In principle, the RFP is a good candidate for a small, relatively low capital cost demonstration reactor since it is stable at high β and is ohmically ignited. It was decided to try to size the CRFP at about 100 MWe which is small enough to have low capital cost but large enough to produce a useful amount of power. This required reducing the dimensions of RFPR by a factor of two (see Figure 16).

Assuming $\tau_E = 200 \tau_{Bohm}$, results (see Table XI) indicate that a 100 MWe CRFP can be ignited, although its performance is diminished from the larger RFPR. While the volume of the CRFP is decreased by a factor of 8, the net electrical output is reduced by a factor of 7.5. Consequently, the CRFP has a slight advantage in power density although its Q_p has dropped from 14.4 to below 10, indicating an undesirable increase in recirculating energy. A further reduction in size causes a rapid decrease in Q_p due to radial energy losses via thermal conduction. Indeed, to overcome large losses in the CRFP shown, it was necessary to increase the magnetic field strength by 25%.

Gross MHD stability was maintained throughout the burn. Suydam's criterion was violated on centerline as was the case for the RFPR simulations. However, as was shown earlier, inclusion of turbulent transport in regions where Suydam's criterion is violated does not greatly change the reactor's performance.

Overall, the CRFP performance was considered marginal. Consequently,

Figure 16. Relative sizes of RFP experiments and reactors.

Table XI. Comparison of CRFP Parameters with RFPR

<u>PARAMETER</u>	<u>RFPR</u>	<u>CRFP</u>
First wall radius, r_w (m)	1.5	.75
Major radius, R (m)	12.7	6.35
Burn time, τ_B (s)	21.6	10.3
Cycle time, τ_C (s)	26.6	15.3
Toroidal field on centerline, $B_{\phi 0}$ (T)	6.93	8.5
Toroidal current, I_ϕ (MA)	20.0	12.0
Average plasma density, n ($1/m^3$)	$2.0(10)^{20}$	$3.8(10)^{20}$
•14.1 MeV fusion neutron energy, W_N (MJ/m)	676.5	173.0
Total thermal power, P_{TH} (MWt)	3000	628
Net electrical power, P_E (MWe)	750	100
• $Q_p = (\text{fusion energy})/(\text{ohmic \& field energy})$	14.4	9.7

it was decided to try to improve performance by refueling. These studies are described in Chapter 6.

G. Conclusions

Generally, the one-dimensional model behaves similarly to the point model. Differences between the results of the two models are mostly caused by profile effects.

One significant difference is that local ignition and radial burn propagation allow the one-dimensional model to ignite with lower currents. This has led to a new optimized RFPR operating with better performance (higher Q_p) than was exhibited by the point model. The lower burnup exhibited by the one-dimensional model (Case A, Table VII) is also caused by profile effects. Lower plasma edge temperature leads to lower burnup in that region causing a smaller total fractional burnup.

Stability studies indicate that gross plasma stability can be maintained with proper field programming for the ideal $m=0$ and $m=1$ modes. Suydam instabilities occur over a substantial region of the plasma, but tend to self-stabilize when anomalous transport is included. This transport flattens the density profiles and causes the plasma to approach the marginally stable Suydam limit. This results in stochastic violation of Suydam's criterion in the inner plasma regions. However, this has little effect on gross plasma parameters.

Parametric transport studies indicate that energy transport with f_B (eq. 3.25) as low as 0.01 could be tolerated with the present device size. Confinement poorer than this would require a larger device. Conversely, assuming $f_B = 0.05$, the RFPR dimensions can be cut in half to make a small demonstration plant of about 100 MWe.

V. STARTUP

A. Introduction

One of the most important unsolved problems for the RFP is how to set up the required profiles. Currently, there are two general types of startup for an RFP that have been tried experimentally. Both methods have problems being scaled to a reactor.

One method is to use self-reversal which involves increasing the toroidal current until the toroidal field reverses and relaxes into a stable state. Unfortunately, the plasma is unstable and turbulent in the pre-reversed state and consequently it loses a significant amount of energy. The second method is to use field programming to set up the profiles on a time scale shorter than an MHD growth time. However, this requires prohibitively large electric fields for a reactor size plasma.

Baker and Phillips⁶⁶ have shown that diffusive startup processes generally require large energy losses (30% to 60%) to reach an equilibrium in the post-implosion phase. Experimentally, these processes result in high β plasmas after an equilibrium is found. Unfortunately, high β plasmas do not have large currents parallel to the field which are necessary to heat the plasma to ignition.

Consequently, it is desirable, and perhaps an absolute necessity, to find a non-diffusive startup technique with low energy losses (a few percent). As mentioned earlier, Newton¹¹ has suggested a convective startup technique that satisfies the necessary stability criterion that the pitch of the field lines (Eq. 3.29) does not have a minimum.⁸ We have extended this technique using the one-dimensional model and found a

startup scenario that maintains ideal MHD stability to both global and local modes during the startup and burn phases.⁶⁷

B. Modeling Technique

It can be shown¹¹ that for an ideal (non-resistive) plasma conservation of toroidal and poloidal magnetic flux implies that pitch is also conserved. This means that if an ideal plasma is expanded or compressed the pitch function maintains its shape. Due to its high conductivity, this is also true to zeroth order for a resistive plasma. Consequently, the idea of convective startup is to "freeze" the pitch of the field lines in the plasma at the wall in order to control the shape of the pitch profile during startup.

As discussed in Appendix C, a natural boundary condition for a Lagrangian code is the specification of the plasma boundary. Compressions and expansions are simple to do. Since convective startup is a plasma compression, we have chosen to model it by initially starting the plasma boundary at ten times the radius of the vessel (see Figure 17) and then moving the boundary inward as the simulation moves forward in time. The equation chosen for the boundary is:

$$r_p = \begin{cases} 10a(0.9((\tau_R - t)/\tau_R)^4 + 0.1) & t < \tau_R \\ a & t \geq \tau_R \end{cases} \quad (5.1a)$$

$$t \geq \tau_R \quad (5.1b)$$

where a is the vessel radius, τ_R is the rise time and t is the present time in the simulation.

An additional consideration is that it is desirable to have profiles that maintain stability through both the heating and burn phases. We

FSL-80-159

Figure 17. Illustration of boundary for startup simulation and vessel boundary.

had a good idea what post-implosion profiles were necessary to do this from the results of Chapter 4. Therefore, the final profiles desired were known but it was necessary to predict what initial profiles would properly evolve into the desired ones.

As noted earlier, resistive effects during startup are first order compared to adiabatic compression. Consequently, in order to find initial conditions, desired post-implosion profiles were chosen and the plasma was then adiabatically expanded to 10 times the wall radius. This produces initial profiles which evolve into post-implosion profiles similar to the desired ones.

C. Results

Using the programming procedures described above, a startup scenario has been found which exhibits ideal MHD stability to both local (Suydam) and gross modes during the startup and burn phases. Startup is achieved in 1 msec. The required feed gas rate and toroidal and poloidal fields at the wall are shown in Figure 18.

Figure 19 shows the radial profiles for temperature, density and magnetic fields during startup. The density initially peaks on-axis but the peak moves off-axis as the profiles build in. This was done intentionally to keep the pressure peaked off-axis. Peaking the pressure off-axis stabilizes Suydam modes (Eq. 3.28) in the central region which is weakly sheared.

Figure 20 shows the average temperatures, β_0 , fractional burnup and wall loading as a function of time during the plasma burn. Temperatures and wall loading are qualitatively similar to the optimized burn (Figure

Figure 18. Feed gas rate (S_w) and toroidal and poloidal fields (B_ϕ and B_θ respectively) at the wall during startup.

Figure 19. Field, density and temperature profiles during startup.

1. $t = 0.0s$	3. $t = 6 \times 10^{-4}s$
2. $t = 4 \times 10^{-4}s$	4. $t = 1.0 \times 10^{-3}s$

Figure 20. Time dependence of average temperature, β_θ , fractional burn-up and wall load for the stable RFP startup and burn.

13). The fractional burnup is somewhat lower as is the plasma Q_p (see Table XII). The off-axis density peak requires larger currents for ignition which is the principle reason for the reduced Q_p . However, this Q_p includes startup losses whereas the others do not. Hence, the effective plasma performance is improved.

D. Stability Considerations

In order to maintain ideal MHD stability during the pre-reversed stage it is necessary to peak the toroidal current off-axis. Peaking the current off-axis causes the pitch profile to strongly peak on-axis. This has a stabilizing effect for ideal MHD modes.

Figure 21 is a plot of a typical "g" function⁸ in Equations 3.30 and 3.31b. From the form of Equations 3.30, 3.31a and 3.31b it is apparent that the first term in $\delta w (\frac{\pi}{2} \int_0^{r_p} f(\frac{d}{dr})^2 dr)$ is always positive (and therefore stabilizing) since f is always positive. Therefore, the g function gives an indication of which regions of the plasma make negative and positive contributions to δw .

Figure 21 is read by selecting an axial wave number k and then drawing a horizontal line. The cross hatch regions are the radius positions where g is positive while in the plain regions g is negative. Note that there is a shaded region for $|k|$ small. This region has positive g if the current is peaked off axis and negative g if it is peaked on-axis. Therefore, peaking the current off-axis helps stabilize long wavelength instabilities. Simulations with the one-dimensional code have shown that this effect is sufficient to stabilize the plasma in the pre-reversed state.

Table XII. Comparison of Energy Balance for Zero- and One-Dimensional Burn Models

PARAMETER (MJ/m)	ZERO-D	ONE-D ^a CASE A	ONE-D ^b CASE D (OPTIMIZED)	ONE-D ^c CASE G
Burn time τ_B (s)	21.6	21.6	22.0	20.0
Cycle time τ_C (s)	26.6	26.6	27.0	25.0
Toroidal field on centerline, $B_{\phi 0}$ (T)	6.93	6.93	5.80	7.50
Toroidal current, I_{ϕ} (MA)	20.0	20.0	12.9	22.3
Average plasma density, $n(1/m^3)$	$2.0(10)^{20}$	$2.0(10)^{20}$	$2.1(10)^{20}$	$2.7(10)^{20}$
Radiation energy, W_{RAD}	28.1	14.3	19.3	22.0
Ohmic heating energy, W_{OHM}	7.1	8.9	8.93	17.6
Plasma energy loss (conduction), W_{COND}	147.5	93.0	146.7	159.2
Plasma expansion energy, W_{PV}	0.7	1.1	0.0	0.0
Magnetic-field energy inside first wall at end of burn cycle, W_{Bf}	51.6	45.0	30.6	68.6
Fusion neutron energy, W_N (14.1 MeV)	676.5	410.0	646.2	676.0
$Q_p = (\text{fusion energy})/(\text{ohmic} + \text{field energy})$	14.4	9.5	20.4	9.8

^aradius forced to follow zero-dimensional case^bfixed plasma radius, Bohm conduction^cprogrammed startup

Figure 21. Regions of positive and negative g as a function of k and r for a decreasing non-reversed pitch profile.

Figures 22 and 23 show the current profiles and pitch profiles initially, at the end of startup, and at the end of the burn. Note that the toroidal current diffuses rapidly into the central plasma. This results in a correspondingly rapid decrease in the pitch profile (Figure 23) on axis. The reason is that

$$\frac{\partial J_\phi}{\partial t} = \eta \left(\frac{\partial^2 J_\phi}{\partial r^2} + \frac{1}{r} \frac{\partial J_\phi}{\partial r} \right) \quad (5.2)$$

which means that currents rapidly diffuse inward due to the $1/r$ dependence of the second term in Eq. 5.2. Thus, the startup must proceed fast enough that reversal is achieved before too much current diffuses in.

Since the current is not an input parameter to the one-dimensional code, peaking the toroidal current off-axis was accomplished by flattening the initial toroidal field on-axis. The toroidal field profile that provided stability was

$$B_\phi = B_{\phi 0} \left[\frac{2r/r_p}{1 + r/r_p} J_0(\alpha r) + \frac{1 - r/r_p}{1 + r/r_p} \right] \quad (5.3)$$

where $\alpha = 2.405/0.77 r_p$. The dashed curves in Figure 23 are initially unstable pitch profiles. Curve A is the pitch profile corresponding to

$$B_\phi = B_{\phi 0} \left[\frac{4r/r_p}{(1 + r/r_p)^2} J_0(\alpha r) + \frac{(1 - r/r_p)^2}{(1 + r/r_p)^2} \right] \quad (5.4)$$

and curve B is the pitch profile corresponding to

$$B_\phi = B_{\phi 0} J_0(\alpha r) \quad (5.5)$$

An unstable eigenfunction for curve A is in Figure 24. Note that it peaks in the central part of the plasma where g is negative. Eigenfunctions for curve B are similar.

Figure 22. Current density profiles as a parameter of time. Numbers in parentheses indicate time.

Figure 23. Pitch profiles as a parameter of time compared to unstable profiles.
 Curve A: marginally unstable
 Curve B: Bessel function model²²

Figure 24. Unstable perturbation function for Curve A in Figure 23.

The stability limits discussed here are not predicted by previous "rules of thumb."⁸ They are profile dependent and require the complete energy check (described in Chapter 3) to be detected. Due to current diffusion, a profile's stability characteristics are time dependent also. Thus, the stability characteristics of convective startup depend both on the initial conditions and the startup procedures.

For the 10 times the radius plasma expansion, stability constraints required the pitch on axis to be greater than 6 m and the center-line to peak-current ratio to be less than 40%. These specific limits are valid for the profiles we used and are not necessarily valid in general.

Qualitative verification of the stability characteristics described here has been achieved on ZT-40.⁶⁶ Pitch programming experiments work much better when the toroidal field leads (in time) the poloidal field. This places a seed toroidal field in the plasma to provide stability which is precisely what the simulations show is necessary.

E. Conclusions

A startup scenario has been found which theoretically predicts ideal MHD stability for both local and gross modes during the startup and burn phases for an RFP reactor. The toroidal current density must be peaked off-axis in the pre-reversed phase in order to maintain ideal MHD stability. For the startup scenario shown, the pitch on center-line must be greater than 6. These trends have been verified qualitatively on ZT-40.

Density and pressure are peaked off-axis in order to stabilize Suydam modes in the weakly sheared plasma interior during the burn phase. Density profiles diffuse little due to high temperatures and classical

resistivities. The off-axis density peak requires larger currents for ignition and hence has a lower Q_p .

VI. REFUELING

A. Introduction

Previous work using a global model¹² has indicated that substantial improvement in reactor performance can be achieved by refueling the plasma with cold gas. This happens because batch burns are extinguished by fuel depletion, rather than ash buildup or field decay. Refueling extends the burn time leading to more fusion energy per pulse. Extending the burn time increases Q_p since the principle energy investment is the magnetic field energy placed in the plasma during the startup phase, which is independent of the burn time. Thus, refueled burns produce more fusion energy than batch burns with the same initial energy investment.

However, refueling introduces new problems of fuel penetration and stability. It was anticipated⁶⁹ that stability could be a particularly acute problem for the RFP since the plasma is supported by resistively decaying currents. Since the global model assumes that profiles maintain their qualitative shape throughout the entire burn, it is inadequate to address the combined refueling and stability problem.

As was discussed in Chapter 4, performance of the batch burn Compact Reverse Field Pinch (CRFP) was significantly degraded compared to the large reactor. This fact, along with the smaller radius which reduces refuel penetration problems, make the CRFP a logical candidate for a refueling study. This chapter considers refueling the CRFP with both gas puffing and pellets.

Gas puffing and pellet refueling represent the two extremes of

refuel penetration. In the gas puffing case, refuel ions are assumed to immediately mix with the fuel. This is modeled by adding refuel ions in proportion to the density profiles in the plasma. There is substantial experimental evidence that gas refueled tokamaks⁷⁰ behave in this manner.

If anomalous mixing does not occur, it is necessary to use injection to achieve plasma penetration by the refuel ions. The most energy efficient manner to do this is pellet injection. For these simulations, the fuel is assumed to equilibrate where it is deposited in the plasma.

B. Background

For the refueling simulations, the following constraints are imposed:

$$\langle P_{\text{wall}} \rangle \leq 2.7 \text{ MW/m}^2 \text{ (14.1 MeV neutrons)} \quad (6.1)$$

$$\tau_b = t(T_i = 5 \text{ KeV}) \text{ s.} \quad (6.2)$$

$$\tau_c = \tau_b + 5 \text{ s.} \quad (6.3)$$

along with gross ideal MHD stability.

The first constraint is chosen to be consistent with the global studies.⁵¹ It is set by neutron wall damage and leads to an estimated first wall lifetime of about 5 years. The temperature at which the burn is extinguished is 5 KeV since fusion power is small below this temperature. The cycle time is chosen so the down time between pulses is 5 s. This corresponds to an estimate of the time required to pump down the chamber and get it ready for another pulse.⁵¹

Since Q_p increases with burn time, the optimum mode in which

to operate an RFP is steady-state (i.e. constant power and temperature). However, this is not possible for a divertorless reactor with wall recycle (i.e. no particles leave the reactor chamber during the burn). This is easily demonstrated using the global balance equations for energy,

$$\frac{3}{2} N \frac{\partial T}{\partial t} = \frac{-3}{2} \frac{NT}{\tau_E} + K_1 K_2 N_i^2 - \frac{3}{2} ST, \quad (6.4)$$

electron line density,

$$\frac{\partial N_e}{\partial t} = \frac{N_e}{\tau_p} + \frac{S}{2}, \quad (6.5)$$

and ion line density,

$$\frac{\partial N_i}{\partial t} = \frac{N_i}{\tau_p} + \frac{S}{2} - K_1 N_i^2, \quad (6.6)$$

where N_e is the electron line density, N_i is the ion line density, N is the total line density, T is the average temperature, S is the line density source rate, τ_E is the energy confinement time, τ_p is the particle confinement time, K_1 is the fusion reactivity coefficient, and K_2 is the birth energy of an alpha particle (3.5 Mev). These equations are adequate to model an RFP plasma provided the plasma is above ignition ($T_i \geq 15$ kev) and the ion temperature is about the same as the electron temperature ($T_i \leq 60$ kev). An ignited plasma allows ohmic heating to be neglected while moderate temperatures allow a one-temperature plasma model.

Steady-state requires the LHS of Equation 6.4 to be zero (constant temperature) and $K_1 K_2 N_i^2$ to be constant (constant power). Also, divertorless operation with wall recycle sets $\tau_p = \infty$. Consequently, Equation 6.5

becomes

$$N_e = N_0 + \frac{1}{2} \int_0^t S dt \quad (6.7)$$

and Equation 6.6 becomes

$$S = 2K_1 N_0^2 \quad (6.8)$$

Substituting Equation 6.8 into Equation 6.4 yields

$$0 = \frac{-3}{2} \frac{NT}{\tau_E} + K_1 K_2 N_i^2 - 3 K_1 N_i^2 T \quad (6.9)$$

Since the last two terms of Equation 6.9 are constant, N/τ_E must be constant, which is generally not the case. Even if $\tau_E \propto N$ a steady-state will not persist due to limitations in β . Equations 6.7 and 6.8 show that N_e grows linearly in time and since the plasma pressure, p , is proportional to $N_e T$, the plasma pressure also grows linearly in time. Increasing p increases β_θ until the stability limit ($\beta_\theta \sim .5$) is exceeded and the plasma disrupts. Steady-state operation with a divertor is beyond the scope of this study but will be commented on briefly in the next chapter.

Since steady-state operation with wall recycle and no divertor is not possible, the optimal scenario is to try to extend the plasma burn as long as possible. Previous global simulations performed best when a low-density plasma was ignited and allowed to reach a high temperature (~ 55 keV) before refueling was started (see Figure 25). It was found that, using a constant refuel rate, the wall load could be kept fairly constant (thus minimizing thermal fatigue) as the plasma temperature

Figure 25. Temperature and density as a function of time for the global model refuel.

decreased (Figure 26). This gave burn times of about 100 seconds.

For these "slow" refuels, the first and second terms on the RHS of Equation 6.4 dominate. Thus, the energy thermally conducted out of the plasma is slightly larger than the fusion power input leading to a slowly decreasing temperature. Since a constant refuel rate produced long burns with a relatively constant wall load for the global simulations, it was also used for the one-dimensional simulations.

C. Gas Puffing Simulations

Tokamak experiments⁷⁰ have shown that an effective way to build up the plasma density is to introduce cold gas into the plasma. These experiments change the density substantially on a millisecond timescale. In tokamaks, the gas quickly redistributes itself (on a fast timescale compared to resistive diffusion) in the plasma along the original density profile. In order to simulate this with the one-dimensional model, a volumetric source rate the same as the density profile was introduced:

$$S(r) = S_0 J_0(\alpha r) \quad (6.10)$$

where $\alpha = 2.405/a$.

Unfortunately, the CRFP does not operate at temperatures as high as those in the global simulations due to different thermal conduction assumptions. As mentioned previously, the one-dimensional code assumed Bohm-like energy confinement compared to classical with a β limit¹² assumed by the global model. This makes $\tau_E \propto 1/T$ for the one-dimensional model, which prohibits the attainment of high temperatures with a reasonable power density. Thus, hundred second burn times are not attainable.

Figure 26. Total wall load as a function of time for the global model refuel.

The refueling scenario used is to choose an initial density that gives a wall load near 1.0 MW/m^2 for temperatures in the 5-10 keV range. After the plasma heats to 10 keV, ($t = 5.2 \text{ s}$) it is slowly refueled to gradually increase the power level. Figure 27 shows the average temperatures, β_0 , fractional burnup, and wall load.

It was found that the plasma performed better when it was allowed to burn near ignition for a substantial length of time before igniting. This both lengthens the burn time and levels the wall load allowing a substantial amount of ash to build up in the plasma before ignition. Since this dilutes the plasma with ash prior to ignition, the peak temperature and peak wall load are less than if the plasma had been allowed to quickly ignite. The longer burn time increases Q_p by 75% (see Table XIII).

The refuel rate is 11.5% per second of the initial fill density. This is a slow refuel rate (r.e. Equation 6.4) and does not lead to MHD instabilities, although ohmic dissipation is considerable (see Figure 28) due to low operating temperatures. Ohmic dissipation leading to instability can be a substantial problem for pellet refueling, which is discussed in the next section.

D. Pellet Refueling

Although tokamak experiments exhibit anomalous mixing of refuel gas with the fuel, it is unclear whether this will occur in RFPs, or at thermonuclear temperatures. One possible solution is to deposit fuel in the plasma directly by injecting high-velocity pellets of deuterium-tritium ice. Previous studies^{71,72} have considered this for refueling with low-velocity pellets in a tokamak with a divertor, but this is the

Figure 27. Average temperatures, fractional burnup, β_θ and wall load for a gas refueled CRFP.

Table XIII. Summary Description of Design Parameters

<u>PARAMETER</u>	<u>CRFP</u>	<u>GAS REFUELED CRFP</u>
First wall radius, r_w (m)	.75	.75
Major radius, R (m)	6.35	6.35
Burn time, τ_B (s)	10.3	21.0
Cycle time, τ_C (s)	15.3	26.0
Toroidal field on centerline, $B_{\phi 0}$ (T)	8.5	8.4
Toroidal current, I_ϕ (MA)	12.0	13.2
Average plasma density, n ($1/m^3$)	$3.8(10)^{20}$	$2.7(10)^{20}$
•14.1 MeV fusion neutron energy, W_N (MJ/m)	173.0	333.0
Total thermal power, P_{TH} (MWt)	628	820
Net electrical power, P_E (MWe)	100	240
• Q_p - (fusion energy)/(ohmic & field energy)	9.7	17.2
Refuel source rate (m-sec) $^{-1}$	-	$3.1(10)^{19}$

Figure 28. Density and field profiles as a function of time for the gas refueled CRFP.

1. $t = 0.0 \text{ s}$	4. $t = 12.6 \text{ s}$
2. $t = 4.2 \text{ s}$	5. $t = 16.8 \text{ s}$
3. $t = 8.4 \text{ s}$	6. $t = 21.0 \text{ s}$

first study that includes magnetic field diffusion, pressure balance, and stability effects, which are significant for an RFP.

The ablation model of Parks et al.⁷³ was used to model dissipation and particle deposition. This model has since been extended to include finite β magnetic field effects by Gilliard and Kim,⁷² but the modifications do not substantially change the pellet performance for RFP conditions.⁷² The ablation model is applicable since the regression of the pellet surface is slower than the sonic velocity. The plasma and refuel gas are assumed to immediately equilibrate in-situ since the classical particle transport time of the cold gas is longer than the classical equilibration time (seconds versus milliseconds).⁷¹

In order to get significant (1/2 the plasma radius) fuel penetration, pellet velocities must be 10^4 - 10^5 m/sec. Present techniques are thought to be able to accelerate pellets to velocities of 10^2 - 10^3 m/sec.^{74,75} Higher velocities (10^4 - 10^5 m/sec) require a more advanced acceleration technique such as laser ablation. For the pellets considered in this study, approximately 10 kJ is required to accelerate the pellet. Simple scalings using CO₂ lasers indicate that this is conceivable.⁷⁶

The refueling scenario uses the same initial conditions as the gas puffing runs with pellets injected into the plasma at a rate of 2 pellets per second after the plasma has been heated to near ignition. The constraints applied are the same as for the gas puffing case (Eqs. 6.1-6.3). Results of the simulations are summarized in Table XIV. Q_p increased by 34% compared to the batch burn case.

The pellet size was chosen so that about 5% of the initial plasma particles are in the pellet. It is desirable to size the pellets as

Table XIV. Summary Description of Design Parameters

PARAMETER	CRFP	PELLET REFUELED CRFP
First wall radius, r_w (m)	.75	.75
Major radius, R (m)	6.35	6.35
Burn time, τ_B (s)	10.3	18.1
Cycle time, τ_C (s)	15.3	23.1
Toroidal field on centerline, $B_{\phi 0}$ (T)	8.5	8.4
Toroidal current, I_ϕ (MA)	12.0	13.2
Average plasma density, n ($1/m^3$)	$3.8(10)^{20}$	$2.7(10)^{20}$
• 14.1 MeV fusion neutron energy, W_N (MJ/m)	173.0	256.5
Total thermal power, P_{TH} (MWt)	628	706
Net electrical power, P_E (MWe)	100	160
• $Q_p = (\text{fusion energy}) / (\text{ohmic \& field energy})$	9.7	13.2
Pellet radius (cm)	-	0.195
Pellet velocity (m/sec)	-	$6.2(10)^4$
Pellet injection rate (sec^{-1})	-	0.5
Effective refuel rate (m-sec^{-1})	-	$2.8(10)^{19}$

large as possible since larger pellets penetrate farther into the plasma than smaller pellets injected at the same velocity. Further increases in the pellet size reduce the injection rate to the point that only a few pellets are injected during an entire burn. Consequently, the time dependence of the power density is erratic and results in an unpredictable burn (i.e. small changes in parameters cause large changes in the burn if only a few pellets are injected). The pellet velocity was chosen so that the maximum pellet penetration distance was approximately 90% of the plasma radius.

Figure 29 shows the average temperature, β_0 , fractional burnup and wall load for the pellet refueling case. As in the gas puffing cases, the scenario that performed best was to burn the plasma near ignition for several seconds before having it ignite. As before, this smooths the wall load and results in longer burn times with higher Q_p .

Although Q_p was higher for the pellet case than the batch burn case, the pellet case did not perform as well as the gas refueled case (refer to Tables XIII and XIV). The principle reason the gas refueled case performed better is that it maintained the density peak on the center-line whereas pellet refueling moved the peak off-axis (see Figure 30). Since the power density is proportional to the density squared, it is optimal to peak the density on the center-line where the temperature is highest (i.e. $\langle \phi v \rangle$ is peaked on center-line). This resulted in a longer burn time for the gas refueled case. A comparison of the startup simulation and the optimized RFPR shows the same behavior in that the off-axis density peak startup simulation had a lower Q_p than the on-axis density peak optimized RFPR (see Table XII).

Figure 29. Average temperatures, fractional burnup, β_θ and wall load for a pellet refueled CRFP.

Figure 30. Plasma density and magnetic field profiles as a function of time for the pellet refueled CRFP. The curves correspond to:

1. $t = 0.0$ s	4. $t = 11.0$ s
2. $t = 3.6$ s	5. $t = 14.6$ s
3. $t = 7.4$ s	6. $t = 18.1$ s

Pellet refueling also peaks the pressure off-axis which reduces the Suydam unstable region in the plasma (see Figure 31). This results in less turbulence. Global stability is maintained although significant ohmic dissipation of magnetic fields occurs (see Figure 30).

However, magnetic field decay and stability are functions of the pellet size and velocity. It is necessary to deposit the refuel ions within a half plasma radius of the center of the plasma in order to get improved burn performance and to maintain stability. Too little pellet penetration results in poor burns and deterioration of the plasma currents near the edge. This causes the toroidal field reversal point to diffuse inward, resulting in an instability. Too much penetration results in a strongly peaked center-line density which both peaks the pressure on-axis and causes a pitch minimum due to local current decay. As shown by Robinson,⁸ profiles with a pitch minimum are always unstable.

To illustrate these effects, burns were done using the initial conditions, pellet size, and pellet refuel rates of Table XIV. For the under-penetration case (Case 2) a pellet velocity of 10^3 m/sec was used while a velocity of $2.0(10)^5$ m/sec was employed for the over-penetration case (Case 3). Typical density deposition profiles for these pellets are shown in Figure 32. Both cases caused MHD instabilities.

Figure 33 shows the density buildup and field decay for Case 2. The increased density near the plasma edge lowers the edge temperature and hence causes current dissipation in this region. Consequently, the reversal point diffuses inward which results in a current driven instability 14.0 seconds after the discharge is formed.

Figure 31. Suydam factor for gas refueled CRFP (curve A) and pellet refueled CRFP (curve B) approximately 8 seconds into the burn.

Figure 32. Typical density deposition profiles for pellet refueling.

1. $V = 6.2 \times 10^4 \text{ m/sec}$
2. $V = 1.0 \times 10^3 \text{ m/sec}$
3. $V = 2.0 \times 10^5 \text{ m/sec}$

Figure 33. Magnetic fields and density as a function of time for the under-penetration case (case 2).

1. $t = 0.0$ s
2. $t = 3.6$ s
3. $t = 7.2$ s
4. $t = 10.8$ s
5. $t = 14.5$ s

Figure 34 shows the density buildup and field decay for Case 3. The first pellet injected reaches the center-line causing a pitch minimum to occur 5.3 seconds into the discharge. Extremely high center-line densities cause localized current decay on-axis which reduces the pitch on-axis. The pitch minimum is unstable to a current driven instability which terminates the burn.

E. Conclusions

Results have shown that although a steady state RFP is not possible for operation in a divertorless mode with wall recycle, energy gain factors can be significantly improved by refueling. Since batch burns are extinguished by fuel depletion rather than excessive energy losses, refueling extends the burntime and raises Q_p . The best refueling scheme appears to be gas puffing, provided the cold gas anomalously mixes with the plasma. There is experimental evidence for this with tokamaks at low (sub-KeV.) temperatures, but the effectiveness of gas puffing for a high-temperature RFP is unknown.

If anomalous penetration is not possible, pellet refueling is another alternative. However, the pellets requires high velocities (6.2×10^4 m/sec) to achieve adequate penetration and would necessitate using an advanced pellet acceleration technique like laser ablation. Pellet injection can also cause MHD instabilities if either under-penetration or over-penetration occurs. Under-penetration causes excessive current dissipation near the plasma edge, eventually resulting in inward diffusion of the toroidal field reversal point which leads to a current driven instability. Over-penetration preferentially dissipates the current on-axis causing a pitch minimum which also results in a current driven mode. Both of these instabilities terminate the burn.

Figure 34. Magnetic fields and density as a function of time for the over-penetration case (case 3).

1. $t = 0.0$ s
2. $t = 7.3$ s
3. $t = 18.1$ s

Gas puffing increased Q_p by 75% while pellet refueling increased Q_p by 34% over the batch burn case. Gas puffing worked better since it concentrated fuel in the hottest plasma region (on center-line) while pellet refueling peaked the density off-axis where the plasma is cooler. Both refueling techniques improve the CRFP performance enough to make it an attractive concept for a scale up to a more efficient reactor.

VII. CONCLUSIONS AND PROPOSED EXTENSION

A. Introduction

The work reported in this thesis represents the first comprehensive one-dimensional study of the RFP reactor regime. This project has both developed a one-dimensional transport and stability code (RFPBRN) and applied it to several reactor problems. Thus, the product of this thesis is two-fold in that a useful new tool was developed and it was applied to learn new physics and gain insight into practical reactor problems. The project has also generated some ideas for new work which will be discussed later in this chapter.

B. Conclusions

The "batch burn" studies (reported in Chapter 4) verify the accuracy of previous global-model studies and also define a new operating point for the RFPR reactor. Center-line ignition, which results from the one-dimensional inclusion of temperature and current profiles, allows a significant reduction in the ohmic current required to ignite the plasma. The resulting reduction in field energy requirements leads to an approximate doubling of Q_p .

Parametric variation of the transport assumptions indicate that the RFPR design looks attractive for energy confinement times as poor as 100 Bohm times. Yet worse transport scaling would require the reactor to have a larger minor radius, resulting in a larger machine, both in size and power level.

Inclusion of turbulent transport due to Suydam instabilities had little effect on global parameters, but principally resulted in a

flattening of the density profile. The reason energy confinement was not greatly affected is two-fold. First of all, the turbulence is confined to the weakly sheared central plasma region. Secondly, the energy transport is dominated by the already anomalous thermal conductivity.

The major effect is that the turbulent transport flattened the density and pressure profiles which forced the plasma towards the marginally stable limit, resulting in a stochastic, pointwise violation of the Suydam criterion. In most cases, these modes were confined to the weakly sheared central plasma region which causes them to saturate and only result in enhanced transport,⁷ as the model assumed.

Global stability was easily maintained for all of the "batch burn" cases provided that the magnetic fields were programmed to keep the plasma at the wall. The Bohm-like thermal conductivity kept β_θ from getting too high while high temperatures prevented instabilities due to field diffusion. However, both of these characteristics are dependent on having the proper transport scaling.

In order to determine the feasibility of a small, low capital cost RFP reactor, a study was done on a Compact RFP (dimensions were 1/2 those of the large reactor). The reactor performed reasonably well, but the optimized Q_p was only about 1/2 that of the larger machine. Further size reduction lowers Q_p to an unacceptable value. Consequently, a refueling study was done to try to increase Q_p .

Although a steady-state does not exist for a divertorless plasma with wall recycle, previous global studies indicated that higher Q_p values could be achieved by refueling. Refueling increases Q_p because burns are quenched by fuel depletion, not large energy loss rates. Refueling extends burn

times thus increasing Q_p . Two refueling techniques were considered: gas puffing and pellet injection. Gas puffing assumes the limiting case of complete mixing while pellet injection assumes the other limit of no mixing.

The gas puffing simulations assume that anomalous mixing of the fuel and background plasma occurs, as is the case in the Alcator experiment. Gas puffing leads to longer burn times and a flatter wall load. Q_p was increased by 75%.

If anomalous mixing does not occur, fuel penetration can be achieved by high velocity injection of solid D-T pellets. Using this technique with in-situ fuel deposition (no anomalous mixing) increases Q_p by about 30%. This also results in a longer burn time and flatter wall load. Since pellet refueling peaked the density off-axis, the pellet refueled case did not have as high a Q_p as the gas puffing case did.

An interesting result of this work is that pellet injection peaks the pressure off-axis (as well as the density) which reduces Suydam instabilities (resulting in decreased plasma turbulence. Unfortunately, pellet refueling requires high pellet velocities (6.2×10^4 m/sec) in order to achieve adequate plasma penetration. This requires an advanced pellet acceleration technique like laser ablation.

Both under-penetration and over-penetration can cause MHD instabilities. Under-penetration does not provide fuel to the hot inner plasma regions (so Q_p is not improved) and also causes preferential resistive decay of the current in the outer plasma regions. This causes the toroidal field reversal point to move inward which results in a current driven MHD instability.

Similarly, over-penetration causes extremely large densities on-axis

which preferentially lowers the current on-axis. This results in a pitch minimum which also is unstable to a current driven MHD mode. The large density on axis is due to the $1/r$ dependence of the flux surface volume. However, this problem is easily remedied by injecting slightly off-axis.

Gas puffing appears to be the preferential refueling technique, provided that anomalous mixing occurs. Under-penetration of the pellets is the major drawback to pellet refueling since this doesn't increase Q_p and causes disruptive MHD instabilities. High pellet velocities that require an advanced acceleration technique are needed to overcome under-penetration.

One of the most significant contributions of this work is that a scenario has been found which maintains ideal MHD stability to global and local modes during the entire startup and burn of the RFP reactor. This is the first time that this has been shown. This is an important contribution since energy losses due to instabilities during startup are a major stumbling block to the RFP reactor.

Startup is achieved using a pitch convection technique. This technique is modeled as a plasma compression. The toroidal and poloidal fields at the wall along with the plasma feed rate at the wall are monitored to determine the proper programming profiles that would give the same behavior as the simulation.

In order to maintain stability it is necessary to peak both the density and the toroidal current off-axis. Peaking the density off-axis causes the pressure to be peaked off-axis which stabilizes Suydam modes in the weakly sheared central plasma region. Peaking the toroidal

current off-axis stabilizes current driven modes in the pre-reversed state. These instabilities are not predicted by analytic stability limits and required use of the numeric energy principle check to be found. The pitch programming stability characteristics have been verified qualitatively in the ZT-40 experiment.

C. Future Work

As is the case in many research projects, this work has suggested as many questions as it answered. Some of these questions and relevant extensions of the thesis are discussed here. Work has already started on some of the extensions.

Perhaps the most important uncertain element for the RFP is that the transport is poorly understood. Although the transport model chosen here agrees qualitatively with experiment, there is insufficient data to determine an accurate scaling. Consequently, improvement in the transport assumptions will largely depend how successful current experiments (ZT-40, η -BII and HBTX-IA) are at burning through impurities so transport scalings can be determined.

One likely candidate for enhanced transport is resistive "g" modes. These are the resistive MHD equivalent of a Suydam mode and are unstable (but with slow growth rates) in a decreasing pressure gradient. G modes are localized around rational surfaces like Suydam modes. Since the RFP has an infinite nesting of $m=1$ rational surfaces around the reversal point, island formation and overlap is a good candidate for transport.⁷⁷ This should provide a mechanism for transporting both particles and energy through the outside region of the plasma. A linear resistive MHD code⁷⁸

currently exists at Los Alamos which could be used to provide growth rate scalings for g modes which can be used to derive transport coefficients in a manner similar to that used for Suydam modes.⁶

One apparent paradox of the RFP is that experimentally, particle confinement appears to be much longer than energy confinement. A possible explanation of the RFP observations is that the plasma density may be refueled by a reflux of neutrals from the wall. This appears to be the case in Tokamaks.⁷⁹ Consequently, a neutral recycle model (several of which already exist)^{80,81} would be a useful addition to the code.

Anomalous particle transport coupled with recycle or refueling and a divertor may allow the RFP to operate in a steady-state or near steady-state mode. Two and a half dimensional MHD simulations indicate that a poloidally asymmetric^{82,83} steady-state RFP configuration may exist (called an ohmic state) although the one-dimensional equivalent does not. This lends itself to the possibility of running in a tokamak-like mode (pulse times of hundreds of seconds or longer), which would be interesting to explore.

Another useful addition to RFPBRN would be an impurity line radiation model. Including impurities would allow the modeling of low temperature experiments, which have their energy losses dominated by line radiation. This is already being undertaken.

There are also some interesting extensions for the stable startup simulations. Robinson⁸⁴ has indicated that in order for a pitch profile to be stable to resistive tearing modes, it must be flat on-axis. Since the ideal MHD stable startup pitch profiles are strongly peaked on axis, tearing is a strong possibility. This indicates that pitch profiles

stable to both ideal MHD and resistive MHD modes would probably need to be flat on axis with a safety factor (q) near 1. However, this would lower the poloidal field in the plasma and could cause problems with the pressure driven stability limit. This would lead to a trade-off between the aspect ratio and toroidal current, which is a completely new constraint for an RFP.

As mentioned previously, one problem for the stable RFP startup scenario is that it requires the toroidal current to be peaked off-axis, which is a condition that resistively decays. This could be a serious problem if the resistivity is anomalous. One possible approach is to put the current in external conductors rather than in the plasma in the pre-reversed state. This yields a configuration similar to a tokapole.^{85,86} Interestingly enough, tokapole experiments⁸⁵ have shown field enhancement similar to an RFP.

Simulation of this requires using 1-1/2 dimensional transport⁸⁷ coupled with a two dimensional stability check. RFPBRN is currently being rewritten to do 1-1/2 D transport, which is one dimensional transport across flux surfaces coupled with a two dimensional spatial pressure balance. It is anticipated that the stability will be checked by modifying a currently operating code (Baker-Mann code)⁸⁸ at Los Alamos.

D. Summary

Overall, the results of this work have been encouraging for the RFP concept. Ohmic ignition with favorable energy gain was achieved for a wide range of parameters using transport assumptions far more stringent than classical. Global MHD stability was easily maintained provided the fields were programmed to keep the plasma at the wall.

Also, a theoretically ideal MHD stable startup scenario has been found which suggests that the energy losses that presently occur during startup may be substantially reduced. Since these losses are one of the biggest problems for an RFP reactor, this result is particularly encouraging.

The potential for a small, near-term machine was demonstrated as well. Thus, the gamble of building a large, expensive demonstration device may be avoided. Using this device, the advantages of refueling were also shown. For all of these reasons, the RFP should be seriously considered as an alternative fusion device.

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TRANSPORT

A. Equations

The working equations are:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot \vec{\Gamma}_i = S_i - 2f(1-f) \langle \sigma v \rangle n_i^2 \quad (A.1)$$

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot \vec{\Gamma}_\alpha = f(1-f) \langle \sigma v \rangle n_i^2 F \quad (A.2)$$

$$\frac{3}{2} n_j K_B \frac{dT_j}{dt} + P_j \nabla \cdot \vec{\Gamma}_j = -\nabla \cdot \vec{q}_j + Q_j \quad (A.3)$$

(j=e,i+α)

and

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times [-\vec{v}_e \times \vec{B} + \frac{1}{en_e} \vec{R}_{Th} + n_{11} J_{11} \hat{e}_{11} + n_{\perp} J_{\perp} \hat{e}_{\perp}] \quad (A.4)$$

where

$$\vec{\Gamma}_j \equiv n_j \vec{v}_j \quad (\text{particle flux})$$

$$f \equiv \text{tritium fraction}$$

$$F \equiv \text{alpha confinement fraction}$$

$$\vec{q}_j \equiv \text{thermal flux}$$

$$Q_j \equiv \text{energy source term}$$

$$\vec{R}_{Th} \equiv \text{thermal force.}$$

Assuming toroidal and poloidal symmetry (1-D transport) and $\vec{v}_{\perp i} = \vec{v}_{\perp e} = \vec{v}_{\perp \alpha}$; $\vec{v}_{\perp j}$ can be calculated from the Generalized Ohm's Law:

$$\vec{v}_{\perp j} = \frac{-n_{\perp} \nabla p}{B^2} + \frac{\vec{E} \times \vec{B}}{B^2} - \frac{1}{en_e} \vec{R}_{Th} \times \vec{B} \quad (A.5)$$

Heat flux is expressed as:

$$\vec{q}_j = -K_{\perp j} K_B \vec{\nabla} T_j \quad (A.6)$$

Source terms are:

$$Q_e = \frac{-\vec{R} \cdot \vec{J}}{n_e} - P_{Br} - P_{cyc} - \frac{3}{2} S_e K_B T_e + Q_{\alpha e} - 3 \frac{m_e}{m_i} \frac{n_e K_B}{\tau_{ei}} (T_e - T_i) \quad (A.7)$$

and

$$Q_i = 3 \frac{m_e}{m_i} \frac{n_e K_B}{\tau_{ei}} (T_e - T_i) + \frac{3}{2} K_B S_i (T_S - T_i) + Q_{\alpha i} \quad (A.8)$$

where

$$Q_{\alpha e} = f(1-f) \langle \sigma v \rangle n_i^2 (E_\alpha + \frac{3}{2} K_B T_i) F f_e \quad (A.9)$$

and

$$Q_{\alpha i} = f(1-f) \langle \sigma v \rangle n_i^2 (E_\alpha + \frac{3}{2} K_B T_i) F f_i \quad (A.10)$$

The force term is expressed as:

$$\vec{R} = \vec{R}_n + \vec{R}_{Th} = en_e (\eta_{11} \vec{J}_{11} + \eta_{\perp} \vec{J}_{\perp}) - \frac{3}{2} \frac{n_e}{\omega_e \tau_e} \frac{\vec{B} \times \nabla T_e}{|\vec{B}|} \quad (A.11)$$

Transport coefficients are assumed to scale classically⁶¹ except for thermal conductivity and perpendicular resistivity. Thermal conduction is assumed to be Bohm-like everywhere while perpendicular resistivity is assumed Bohm-like in Suydam unstable regions. Classical slowing down is used for alpha particles.⁸⁹ $\langle \sigma v \rangle$ is calculated using Hively's analytic forms.⁹⁰

Resistivities are expressed as:

$$\eta_{11} = 0.51 m_e / (e^2 n_e \tau_e) \quad (A.12)$$

$$\eta_{\perp} = m_e / (e^2 n_e \tau_e) + f_T f_S C_S B / (16 e n_e) \quad (A.13)$$

where

$$f_S = \begin{cases} (\Sigma-1)/\Sigma & \Sigma > 1 \\ 0 & \Sigma < 1 \end{cases} \quad (\text{A.14})$$

and

$$\Sigma = - \frac{dp}{dr} / \left(\frac{B_\theta^2}{8r} \frac{dp}{dr} \right)^2 ; \quad P \equiv \frac{rB_\phi}{B_\theta} \quad (\text{A.15})$$

Thermal conductivities are expressed as:

$$K_{\perp e} = 4.66 \frac{n_e K_B T_e}{m_e \omega_e^2 \tau_e} + \frac{n_e K_B T_e}{16eBf_B} \quad (\text{A.16})$$

$$K_{\perp(i+\alpha)} = 2.0 \frac{(n_i + n_\alpha) K_B T_{i+\alpha}}{m_{i+\alpha} \omega_{i+\alpha}^2 \tau_{i+\alpha}} + \frac{(n_i + n_\alpha) K_B T_{i+\alpha}}{16eBf_B} \quad (\text{A.17})$$

Bremsstrahlung and electron cyclotron radiation are both included:

$$P_{Br} = 4.227 \times 10^{-29} n_e (n_i + 4n_\alpha) (K_B T_e)^{1/2} \quad (\text{A.18})$$

$$P_{cyc} = 3.85 \times 10^{-1} \cdot n_e \cdot K_B T_e \cdot B^2 \cdot f_{LOSS} \quad (\text{A.19})$$

Ion and electron alpha energy splits are:

$$f_i = 1.0 - f_e \quad (\text{ion energy fraction}) \quad (\text{A.20})$$

$$f_e = \frac{1}{\tau_{se} 3500} \int_0^{\tau_{SD}} \{ [5.68 \times 10^2 T_e^{3/2} + (3500)^{3/2}] e^{-3t/2\tau_{se}} - 5.68 \times 10^2 T_e^{3/2} \}^{2/3} dt ; \quad T_e \text{ in kev} \quad (\text{A.21})$$

where

$$\tau_{SD} = \ln[(\tau_{se} + 2.07 \times 10^5 \tau_{si}) / (\tau_{se} + T_i^{3/2} \tau_{si})] \cdot 2/3 \cdot \tau_{se} ; \quad T_i \text{ in kev} \quad (\text{A.22})$$

and

$$\tau_{se} = 1.0 \times 10^{19} T_e^{3/2} / n_e \ln \Lambda \quad (\text{A.23})$$

$$\tau_{si} = 1.76 \times 10^{16} / n_e \ln \Lambda \quad (\text{A.24})$$

The cyclotron frequency is

$$\omega_j = \frac{ZeB}{m_j c} \quad (A.25)$$

and electron and ion relaxation times are

$$\tau_e = \frac{3\sqrt{m_e} (K_B T_e)^{3/2}}{4\sqrt{2\pi} \ln \Lambda e^4 Z^2 n_i} \quad (A.26)$$

$$\tau_{i+\alpha} = \frac{3\sqrt{m_{i+\alpha}} (K_B T_{i+\alpha})^{3/2}}{4\sqrt{\pi} \ln \Lambda e^4 Z^4 n_i} \quad (A.27)$$

Anomalous transport coefficients are:

$$f_B = 200 \quad (\text{base case}) \quad (A.28)$$

$$f_T = \begin{cases} 1/9 & \text{for } j=i-2 \\ 2/9 & \text{for } j=i-1 \\ 3/9 & \text{for } j=i \\ 2/9 & \text{for } j=i+1 \\ 1/9 & \text{for } j=i+2 \end{cases} \quad (A.29)$$

where i is the mesh point that is Suydam unstable. Other constants are:

$$f = .5 \quad (A.30)$$

$$\tau_{ei} = Z \tau_e \quad (A.31)$$

and

$$E_\alpha = 3.5 \text{ Mev} \quad (A.32)$$

B. Transport Boundary Conditions

Extrapolated boundary conditions were used for density and temperature in the plasma edge region. Plasma processes in this region are generally complicated involving neutrals, impurities, field error and turbulence. A detailed two region tokamak code which modeled these effects determined the following boundary conditions give similar behavior to a two region model.⁵⁶

$$\Delta_n \equiv \left(-\frac{1}{n} \frac{\partial n}{\partial r} \right)^{-1} = 10 \text{ cm} \quad (\text{A.33})$$

$$\Delta_{Te} \equiv \left(-\frac{1}{T_e} \frac{\partial T_e}{\partial r} \right)^{-1} = 5 \text{ cm} \quad (\text{A.34})$$

$$\Delta_{Ti} \equiv \left(-\frac{1}{T_i} \frac{\partial T_i}{\partial r} \right)^{-1} = 10 \text{ cm} \quad (\text{A.35})$$

We have carried these conditions over to our model. Actually, the extrapolation lengths used does not have a profound effect on the plasma model as long as it is much less than the plasma radius but greater than the plasma mesh size.

C. Numerics

The differential equations derived in the model section are solved using a 1-D finite difference LaGrangian mesh scheme. Solution is done in two steps. First of all, the differential equations are integrated in time on a fixed mesh. A variable implicit/explicit technique is used to accomplish this. Then the mesh is adjusted (conserving flux and energy) to satisfy the pressure balance. This step is covered in Appendix B.

As an illustration of the differencing technique, the continuity equation in the convective frame (i.e., $\vec{E} \times \vec{B} = 0$) will be used.

$$\frac{\partial n_i}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r D \frac{\partial p}{\partial r} \right) + S_i - 2 f(1-f) \langle \sigma v \rangle n_i^2 \quad (\text{A.36})$$

$$p = n_i T \quad (\text{A.37})$$

where

$$T \equiv T_e \left(1 + \frac{2n_\alpha}{n_i} \right) + T_i + \frac{n_\alpha}{n_i} T_\alpha \quad (\text{A.38})$$

" n_i " will hereafter be referred to as " n " since " i " will be used to denote the " i th" mesh ring. Now consider the half interval values:

$$Dr \frac{\partial p}{\partial r} \Big|_{i+1/2} = \frac{1}{2} (r_{i+1} D_{i+1} + r_i D_i) \left[\frac{n_{i+1} T_{i+1} - n_i T_i}{r_{i+1} - r_i} \right] \quad (A.39)$$

Therefore,

$$\begin{aligned} \frac{\partial n_i}{\partial t} = & \frac{u_i}{2hr_i} (r_{i+1} D_{i+1} + r_i D_i) \left(\frac{n_{i+1} T_{i+1} - n_i T_i}{r_{i+1} - r_i} \right) \\ & - (r_i D_i + r_{i-1} D_{i-1}) \left(\frac{n_i T_i - n_{i-1} T_{i-1}}{r_i - r_{i-1}} \right) + S_i - 2f_i(1-f_i) \langle \sigma v \rangle n_i^2 \end{aligned} \quad (A.40)$$

where

$$u_i \equiv \frac{\partial r'}{\partial t} \quad r' \equiv \text{fixed mesh and } r \equiv \text{Lagrangian mesh} \quad (A.41)$$

$$h \equiv \text{distance between points on fixed mesh.} \quad (A.42)$$

Therefore,

$$\frac{\partial n_i}{\partial t} = a n_{i+1} + b n_i + c n_{i-1} + d \quad (A.43)$$

where

$$a = \frac{u_i}{2hr_i} \frac{(r_{i+1} D_{i+1} + r_i D_i)}{(r_{i+1} - r_i)} T_{i+1} \quad (A.44)$$

$$b = \frac{-u_i}{2hr_i} \left[\frac{(r_{i+1} D_{i+1} + r_i D_i)}{(r_{i+1} - r_i)} + \frac{(r_i D_i + r_{i-1} D_{i-1})}{(r_i - r_{i-1})} \right] T_i \quad (A.45)$$

$$c = \frac{u_i}{2hr_i} \frac{(r_i D_i + r_{i-1} D_{i-1})}{(r_i - r_{i-1})} T_{i-1} \quad (A.46)$$

$$d = S_i - 2f_i(1-f_i) n_i^2 \langle \sigma v \rangle \quad (A.47)$$

Using "j" to denote the time step and "i" to denote the spatial grid, the equation is forward differenced in time.

$$\begin{aligned} \frac{n_{ij+1} - n_{ij}}{\Delta t} = & \gamma (a_{j+1} n_{i+j+1} + b_{j+1} n_{ij+1} + c_{j+1} n_{i-j+1} + d_{j+1}) \\ & + (1-\gamma) (a_j n_{i+j} + b_j n_{ij} + c_j n_{i-j} + d_j) \end{aligned} \quad (A.48)$$

where γ is the implicit fraction.

Rearranging gives:

$$\begin{aligned}
 (1-\gamma) (a_j n_{i+1j} + b_j n_{ij} + c_j n_{i-1j} + d_j) + \frac{n_{ij}}{\Delta t} + \gamma d_{j+1} \\
 = -\gamma a_{j+1} n_{i+1j+1} + \left(\frac{1}{\Delta t} - \gamma b_{j+1} \right) n_{ij+1} - \gamma c_{j+1} n_{i-1j+1}
 \end{aligned} \quad (A.49)$$

This can be written in the form of a tri-diagonal matrix equation:

$$\text{knowns} \begin{pmatrix} \vdots \\ j \end{pmatrix} = \begin{pmatrix} & & 0 \\ & \text{---} & \\ 0 & \text{---} & \end{pmatrix} \begin{pmatrix} \vdots \\ j+1 \end{pmatrix} \quad (A.50)$$

Consequently all that is necessary to do is to invert the tri-diagonal matrix. However, the $j+1$ coefficients are not known so they have to be predicted. This is done by a linear projection of the $j-1$ and j coefficients. The coefficients are then iterated on implicitly four times. Matrix inversion is done by Gaussian elimination.

APPENDIX B

ALPHA PARTICLE PROMPT LOSS MODEL

Using a straight cylinder approximation, the Lagrangian for a collisionless particle is:

$$L = \frac{1}{2} m_{\alpha} \dot{r}^2 + \frac{1}{2} m_{\alpha} r^2 \dot{\theta}^2 + \frac{1}{2} m_{\alpha} \dot{z}^2 + q_{\alpha} r \dot{\theta} A_{\theta}(r) + q_{\alpha} \dot{z} A_z(r) \quad (B.1)$$

Note that

$$\frac{\partial L}{\partial z} = 0 \quad (B.2)$$

which implies that

$$\frac{\partial L}{\partial \dot{z}} = m_{\alpha} \dot{z} + q_{\alpha} A_z(r) \equiv m_{\alpha} P_z = \text{constant} \quad (B.3)$$

where P_z is the canonical linear momentum.

Similarly,

$$\frac{\partial L}{\partial \theta} = 0 \quad (B.4)$$

implies that

$$\frac{\partial L}{\partial \dot{\theta}} = m_{\alpha} r^2 \dot{\theta} + q_{\alpha} r A_{\theta}(r) \equiv m_{\alpha} P_{\theta} = \text{constant} \quad (B.5)$$

where P_{θ} is the canonical angular momentum.

Since the particles are collisionless, kinetic energy is also conserved:

$$E = \frac{m_{\alpha}}{2} (\dot{r}^2 + r^2 \dot{\theta}^2 + \dot{z}^2) = \text{constant} \quad (B.6)$$

Substituting B.3 and B.5 into B.6 and solving for $\dot{r}$ yields:

$$\dot{r}^2 = (E - U_{\text{eff}}) 2/m_{\alpha} \quad (B.7)$$

where

$$U_{\text{eff}} = \frac{m_{\alpha}}{2} \left[\left(\frac{P_{\theta}}{r} - \frac{q_{\alpha} A_{\theta}(r)}{m_{\alpha}} \right)^2 + \left(P_z - \frac{q_{\alpha} A_z(r)}{m_{\alpha}} \right)^2 \right] \quad (B.8)$$

Thus the particle's radial velocity can be characterized as

$$\dot{r}^2 = \dot{r}^2(p_\theta, p_z, E, A_\theta(r), A_z(r)) \quad (\text{B.9})$$

Obviously, $\dot{r}^2 < 0$ is unphysical so particles can never reach that radius. Hence, the alpha confinement criterion is

$$\dot{r}^2(r_{\text{wall}}) < 0 \rightarrow \text{particles are confined} \quad (\text{B.10})$$

and

$$\dot{r}^2(r_{\text{max}}) > 0 \rightarrow \text{particles are lost} \quad (\text{B.11})$$

where

$$r_{\text{max}} \equiv r(U_{\text{eff}} = U_{\text{max}}) \text{ and } r_0 < r_{\text{max}} < r_{\text{wall}} \quad (\text{B.12})$$

Thus, the alpha confinement fraction is determined by sweeping phase space at each mesh point for a 3.5 Mev alpha and seeing if $\dot{r}^2(r_{\text{wall}}) < 0$. If this is not satisfied, $U_{\text{max}}(r)$ for $r_0 < r < r_{\text{wall}}$ is found and $\dot{r}^2(r_{\text{max}}) > 0$ is checked. If this is true then particles are lost. The confinement fraction is then the ratio of particles confined to total particles tested at each mesh point.

APPENDIX C

MESH ADJUSTMENT

A. Pressure Balance

Starting with:

$$\nabla P = \vec{J} \times \vec{B} \quad (C.1)$$

$$\vec{J} = \mu_0^{-1} \nabla \times \vec{B} \quad (C.2)$$

$$\text{and } \vec{B} = B_z \hat{e}_z + B_\theta \hat{e}_\theta \quad (\text{straight cylinder}), \quad (C.3)$$

substituting Eqs. C.3 and C.2 into Eq. C.1 yields

$$\nabla P = -\mu_0^{-1} \left[\frac{B_\theta}{r} \frac{\partial}{\partial r} (r B_\theta) + B_z \frac{\partial B_z}{\partial r} \right] \quad (C.4)$$

Integrating gives:

$$P + \frac{1}{2\mu_0} (B_\theta^2 + B_z^2) = \frac{2}{r^2} \int_0^r (P + \frac{1}{2\mu_0} B_z^2) r' dr' \quad (C.5)$$

As mentioned previously, solutions are done in a two step process.

Defining "x" as the fixed mesh variable, after the transport is done the mesh is out of pressure balance. The unbalanced variables are denoted by the "primes". Defining

$$v(x) \equiv n'_{ion}/n_{ion} \equiv n'/n \quad (C.6)$$

the following relations can be derived using the ideal gas law ($\gamma=5/3$) and flux conservation:

$$T = T'/v^{2/3} \quad (C.7)$$

where $T \equiv (n + n_\alpha)/n \cdot T_i + n_e/n T_e$

$$B_\theta = r B'_\theta / (r' v) \quad (C.8)$$

and

$$B_z = B'_z / v. \quad (C.9)$$

The unprimed variables satisfy pressure balance.

"u" is defined as the magnitude of the Jacobian of the transformation from the fixed mesh to the Lagrangian mesh,

$$u \equiv \left| \frac{dx}{dr} \right| = \frac{rn}{xn_0} \quad (C.10)$$

where n_0 is the density in x space. Using these definitions the radius is expressed as:

$$r(x) = \left\{ 2 \int_0^x \frac{n_0(x)}{n(x)} x dx \right\}^{1/2} \quad (C.11)$$

Substituting Eqs. C.6 through C.10 into Eq. C.5 yields

$$\frac{n'T'}{v^{5/3}} + \frac{1}{2\mu_0 v^2} \left(\frac{r^2 B_\theta'^2}{r'^2} + B_z'^2 \right) = \frac{2}{r^2} \int_0^x \left(\frac{n'T'}{v^{5/3}} + \frac{B_z'^2}{2\mu_0 v^2} \right) \frac{x n_0 v}{n'} dx. \quad (C.12)$$

Defining

$$S_{r_i} \equiv 2 \int_0^{x_i} \frac{n_0}{n'} v x dx \quad (C.13)$$

and

$$S_{p_i} \equiv \int_0^{x_i} \left(\frac{n_0 T'}{2v^{2/3}} + \frac{B_z'^2 n_0}{4\mu_0 n' v} \right) x dx \quad (C.14)$$

Eq. C.11 can be rewritten as:

$$r_{i+1}(x) = \sqrt{S_{r_i} + \frac{n_{0i}}{n'_{i+1}} v_i x_i + \frac{n_{0i+1}}{n'_{i+1}} v_{i+1} x_{i+1}} \quad (C.15)$$

and Eq. C.12 can be rewritten as:

$$\begin{aligned}
& \frac{n_{i+1} T_{i+1}}{v_{i+1}^{5/3}} + \frac{1}{2\mu_0 v_{i+1}^2} \left[\frac{r_{i+1}^2}{r_{i+1}'^2} B_{\theta i+1}'^2 + B_{z i+1}'^2 \right] - \frac{n_{o i+1} T_{i+1} h_{x i+1}}{r_{i+1}^2 v_{i+1}^{2/3}} \\
& - \frac{B_{z i+1}'^2 n_{o i+1} h_{x i+1}}{2\mu_0 n_{i+1}' v_{i+1} r_{i+1}^2} = \frac{4}{r_{i+1}^2} \left[S_{p i} + \frac{n_{o i} T_i h_{x i}}{4v_i^{2/3}} + \frac{B_{z i}'^2 n_{o i} h_{x i}}{8\mu_0 n_i' v_i} \right] \quad (C.16)
\end{aligned}$$

where the solution is assumed known through the " i^{th} " mesh point and is to be solved for the $i+1^{th}$ mesh point. Note that Equations C.15 and C.16 are two transcendental equations with two unknowns: v_{i+1} and r_{i+1} .

The equations are solved iteratively. In the first step, r_{i+1} is assumed to be equal to r_{i+1}' so Equation C.16 can be written as

$$C_a v_{i+1}^{-5/3} + C_b v_{i+1}^{-2} + C_c v_{i+1}^{-2/3} + C_d v_{i+1}^{-1} = D \quad (C.17)$$

Multiplying Eq. C.17 by v_{i+1}^y gives

$$C_a v_{i+1}^{-5/3+y} + C_b v_{i+1}^{-2+y} + C_c v_{i+1}^{-2/3+y} + C_d v_{i+1}^{-1+y} = D v_{i+1}^y \equiv G. \quad (C.18)$$

or

$$v_{i+1} = (G/D)^{1/y} \quad (C.19)$$

Equation C.19 is the recursive relation that will be used to calculate a new v_{i+1} . Note that $G=G(v_{i+1})$ so choose y such that G is relatively insensitive to the previous choice of v_{i+1} . We know that $v_{i+1} \approx 1$ since a large expansion or contraction in one time step greatly changes n and T which the variable time step check will not allow. Therefore, choose y such that $\partial G / \partial v_{i+1} = 0$ for $v_{i+1} = 1$. This yields

$$y = (5/3 C_a + 2C_b + 2/3 C_c + C_d) / (C_a + C_b + C_c + C_d). \quad (C.20)$$

Therefore, the solution sequence is as follows:

1. Assume $r_{i+1} = r_{i+1}^1$, $v_{i+1} = 1$
2. Calculate C_s , C_b , C_c and C_d
3. Calculate y (Eq. C.20)
4. Calculate v_{i+1} (Eq. C.19)
5. Calculate r_{i+1} (Eq. C.15)
6. Repeat steps 2-5 using new approximation for r_{i+1} and v_{i+1} until the solution converges to within a specified tolerance.

B. Boundary Conditions

As noted in the previous section, determining the pressure balance at a mesh point requires knowing what the profiles look like at all previous mesh points. This works for all mesh points except at the origin. As first explained by Grad,⁸⁷ the expansion factor (v_1) at the origin is dependent on the boundary conditions. Thus, the expansion factor at the origin is iterated on until the boundary conditions are satisfied.

The model currently can handle two types of boundary conditions. The one most often used is a specified radius boundary condition. Since $r \propto \sqrt{\langle v \rangle}$ (where brackets denote average) a rough estimate of the expansion factor is:

$$v_1 = (a/r_L)^{1/2} \quad (C.21)$$

where "a" is the required plasma radius and r_L is the radius from the previous iteration. If this technique does not converge rapidly, a bisection routine is used.

Another possible boundary condition is a specified toroidal current. Rewriting Equation C.12 yields

$$B_{\theta}^2 = \frac{4\mu_0}{r^2} \int_0^x \left(\frac{P'}{v^{2/3}} + \frac{B_z'^2}{2\mu_0 v} \right) \frac{x n_0}{n_1'} dx - 2\mu_0 P' / v^{5/3} - \frac{B_z'^2}{2\mu_0 v^2}. \quad (C.22)$$

This gives a relation for toroidal current I:

$$\begin{aligned} I &= \frac{2\pi}{\mu_0} r B_{\theta} \\ &= \left\{ \frac{16\pi^2}{\mu_0} \int_0^x \left(P' v^{-2/3} + \frac{B_z'^2 v^{-1}}{2\mu_0} \right) \frac{n_0}{n_1'} x dx \right. \\ &\quad \left. - \frac{8\pi^2}{\mu_0} P' r^2 v^{-5/3} - \frac{2\pi^2}{3} \frac{r^2}{v^2} B_z'^2 \right\}^{1/2}. \end{aligned} \quad (C.23)$$

Estimating $v(x) \cong v_1$ and $r(x) = \sqrt{v_1} r'(x)$, as in Equation C.21 gives

$$I^2 \cong A v_1^{-2/3} + B v_1^{-1} + C v_1. \quad (C.24)$$

However, if the toroidal field is small at the plasma edge then $C \cong 0$ so

$$I \sim I' v_1^{-\epsilon} \quad (C.25)$$

where

$$1/3 < \epsilon < 1/2$$

or

$$v_1 \sim (I'/I)^{1/\epsilon}. \quad (C.26)$$

APPENDIX D

PARTICLE BALANCE AND ENERGY BALANCE CHECKS

Due to the implicit nature of the code and since it is cast in a tri-diagonal form rather than a banded matrix form, particles and energy are not explicitly conserved. Thus, a check on the degree of violation of energy and particle conservation serves as an internal check of the accuracy of the code's numerics. It is also an extremely useful debugging tool.

The density check is

$$N_E = N_i(t) + \int_0^t (2R_F + R_C) dt - N_i(t=0) \quad (D.1)$$

where

$$N_E \equiv \text{line density error} \quad (D.2)$$

$$N_i \equiv 2\pi \int_0^a n_i r dr \quad (\text{ion density})$$

$$R_F \equiv 2\pi \int_0^a f(1-f) n_i^2 \langle \sigma v \rangle r dr \quad (\text{fusion losses}) \quad (D.3)$$

and

$$R_C \equiv 2\pi a v n_i \Big|_{r=a} \quad (\text{transport losses}). \quad (D.4)$$

The energy check is

$$W_E = W_B + W_P - W_B(t=0) - W_P(t=0) + \int_0^t (P_R + P_{th} + P_C + P_{\alpha L} - P_{EM} - P_F) dt \\ + \int_{a(t=0)}^{a(t)} F n dr \quad (D.5)$$

where

$$W_B = \frac{\pi}{\mu_0} \int_0^a B^2 r dr \quad (D.6)$$

$$W_P = 3\pi \int_0^a [(n_i + n_\alpha)T_i + (n_i + 2n_\alpha)T_e] r dr \quad (D.7)$$

$$P_R = 2\pi \int_0^a (P_{\text{Brem}} + P_{\text{cyc}}) r dr \quad (D.8)$$

$$P_{TH} = -2\pi a \left(k_i \frac{\partial T_i}{\partial r} + k_e \frac{\partial T_e}{\partial r} \right) \Big|_{r=a} \quad (D.9)$$

$$P_C = 3\pi a v \left[(n_i + n_\alpha)T_i + (n_i + 2n_\alpha)T_e \right] \Big|_{r=a} \quad (D.10)$$

$$P_{\alpha L} = 2\pi \int_0^a f(1-f) n_i^2 \langle \sigma v \rangle (1-F) r dr \cdot 3.5 \text{ Mev} \quad (D.11)$$

$$P_{EM} = \frac{-2\pi a}{\mu_0} \left(\vec{E} \times \vec{B} \right) \Big|_{r=a} \quad (D.12)$$

$$P_F = 2\pi \int_0^a n_i^2 \langle \sigma v \rangle F(1-f) f r dr \cdot 3.5 \text{ Mev} \quad (D.13)$$

and

$$F_R = 2\pi a \left(P + B^2/2\mu_0 \right)$$

All of the variables are defined consistently with those in Appendix A.

APPENDIX E

RFPBRN USER'S GUIDE

The one-dimensional transport and stability code RFPBRN is written in Fortran and is available on the MFECC (Magnetic Fusion Energy Computer Center) computer system through filem. This guide will assume the user is familiar with the system and only simple system commands will be given to show how to retrieve and execute the program. Further system documentation is available on line through the DOCUMENT utility.

The fortran source file and a data file is available through FILEM on the 7600 using the command:

```
FILEM RDS 1346 RFPBRN RIN.
```

To send the files to the cray the NETOUT command is used:

```
NETOUT RFPBRN RIN U. XXXX
```

where XXXX is the desired user number. The source file is compiled using OLDCFT:

```
OLDCFT I=RFPBRN,L=0
```

which generates the compiled binary file \$BLD. The binary file is then loaded along with the fortran library (FORTLIB), the plotting library (TV80LIB), and a cray utility library (OMNILIB):

```
LDR B=$BLD, X=XRFPBRN, LIB=(FORTLIB, OMNILIB, TV80LIB).
```

The program is then executed by typing

```
XRFPBRN / T V
```

where "T" is the execution time desired and "V" is the point value. A typical burn takes about five minutes to execute although this time varies considerably depending on the options chosen.

The program runs in batch mode with no interaction from the terminal. Output is contained in a printed file named OUT and a plot file named F105xx0x where xx can be any two letters. Input is through the file RIN.

Input variables and their description are in table E1. These variables are principally used in subroutine TINIT to initialize the density, temperature and field profiles. Modifying the code to do profiles other than the RFP would require modifying TINIT. A sample input file is shown in table E2.

Output variables and their description are in table E3. All printed output is put in file OUT and can be retrieved using the text editor or via the NETOUT command. Plotted output requires use of the NETPLOT command. A sample of the printed output is in table E4.

An abbreviated flow chart for RFPBRN is in Figure 34. The code is modularized to make it easy to modify the physics. Questions about how to do this should be referred to the author.

Table E-1. Input Variables for RFPBRN

Variable	Value	Remarks
A		Plasma radius + extrapolation length
RM		Field reversal radius
ATH		Toroidal field on center-line
TEM0		Initial temperature
DENO		Initial average density
RR		Major radius
L	< 201	Number of mesh points
M		Maximum number of timesteps
DT		Initial timestep
TMAX		End of simulation time
JPT		Number of times plot routine is called
ISTAB		Ratio of number of mesh points to number of instability trial functions
FLUX	0	Plasma radius specified (B.C.)
	1	Toroidal flux conserved (B.C.)
FRA		Implicit fraction
WALLFT		(Plasma radius)/A
TWALL		Resistive diffusion time for wall
IFRAME	0	Lagrangian frame
	1	Convective frame
TSTAB		Time stability routine is first called
ALFACT		Tritium fraction
BOHMFT	200	Bohm factor for energy transport
BOHMLC	0	No Bohm energy transport
	1	Bohm energy transport
BOHMPT		Perpendicular resistivity Bohm fraction
IISTAB	1	Singular instability trial functions
	2	Finite instability trial functions
TREE		Refuel start time
	0	No refuel
REFOP	1	Gas puffing
	2	Pellet refuel
DTBIG		Maximum timestep
PELVEL		Pellet velocity
PELRAD		Pellet radius (cm)
PELINC		Inverse pellet injection rate

Table E2. Input file

```

8.500E-01 6.500E-01 8.400E+00 1.400E+00 2.700E+20 6.35
201 999999 2.000E-6 1.800E+1 5 8
1 1 0.50.8824 1.00999.9E+140999.9E+140
1.00E+001.500 0.00 0.00 0 4.00E+02
0.50 1.00 1.00 1.00 1.00 1.00 1.00
2.00E+02 1.00 0.00E-08 2.00E+05 0.00E-04 1
5.20 0.20 6.200E+04 1.950E-01 5.000E-01
-1.0

```

Table E-3. Output Variables for RFPBRN

Variable	Remarks
TIME	Time in burn simulation
L	Number of mesh points
BETAPL	Minimum average poloidal beta
LLL	Number of time steps
PFUS	Alpha particle fusion power
PIN	Ohmic heating power
Q	Fusion Energy (17.6 Mev)/Ohmic Energy
TI	Average ion temperature
TE	Average electron temperature
IMAXI	Mesh point of most rapidly changing ion temperature
FMAXI	Maximum fractional change in ion temperature
IMAXE	Mesh point of most rapidly changing electron temperature
FMAXE	Maximum fractional change in electron temperature
ERAD	Radiation energy lost from the plasma
ETH	Thermal conduction energy lost from the plasma
EC	Plasma energy lost by plasma convection
RINJ	Gas puffing line density source rate
EALOSS	Prompt alpha loss energy
EEM	Poynting's vector energy into plasma
EFUS	Alpha particle fusion energy
ELOSS	Energy lost from plasma
EGAIN	EEM + EFUS
PVWORK	$\int p dv$ work done by plasma
WERR	Energy error
ENPER	Energy error fraction
ANLOSS	Fuel depletion rate
DAN	Total fuel depletion
ANERR	Line density error
DENPER	Line density error fraction
WB(3)	Field energy in plasma
WP(3)	Kinetic energy in plasma
DT	Time step size

Variable	Remarks
THETA	Plasma theta value
FAVE	Plasma F value
PWALL	Total wall load
RATE	Startup gas feed rate
TEWALL	Electron temperature at wall
TIWALL	Ion temperature at wall
BZWALL	Toroidal field at wall
BTHWALL	Poloidal field at wall
RADIUS	Radius in plasma
NION	Ion density
NALPHA	Alpha particle density
TION	Ion temperature
TELECT	Electron temperature
U	$\left \frac{dx}{dr} \right $
BPOL	Poloidal field
BZ	Toroidal field
VXA	Mesh expansion parameter
SIG	$(\Sigma-1)/\Sigma$ (Suydam factor)
BETA	Local beta
PARTS	Refuel ions in pellet
PELPAR	Refuel ions deposited in mesh ring
PELDEN	Refuel ion density
OM2	$\delta w/(\xi, \rho_0 \xi)$
NENTI	Toroidal mode number
M	Poloidal mode number
GEES	Value of "g" function
DENOM	$(\xi, \rho_0 \xi)$
DISP	$\xi/\sqrt{F}$
CCS	Trial function's weighting constant
SQUIG	ξ

Table E-4. Output sample

```

ltime= 4.224e+00 l= 201 dt= 2.000e-01 betapl= 1.029e-01 lll= 100
pfus= 2.370e+06 pin= 4.106e+05 q= 7.259e+00 ti= 9.803e+00 to= 1.037e+01
lmaxi= 198 fmaxi= 1.929e-02 lmaxo= 198 fmaxo= 1.880e-02
erad= 9.968e+05 eth= 5.745e+06 ec= 0. rinj= 0. ealoss= 0.
eem= 1.788e+05 efus= 5.204e+06 eloss= 6.742e+06 egain= 5.383e+06
pvwork= 3.316e+02 werr= 5.330e+04 enper= 1.850e-03 anloss= 8.453e+18
dan= 1.856e+19 anerr= -1.687e+16 denper= -3.535e-05 wp(3)= 1.978e+07 wp(3)= 2.294e+06
theta= 1.561e+00 bthwall= 3.435e+00 favo= -6.459e-01 pwall= 2.678e+00
ratio= 1.381e+14 tewall= 5.041e+00 tiwall= 4.747e+00 bzwall= -1.423e+00

radius nion nalpha nion ticon telec u
0.00000 4.479e+20 2.165e+19 12.53977 13.12067 0.98492
0.03813 4.456e+20 2.132e+19 12.51852 13.09931 0.98310
0.07628 4.426e+20 2.074e+19 12.45475 13.03510 0.98316
0.11442 4.376e+20 1.967e+19 12.31919 12.92600 0.98313
0.15257 4.306e+20 1.858e+19 12.20413 12.78211 0.98310
0.19071 4.215e+20 1.709e+19 12.02054 12.59620 0.98323
0.22885 4.105e+20 1.541e+19 11.50096 12.37326 0.98337
0.26698 3.973e+20 1.361e+19 11.54789 12.11559 0.98367
0.30509 3.822e+20 1.177e+19 11.26388 11.82567 0.98422
0.34318 3.651e+20 9.944e+18 10.95147 11.50596 0.98515
0.38122 3.462e+20 8.198e+18 10.61289 11.15375 0.98664
0.41919 3.256e+20 6.581e+18 10.24967 10.78588 0.98894
0.45705 3.035e+20 5.130e+18 9.86341 10.38846 0.99243
0.49474 2.800e+20 3.869e+18 9.45343 9.96643 0.99767
0.53220 2.555e+20 2.808e+18 9.01835 9.51799 1.00537
0.56931 2.301e+20 1.946e+18 8.55424 9.03866 1.01622
0.60597 2.038e+20 1.271e+18 8.05298 8.51927 1.03023
0.64210 1.761e+20 7.623e+17 7.49815 7.94172 1.04567
0.67775 1.464e+20 4.004e+17 6.85457 7.26802 1.05664
0.71329 1.136e+20 1.661e+17 6.03647 6.40650 1.04364

0.71687 1.101e+20 1.490e+17 5.93790 6.30238 1.04558
0.72046 1.065e+20 1.330e+17 5.83486 6.19348 1.04168
0.72407 1.029e+20 1.181e+17 5.72679 6.07921 1.03740
0.72769 9.927e+19 1.042e+17 5.61303 5.95884 1.03212
0.73134 9.555e+19 9.132e+16 5.49275 5.83152 1.02501
0.73501 9.176e+19 7.943e+16 5.36497 5.69617 1.01801
0.73871 8.790e+19 6.851e+16 5.22842 5.55146 1.00912
0.74244 8.393e+19 5.854e+16 5.08151 5.39370 0.99872
0.74622 7.986e+19 4.947e+16 4.92213 5.22660 0.98581
0.75006 7.562e+19 4.128e+16 4.74731 5.04097 0.96804

radius bpol bz vxu sig bccu
0.00000 0. 7.640e+00 1.00064 0.00000 0.00000
0.03813 5.497e-01 7.602e+00 1.00061 121.03355 0.08005
0.07628 1.091e+00 7.487e+00 1.00062 29.19045 0.08407
0.11442 1.616e+00 7.299e+00 1.00063 12.89307 0.08510
0.15257 2.117e+00 7.039e+00 1.00063 7.16953 0.08537
0.19071 2.586e+00 6.712e+00 1.00064 4.50873 0.08570
0.22885 3.017e+00 6.324e+00 1.00065 3.05673 0.08600
0.26698 3.404e+00 5.879e+00 1.00065 2.17663 0.08627
0.30509 3.742e+00 5.386e+00 1.00064 1.60172 0.08647
0.34318 4.026e+00 4.850e+00 1.00063 1.20403 0.08652
0.38122 4.255e+00 4.279e+00 1.00059 0.91596 0.08633
0.41919 4.426e+00 3.680e+00 1.00052 0.69961 0.08570
0.45705 4.539e+00 3.059e+00 1.00041 0.53344 0.08472
0.49474 4.593e+00 2.420e+00 1.00022 0.40571 0.08293
0.53220 4.586e+00 1.770e+00 0.99985 0.31013 0.08024
0.56931 4.517e+00 1.118e+00 0.99961 0.24262 0.07626
0.60597 4.384e+00 4.808e-01 0.99925 0.19917 0.07057
0.64210 4.192e+00 -1.166e-01 0.99901 0.17728 0.06200
0.67775 3.954e+00 -6.471e-01 0.99903 0.17439 0.05206
0.71329 3.693e+00 -1.088e+00 0.99932 0.19550 0.03547
0.71687 3.667e+00 -1.126e+00 0.99971 0.19960 0.03694
0.72046 3.641e+00 -1.164e+00 0.99982 0.20443 0.03530
0.72407 3.614e+00 -1.200e+00 0.99994 0.20689 0.03379
0.72769 3.588e+00 -1.235e+00 1.00007 0.21611 0.03217
0.73134 3.562e+00 -1.269e+00 1.00020 0.22330 0.03051
0.73501 3.536e+00 -1.302e+00 1.00035 0.23168 0.02832
0.73871 3.511e+00 -1.334e+00 1.00051 0.24159 0.02708
0.74244 3.485e+00 -1.365e+00 1.00067 0.25359 0.02530
0.74622 3.460e+00 -1.395e+00 1.00082 0.26864 0.02348
0.75006 3.435e+00 -1.423e+00 1.00097 0.00000 0.02155

```

Table E-4 continued

```

parts= 1.413e+21
radius polpar polcen
7.501e-01 1.138e+20 3.135e+20
7.463e-01 2.271e+20 3.169e+20
7.425e-01 2.179e+20 3.100e+20
7.387e-01 2.043e+20 2.955e+20
7.350e-01 1.860e+20 2.730e+20
7.313e-01 1.626e+20 2.418e+20
7.276e-01 1.337e+20 2.012e+20
7.240e-01 9.863e+19 1.500e+20
7.204e-01 5.676e+19 8.718e+19
7.168e-01 1.265e+19 1.961e+19
om2=-3.774e+08 ncntl= 12 m= -1cccs= 1.000e+45denom= 2.809e-07
n=0 disp= 0. ccsc= 1.000e+00 squig= 0.
n=3.808e-03 disp= 1.595e-02 ccsc= 0. squig= 6.171e+00
n=7.618e-03 disp= 3.190e-02 ccsc= 0. squig= 3.987e+00
n=1.143e-02 disp= 4.786e-02 ccsc= 0. squig= 2.843e+00
n=1.525e-02 disp= 6.381e-02 ccsc= 0. squig= 2.106e+00
n=1.906e-02 disp= 7.974e-02 ccsc= 0. squig= 1.593e+00
n=2.287e-02 disp= 9.566e-02 ccsc= 0. squig= 1.224e+00
n=2.669e-02 disp= 1.115e-01 ccsc= 0. squig= 9.530e-01
n=3.050e-02 disp= 1.274e-01 ccsc= 0. squig= 7.563e-01
n=3.432e-02 disp= 1.432e-01 ccsc= 0. squig= 6.076e-01
n=3.813e-02 disp= 1.590e-01 ccsc= 0. squig= 4.946e-01
n=4.195e-02 disp= 1.748e-01 ccsc= 0. squig= 4.077e-01
n=4.576e-02 disp= 1.905e-01 ccsc= 0. squig= 3.399e-01
n=4.958e-02 disp= 2.062e-01 ccsc= 0. squig= 2.864e-01
n=5.339e-02 disp= 2.218e-01 ccsc= 0. squig= 2.436e-01
n=5.721e-02 disp= 2.373e-01 ccsc= 0. squig= 2.090e-01
n=6.102e-02 disp= 2.528e-01 ccsc= 0. squig= 1.802e-01
n=6.483e-02 disp= 2.682e-01 ccsc= 0. squig= 1.575e-01
n=6.865e-02 disp= 2.836e-01 ccsc= 0. squig= 1.381e-01
n=7.246e-02 disp= 2.989e-01 ccsc= 0. squig= 1.216e-01
n=7.628e-02 disp= 3.141e-01 ccsc= 0. squig= 1.081e-01
n=8.009e-02 disp= 3.292e-01 ccsc= 0. squig= 9.639e-02
n=8.391e-02 disp= 3.442e-01 ccsc= 0. squig= 8.635e-02
n=8.772e-02 disp= 3.592e-01 ccsc= 0. squig= 7.772e-02
n=9.154e-02 disp= 3.741e-01 ccsc= 0. squig= 7.023e-02
n=9.535e-02 disp= 3.888e-01 ccsc= -1.294e-03 squig= 6.370e-02
n=9.918e-02 disp= 4.035e-01 ccsc= 0. squig= 5.798e-02
n=1.030e-01 disp= 4.181e-01 ccsc= 0. squig= 5.295e-02
n=1.068e-01 disp= 4.325e-01 ccsc= 0. squig= 4.850e-02
n=1.106e-01 disp= 4.469e-01 ccsc= 0. squig= 4.456e-02
n=1.144e-01 disp= 4.611e-01 ccsc= 0. squig= 4.104e-02
n=1.182e-01 disp= 4.752e-01 ccsc= 0. squig= 3.790e-02
n=1.221e-01 disp= 4.892e-01 ccsc= 0. squig= 3.508e-02
n=1.259e-01 disp= 5.031e-01 ccsc= 0. squig= 3.255e-02
n=1.297e-01 disp= 5.168e-01 ccsc= 0. squig= 3.026e-02
n=1.335e-01 disp= 5.304e-01 ccsc= 0. squig= 2.819e-02
n=1.373e-01 disp= 5.439e-01 ccsc= 0. squig= 2.631e-02
n=1.411e-01 disp= 5.573e-01 ccsc= 0. squig= 2.460e-02
n=1.449e-01 disp= 5.704e-01 ccsc= 0. squig= 2.304e-02
n=1.488e-01 disp= 5.835e-01 ccsc= 0. squig= 2.161e-02
n=1.526e-01 disp= 5.964e-01 ccsc= 0. squig= 2.031e-02
n=1.564e-01 disp= 6.091e-01 ccsc= 0. squig= 1.911e-02
n=1.602e-01 disp= 6.217e-01 ccsc= 0. squig= 1.801e-02
n=1.640e-01 disp= 6.342e-01 ccsc= 0. squig= 1.699e-02
n=1.678e-01 disp= 6.464e-01 ccsc= 0. squig= 1.605e-02

```

Figure 34. RFPBRN Flow Chart

VITA

Richard Alan Nebel was born on July 19, 1953 in Springfield, Illinois. He attended the Springfield public schools and graduated from Springfield High School in 1971. In September of 1971 he entered the University of Illinois at Urbana-Champaign, and in May of 1975 received a B.S. degree in General Engineering, graduating with highest honors.

He entered the graduate program in Nuclear Engineering as an ERDA trainee in June, 1975 and completed his M.S. degree in May, 1976. He received a University Fellowship the following year. Throughout the remainder of his graduate work, he served as a research assistant in the Fusion Studies Laboratory, except for the summer of 1979 when he was employed as a research assistant with the Plasma Theory Group at Los Alamos Scientific Laboratory. He is an associate member of the Sigma Xi research society and received an award for the best student fusion paper at the spring 1980 meeting of the American Nuclear Society. Starting in the fall of 1980 he will be employed as a post-doctoral fellow in the plasma theory group at Los Alamos.