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ELECTRIC PROPULSION APPLICATIONS
FOR SNAP SYSTEMS

AEC Research and Development Report

ATOMICS INTERNATIONAL

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ELECTRIC PROPULSION APPLICATIONS
FOR SNAP SYSTEMS

By
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ABSTRACT

The application of SNAP systems to electric propulsion has been investigated. A review is given of the basic analysis involved in establishing optimum power levels and payload capabilities for electric spacecraft, and several typical missions are analyzed to determine the usefulness of the SNAP systems which are under development or systems which are based on the current SNAP technology.

In general, it is found that SNAP power units in the range of 60 to 180 kw offer significant mission capability when used in conjunction with initial spacecraft weights of about 10,000 lb in a low-level orbit.

I. INTRODUCTION

Considerable effort has recently been directed toward the development of electrical propulsion engines for application in space. Assuming that an early demonstration is achieved of long-term engine operation in the environment of space, the early use of these systems for spacecraft propulsion is still contingent upon the development of sufficiently lightweight, long-lived sources of electrical power. Compared with the requirements for spacecraft auxiliary power, the power requirements for most propulsion missions are relatively high, covering the SNAP 8 range of 30 to 60 kw and extending far into the megawatt range as more advanced missions, involving large space ships, are considered. At these power levels, the use of nuclear power becomes necessary to achieve systems of reasonable size and weight.

Mission analysis performed by the Jet Propulsion Laboratory¹ has indicated that a wide variety of missions in the solar system (excluding planetary landings or manned missions) can be accomplished with an advanced nuclear-electric system operating upwards of 300 kw at a specific weight in the range of 10 to 15 lb/kw. Only a modest sized chemical vehicle, such as the Atlas-Centaur or three-stage Saturn, would be required as initial booster for such a propulsion system. However, it is not expected that systems of this high performance will be operationally available in this decade. Thus it is also of interest to examine the propulsion potential of less advanced systems operating in the medium power range which can be operational at an earlier date.

Presented in this report is a study of some typical missions for electric propulsion, with attention restricted to a power range which can be covered by the SNAP systems currently under development or by modest extensions of this technology. The purpose here is to establish the suitability of SNAP power levels for electric propulsion and to determine payload capability for SNAP propulsion systems in certain orbital and planetary applications.

An estimate of powerplant specific weights will be essential to this study. Reproduced in Figure 1 is a curve, taken from Reference 2, of weight vs power for a medium power SNAP system. This curve reflects a preliminary design estimate of what can be achieved with an uprated SNAP 8 reactor coupled to a mercury Rankine conversion cycle. The weight variation shown will form the basis for selecting optimum power levels in many of the application studies which follow.

Figure 1. Variation of Weight With Power for Medium Power SNAP System

II. BASIC ANALYSIS

A. ENGINE DESCRIPTION

The electric propulsion engine is a rocket engine in which thrust is produced by the conversion of electrical energy from the vehicle powerplant into kinetic energy of the expellant. Although there are various types of electric engines, the distinguishing characteristic of all is the heavy weight penalty associated with the production of electrical power. Electric propulsion devices are thus termed "power-limited" and constrained to low-power, low-thrust operation. Thrust-to-weight ratios for electric spacecraft of the near future will be in the range of 10^{-5} to 10^{-3} .

The feature of electric propulsion which makes it attractive for space applications is the high expellant velocities which can be obtained by electrical means as compared with the more conventional methods of chemical combustion or direct nuclear heating. As a result of the higher velocities achieved, the specific impulse (pounds of thrust per pound of propellant expended per second) is higher for electric engines and the propellant consumption is lower. As will be seen later, the real payoff for electric propulsion comes when considering relatively high energy, long-term space missions, in which propellant requirements for chemical rockets become prohibitively large.

According to the method of electrical/kinetic energy conversion, electric propulsion engines may be divided into three general categories: electrothermal, electrostatic, and electromagnetic. The first of these, the electrothermal engine, has the least potential in terms of specific impulse and will be suitable for a rather narrow range of low energy missions. In this type engine, energy is supplied to an expellant gas by an electric arc (common) or a resistance heater (less common); and thrust is derived from expansion of the gas through the rocket nozzle. The specific impulse is limited by enthalpy considerations to a maximum of about 2000 or possibly 2500 sec.

Major effort in the field of electric propulsion has been directed at the ion engine, in which thrust is obtained by the electrostatic acceleration of a stream of ions. There are two common types of ion engines as defined by the method of ion generation: the contact and bombardment source engines. In the former type, an expellant having a low ionization potential is passed through a heated

surface of a material with a high work function. Upon contact between expellant and surface (commonly cesium and porous tungsten), ionization occurs. The positive ions which are boiled off the surface are then accelerated by a strong electrostatic field and leave the engine producing thrust.

In the bombardment source engine, electrons are emitted from a hot filament, accelerated toward the first electrode, and then reflected by a ferromagnetic ring between first and second electrodes. A cloud of energetically oscillating electrons is thus established. The neutral expellant gas is now introduced, and ionization takes place upon collision between the neutral atoms and electrons. The heavy expellant ions pass through the magnetic field and are electrostatically accelerated out of the engine.

The operating range of specific impulse for ion engines is from about 2000 sec on up, but there may be considerable difficulty in achieving efficient operation at the lower end of the spectrum, from 2000 to 4000 sec. Electric engine efficiency as a function of specific impulse³ is shown in Figure 2. This curve represents an estimate of engine performance to be achieved in the latter half of the decade, based on an extrapolation of present day technology. More than one type of engine would be required to achieve this performance over the entire impulse range shown; however, the curve should be representative of ion engine capability down to about 2000 sec. Since engine efficiency will affect system design, the variation shown by Figure 2 has been included in some of the application studies which follow.

Since ion engines require dc voltage in the kilovolt range at the accelerating electrodes, there will be additional losses in transforming and rectifying the output of the nuclear powerplant. It is expected that overall power conditioning efficiencies of 85 to 90% will be achieved.

The third type of electric propulsion device, which need be mentioned only briefly, is the MHD engine. In comparison with the ion engine, the MHD engine has the advantages that (a) the ejected plasma is electrically neutral, which eliminates the problem of beam neutralization, and (b) the minimum specific impulse is somewhat lower. However, work on MHD propulsion is less advanced than work on either the arc jet or ion machine, and this type engine is not being considered for the early flights.

Figure 2. Variation of Electrical Propulsion Motor Efficiency With Specific Impulse

B. BASIC EQUATIONS

The rocket equation is obtained from a momentum balance between rocket ship and exhaust propellant. With no external forces acting, the rate of change of momentum of the system plus the net rate of outflow of momentum from the system equals zero:

$$\frac{d(M\vec{V})}{dt} + \vec{V}_g \dot{M}_g = M \frac{d\vec{V}}{dt} + \vec{V} \frac{dM}{dt} + \vec{V}_g \dot{M}_g = 0 , \quad \dots(1)$$

where

M = spacecraft mass

$\vec{V}$ = spacecraft velocity

t = time

$\vec{V}_g$ = expellant velocity

$\dot{M}_g$ = expellant flow rate.

Substituting

$$\vec{V}_g - \vec{V} = \vec{c} \quad \dots(2)$$

and

$$\dot{M}_g = -\dot{M} , \quad \dots(3)$$

we get

$$M \dot{\vec{V}} = \dot{M} \vec{c} , \quad \dots(4)$$

where

$\vec{c}$ = expellant velocity relative to spacecraft.

Integrating Equation 4, the conventional rocket equation for field-free flight is obtained:

$$\frac{M_1}{M_0} = e^{-\Delta V/c} , \quad \dots(5)$$

where

M_0 = initial spacecraft mass

M_1 = terminal spacecraft mass.

The velocity increment, ΔV , is the true velocity change in field-free space or, for a spacecraft operating in a gravity field, the velocity equivalent of a change in energy level. The exponential form of Equation 5 represents a serious limitation on mission energy for conventional propulsion systems, which are constrained to operation at relatively low values of the exhaust velocity, c . The electric engine has the advantage that the exhaust velocity may be chosen for the mission.

Equation 5 is used for chemical or direct nuclear rockets and may also be used for the electric propulsion engine when a single value of c is being considered. In a more general analysis, the exhaust velocity is considered variable but constrained by constant engine power according to the relationship

$$p = \epsilon P = -\frac{1}{2} \dot{M} c^2, \quad \dots(6)$$

where

p = power delivered to expellant

P = powerplant output

ϵ = overall engine efficiency.

Combining Equations 4 and 6 and integrating, the rocket equation for power-limited flight is obtained:

$$\frac{1}{M_1} = \frac{1}{M_0} + \frac{1}{2\epsilon P} \int_{t_0}^{t_1} a^2 dt. \quad \dots(7)$$

The acceleration, a , in Equation 7 is defined to be the rocket thrust divided by vehicle mass and is thus the acceleration in field-free space or the so-called "thrust acceleration" in a gravity environment. A particular space mission will be characterized by an optimum program of thrust acceleration, as determined from trajectory analysis, and a corresponding minimum value of $\int a^2 dt$ and maximum terminal mass.*

The engine specific impulse is related to exhaust velocity of the propellant by

$$I_{sp} = \frac{c}{g_0}, \quad \dots(8)$$

---

*The term "mission," used in this sense, includes specification of the travel time.

where

$$g_o = \text{dimensional conversion factor} = 32.2 \text{ lb}_m\text{-ft/lb}_f\text{-sec}^2.$$

The powerplant weight is related to the power by

$$M_W = \alpha P = \frac{\alpha P}{\epsilon} \quad , \quad \dots(9)$$

where

M_W = powerplant weight

α = powerplant specific weight.

Inspection of Equations 4, 6, 8, and 9 reveals that, for a given mission, there will be an optimum specific impulse or specific impulse program. For a particular thrust or acceleration requirement, increasing the specific impulse will decrease the fuel consumption but increase the power and thus the powerplant weight. Optimum specific impulse thus represents a balance between power and propellant requirements. Commonly, it is desired to maximize payload for a mission. Assuming constant values for α and ϵ , it may easily be shown that this is accomplished with the following fractions:

$$\left(\frac{M_W}{M_o} \right)_{\text{opt}} = \gamma - \gamma^2 \quad , \quad \dots(10)$$

$$\left(\frac{M_P}{M_o} \right)_{\text{opt}} = \gamma \quad , \quad \dots(11)$$

$$\left(\frac{M_L}{M_o} \right)_{\text{max}} = (1 - \gamma)^2 \quad , \quad \dots(12)$$

where

$$\gamma^2 = \frac{\alpha}{2} \int a^2 dt \quad , \quad M_W + M_P + M_L = M_o$$

M_P = propellant weight

M_L = payload weight.

Actually, the powerplant specific weight, α , is usually a function of power; and the engine efficiency is dependent on specific impulse and thus is also a function of power. Typically (as in most of the application studies made), specific weight decreases with power and the efficiency increases with specific impulse. Since both of these effects favor increased power, the optimum powerplant fraction will often be higher than indicated by Equation 10.

C. TRAJECTORY ANALYSIS

Assuming that there are just two forces acting on the rocketship, gravity and thrust, the equation of motion is

$$\frac{d^2 \vec{r}}{dt^2} = -\nabla U(\vec{r}, t) + \vec{a} \quad , \quad \dots (13)$$

where

$\vec{r}$ = spacecraft position vector

U = gravitational potential,

which simply states that the total acceleration is the vector sum of the accelerations due to gravity and thrust. For plane motion in a central force field, the polar coordinate form of Equation 13 is

$$\frac{d^2 r}{dt^2} = \frac{h^2}{r^3} - \frac{\mu}{r^2} + a_r \quad \dots (14)$$

$$\frac{dh}{dt} = r a_\theta \quad , \quad \dots (15)$$

where

$h = r^2 \frac{d\theta}{dt}$ = angular momentum per unit mass

μ = gravitational constant = universal gravitational constant multiplied by the mass of the gravitating source

a_r = radial component of thrust acceleration

a_θ = circumferential component of thrust acceleration

θ = angular position.

These equations are of general use because most missions, at least in the preliminary stage, are analyzed as a series of two-body problems.

Applying variational methods to Equations 14 and 15, it has been found⁴ that the necessary conditions for $\int a^2 dt$ to be minimum are

$$\frac{d a_{\theta}}{dt} - \frac{a_{\theta}}{r} \frac{dr}{dt} + 2 \frac{h a_r}{r} = 0 \quad , \quad \dots (16)$$

$$\frac{d^2 a_r}{dt^2} - \frac{a_{\theta}^2}{r} - \frac{a_r}{r^4} (2\mu r - 3h^2) = 0 \quad , \quad \dots (17)$$

if the terminal angular position is unspecified (or, equivalently, the firing position from the initial orbit is unimportant). Solution of Equations 14 through 17, with the necessary six boundary conditions, will yield the desired optimum trajectory.

The six boundary conditions are the initial values of r , $\dot{r}$, and h plus three terminal conditions, whose form will depend on the type of mission (e.g., flyby or capture) and whether, for the capture, the final orbit should be circular or elliptical. Unfortunately, there is no direct numerical procedure for solving these equations subject to both initial and terminal conditions. Thus three additional initial conditions must be assumed (usually a_r , a_{θ} , and $\dot{a}_r$) and the applicable terminal conditions satisfied by trial and error.

Extensive computer studies of interplanetary trajectories, involving equations similar to those given above, have been performed by W. Melbourne; and the results, tabulated in References 5 and 6, are used in the following section of this report.

III. ELECTRIC PROPULSION MISSIONS

A. ORBITAL MISSIONS

The first mission which has been considered is that of establishing a satellite in a 24-hr synchronous orbit. The major application for such a satellite will probably be to active communications, including radio and TV relays and possibly hemispheric or global broadcasting to home receivers. Other possible applications include the establishment of astronomical or stationary meteorological observatories.

In a 24-hr mission, the Atlas-Centaur (improved model) can deliver about 2000 lb of useful payload, which is unacceptable where kilowatts of power are required at the 24-hr level. As an alternative to using a larger chemical booster, which would be economically unattractive, increased payload may be obtained by transferring from a low-level to a synchronous orbit with electric propulsion. Where long missions are planned, the travel time required for the transfer will represent only a small penalty as compared with the payload advantage gained.

The assumed starting point for the transfer is a 600 n.mi circular orbit at an inclination of 30° to the earth's equator, which corresponds approximately to an easterly launch from Cape Canaveral. The electric propulsion portion of the total mission will consist of a planar transfer by slow spiral up to the 24-hr level followed by orbital pitching to the desired synchronous position.

The planar transfer is accomplished with constant thrust directed tangential to the vehicle flight path. Power and propellant requirements may be determined from energy considerations.⁵ The thrust acceleration as a function of time is

$$a = \frac{a_o}{1 - \frac{a_o}{I_{sp} g_o} t}, \quad \dots(18)$$

with the initial acceleration given by

$$a_o = - \frac{\dot{M}c}{M_o} = \frac{2p}{M_o c} \quad \dots(19)$$

Orbital velocity is given by

$$V^2 = \frac{\mu}{r} \quad \dots (20)$$

The total energy per unit mass of an orbiting body is the sum of its kinetic and potential energies:

$$E = \frac{1}{2} V^2 - \frac{\mu}{r} = - \frac{\mu}{2r} \quad \dots (21)$$

The power input to the system may now be equated to the rate of change of orbital energy:

$$V a = \frac{\mu \frac{dr}{dt}}{2r^2} \quad \dots (22)$$

Rearranging and combining with Equations 18 and 20,

$$\frac{dr}{dt} = \frac{2r^{3/2} a_o}{\mu^{1/2} \left(1 - \frac{a_o}{I_{sp} g_o} t \right)} \quad \dots (23)$$

The transfer time is obtained by integrating between initial and final orbital radii:

$$T_P = \frac{1 - e^{-\nu \left(1 - \sqrt{r_o/r_1} \right)}}{\nu} \times \frac{\sqrt{\mu/r_o}}{a_o} \quad \dots (24)$$

where

$$\nu = \frac{1}{I_{sp} g_o} \sqrt{\mu/r_o}$$

r_o = initial orbital radius

r_1 = final orbital radius.

The propellant expenditure is

$$\Delta M_P = -\dot{M} T = \frac{2P}{c} T \quad \dots (25)$$

Equations 24 and 25 are used to describe the planar transfer to the 24-hr orbit. Once the 24-hr level has been reached, a thrust normal to the orbital plane will be used to pitch the orbit through about 30° until the satellite is directly over the equator. The rate of orbital inclination due to a normal disturbing acceleration is given in Reference 7 as

$$\frac{di}{dt} = \frac{r \cos(\omega + \eta)}{nb^2 \sqrt{1 - e^2}} a \quad , \quad \dots(26)$$

where

i = inclination angle between initial and final orbits

ω = longitude of peri-apsis (angle between peri-apsis and ascending node, as seen from the center of attraction)

η = true anomaly (angular position, measured from peri-apsis)

$n = 2\pi/P$

P = period of orbit

b = semimajor axis

e = eccentricity of orbit,

which, for a circular orbit (letting $\omega = 0$), becomes

$$\frac{di}{dt} = \frac{a \cos \eta}{nr} \quad . \quad \dots(27)$$

A choice exists in selecting the limits of integration for Equation 27, since thrust is most effective in the vicinity of $\eta = 0^\circ$ and $\eta = 180^\circ$. Propellant requirements may thus be reduced by coasting in regions which are far from the nodes of intersection of initial and final orbits. However, calculations for the present report are based on maximum inclination rate, which is obtained with a continuous normal thrust,⁸ reversed in sense twice per revolution at points 90° from the nodes. For this case, the inclination per revolution is found to be

$$\frac{\Delta i}{\text{rev.}} = 4 \frac{a}{g_r} \quad , \quad \dots(28)$$

where

g_r = acceleration of gravity at radius r .

Combining Equations 18 and 28 and integrating once again, the transfer time is obtained:

$$T_i = \frac{g_o I_{sp}}{a_o} (1 - e^{-\Delta i/B}) , \quad \dots(29)$$

where

$$B = \frac{2r^{1/2} g_o^{1/2} I_{sp}}{\pi R_o}$$

g_o = acceleration of gravity at sea-level

R_o = radius of earth.

Due to the averaging effect of the first integration (over one revolution), the time given by Equation 29 is somewhat approximate: the effect of true anomaly on inclination rate has been blurred over. For present purposes, where a relatively large number of revolutions will be required to complete the mission, this feature is not important.

It should be pointed out that the overall transfer described above, from a low-level to a synchronous orbit, represents a possible solution but not an optimum one. Some additional savings may be realized with a transfer in which the thrust is programmed to achieve orbital inclination during ascent. Payloads as determined are thus somewhat conservative.

Both ion engines and arc jets were considered for the orbital transfer. Ion engine efficiency is taken from Figure 2; a constant efficiency of 50% was assumed for the arc jet engines. Power conditioning losses of 15% were assumed for both systems.

Shown in Figure 3 is terminal weight in the synchronous orbit as a function of travel time for ion systems of various power levels. To determine gross payload capabilities, the weight variation of a medium power nuclear system using a mercury Rankine conversion cycle (see Figure 1) was extracted from Reference 2. Figure 4 was obtained by subtracting powerplant weights from the terminal weights.

The curves of Figures 3 and 4 are based on an Atlas-Centaur launch capability of 8800 lb in a 600 n. mi orbit. It can be noted that reasonable payloads in the synchronous orbit are obtained with systems utilizing launch vehicles of this class.

Figure 3. Transfer to a 24-Hr Synchronous Orbit With Ion Propulsion

Figure 4. Transfer to a 24-Hr Synchronous Orbit With Medium Power Turbogenerator System

Note also that, over a broad range of mission times and payload weights, the optimum electrical power level varies between 90 and 150 kw. Thus, the 90 kw system is optimum for payloads of 3000 to 4000 lb, 120 kw is optimum for 2000 to 3000 lb, and 150 kw is optimum for 1000 to 2000 lb.

Orbital transfer capabilities with an arc jet are shown by the dotted lines in Figure 4. For a particular power level, the arc jet may always be used in place of the ion engine if decreased travel time (at the expense of payload capability) is desired. This is due to the higher thrust of the arc jet. However, where a choice of power level may be made in planning the mission, the arc jet appears less promising: the higher thrust may often be obtained with less weight penalty by increasing the power level of the ion system. Thus it may be seen from Figure 4 that the ion system is superior except where quite small payloads are considered. These conclusions, of course, are based on assumptions already noted concerning arc jet and ion engine operating efficiencies.

A second application for nuclear-electric propulsion which has been investigated is the low-altitude orbital sustainer. Orbital lifetimes of one year or greater are desired in many satellite applications. For typical satellite configurations, a one-year lifetime corresponds to an initial orbital altitude somewhere in excess of 200 mi, at lesser orbits, some form of sustaining power is required to achieve the desired lifetime.

For near-earth satellites, the predominant contribution to orbital decay comes from aerodynamic drag. This effect is shown in Figure 5 for orbits ranging from 70 to 170 n.mi. Calculation of drag pressure was based on the atmospheric density variation given by the ICAO standard atmosphere extension and an assumed drag coefficient of 2.0.⁹

Assuming that a continuous sustaining thrust is employed to balance the drag force, power and propellant requirements for maintaining a constant orbit were calculated and are shown in Figure 6. Obviously, intermittent thrust could also be used if the power level is such as to give the same total impulse. Propellant requirements for a typical chemical sustainer ($I_{sp} = 300$ sec) and cold gas system ($I_{sp} = 100$ sec) are shown for comparison. It may be noted that the latter systems represent reasonably lightweight solutions to orbital sustenance at altitudes above about 140 n.mi. No general statement can be made as to where these systems become optimum, however, since propellant requirements for electric

Figure 5. Drag Pressure for Circular Orbits

Figure 6. Net Power Required for Orbital Sustenance (Power in Propellant Stream) Based on 100 ft² of Drag Area

propulsion will always be less, whereas the weight penalty for electrical power will be a function of how much power is required aboard the satellite for auxiliary purposes.

Since Figure 6 is based on power delivered to the exhaust propellant, some account must be taken of losses in the power conditioning equipment and engine proper. An overall efficiency of 50% would be typical. On this basis, the following systems are suitable for orbital altitude sustenance:

Altitude (n. mi)	Power (kw)	Propellant Weight (lb)
80	100	1000
100	3	400
120	0.3	200

The 24-hr satellite and the orbital sustainer are only two of a variety of missions which may be accomplished with nuclear-electric propulsion systems. The interested reader is referred to Reference 10 for a review of propulsion requirements for several orbital maneuvers, including orbital maintenance; corrections to orbital position, radius, and inclination; planar and non-planar transfers; and combination maneuvers with optimum steering programs.

B. PLANETARY MISSIONS

Thrust-to-weight ratios for ion propelled spacecraft of the near future will be in the range of 10^{-5} to 10^{-3} . Thus, ion propulsion must commence from a low earth orbit into which the spacecraft has been placed by a conventional high thrust device. Starting from this orbit, the planetary mission is most conveniently analyzed by splitting it into three parts:

- 1) the geocentric escape phase, during which the spaceship is considered to be under sole influence of the earth's gravitation
- 2) the heliocentric transfer, in the gravitational field of the sun, from the solar orbit of the earth to the target planet
- 3) the planetocentric phase, during which the spacecraft is considered to have left the sphere of influence of the sun and to be influenced solely by the target planet.

Optimum variable-thrust programs for the first phase, geocentric escape, have been investigated in the literature.^{4,11} It has been found in general that there is negligible difference between an optimum program and one in which a suitably chosen constant thrust is directed either circumferentially or tangential to the vehicle flight path. Using one of the latter methods, the geocentric phase consists simply of a slow outward spiral to the point when escape energy is achieved.

During the succeeding heliocentric portion of the journey, the ion thrust is programmed (at constant power) to achieve an Irving and Blum optimum trajectory. One of two programs would be used, depending on the type of mission. For a planetary probe (flyby), only the terminal position of the spacecraft at terminal time need be specified. If planetary rendezvous is required, then there is the additional restriction that the kinematic condition of the spacecraft must match that of the target planet upon arrival. With minimal terminal control, the spacecraft will then be captured in a highly eccentric planetary orbit, or, if a low-level circular orbit is desired ("orbiter" mission), a slow inward spiral will be achieved with constant tangential thrust.

As an alternative to the variable thrust program, a constant thrust transfer, with an optimized coast period, may be used. This type transfer may actually be thought of as an approximation to the optimum program, in which a low-thrust (high specific impulse) portion of the latter is replaced by the coast period. Constant thrust will probably be employed on early shots because of the difficulty in designing an ion engine for efficient operation over a range of specific impulses. For equal payloads in a Mars capture mission, the transfer time would exceed optimum by only about 5%.⁶

Certain modest planetary missions have been investigated for SNAP 8 type systems operating in the range of 60 to 180 kw. Powers in this range are suitable for Mars and Venus missions. The analysis was directed mainly at Mars capture and orbiter missions, Venus missions would be slightly easier. Calculations were based on initial weights of 7000, 10000, and 15000 lb in a 600 n. mi orbit, which correspond to payload capabilities for possible configurations of the Centaur or Saturn launch systems (see Table I).

The analysis follows the methods and computer results of Reference 5. Time for geocentric escape is given by

$$T_e = \frac{1 - e^{-\nu}}{\nu} \times \frac{\gamma(a_o)}{a_o} \times \sqrt{\mu/r_o} \quad \dots(30)$$

Note that this equation is identical to Equation 24 with the exception that an empirical correction factor, $\gamma(a_o)$, has been added. Derivation of Equation 24 was based on the assumption that the spiral path of the spacecraft is an osculating ellipse; the correction term takes account of the fact that this is not true over the last couple revolutions, when escape energy is approached, and the craft moves rapidly outwards. For an initial acceleration of $a_o = 10^{-4}g$, $\gamma = 0.92$.

TABLE I
CAPABILITY OF CHEMICAL BOOSTERS

Launch System	Useful Payload in 600 n. mi Orbit (lb)
Atlas-Centaur	7,000*
Titan-Centaur	10,000
Advanced Centaur† or Saturn C-1	15,000

*Early model Atlas-Centaur; capability of improved version estimated at 8800 lb.

†See Reference 1.

Requirements for the heliocentric portion of the trajectory are based on Equation 7 and the computer results of Reference 5. These results for the Mars capture mission are reproduced in Figure 7. Values for $\int a^2 dt$ as shown are for the variable thrust program and optimum true anomaly at planetary rendezvous.

The final portion of an orbiter mission is the terminal planetocentric spiral, which may be described by Equation 30 if the sign of ν is reversed and time is measured positive from the terminal planetary orbit.

For the missions studied, terminal spacecraft weight as a function of travel time and power level is shown in Figures 8 through 13. These curves are based on an optimum apportionment of flight times among the geocentric escape, the heliocentric transfer, and (for the orbiter missions) the planetocentric spiral. Variable thrust was used in the heliocentric transfer. During this portion of the trajectory, an average engine efficiency of 70% was assumed. For the constant thrust portions, engine efficiency was taken from Figure 2. A power conditioning efficiency of 85% was assumed throughout.

Figure 7. Rocket Equation Parameter for Optimum Rendezvous Trajectory Between Earth and Mars

Figure 8. Variation of Terminal Weight With Flight Time for a Mars Capture Mission

Figure 9. Variation of Terminal Weight With Flight Time for a Mars Capture Mission

Figure 10. Variation of Terminal Weight With Flight Time for a Mars Capture Mission

Figure 11. Variation of Terminal Weight With Flight Time for a Mercury Capture Mission

Figure 12. Variation of Terminal Weight With Flight Time for a Mars Orbiter Mission

Figure 13. Variation of Terminal Weight With Flight Time for a Mars Orbiter Mission

Using the powerplant weights shown in Figure 1, gross payload capability for a range of power levels is shown in Figures 14, 15, and 16. Gross payload is defined to be the entire terminal weight less the weight of the power plant. It therefore includes the ion engine, tankage, communication equipment, guidance and control equipment, scientific payload, and associated structure. The minimum weight required for guidance, control, communication equipment, and spacecraft structure has been estimated at 1000 to 1500 lb.¹ The ion engine and tankage will weigh about 1.5 lb/kw. Based on these estimates, a reasonable objective for gross payload is in the range of 2000 to 3000 lb.

A summary of payload capability for the various missions is given in Table II. Note that, for missions in which useful payloads are delivered, the optimum powerplant fraction falls in the range of 35 to 45%. This is somewhat high in comparison with many previous analyses. If constant values of engine efficiency and powerplant specific weight are assumed, the optimum powerplant fraction (Equation 10) will generally be between 20 and 25%. It has been shown in Reference 12 that 30 to 35% is a more reasonable estimate where specific weight is assumed constant, but engine efficiency is allowed to vary in a manner similar to Figure 2. In the present analysis, optimum power levels are pushed still higher by inclusion of the powerplant weight variation shown in Figure 1. Thus, the effect of including practical estimates for engine efficiency and powerplant weight has been to nearly double the optimum powerplant fraction.

TABLE II
INTERPLANETARY MISSION CAPABILITY

Booster	Mission	Gross Payload (lb)	Flight Time (days)	Optimum Power Level (kw)	Powerplant Fraction
Atlas-Centaur	Mars capture	2000	400	70	0.55
	Mars orbiter	no mission	-	-	-
Titan-Centaur	Mars capture	2000	240	130	0.48
		3000	300	110	0.44
	Mars orbiter	2000	290	130	0.48
		3000	370	110	0.44
Saturn C-1	Mars capture	5000	280	160	0.37
	Mars orbiter	5000	350	160	0.37
	Mercury capture	2000	270	170	0.38
		3000	340	160	0.37

Figure 14. Variation of Gross Payload With Power Level for a Mars Capture Mission

Figure 15. Variation of Gross Payload With Power Level for a Mars Orbiter Mission

Figure 16. Variation of Gross Payload With Power Level for a Mercury Capture Mission

It may be seen from Table II that useful missions are accomplished with systems operating in the power range between 60 and 180 kw. An initial orbital weight of 7000 lb was not found to be really adequate for any of the missions studied, but there is substantial mission capability if the initial weight is around 10,000 lb.

In conjunction with the 10,000 lb initial weight, the 60 kw powerplant can deliver about 3000 lb in a (one-year) Mars capture mission or about 2000 lb in an orbiter mission. Since the gross payload which is required for a useful mission is probably in the range of 2000 to 3000 lb (but may be higher), both of these missions must be considered to be marginal. However, with a modest increase in power level, the picture is much improved. Thus, a 120 kw system can deliver 3000 lb in a Mars orbiter mission and close to 4000 lb in a capture mission. It is believed these payload capabilities, as shown in Figure 14 and Table II, represent a significant incentive for the development of a powerplant with an output of around 120 kw.

Finally, if a booster such as the Saturn C-1 or Advanced Centaur is used, substantial payloads for the Mercury and Mars missions may be realized. With an initial weight in orbit of 15,000 lb, the 120-kw system provides adequate capability, although a power level of 160 kw is more nearly optimum. With either of these systems, payloads of about 3500 and 5000 lb, respectively, can be delivered in Mercury capture and Mars orbiter missions.

NOMENCLATURE

- a = thrust acceleration; a_r is radial component, a_θ is circumferential component; subscript o refers to initial acceleration
- b = semimajor axis of orbit
- B = constant defined in Equation 29
- c = rocket expellant velocity relative to spacecraft
- e = eccentricity of orbit
- E = total energy per unit mass
- g = acceleration of gravity; g_o at sea level, g_r at radius r
- h = angular momentum per unit mass
- i = inclination angle between orbits
- I_{sp} = specific impulse
- M = mass; weight in Equation 9; subscript o refers to initial spacecraft mass, l to final spacecraft mass, g to expellant, L to gross payload, P to propellant, W to powerplant; without subscript, total spacecraft mass
- $n = 2\pi/P$, where P is the orbital period
- p = power in expellant stream
- P = powerplant output power
- r = distance from gravitational source or orbital radius; subscript o refers to initial orbit, l to final orbit
- R_o = radius of the earth
- t = time
- T = trip time; subscript e refers to geocentric escape, i to orbital inclination, P to transfer between coplanar orbits
- U = gravitational potential
- V = velocity; subscript g refers to expellant; without subscript, spacecraft velocity
- α = powerplant specific weight (weight/power)
- γ = dimensionless parameter used in Equations 10, 11, and 12
- $\gamma(a_o)$ = empirical correction factor to geocentric escape time
- ϵ = engine efficiency
- η = true anomaly (angular position in orbit, measured from peri-apsis)
- θ = angular position
- μ = gravitational constant, equals the universal gravitational constant multiplied by the mass of the gravitating body
- ν = dimensionless factor defined in Equation 24
- ω = longitude of peri-apsis (angle in the plane of the orbit between peri-apsis and the ascending node)

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