MODELS FOR GEOTHERMAL WELLS

by

Efstathios E. Michaelides

B.A., Oxford University, 1977

M.S., Brown University, 1979

Thesis

submitted in partial fulfillment of the requirements for
the Degree of Doctor of Philosophy at Brown University

June, 1980

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

MASTER

This dissertation by Efsthathios E. Michaelides
is accepted in its present form by the Division of Engineering
as satisfying the
dissertation requirement for the degree of Doctor of Philosophy

Date *May. 23, 1980* *J. Weston* .

Recommended to the Graduate Council

Date *May. 23, 1980* . . . *Richard A. Dobbins*
Date *May. 23, 1980* . . . *Paul C. Merriam* .

Approved by the Graduate Council

Date

Curriculum Vitae

Efstathios E. Michaelides

Place and time of birth: Thessaloniki, Greece, 13 February 1955.

Institutions attended:

A' High School for Boys, Thessaloniki, Greece. (1967-1973)

Sevenoaks School, Kent, England. (1973-1974)

Oxford University, England. (1974-1977)

Brown University, Rhode Island (1977-1980)

Awards:

Schillizzi Scholarship, St. John's College, Oxford.

Sigma Xi: Associate Member

ABSTRACT

Geothermal energy is now used in twelve countries and geothermal plants exceed 5000 MW installed capacity. The rate of increase of this capacity is about 15% per annum in the U.S. Resources of relatively low temperature are being exploited and sophisticated methods of energy conversion are being used. The prediction of the wellhead conditions and the mass flow rate is essential to the design of the plant. The accurate knowledge of these conditions is used for the precise sizing of the plant components and to avoid any defensive design. In addition, the efficiency of the plant can be determined and an economic appraisal for the whole project can be made.

This thesis examines the problem of two-phase flow pressure loss in order to give an answer to the problem of determination of the wellhead conditions. For this purpose two models have been developed, the first based on the pattern structure of the flow and the second on the mixing length theory. The void fraction correlations and the transition conditions are presented in the first model as a means of estimating the pressure loss. Heat losses, and the effect of impurities are examined in detail. An expression for the critical flow conditions is also derived. The model is used to predict the available power at the wellhead under various conditions and an answer to the problem of well pumping is given. For the second model an outline of the mixing length theory and the boundary layer coordinates is given; a density distribution in the geothermal well is assumed and the equations for the pressure loss are derived by means of the entropy production function. Finally a comparison of the two models is made and their predictive power is tested against known well data. A brief comparison with the Denver Research Institute is also made.

Table of Contents

- I. Introduction
 - I.1 Introduction on geothermal energy and the wells; statement of the problem.
 - I.2 A survey of two-phase flow theories.
- II. The pattern model
 - II.1 Theoretical development of the model and its assumptions.
 - II.2 Criteria for the transitions between flow patterns.
 - II.3 The calculation of pressure losses.
 - II.4 Critical flow in geothermal wells.
 - II.5 Heat transfer in geothermal wells.
 - II.6 The effect of salts and gases on the brine properties.
 - II.7 The computer program.
 - II.8 Influence of varying the well characteristics.
 - II.9 Pumping of the well versus diameter stepping.
- III. The mixing length model
 - III.1 Model of the geothermal well.
 - III.2 The choice of the density function.
 - III.3 The mixing length theory.
 - III.4 Transformation of the equations into boundary layer coordinates.
 - III.5 The equations of two-phase flow.
 - III.6 Entropy production in the pipe.
 - III.7 The solution of the equations.

Table of Contents, cont.

IV. Conclusions

IV.1 Comparison of results.

IV.2 Conclusions.

References - Bibliography

Appendices

Acknowledgements

I. INTRODUCTION

I.1 Introduction on geothermal energy and the wells; statement of the problem.

Geothermal energy, although a curiosity some years ago, is now advocated as one of the viable alternative sources for the future. Geothermal power plants are being constructed at a fast rate in many countries and it is estimated that by the year 1985 there will be 3500 MW installed capacity in the U. S. alone.

Geothermal plants utilize the high temperature - high pressure resources of water enclosed in pockets of permeable rock under the earth's surface at depths of 1-3 km. Under these conditions water exists as a solution of various salts (NaCl , KCl , CaCl_2 , CaCO_3) and gases (CO_2 , H_2S , N_2 , CH_4) at a pressure higher than the saturation pressure of the solution. Consequently, in the early stages, the flow is liquid as far as the flashing level, where the prevailing static pressure becomes equal to the saturation pressure of the solution and evaporation starts with the formation of bubbles (bubbly flow regime). These travel inside the liquid matrix and are gradually multiplied with the evaporation of more liquid, their velocities being higher than the average liquid velocity because of the lifting buoyancy effect. As their number increases, the bubbles start to coalesce and to form big Taylor bubbles, which occupy most of the cross-sectional area of the pipe, and have the shape of a bullet. The new flow regime is called slug flow. Flow instabilities develop at the liquid film which exists

along the pipe wall, and this causes a distortion of the Taylor bubbles and the breaking up of the liquid slugs. The vapor assumes irregular shapes and its average velocity is further increased relative to the liquid velocity. This new flow regime is called froth, or churn, flow. Finally, the irregularly shaped bubbles coalesce and form a cylinder of vapor surrounded by an annulus of liquid. In this case the liquid phase travels next to the wall in a thin film and the vapor is carried by the central part of the pipe with a few liquid droplets entrained in it. This is the annular-flow regime which is stable and prevails as far as the wellhead. All flow regimes just mentioned are shown schematically in Fig. I.1.

The condition of the geothermal fluid at the wellhead must be known in order to design a geothermal plant and to calculate the available power. It is important to be in a position to make reliable predictions of this state before the completion of the well, not merely in order to calculate the efficiency of the plant, but also to avoid the financial burden of defensive design. The state of the fluid at the wellhead is characterized by the temperature, pressure, salinity, dryness fraction and gas content. These parameters, and the mass flow rate, determine the available power at the wellhead and hence the size of the geothermal installation.

A good prediction of the wellhead conditions can be made, if the bottomhole conditions are known accurately, and if there exist reliable analytical methods for the calculation of the variation

Fig. I.1 Flow patterns in vertical pipe flow.

of pressure in the single and two-phase flow regimes. The single-phase flow regime is well understood and existing theories provide an accurate determination of the pressure variation, but this is not the case in the two-phase flow regimes. Here, the calculation of pressure drops obtained by alternative theories differ by as much as 80% in some cases (Levy, 1962), and no reliable quantitative prediction is yet possible. We need a reliable model which would fully account for the physical processes taking place in the well and which would yield predictions on the state of the fluid at the wellhead with the required degree of accuracy.

In this thesis two different ways of geothermal well modeling are presented. The first one takes into account all the flow regimes encountered in the well and the transitions between them. The pressure loss is calculated by means of the void fraction and the quality of the fluid. The estimation of the void fraction is essential in the procedure, and correlations known from the literature for the different flow regimes have been used in the calculations. The second model is based on the mixing-length theory which was first successfully applied to single-phase flow by Prandtl (1925). A density profile across the well is assumed for this model, and the velocity profile is derived from it. The entropy production calculated for the model constitutes the main step for the prediction of the static pressure drop. A description of the two models will be given in Chapters II and III, and their predictive powers will be tested by comparisons with experimentally obtained data under known conditions.

I.2. A survey of two-phase flow theories

The first reliable theory on two-phase flow was developed by Martinelli (1944) who proposed a correlation for the pressure drop in horizontal pipes for a two-component (air-water) fluid. He made use of similarity laws to bypass the difficulties encountered by the existence of various flow regimes, and did not consider the flow structure or the local variation of parameters. This correlation was successful in many engineering applications and was used under conditions beyond the range of its applicability with various degrees of accuracy.

For the pipe-flow case, one is mainly interested in the frictional pressure losses (or entropy production) and the average properties of the fluid, especially its density when vertical flow is considered. Consequently, the necessary parameters in two-phase flow are the friction factor at the wall, which is different than the single-phase flow friction factor, and the void fraction in the pipe which will give the average density. The void fraction is the ratio of the volume flow rate of the vapor to the total volume flow rate of the fluid. Once these two quantities are accurately known, all the average properties of the fluid are predicted and the pressure distribution can be deduced as well. Therefore, the two-phase flow theories that were developed aim at estimating these two parameters either analytically or by means of a correlation of existing data.

The fact that the Martinelli correlations had limited applications, and were inaccurate in many cases outside their applicability limits, gave rise to other two-phase flow models which rely more on analytical considerations and less on empirical correlations. Additional data provided with more information about the two-phase flows, and the second generation of models appeared, which can be characterized as separated models. They take an average velocity for each phase and apply the conservation equations separately for each phase. Slip is introduced in all these models to account for the greater velocity of the gaseous phase. The slip ratio is defined as the ratio of the two velocities. Levy (1960), Zuber and Hench and Woods contributed significantly to the development of these models. Levy (1962) calls them "lumped models".

The separated models are an improvement, but in some cases they have proved to be inaccurate and their deviations from experimental results are as high as 80% in the worst cases. In addition to the analytical methods employed, the models require assumptions for the two-phase pressure drop, which must come from experimental data. Their major deficiency, though, lies in the fact that they do not describe the local variation of quantities, such as density and velocity, and even if they successfully predict the pressure distribution over some length of the pipe, they fail to account for phenomena which depend on local quantities.

The flow-pattern models belong to a third generation and they account for the local variation of the parameters. The conservation equations for the two phases are solved in this case under the assumption of predetermined flow-pattern structures (bubbly, slug, annular). However, the resulting equations are complicated, and in many cases require special assumptions to be made which restrict their applicability. They also require an additional assumption for the pressure drop over the pipe length. Models of this category include Griffith's and Wallis's for slug flow, Mendelson's for bubbly flow, Levy's (1964) for annular flow and Bankoff's for variable-density flow. The first model to be presented emanates from considerations similar to those of pattern flows; the equations for the void fraction are taken from the models of this category and the friction factors from the Lotus-Flin theory on pressure drop.

A different approach was chosen by Levy (1962) who employed the mixing-length hypothesis in his model. This hypothesis, which is analyzed in Chapter III, was very successfully used by Prandtl to predict the friction factors in single-phase flows. The shear stress is taken to be

$$\tau = \ell_u^2 \frac{d\bar{u}}{dy} \left| \frac{d\bar{u}}{dy} \right| \quad (\text{I.2.1}).$$

Levy takes ℓ_u from the Van-Driest theory

$$\ell_u = Ky \left[1 - \exp(-y/F_1) \right] , \quad (\text{I.2.2})$$

and introduces a density distribution characterized by a density mixing length, which is assumed equal to ℓ_u , and which gives rise to a density distribution equation of the form

$$\tilde{\rho} = A\tilde{\rho}u + B. \quad (I.2.3)$$

This model is a significant contribution to the two-phase flow theories and to the understanding of local phenomena; it gives qualitatively accurate results and presents an entirely new approach to two-phase flow phenomena. It is believed that a series of improvements (especially in the density and velocity distributions) can be made in order to increase the model's quantitative predictions; this task requires extensive experimentation for the determination of the relevant parameters.

There is no provision in the above model for the gravitational effects and hence its use is limited to horizontal flows only. For this reason it is inapplicable to the case of geothermal wells, where the lifting of the fluid plays an important role. A new approach to the subject, which incorporates vertical flows, is presented in Chapter III. This is based on the mixing-length theory and a different density-distribution function. This model will describe the local time-average conditions in the well, and its special features are further believed to give answers to the deposition problem in the wells.

II. THE PATTERN MODEL

II.1 Theoretical development of the model and its assumptions.

The common feature in all approaches to two-phase flow through pattern modeling is the equation to calculate the void fraction. This is done in two ways: either a correlation of experimental data is used, or an analytical expression is derived for the void fraction, ϕ , or the slip ratio K . The void fraction is given by the following equation if K is determined:

$$\phi = \frac{1}{1 + (1 - \gamma) K \rho_\ell / \rho_v \gamma} \quad . \quad (II.1.1)$$

The flow patterns which are met in geothermal wells are: liquid flow below the flashing level, bubbly, slug, froth and annular flow above it. A brief description of the equations governing these flow regimes will be presented here as they were used for the completion of the geothermal well model and the resulting calculations.

A. Liquid flow

The liquid flow equations have been tried successfully in many situations and one is confident in using them. It is a single-phase flow with uniform density ρ_ℓ (the void fraction is zero) and for the friction at the walls of the pipe the Blasius equation may be used:

$$f = 0.079 (\text{Re})^{-0.25} \quad . \quad (II.1.2)$$

where γ is the quality of the fluid.

Here

$$\text{Re} = \frac{\rho_{\ell} \bar{V} D}{\mu_{\ell}}, \quad (\text{II.1.3})$$

is the Reynolds number of the flow, with ρ_{ℓ} denoting the density of the liquid, $\bar{V}$ the average velocity, D the diameter and μ_{ℓ} the viscosity of the liquid.

The pressure drop per unit length of the pipe $\Delta P/\Delta Z$ due to friction is then given by the following expression:

$$\frac{\Delta P}{\Delta Z} = 0.158 \frac{\bar{V}^2 \rho_{\ell}}{D} (\text{Re})^{-0.25}. \quad (\text{II.1.4})$$

The static pressure at the bottom of the well is higher than the saturation pressure, and hence the early stages of the well are characterized by liquid flow. A geothermal well can be justifiably approximated by an adiabatic pipe (Section II.5); therefore, until the flashing point is reached, the properties of the fluid do not change if they are assumed to be pressure independent. The last assumption is true especially for the density and the viscosity. Consequently, the Reynolds number of the flow remains the same for the whole length of the liquid regime. The total pressure gradient is the sum of the frictional, and the gravitational pressure losses, and is given by

$$\frac{\Delta P}{\Delta Z} = 0.158 \frac{\bar{V}^2 \rho_{\ell}}{D} (\text{Re})^{-0.25} + \rho_{\ell} g. \quad (\text{II.1.5})$$

According to the above assumptions, this pressure gradient remains constant in the whole of the liquid-flow regime. Therefore,

the flashing level of the well can be calculated if the bottomhole conditions of the fluid are known. Hence,

$$L_f = L - \frac{P_b - P_s(T_b)}{(\Delta P / \Delta Z)}, \quad (\text{II.1.6})$$

where L is the depth of the well, L_f is the depth of the flashing level, P_b the bottomhole pressure, $P_s(T_b)$ the saturation pressure corresponding to the bottomhole temperature T_b and $\Delta P / \Delta Z$ is given by eqn. II.1.5. Typical lengths of liquid flow in the wells ($L - L_f$) are about 800-900 m depending, of course, on the bottomhole conditions and the mass-flow rate.

B. Bubbly flow

When the flashing level of the geothermal well is passed, the liquid starts evaporating and bubbles are formed inside the pipe; this is the onset of two-phase flow and thereafter the two-phase equations govern the flow. The temperature of the fluid drops in accordance with the equilibrium relation given by the Clausius-Clapeyron equation:

$$\frac{dP}{dT} = \frac{h_{fg}}{T(v_v - v_\ell)}, \quad (\text{II.1.7})$$

where h_{fg} is the heat of vaporization, v the specific volume and T the thermodynamic temperature. The equilibrium assumption is justified in view of the small relaxation time of the process of vaporization and the large characteristic time of the flow

$(D/\bar{V})$, given that the average velocities at this stage are of the order of 2-4 m/s.

The density of the fluid is not uniform any more because of the presence of the bubbles inside the liquid matrix. An average value of the density is given in terms of the void fraction:

$$\bar{\rho} = \phi \rho_v + (1-\phi) \rho_\ell . \quad (\text{II.1.8})$$

The frictional pressure drop depends now on the void fraction and this dependence will be investigated in Section II.3.

It follows from the above that the void fraction is an important parameter for the calculation of the pressure gradient. For this reason it is necessary to establish reliable equations for the prediction of this quantity. An equation derived by H.D. Mendelson (1967) was used to calculate the void fraction in bubbly flow: it gives the velocity of a bubble inside a liquid matrix in a gravitational field as

$$V_b = \left(\frac{2\sigma}{D\rho_\ell} + \frac{gD}{2} \right)^{0.5} . \quad (\text{II.1.9})$$

Preference was given to this equation instead of others which appear in the literature (Dukler, 1977) because it has been recommended (Cole, 1967), and because of its apparent simplicity. In the equation, σ is the surface tension of the liquid and g is the gravitational acceleration. According to this equation, and the definition of the void fraction, the latter is given by

the equation

$$\alpha\phi^2 + \beta\phi + 1 = 0, \quad (\text{II.1.10})$$

where

$$\alpha = \frac{\pi D^2 \cdot V_b \cdot \rho_v}{4\dot{m}\gamma} \quad (\text{II.1.11})$$

and

$$\beta = \frac{(1-\gamma) \rho_v}{\gamma \cdot \rho_\ell} + 1, \quad (\text{II.1.12})$$

with γ denoting the quality. The above equations can be solved to yield a unique value for ϕ , given that $0 \leq \phi \leq 1$.

Having calculated ϕ , the average density of the fluid can be computed and hence the gravitational pressure drop. The frictional pressure drop for two-phase flow is given by the Lottes-Flinn relationship

$$\frac{(\Delta P / \Delta Z)_{TP}}{(\Delta P / \Delta Z)_{SP}} = R = \left(\frac{1-\gamma}{1-\phi} \right)^2. \quad (\text{II.1.13})$$

A justification of this relationship will be given in Section II.3. Therefore, with the determination of ϕ , the bubbly flow regime can be analyzed and the distribution of the flow parameters in it can be determined.

C. Slug flow

When the amount of bubbles in the flow increases and the void fraction exceeds the value 0.3, the bubbles start to coalesce and form bullet-shaped Taylor bubbles, as shown in Fig. I.1. The flow pattern now changes and one observes slugs of liquid con-

taining a few small bubbles alternating with the long Taylor bubbles. The length of the latter can be appreciably large, sometimes up to 7 diameters of the pipe. Even at the bubble, the walls of the pipe are always wetted by a thin layer of liquid brine. The average velocity of the vapor phase in this case has been estimated by Dukler to be related to the average liquid velocity as follows:

$$\bar{V}_v = 1.2 \bar{V}_\ell + 0.35 \sqrt{gD} . \quad (\text{II.1.14})$$

The void fraction in this case is then given in terms of the superficial velocities for the vapor and liquid defined by the following relations:

$$V_v^s = \frac{4\dot{m} \cdot \gamma}{\pi D^2 \rho_v} , \quad (\text{II.1.15})$$

and

$$V_\ell^s = \frac{4\dot{m}(1-\gamma)}{\pi D^2 \rho} , \quad (\text{II.1.16})$$

where $\dot{m}$ is the mass-flow rate through the pipe. It is seen that the superficial velocity of each phase denotes the flow velocity that the same phase would have attained, if it was occupying the whole of the pipe cross-sectional area. Then, the void fraction for slug flow is given by the expression:

$$\phi = \frac{1.06}{1.2 \left(\frac{V_\ell^s}{V_v^s} + 1 \right) + \frac{0.25}{Fr_v}} , \quad (\text{II.1.17})$$

where Fr_v is the superficial Froude number for the vapor phase,

$$Fr_v = \frac{V_v^s}{\sqrt{gD}} \quad (II.1.18)$$

The quality γ of the fluid is calculated from the fact that the pipe is considered adiabatic, so that the total specific enthalpy of the fluid remains the same:

$$(h_1 + gz_1) - (h_2 + gz_2) = 0, \quad (II.1.19)$$

and

$$\gamma = \frac{h(T_b) - h_\ell(T) - gz}{h_{fg}(T)}, \quad (II.1.20)$$

where $h(T_b)$ is the enthalpy of the fluid at the well bottom and $h_\ell(T)$ the specific enthalpy of the liquid at the prevailing temperature. Here z is the elevation above the well bottom. It must be noted that γ is defined as the ratio of the mass flow rate of vapor to the total mass flow rate ($\gamma = \dot{m}_v / \dot{m}$) as in the steady flow applications (turbines) and not as in closed systems ($\gamma = m_v / m$). For this reason the term quality is used instead of the more usual "dryness fraction".

D. Froth (Churn) flow

When the velocity of the Taylor bubbles exceeds a certain limit (Section II.2), instabilities appear at the liquid film surrounding the bubble and the latter loses its regular shape and may break up. This characterizes the onset of a new flow regime which is called

froth (or churn) flow. The vapor pockets inside the liquid matrix assume irregular shapes and the sizes of the bubbles differ considerably. There is no regular geometry by which this flow field may be characterized; it is thought of as a transitory regime between slug and annular flow. In fact, in some cases this regime does not appear distinctly and there is a direct transition from slug to annular flow.

For the calculation purposes the void fraction for froth (churn) flow is taken as the average of the void fractions of no-slip and annular flow.

$$\phi_{FR} = \frac{1}{2} (\phi_{NS} + \phi_{AN}) , \quad (II.1.21)$$

where the no-slip void fraction is obtained from the case of equal vapor and liquid velocities from eqn. (II.1.1)

$$\phi_{NS} = \frac{1}{1 + \frac{1-\gamma}{\gamma} \frac{\rho_v}{\rho_\ell}} , \quad (II.1.22)$$

and the void fraction for annular flow is given by eqn. II.1.25.

E. Annular flow

When the amount of vapor in the pipe becomes relatively high (12-15%), the irregularly shaped vapor formations acquire much higher velocities than the surrounding liquid and start coalescing at the center of the pipe, thus forming a cylindrical path; the liquid phase is pushed away towards the walls of the pipe and moves in an annular configuration bounded by the pipe wall and the vapor cylinder. Some of the liquid may be entrained into the vapor matrix and move with it at the same velocity. The void fraction for this

flow regime can be analytically predicted if the entrainment factor F_e and the liquid annular thickness t are known. Dukler (1977) shows one way to calculate it:

$$\phi = \frac{\left(1 - \frac{4t}{D}\right) \left(\frac{V_v^s}{F_e V_\ell^s}\right)}{\left(1 + \frac{V_v^s}{F_e V_\ell^s}\right)} \quad (II.1.23)$$

Of course the estimation of F_e and t is difficult in any case, and for this reason another equation for the void fraction in annular flow is adopted. This results from the fact that the slip ratio is found to be

$$K = \left(\frac{\rho_\ell}{\rho_v}\right)^{0.5} \quad (II.1.24)$$

With this assumption, the void fraction for the annular flow is given by

$$\phi = \frac{1}{1 + \frac{1-\gamma}{\gamma} \left(\rho_v/\rho_\ell\right)^{0.5}} \quad (II.1.25)$$

Thus the void fraction can be calculated for every flow regime. This quantity and the quality of the fluid are the main parameters needed for the calculation of the pressure drop ratio R for two-phase flow and for the determination of the pressure distribution along the well.

II.2 Criteria for the transitions between flow patterns.

The transition from liquid flow to bubble flow occurs when the static pressure of the fluid becomes lower than the saturation pressure at the prevailing temperature. Since thermodynamic equilibrium is assumed in the pipe, the first bubbles will usually be formed at the level where the static and saturation pressures become equal

$$P = P_s(T) , \quad (II.2.1)$$

because supersaturation is not likely to occur in the presence of impurities. After this level the temperature of the fluid adjusts itself according to the static pressure, and we have the relationship:

$$T = T_s(P) . \quad (II.2.2)$$

Therefore, when the pressure distribution is known for the two-phase flow part of the well, its temperature can be deduced from the phase equilibrium relationship. (Kestin, 1968).

When the bubbly-flow regime is established and the pressure drops further, more liquid is evaporated and new bubbles are formed. The bubbles move with a velocity higher than the average velocity of the liquid and wakes are formed behind them. When the bubbles increase in number they start coalescing with each other and form bigger bubbles which eventually assume the shape of a Taylor bubble, and transition to the slug-flow regime takes

place. The transition does not occur abruptly, but over a sizable length of the pipe. For computational reasons, though, it is desirable to have a sharp mathematical criterion which characterizes the process, and this makes the transition look as if it takes place abruptly. The change of the regime is accompanied by a change of the equation for the calculation of void fractions and, of course, an abrupt transition is going to create a discontinuity in the function of the void fraction, which is physically unjustifiable. For this reason, the void-fraction function is statistically smoothed for the calculations so that all the discontinuities are removed. A similar procedure is adopted in all transitions between flow regimes.

The physical picture for the bubbly-to-slug transition is that the number of bubbles becomes high and they coalesce to form Taylor bubbles. Mathematically this can be described as follows: The average distance between the bubbles becomes very small and equal to the effective length of the wake behind the bubbles, and coalescence occurs immediately. The average distance between the spherical bubbles is obtained from statistical methods by considering all bubbles have the size favored by minimum total energy considerations (Bilicki 1978).

$$a = D \left[\frac{0.886}{\sqrt{\phi}} - 1 \right] . \quad (\text{II.2.3})$$

The effective length of the wake behind the bubble is given by

the following equation:

$$\ell = D \sqrt{0.3\pi} \left[\frac{0.4 \bar{V}_r}{8 \bar{V}_r \sqrt{0.023 \left(\frac{\bar{V}_r D}{\nu_\ell} \right)^{-0.25}}} \right]^{1.5}, \quad (\text{II.2.4})$$

(Bilicki, 1978, Bilicki et al 1980).

$\bar{V}_r$ is the average relative velocity of the vapor with respect to the liquid:

$$\bar{V}_r = \frac{V_v^s}{\phi} - \frac{V_\ell^s}{1-\phi}, \quad (\text{II.2.5})$$

and ν is the kinematic viscosity of liquid. The criterion for the transition is then

$$a \leq \ell. \quad (\text{II.2.6})$$

The first criterion for the slug-to-froth flow transition is similar to the above. There is a wake behind the Taylor bubble and this wake may become equal to the size of the slug. Then there is interference between the two neighboring bubbles and they will tend to meet and coalesce. This way, a vapor formation appears in the center of the pipe which will mark the beginning to annular flow transition. In this case (which is met at higher mass-flow rates in computations) the bubble's shape is not distorted or broken and froth flow is not established before annular flow. This criterion can be written as:

$$L_s \leq \ell, \quad (\text{II.2.7})$$

where L_s is the slug length given for the case when the bubble length equals about five diameters

$$L_s = 5D \frac{1-\phi}{\phi}, \quad (\text{II.2.8})$$

and the wake length is:

$$\ell = D 1.2 \left[\frac{0.05 \bar{V}_r (1-\phi)^{0.7}}{v_\ell^s \sqrt{0.023 \left[\frac{v_\ell^s D}{\nu_\ell} \right]^{-0.25}}} \right]^{1.5}, \quad (\text{II.2.9})$$

(Bilicki, 1978, Bilicki et al 1980).

The transition from slug to froth flow is characterized by instabilities in the liquid film surrounding the Taylor bubble and the breaking of this bubble. In some cases reverse flow is observed in the liquid film, and the liquid material runs downwards. The transition now takes place when the relative velocity of the bubbles, with respect to the liquid, becomes higher than the speed of surface waves formed in the liquid film (Bilicki 1978). This leads to the condition

$$\bar{V}_v - \bar{V}_B \geq \sqrt{\frac{\sigma}{\delta \rho_v}}, \quad (\text{II.2.10})$$

where σ is the surface tension, δ the film thickness and $\bar{V}_B$ the average velocity of the liquid film.

The liquid film thickness is given as:

$$\delta = 1.5 \sqrt{\frac{3 \nu_\ell \bar{V}_B}{g}}, \quad (\text{II.2.11})$$

and the average film velocity is

$$\bar{V}_B = \frac{V_v^S \left(1 - 2 \frac{\delta}{D}\right)^2 - \phi (V_\ell^S + V_v^S)}{4\phi \frac{\delta}{D} \left(1 - \frac{\delta}{D}\right)} . \quad (\text{II.2.12})$$

The last two equations can be solved simultaneously to yield the quantities δ and $\bar{V}_B$. When the film thickness is much smaller than the diameter ($D \gg \delta$), eqn. (II.2.12) can be shortened to

$$\bar{V}_B = \frac{V_v^S - \phi (V_\ell^S + V_v^S)}{4\phi \frac{\delta}{D}} , \quad (\text{II.2.13})$$

and this yields in combination with eqn. (II.2.11) the following expression for the film velocity:

$$\bar{V}_B = \left[\frac{D}{6} \cdot \frac{(V_v^S - \phi (V_\ell^S + V_v^S)) \sqrt{g}}{\phi \sqrt{3\nu_\ell}} \right]^{1/1.5} . \quad (\text{II.2.14})$$

The above equation may be used for the transition criterion to froth flow.

The last transition which is considered is that between froth and annular flow. Froth flow has irregular shapes of vapor formations and is unstable structurally, especially at high mass flow rates. Under these conditions a direct transition between slug and annular flow takes place (Dukler 1977). At low mass flow rates the slugs break up and the froth regime is established in the well, only to give up its place to annular flow. The reason

for this transition is the onset of instabilities which develop in the flow and induce the coalescence of the vapor formations. The transition point for this pattern is located at a cross-section where the average vapor velocity becomes equal to a "Froude number velocity" V_F defined as follows (Bilicki 1978, Bilicki et al 1980):

$$V_F = \phi \sqrt{\frac{gD(\rho_\ell - \rho_v)}{\rho_v}} \quad (\text{II.2.15})$$

and the criterion for froth flow is:

$$\bar{V}_v \geq V_F \quad (\text{II.2.16})$$

Having thus established all the criteria for the transitions from one regime to another, we can construct a map of the flow conditions showing the regions of the various flow patterns and a given well diameter; such a map is shown in Fig. II.1 for a well of 0.178 m of diameter and 4 molal salinity.

II.3 The calculation of pressure losses.

In vertical-pipe flow there are three types of pressure losses to be considered: a) gravitational, b) frictional, and c) accelerational. The gravitational pressure losses arise from the fact that work must be done on the fluid to lift it against the gravity forces. The pressure loss per unit length in this case is

Fig. II.1 Transition maps

$$\left(\frac{\Delta P}{\Delta z}\right)_{GR} = \bar{\rho} g , \quad (II.3.1)$$

where $\bar{\rho}$ is the average density given by the equation,

$$\bar{\rho} = (1-\phi)\rho_\ell + \rho_v\phi . \quad (II.3.2)$$

The frictional pressure losses are due to shear stresses developed on the wall of the pipe because of the liquid viscosity. This shear stress is the result of inter-atomic forces between the liquid particles and the wall; it accounts for the velocity gradient and its value at the wall is given by Newton's equation,

$$\tau_w = \mu(\partial u / \partial y)_w , \quad (II.3.3)$$

where y is the distance from the wall and u is the velocity of the fluid.

This shear stress has been investigated for the case of liquid flow and reliable expressions for its value have been established. One such expression is given by the Blasius equation for the pressure drop in the liquid flow:

$$(\Delta P / \Delta z)_{SP} = 0.158 \rho \bar{V}^2 (Re)^{-0.25} = (1/2)f\rho\bar{V}^2 . \quad (II.3.4)$$

A theory (Lottes-Flinn) was developed to relate this pressure loss to the pressure loss in a two-phase flow regime. This theory considers two-phase flow in a pipe and assumes that the wall is

always wet; this is true in the flow regimes encountered in geothermal wells where a liquid film always wets the pipe wall. In addition, the theory assumes that the phase changes and velocity gradients that exist in the core of the pipe do not affect the flow conditions at the boundary layer, and hence the friction factor f . In the case of two-phase flow the average velocity of the liquid phase is:

$$\bar{V}_\ell = \frac{\dot{m}(1-\gamma)}{A(1-\phi)\rho_\ell}, \quad (\text{II.3.5})$$

where A is the area of the pipe. According to the above, the shear stress at the wall is:

$$\tau_w = \frac{1}{2} f \bar{V}_\ell^2 \rho_\ell \quad (\text{II.3.6})$$

and the two-phase pressure loss becomes

$$\left(\frac{\Delta P}{\Delta z}\right)_{\text{TP}} = f \frac{\dot{m}^2}{2A^2 \rho_\ell} \left(\frac{1-\gamma}{1-\phi}\right)^2. \quad (\text{II.3.7})$$

A comparison of the last equation with equation (II.3.4) will yield the following relation between the frictional pressure drop of two pipes discharging the same amount of mass per second, the one carrying liquid and the other a two-phase mixture

$$\frac{(\Delta P/\Delta z)_{\text{TP}}}{(\Delta P/\Delta z)_{\text{SP}}} = \frac{(1-\gamma)^2}{(1-\phi)^2} = R. \quad (\text{II.3.8})$$

Therefore, a knowledge of the void fraction and an appropriate choice of the pressure loss correlation in single-phase flow (taken as the Blasius equation in this case), will yield the two-phase flow pressure loss. It should be noted that frictional losses in geothermal wells constitute a very small fraction (2-5%) of the total pressure losses, until very high void fractions (close to 90%) are reached.

The accelerational pressure losses are due to the fact that the kinetic energy of the fluid increases as the fluid accelerates. The acceleration is a result of the vapor formation inside the pipe, which has a lower density than the liquid. The average cross-sectional area for liquid is considerably reduced and its velocity increases. The average liquid velocity is then equal to

$$\bar{V}_L = \frac{\dot{m}(1-\gamma)}{A(1-\phi)\rho_L}, \quad (\text{II.3.9})$$

and the average velocity of the vapor is

$$\bar{V}_V = \frac{\dot{m}\gamma}{A\phi\rho_V}. \quad (\text{II.3.10})$$

The average change of kinetic energy and, hence, of pressure loss is given by the following equation:

$$\left(\frac{\Delta P}{\Delta z}\right)_{AC} = \frac{1}{2} \Delta \left[\bar{\rho} \left(\bar{V}_L^2 + \bar{V}_V^2 \right) \right]. \quad (\text{II.3.11})$$

Thus the acceleration pressure loss between two levels a distance Δz apart can be calculated from this equation.

The total pressure gradient is the sum of the expressions in eqns. (II.3.1), (II.3.7) and (II.3.11) and is given as follows:

$$\left(\frac{\Delta P}{\Delta z}\right)_t = \left(\frac{\Delta P}{\Delta z}\right)_{GR} + \left(\frac{\Delta P}{\Delta z}\right)_{TP} + \left(\frac{\Delta P}{\Delta z}\right)_{AC} \quad (II.3.12)$$

or

$$\begin{aligned} \left(\frac{\Delta P}{\Delta z}\right)_t = & \bar{\rho}g + \left(\frac{1-\gamma}{1-\phi}\right)^2 0.158 \rho_\ell \left(\frac{\dot{m}}{A}\right)^2 (Re)^{-0.25} + \\ & + \frac{1}{2} \Delta \left[\bar{\rho} \left(\bar{v}_\ell^2 + \bar{v}_v^2 \right) \right] . \end{aligned} \quad (II.3.13)$$

From the three components of the total pressure gradient, the gravitational is the highest and the others become important when high void fractions are reached (frictional), or when critical flow is approached.

II.4 Critical flow in geothermal wells.

A reduction of the wellhead pressure is observed to yield a higher mass-flow rate, in general. In all the characteristic curves of geothermal wells ($\dot{m}$ vs. P_{wh}) the derivative ($d\dot{m}/dP$) is negative until it reaches a limit and becomes close to zero. Physically, this situation means that a further reduction of the wellhead pressure is not going to yield higher mass-flow rates. This regime is described as choked or critical flow. In one-phase flow choking and sonic conditions are synonymous. This is not the case with two-phase flow where the sonic velocity is not linked to the critical flow.

The momentum equation of the flow in differential form will help to understand the mechanism for the onset of critical conditions:

$$\rho u du + \rho g dz + \frac{1}{2} f \rho u^2 dz + dp = 0 , \quad (II.4.1)$$

where the terms correspond to acceleration, gravity head, friction and static pressure drop and all the quantities are space averages. The evaporating water increases enormously the mean specific volume of the mixture (for example at 1.5 atm the ratio of vapor to liquid specific volume is about 1000) and this increases the mean velocity of the gaseous phase a great deal. Consequently, the acceleration term in eqn. (II.4.1) dominates the others and the static pressure drop is only sufficient to balance this term; thus, the additional pressure loss in the

well is spent in accelerating the existing fluid and not in drawing any more of it upwards. At this point, critical conditions are established in the well and the flow is choked as characterized by the flat portion of the curve in the mass-flow rate versus pressure diagram.

The momentum equation can be written for the two phases as follows:

$$d(G_{\ell}u_{\ell}) + (1-\phi) \rho_{\ell}gdz + F_{\ell}dz + (1-\phi)dP = 0 , \quad (\text{II.4.2})$$

for the liquid, and

$$d(G_vu_v) + \phi\rho_vgdz + F_vdz + \phi dP = 0 , \quad (\text{II.4.3})$$

for the vapor phase, where F_{ℓ} and F_v are the pressure gradients due to the friction, symbolized as such for simplicity.

If the total mass flux is G , then:

$$G_{\ell} = G(1-\gamma) = (1-\phi)\rho_{\ell}u_{\ell} , \quad (\text{II.4.4})$$

$$G_v = G\gamma = \phi\rho_vu_v , \quad (\text{II.4.5})$$

where the velocities and densities are spatially averaged for the two phases. Substituting the above expressions in (II.4.2) and (II.4.3) and adding the resulting equations it follows:

$$d[G(1-\gamma)u_{\ell} + G\gamma u_v] + [\phi\rho_v + (1-\phi)\rho_{\ell}]gdz + (F_{\ell} + F_v)dz + dP = 0. \quad (\text{II.4.6})$$

Given that at the onset of the critical flow G is insensitive to P , eqn. (II.4.6) leads to:

$$G = \frac{-1}{\left[\frac{\partial}{\partial P} \left[(1-\gamma) u_{\ell} + \gamma u_v \right] \right]_z} . \quad (\text{II.4.7})$$

This is the expression for the critical mass flux. Given that it is difficult to calculate the velocities u_{ℓ} and u_v , this form of the equation is of no help for the computation of this flux. For this reason, the denominator of the fraction will be transformed to another form which is easy to evaluate.

The ratio of the two velocities is the slip ratio K :

$$u_v = K u_{\ell} . \quad (\text{II.4.8})$$

This will give for the void fraction the following expression:

$$\phi = 1 / \left[1 + K(1-\gamma) v_{\ell} / \gamma v_v \right] , \quad (\text{II.4.9})$$

and for the mass flux:

$$G = \frac{K u_{\ell}}{K(1-\gamma) v_{\ell} + \gamma v_v} . \quad (\text{II.4.10})$$

The last equation gives for the liquid velocity:

$$u_{\ell} = \frac{G}{K} \left[K(1-\gamma) v_{\ell} + \gamma v_v \right] . \quad (\text{II.4.11})$$

When (II.4.11) and (II.4.8) are substituted in (II.4.7) we obtain another equation for the critical-mass flux in terms of specific volumes, slip ratio and quality:

$$G^2 = \frac{-1}{\left\{ \frac{\partial}{\partial P} \left[\frac{1}{K} \left[K(1-\gamma) v_{\ell} + \gamma v_v \right] \left[1-\gamma + K\gamma \right] \right] \right\}_z} . \quad (\text{II.4.12})$$

It can be assumed that the change of the specific volumes of the liquid and vapor phase with respect to pressure is negligible and in this case only the derivatives $\left(\frac{\partial \gamma}{\partial P}\right)_z$ and $\left(\frac{\partial K}{\partial P}\right)_z$ have to be evaluated. The first will be calculated from the exchange of mass between the two phases and the thermodynamic equilibrium conditions, while the second will be calculated from the momentum transfer. For the latter, the flow pattern that prevails in two-phase flow must be taken into account. The mass transfer equation for the evaporation of the liquid phase yields:

$$\left(\frac{\partial \gamma}{\partial T}\right)_z = - \frac{C_p}{h_{fg}} \quad (\text{II.4.13})$$

and from the chain rule:

$$\left(\frac{\partial \gamma}{\partial P}\right)_z = - \frac{C_p}{h_{fg}} \frac{dT}{dP} . \quad (\text{II.4.14})$$

From the thermodynamic equilibrium between the phases we have the Clausius-Clapeyron equation:

$$\frac{dT}{dP} = \frac{(v_v - v_\ell)T}{h_{fg}} , \quad (\text{II.4.15})$$

and

$$\left(\frac{\partial \gamma}{\partial P}\right)_z = - \frac{C_p (v_v - v_\ell) T}{h_{fg}^2} . \quad (\text{II.4.16})$$

Observations in geothermal wells show that critical conditions are established at the annular pattern, when the mass of the vapor phase increases substantially and "pushes" the liquid

phase to the walls of the pipe. For that matter, the annular-pattern model will be assumed for the calculation of momentum transfer. Then, no momentum exchange occurs between the phases and the eqns. (II.4.2) and (II.4.3) are written as follows:

$$dP = - \rho_{\ell} u_{\ell} du_{\ell} ; \quad (II.4.17)$$

$$dP = - \rho_v u_v du_v = - \rho_v Ku_v [u_{\ell} dK + K du_{\ell}] . \quad (II.4.18)$$

The last two equations yield:

$$\left(\frac{\partial K}{\partial P} \right)_z = \frac{Ku_{\ell} - u_v/K}{u_{\ell}^2} . \quad (II.4.19)$$

Making use of the eqn. (II.1.24) for the slip ratio in annular flow it follows that:

$$\left(\frac{\partial K}{\partial P} \right)_z = 0 . \quad (II.4.20)$$

Taking into consideration eqns. (II.4.16) and (II.4.20), eqn. (II.4.12) becomes:

$$G^2 = \frac{-K^2}{\beta} , \quad (II.4.21)$$

where

$$\begin{aligned} \beta &= \frac{-C_p' (v_v - v_{\ell}) T [(v_{\ell} - Kv_v) K(K\gamma - 1 - \gamma) + K(K-1) (K(1-\gamma)v_{\ell} + \gamma v_v)]}{h_{fg}^2} \\ &= \frac{1}{K^2} \cdot \frac{-v_{\ell} d\gamma/dP}{2 [(1-\gamma) + K\gamma] (K-1)} \end{aligned}$$

and

$$K = (v_v/v_{\ell})^{1/2} .$$

For the case of critical flow occurring at another flow regime with negligible momentum exchange the above equations are modified to:

$$G^2 = - \frac{K^2 - \alpha}{\beta}, \quad (\text{II.4.22})$$

where

$$\alpha = K^2 \frac{\left[K v_{\ell} - v_{v/K} \right]}{(1-\gamma) K u_{\ell} + \gamma u_v}^2 \left[(1-\gamma) u_{\ell} K (k\gamma - 1 - \gamma) + \gamma K (K u_{\ell} (1-\gamma) + \gamma u_v) - (K\gamma + 1 - \gamma) \left[K(1-\gamma) u_{\ell} + \gamma u_v \right] \right]$$

Equation (II.4.21) was used to evaluate the critical mass flow rate for several equilibrium states of the fluid modeled as pure water. The results are shown in Fig. II.2 as the solid line; some available experimental data are displayed together with two other curves representing two different theoretical predictions which appear in the literature (Fauske, Moody). It is evident that there is good agreement between experimental results and eqn. (II.4.21) despite the simplifying assumptions that were used for its derivation. The equation has the additional advantage of being presented in a simple form, a very helpful fact for the computations.

Fig. II.2 Critical mass fluxes

II.5 Heat transfer in geothermal wells.

The subject of heat transfer in one-phase flow has been studied for a long time, and reliable equations have been derived for it. The heat flux due to convection in a pipe is:

$$g = h(T_B - T_W) , \quad (II.5.1)$$

where h is the heat transfer coefficient (Eckert and Drake, 1972), T_B the bulk fluid temperature and T_W the wall temperature. The heat transfer coefficient depends on the properties of the fluid and the flow type. Reliable correlations for this quantity can be found in the literature for all relevant conditions.

For two-phase flow, various theories of heat transfer have been formulated. Rohsenow (1964) takes the two-phase flow heat transfer coefficient to be the sum of two terms, the convective (which is equal to the one-phase flow coefficient) and the pool boiling heat-transfer coefficient:

$$h_{TP} = h_c + h_{PB} . \quad (II.5.2)$$

Kutateladze (1961), on the other hand, takes this relationship to be like the one for the hypotenuse in an orthogonal triangle:

$$h_{TP}^2 = h_c^2 + h_{PB}^2 . \quad (II.5.3)$$

Mikielewicz (1974) proposes yet another form, namely

$$h_{TP}^2 = h_c^2 R^{0.8} + h_{PB}^2 , \quad (II.5.4)$$

where R is the pressure drop coefficient in eqn. (II.3.8) and Bilicki (1979) introduced the following improvement in the above formula:

$$h_{TP}^2 = h_c^2 R^{0.8} + A h_{PB}^2, \quad (II.5.5)$$

where

$$A = 0.2 \left(\frac{q}{h_{fg} \rho_v V} \right)^{0.8} \cdot \left(\frac{\rho_l V D}{\mu_l} \right)^{0.75} \cdot \left(\frac{\rho_v h_{fg} D}{q \sqrt{\frac{\sigma}{g(\Delta \rho)}}} \right)^{0.1}.$$

The pool-boiling heat-transfer coefficient is given by the so-called Stefan's correlation

$$\alpha_{PB} = (2 + 0.03 T_s) \cdot q^{0.666}. \quad (II.5.6)$$

A good presentation of these theories on two-phase flow heat transfer and boiling is given in J. Madejski's (1973) treatise on the subject.

The above theories have been developed for the case of boiling at the wall when the two phases coexist there and influence together the heat transfer process. This situation does not agree with the model of the geothermal well, where the walls of the pipe are always wetted by the liquid. The liquid phase is always in contact with the wall and its characteristics determine the heat transfer process. For the case of the turbulent flow in a liquid, the heat-transfer coefficient is given by the following equation:

$$h = \frac{k}{D} 0.023 \text{ Re}^{0.8} \text{ Pr}^{0.4}, \quad (\text{II.5.7})$$

where k is the conductivity of water.

When the geothermal well is drilled, the steel casing is protected by an annulus of concrete which acts as an insulating material of thickness 5-12 cm. This presents a resistance to the heat flow from the well and lowers significantly the heat losses. In this case the overall heat-transfer coefficient h_t is given by the equation:

$$\frac{1}{h_t} = \frac{1}{h} + \frac{r_1}{k_1 \ln \frac{r_2}{r_1}} + \frac{r_1}{k_2 \ln \frac{r_3}{r_2}} + R_t, \quad (\text{II.5.8})$$

where r_1 is the inner radius of the pipe, r_2 outer radius, r_3 the outer radius plus the concrete thickness, k_1 the conductivity of the casing, k_2 the conductivity of the concrete and R_t the total resistance to heat flow. The heat flux per unit area of the pipe is then

$$q = h_t (T_B - T_w), \quad (\text{II.5.1})$$

where T_B is the bulk temperature of the pipe (uniform temperature when equilibrium is assumed) and T_w is the temperature of the surrounding stratum. The latter can be determined experimentally from temperature measurements, when the well is at a standstill. Using the above scheme a qualitative estimation of the heat losses for a typical well was made.

As such was taken the case of the IID-1 well in Imperial Irrigation District. The surrounding stratum there has the temperature profile shown in fig. II.3. At the wellbottom the temperature is 313°C and gradually decreases, initially with a slope 0.022°C/m for the first 620m and then with 0.3°C/m . The heat losses were calculated for an assumed mass-flow rate of 60 kg/s and the well height and diameter were 1520m and 0.356m (14"), respectively. The thickness of the concrete insulation was 5 cm and no other heat resistance was assumed; the maximum possible amount of heat loss was thus calculated. The total heat losses amounted to 114 kJ/s. Given that the well carries 60 kg/s of fluid this value corresponds to 1.9 kJ/kg enthalpy loss. This number is very small compared to the specific exergy of the fluid (~ 360 kJ/kg) and hence the heat losses may be neglected, and the well can be considered adiabatic. This result is justified by experimental observation of the state of the two-phase mixture at the wellhead. [Ellis, 1977]

The calculations reveal that the major part of the heat resistance is due to the insulation of the pipe with concrete; even in the absence of concrete, the surrounding debris can play this insulating role. Therefore, it can be concluded that geothermal wells may be assumed to operate adiabatically under normal conditions.

Fig. II.3 Temperature profile for IID-1 well

When the mass-flow rate is reduced substantially the heat losses become more important. When the flow-rate becomes very small, the fluid is cooled and only liquid flows through the well.

II.6 The effect of salts and gases on the brine properties.

Geothermal brines are aqueous solutions of various salts and gases. Three categories of salts are encountered: chlorides (NaCl , KCl , CaCl_2), carbonates (Na_2CO_3 , NaHCO_3 , K_2CO_3 , KHCO_3) and sulphates (Na_2SO_4 , K_2SO_4), the most abundant of the salts being NaCl . The main constituent of the gases is CO_2 and to a lesser extent H_2S , NH_3 , N_2 and CH_4 . The composition of salts and gases is different in different wells and varies with time at the same well.

In the early stages of well flow CO_2 is liberated from the mixture causing a rise of the pH of the solution; this upsets the chemical equilibrium of the species and particles of CaCO_3 are separated and deposited on the walls of the pipe, causing "clogging". Such wells must be cleaned and the scale removed.

The thermodynamic properties of the solutions are modified according to the composition of brines. An accurate determination of these properties would have involved the calculation of the effect of all the salts and gases, a work that is extremely complicated given the numerous ionic interactions and

chemical equilibria. For this reason, the equivalent NaCl content is sometimes used to obtain a reasonably good approximation to the properties of brines. This equivalent NaCl content is the amount of NaCl in the solution that will give the same effect on the properties as the amount of all the salts combined. This approximation is supported by the fact that the major constituent of salts are chlorides (more than 95%) and the effect of all of them on the properties is similar to the effect of NaCl. Given that the latter is the most abundant in brines, the real NaCl composition is not very much different from the equivalent NaCl composition. The effect of the gases on the properties of brines can be neglected since their fraction is never more than 0.5% by weight.

The correlations for the enthalpy and entropy of water and steam are derived from the Keenan et al steam tables. For that purpose the existing equations of the tables were not used because of their complexity. Instead the data are correlated as functions of temperature and presented here in the form of polynomials. The least-squares method was used for the calculation of the coefficients. The accuracy of the correlations is $\pm 1\%$.

For the purpose of computations the brines will be modeled as solutions of NaCl in water, and hence their properties identified with those of NaCl solutions.

In the case of the model described above the properties of interest for the determination of the flow in geothermal wells are the density, the viscosity, the enthalpy, the entropy and the saturation pressure at a given temperature. Given the structure of the developed model it is desirable to obtain these properties as functions of temperature and molality.

It is imperative for the economics of the calculations that the formulation of the property functions be simple. However, the ability of the simple functions to represent accurately the data, is limited and one must make a trade-off between simplicity and accuracy. For that reason, in this study the properties of NaCl solutions were not taken directly from known correlations (Haas, Silvester and Pitzer). A simpler form of the functions was adopted instead while keeping the accuracy within acceptable limits (2%). These functions for the properties of salt solutions are given as follows:

a) Density.

For the determination of the density of NaCl solutions the correlation developed by Haas (1971) was adopted, being both accurate (claimed accuracy 0.5% up to 300°C) and simple. The density is given by the following expression:

$$\frac{\rho}{\text{kg/m}^3} = \frac{1000 + m M_2}{1000 v_o + m \phi} \quad (\text{II.6.1})$$

where m is the molality, M_2 the apparent molecular weight of NaCl (58.44 kg/kmol) and v_o the specific volume of pure water. The auxiliary function is defined as

$$\phi = \phi^* + k \sqrt{m}, \quad (\text{II.6.2})$$

with

$$\phi^* = C_o + C_1 v_o + C_2 v_o^2, \quad (\text{II.6.3})$$

$$k = \frac{(C_3 + C_4 v_o) v_c^2}{(v_c - v_o)^2}, \quad (\text{II.6.4})$$

$$v_o = \frac{v_c + C_5 \theta^{1/3} + C_6 \theta + C_7 \theta^4}{1 + C_8 \theta^{1/3} + C_9 \theta}. \quad (\text{II.6.5})$$

The symbol v_c denotes the critical specific volume of water (3.1975 dm³/kg), θ is the difference of the saturation temperature of brine from the critical water temperature and the constants C_i are given below:

$C_o = -167.219$	$C_5 = +0.315154$
$C_1 = +448.55$	$C_6 = -1.203374 \times 10^{-3}$
$C_2 = -261.07$	$C_7 = +7.48908 \times 10^{-13}$
$C_3 = -13.644$	$C_8 = +0.1342489$
$C_4 = +13.97$	$C_9 = +3.946263 \times 10^{-3}$

b) Enthalpy.

The enthalpy of the liquid solution and its vapor,

as well as the latent heat of vaporization, are needed for the calculations of flashing and the evaluation of the vapor quality. Given that the solute is not volatile, its partial pressure must be very low in the solution. Therefore, the vapor can be assumed to be pure steam but slightly superheated, since the saturation temperature for a given pressure is elevated (see section II.6.e). The enthalpy of the vapor is then independent of the salt content and an accurate correlation for this is (Keenan et al, 1968)

$$h(T)/(kJ/kg) = -0.81257(T/^\circ C)^2 + 3.65228(T/^\circ C) + 2388. \quad (II.6.6)$$

This equation is accurate to within 0.5% up to 300°C.

For the enthalpy of the liquid solution a correction to the ideal solution enthalpy was adopted as follows:

$$h'(T,m)/(kJ/kg) = \gamma_1 h'_1(T) + \gamma_2 h'_2(T) + m \phi(T,m) \quad (II.6.7)$$

where $h'(T,m)$ is the enthalpy of solution at temperature $T^\circ C$ and molality m , γ the mass fraction, ϕ the correction factor, and the subscripts 1 and 2 refer to pure water and pure NaCl.

The correction function $\phi(m,T)$ is given by the double power series:

$$\phi(m,T)/(kJ/kg) = \sum_{i=0}^3 \sum_{j=0}^2 a_{ij} \left(\frac{T}{T^*} \right)^i \left(\frac{m}{m^*} \right)^j \frac{4.184}{1000 + mM_2}, \quad (II.6.8)$$

$$T^* = 1^\circ C, \quad m^* = 1 \text{ molal}$$

where the coefficients a_{ij} are given as follows,

$$\begin{array}{lll} a_{00}=+9633.6 & a_{01}=+4080.0 & a_{02}=+286.49 \\ a_{10}=+166.58 & a_{11}=+68.577 & a_{12}=-4.6865 \\ a_{20}=-0.90963 & a_{21}=-0.36524 & a_{22}=+0.249667 \times 10^{-1} \\ a_{30}=+0.1796 \times 10^{-2} & a_{31}=+0.71924 \times 10^{-3} & a_{32}=-0.4900 \times 10^{-4} \end{array}$$

The reference point for the calculation of enthalpies was taken as 0°C and 0 molality. Enthalpy differences were compared with data from other correlations (Silvester and Pitzer, Haas) and the agreement is within 1.5% up to 300°C and $m=6$.

The latent heat of vaporization h_{fg} also enters the computations. This is given by the equation:

$$h_{fg}(T,m) = h''(T) - h'(T,m) .$$

Whenever the specific heat capacity of the solution is needed, it may be obtained from the equations:

$$C'_p = \left(\frac{\partial [h'(T,m)]}{\partial T} \right)_m , \quad (\text{II.6.9})$$

$$C''_p = \left(\frac{\partial [h''(T)]}{\partial T} \right)_m . \quad (\text{II.6.10})$$

c) Entropy.

All known geothermal waters are low molality solutions (up to 4 molal) and, hence, may be approximated

as ideal solutions, with their entropy being:

$$S(T, P, m) = n_1 s_1(T, P) + n_2 s_2(T, P) - n_2 R \ln \frac{n_2}{n_1} \quad (\text{II.6.11})$$

The first two terms are the entropy of the pure water and NaCl respectively and the last term the entropy of mixing.

The entropy of water is taken from an equation fitted to the Keenan et al (1968) tables:

$$s_1(T, P_s) = (0.299 \times 10^{-7} T^3 - 0.2306 \times 10^{-4} T^2 + 0.15049 \times 10^{-1} T + 0.1894 \times 10^{-2}) \cdot 18 \text{ kJ/kmol} \cdot \text{K} \quad (\text{II.6.12})$$

where T is measured in °C. The entropy of the pure salt was evaluated from specific heat data and was fitted to the following equation:

$$s_2(T)/\text{kJ/kg} \cdot \text{K} = 4.184 (-0.1254 \times 10^{-2} T_o^2 + 1.70578 T_o - 304.63 \ln T_o + 1336.6) \quad (\text{II.6.13})$$

and T_o is in kelvin. If the molality of the solution is m, then the first equation becomes for the specific entropy of the solution:

$$s(T, P_s, m)/(\text{kJ/kg} \cdot \text{K}) = \left\{ 55.56 s_1(T, P_s) + m[s_2(T_o) + R \ln \frac{55.56}{m}] \right\} \frac{1}{1000 + m \cdot 58.44} \quad (\text{II.6.14})$$

d) Viscosity.

A sufficiently simple and accurate correlation for the viscosity of the liquid NaCl solutions are given by Kestin and Khalifa for the viscosity of NaCl solutions as follows:

$$\log \left[\mu_w(T) / \mu_w(20) \right] = (20-T) [1.2378 - 1.303 \times 10^{-3}(20-T) + 3.06 \times 10^{-6}(20-T)^2 + 2.55 \times 10^{-8}(20-T)^3] / (96+T), \quad (\text{II.6.15})$$

with $\mu_w(20) = 1002 \text{ } \mu\text{Pas}$ and T is in $^{\circ}\text{C}$. The viscosity of a solution of molality m is then:

$$\log \left[\mu_w(T, m) / \mu_w(T) \right] = A(m) + B(m) \log \left[\mu_w(T) / \mu_w(20) \right], \quad (\text{II.6.16})$$

with $A(m) = 0.3224 \times 10^{-1}m + 0.3624 \times 10^{-2}m^2 - 0.1879 \times 10^{-3}m^3$,

$$B(m) = 0.3961 \times 10^{-1}m + 0.102 \times 10^{-1}m^2 - 0.702 \times 10^{-3}m^3.$$

The accuracy of the correlation is within 1.5% up to 300°C .

e) The elevation of the saturation temperature.

When the saturation pressure of the salt solution is given and its molality is known, the saturation temperature of the solution is higher than that of pure water by an amount:

$$\Delta T = \frac{1.8 \cdot R T_o^2}{h_{fg}} \cdot \frac{n_2}{n_1}, \quad (\text{II.6.17})$$

where R is the universal gas constant, T_0 is measured in kelvins, and n_2 is the moles of salt dissolved into n_1 moles of water. This is a modification of the well-known Raoult's law and the empirical constant 1.8 is due to the fact that the salt dissociates in the solution.

Conversely, if the saturation temperature T_0 is known, the saturation pressure elevation is given as follows (Henry's law):

$$\Delta P = \frac{1.8 RT_0}{v''_0 - v'_0} \frac{n_2}{n_1}, \quad (\text{II.6.18})$$

where v''_0 and v'_0 are the specific volumes of vapor and liquid pure water.

The elevation of the boiling point for a solution can be represented on a P, T phase diagram with separate lines corresponding to the various molalities and the quantities ΔT and ΔP can be read from this diagram.

II.7 The computer program.

The purpose of establishing a model for the geothermal well flow is to determine sufficiently accurate values of the properties which characterize the state of the geothermal fluid at the wellhead. When these properties are known, the specific exergy of the fluid can be computed and hence, the power, a given installation is capable to deliver. The reason for requiring accurate data for the wellhead conditions is not merely to predict the efficiency or utilization factor of the plant, but to design it. Defensive design is more expensive and the resulting installations are seldom efficient. Therefore, an accurate knowledge of the geothermal well flow and the quantitative prediction of the wellhead conditions are needed to secure a reliable plant design.

The geothermal well flow model based on the flow regimes can be used for the prediction of wellhead conditions; furthermore, available data on geothermal wells can be used as a test for the model. For that reason a computer program has been developed in fortran IV language to facilitate the calculations, predictions and tests. A flow chart of the program is given in Fig. II-4 where the main steps of the program are presented in a diagrammatic form. The whole program is given in Appendix 1 with sufficient documentation. The structure of the program is such

Fig. II.4 Flow chart of the geothermal well program

as to allow variation of all parameters which characterize the well flow: the bottomhole conditions, the structure of the surrounding stratum, the diameter, the drawdown factor and all other variables can be changed according to the characteristics of the well. Provision has been taken to allow for the variation of these parameters by including a separate loop in the program.

The program employs the step-by-step method for the calculation of the fluid properties in the geothermal well. At each step the thermodynamic properties of the fluid are estimated, then the pertinent flow regime is determined and this leads to the calculation of the void fraction and eventually of the pressure gradient. The step of the calculation is taken in terms of pressure rather than the (seemingly more appropriate) elevation. This choice is made for the following reasons: a) The properties of the fluid are given directly as a function of pressure; b) It assures more uniform variation of the fluid properties in the well for a given number of calculation steps; c) It minimizes the number of steps (programming efficiency) for a given accuracy. Thus, a step of pressure is defined in the original data set and the elevation at the i -th station is computed from the calculated pressure gradient (dP/dz) as follows:

$$\Delta z = \frac{\Delta P}{(dP/dz)_i} . \quad (11.7.1)$$

The pressure and temperature of the fluid are computed at the next step (i+1) and two tests are performed: a) To determine whether the flow is critical or not, and b) to determine if the wellhead has been reached. If the answer in both tests is negative, the calculations continue for the next step. Otherwise, the calculations stop and the output contains the appropriate message for the stopping reason.

An explanation of the symbols used in the program is provided in Appendix I, where all the statements and comments are given in capital letters and other explanations in low case letters bounded by asterisks. This Appendix is itself a computer output of the program file in script language.

II.8 Influence of varying the well characteristics.

The establishment of a method to quantitatively predict the state of the geothermal fluid at the wellhead is helpful for the determination of the effect of various parameters in the well flow, and the exergy available to the geothermal plant. Thus, the computer program, as developed before, will be used to predict the influence of various parameters, such as the bottomhole temperature and pressure, on the state of the fluid at the wellhead and especially on the power that could be extracted from it. The study of the effect of these parameters is essential in design

problems related to rock fracturing, downhole pumping and stepping of the well diameter.

The main factors that influence the exergy of the fluid at the wellhead are: the bottomhole pressure and temperature, the drawdown factor, the salinity of the fluid, the magnitude of heat loss from the well, the diameter and the depth of the well. Therefore, the rate of exergy of the fluid (available power) at the wellhead can be written as follows:

$$\dot{W} = \dot{W} (P_b, T_b, k, m, \dot{Q}, D, L) . \quad (\text{II.8.1})$$

The effect of each one of these parameters will be examined separately. For this purpose one of the parameters is varied at a time while the others are kept constant. All the results are shown in rate of exergy versus mass flow rate diagrams. The curves show an abrupt end which corresponds to choking in the well at the given conditions. The key quantity in all figures is the specific exergy, e , of the fluid at the wellhead which is related to the quantities plotted by the equation

$$\dot{W} = \dot{m} e \quad (\text{II.8.2})$$

and is itself a function of the wellhead temperature, pressure and quality.

II.8.a. The influence of the bottomhole pressure.

The effect of increasing the bottomhole pressure from 14 to 16 and 18 MPa is depicted in Fig. II.5. The other parameters of the well are being kept at the following values:

$$T_b = 300^{\circ}\text{C}$$

$$k = 0.02 \text{ MPa s/kg}$$

$$L = 1600 \text{ m}$$

$$m = 1 \text{ molal}$$

$$D = 0.2 \text{ m}$$

$$\dot{Q} = 0 \text{ kW}$$

It is seen that a modest increase of the bottomhole pressure by 30% results in the doubling of the rate of exergy at the wellhead. This effect can be achieved by the installation of a downhole pump, a subject which will be examined in Section II.9. It can also be observed that the specific exergy at the wellhead increases with P_b due to the lower frictional losses resulting from the sustainment of the single phase flow regime.

II.8.b The influence of the bottomhole temperature.

The effect on exergy from varying the bottomhole temperature in the wells is examined. The geothermal fluid is taken to be at 200, 250 and 300°C in the reservoir

Fig. II.5 Effect of the bottomhole pressure

and its rate of exergy at the wellhead is depicted in Fig. II.6. The other parameters of the well are being kept constant at the following values:

$$P_b = 16 \text{ MPa}$$

$$k = 0.02 \text{ MPa}\cdot\text{s/kg}$$

$$L = 1600 \text{ m}$$

$$m = 1 \text{ molal}$$

$$D = 0.2 \text{ m}$$

$$\dot{Q} = 0.0 \text{ kW}$$

It is seen that the specific exergy of the fluid increases substantially with T_b , due to the fact that the fluid leaves the well at a higher temperature. The mass flow rate and the rate of exergy also increase substantially, the latter increasing by 500% with 100°C difference in T_b .

II.8.c The influence of the drawdown factor.

The drawdown factor, which is the effect of the permeability of the rock surrounding the reservoir, takes the values 0, 0.02 and 0.04 MPa·s/kg. Then the pressure drop in the reservoir is given as:

$$\Delta P_b = \dot{m} k \quad (\text{II.8.3})$$

Fig. II.6 Effect of the bottomhole temperature

The other parameters of the well are fixed as follows:

$$P_b = 15 \text{ MPa}$$

$$T_b = 295^\circ\text{C}$$

$$L = 1520 \text{ m}$$

$$\dot{m} = 0 \text{ molal}$$

$$D = 0.178 \text{ m}$$

$$\dot{Q} = 0.0 \text{ kW}$$

These conditions are similar to those encountered at well IID-1. The results are depicted in Fig. II.7 where it is seen that decreasing the drawdown factor by 50% results to an exergy increase of 35%. This effect can be achieved by the fracture of the rocks (hydraulic or other) surrounding the reservoir.

II.8.d. The influence of the fluid molality.

The effect of changing the amount of salts dissolved in the geothermal fluid is shown in Fig. II.8. A qualitative discussion of the effect of salts in the geothermal fluids was made in section II.6, and here the quantitative results are depicted for a well characterized by the following parameters:

$$P_b = 15 \text{ Mpa}$$

$$T_b = 295^\circ\text{C}$$

$$L = 1520 \text{ m}$$

Fig. II.7 Effect of the drawdown factor

Fig. II.8 Effect of salinity

$$k = 0.023 \text{ MPa}\cdot\text{s/kg}$$

$$D = 0.178 \text{ m}$$

$$\dot{Q} = 0.0 \text{ kW}$$

There is no significant effect on the critical mass flow rate and it is seen that the wells choke at almost the same critical flow. There is, however, a substantial decrease in the rate of exergy (available power) with increasing molality. The rate of exergy drops by 42% when the molality is increased from 0 to 2.

II.8.e. The influence of heat losses.

The heat losses to the surroundings were varied as a percentage of the static enthalpy at the reservoir. The rate of heat lost was taken uniform along the length of the pipe and the other parameters of the well flow were fixed at the following values:

$$P_b = 22.5 \text{ MPa}$$

$$T_b = 168^\circ\text{C}$$

$$L = 2430 \text{ m}$$

$$k = 0.021 \text{ MPa}\cdot\text{s/kg}$$

$$m = 0.0 \text{ molal}$$

$$D = 0.391 \text{ m}$$

These conditions roughly correspond to the conditions met at an East Mesa well. The results are shown in

Fig. II.9. It is apparent that heat losses decrease the rate of exergy (available power) and a 6% heat loss results in a 21% decrease in the exergy. Heat losses are avoided in the wells by installing a concrete insulation (something that protects the steel from fracturing).

II.8.f. The influence of diameter.

The effect of increasing the diameter all the way along the well from 0.10 to 0.20 and 0.30 m is depicted in Fig. II.10. The other flow parameters are kept fixed at the following values:

$$P_b = 22.5 \text{ MPa}$$

$$T_b = 168^\circ\text{C}$$

$$L = 2430 \text{ m}$$

$$k = 0.021 \text{ MPa}\cdot\text{s/kg}$$

$$m = 0.0 \text{ molal}$$

$$\dot{Q} = 0.0 \text{ MW}$$

These conditions correspond to a well at East Mesa. It is shown that a remarkable increase of mass flow rate and exergy is accomplished with wider wells. A doubling of the diameter results in a doubling of the critical mass flow rate and a tripling of the exergy (available power). This effect is more pronounced at higher temperature wells, as it will be shown in section II.9 where

Fig. II.9 Effect of heat losses

Fig. II.10 Effect of diameter change

stepping of the well diameter is discussed.

II.9 Pumping of the well versus diameter stepping.

A high fraction of the geothermal power cost is in the drilling of the wells. It is a lump-sum cost spent at the early stages of the geothermal field exploitation, and is not related to the amount of fluid that can be extracted. Because of this high initial cost, it follows that the well owner will try to obtain as much fluid as possible from a given geothermal well. Increasing the mass flow rate is limited by the choking of the wells or by clogging at places where CaCO_3 is precipitated. Choking can be avoided if flashing occurs at a high level in the well or if the flow is all in the liquid phase. Both results can be achieved by artificially increasing the pressure at a point below the flashing level by the use of a pump. This point will have to be located sufficiently lower than the flashing level so that cavitation does not cause damage to the pump. In addition, pumping will not allow CO_2 to escape from the brine, thus maintaining the chemical balance of the solution and reducing substantially the amount of CaCO_3 deposited.

The pressure at the reservoir is essentially hydrostatic at zero mass flow rate as given by the following equation:

$$P_b = \rho_o g L \quad . \quad (II.9.1)$$

When the reservoir is tapped the geofluid flows through the permeable formation, pressure gradients are formed and at the well-bottom its pressure has been reduced by an amount indicated by the drawdown factor k

$$\Delta P_d = k \dot{m} . \quad (II.9.2)$$

As the fluid rises up the well its pressure is further reduced by the friction and gravitational losses:

$$\Delta P_f = \int_0^z f \cdot \frac{1}{2D} \frac{\dot{m}}{\rho A}^2 dz , \quad (II.9.3)$$

$$\Delta P_g = \int_0^z g \rho dz . \quad (II.9.4)$$

At a certain point in the well the reduction in pressure is sufficient for the saturation pressure of the brine to become equal to the static pressure and flashing occurs there:

$$P_s (T) = P_f = P_b - \Delta P_d - \Delta P_f - \Delta P_g . \quad (II.9.5)$$

Above the flashing level vapor is formed in the well and the specific volume of the fluid increases substantially; if the well diameter is constant, the mean velocity of the fluid rises and the static pressure is converted to mechanical energy.

An increase of the mass flow rate will result in an increase of ΔP_d and ΔP_f , thus bringing the flashing level lower. The onset of two-phase flow, which is characterized by increased

frictional losses, (eqn. II.3.8) is followed by choking whenever the mass flow rate is high enough. Avoiding choking inside the well, while maintaining high mass flow rates, is the reason for installing a pump at the early stages of well flow. By these means the pressure at any point in the well is increased and the flashing level is higher, as characterized by the equation

$$P_f = P_b + \Delta P_p - \Delta P_d - \Delta P_f - \Delta P_g . \quad (\text{II.9.6})$$

The installation of a downhole pump requires the expense of useful work of pumping:

$$\dot{W}_p = \frac{\dot{m}}{\eta_p} \frac{\Delta P_p}{\rho} \quad (\text{II.9.7})$$

where η_p is the efficiency of the pump. Furthermore, it requires maintenance procedures which may be very costly: the installation of a component with moving parts at a depth of one kilometer in adverse environmental conditions is conducive to malfunctioning and access for preventive maintenance or repairing requires the well to be shut down for days. This is a great disadvantage of downhole pumping and does not encourage utilities to embark on it.

The thermodynamic advantages of downhole pumping can be estimated without accounting for any operational or economic effects. For that reason an exergy balance can be made for the

work consumed by pumping and the work gained as a result of it. It is assumed that the pump receives work from the geothermal plant; then the net specific work obtained by the combination pump-geothermal plant is:

$$w_n = - \Delta P_p / \eta_p \rho + \eta_u e_{wh} , \quad (II.9.8)$$

where e_{wh} is the specific exergy at the wellhead and η_u the utilization factor for the plant. The net power produced by the plant is:

$$\dot{W}_n = \dot{m} w_n , \quad (II.9.9)$$

which shows that the advantage of pumping lies in the large increase in the mass flow rate through the bore.

Another way of avoiding choking in the well is to increase the diameter. It is observed that near choking the frictional and accelerational components of the pressure drop become relatively high. As a result, the pressure gradient increases substantially, and the equilibrium temperature drops at a fast rate; the amount of vapor released under these circumstances forces the volumetric flow rate to attain high levels, and the consequence is a further increase in velocity and pressure drop due to acceleration. This mechanism leads to choking at the upper levels of the well and any reduction at the wellhead pressure results in insignificant increases in the mass flow rate (see section II.4).

An increase in diameter acts against choking in two ways:

a) It decreases the frictional pressure losses; b) It provides more area for the fluid to move, thus decreasing the velocities, and consequently the accelerational pressure loss. The second effect is very pronounced at near-choking conditions. Doubling of the diameter results in quadrupling the area of the pipe and decreases the mean velocity to one quarter of its value, other things being equal. Since the pressure loss is proportional to the square of the velocity, it follows that this item is reduced by a factor of sixteen. All these considerations are summarized in the following calculations:

$$A = \pi \frac{D^2}{4} ; \quad (\text{II.9.10})$$

$$u = \frac{\dot{m}}{\rho A} = \frac{4\dot{m}}{\rho \pi D^2} ; \quad (\text{II.9.11})$$

$$\Delta P_a = \frac{1}{2} \rho u^2 = \frac{2\dot{m}^2}{\rho \pi^2 D^4} . \quad (\text{II.9.12})$$

If

$$D' = 2 D$$

then

$$\Delta P'_a = \frac{1}{8} \frac{\dot{m}^2}{\rho \pi^2 D^4} = \frac{1}{16} \Delta P . \quad (\text{II.9.13})$$

Thus, the increase in diameter delays the rapid evaporation of the liquid and delays choking. It is evident that the increase in

diameter need not take place over the whole length of the well, but only over the upper part of it (stepped well). The widening of the cross-sectional area of the well will move choking to another point in the well where the pressure losses become substantial and conditions favorable for choking are established. Another widening (step of diameter) may then take place and repeated. The profile of such a well is depicted in Fig. II.11.

It is seen that well pumping and stepping result in the same effects, namely they prevent choking at low mass flow rates and increase the exergy available to the geothermal plant. Installation of either system is expensive and increases substantially the capital cost of the plant, but stepping of the well has some advantages with respect to pumping: a) It does not involve any moving components and hence requires little maintenance; because of this the operation of the plant is not disrupted by pump failure or malfunction; b) The technology of stepping wells is known and can be achieved by existing equipment, while the technology of downhole pumps at the geothermal well conditions is new and has a lot to be desired. The only limitation to drilling wider wells is the size of the bits used; at the present, this size is limited to 36" (0.915 m), a number appreciably higher than the usual size of the well diameters (7"). The additional cost of drilling a wider well is low. For example,

Fig. II.11 A stepped well

a 30" diameter well costs only 30% higher than a 14" well.

The merits of each system can be examined from the energetic point of view. The previous model and the computer program can be used in order to estimate the exergy of the fluid at the wellhead. A typical geothermal site has been chosen for the comparison, namely well 1 at the Imperial Irrigation District (IID-1). The natural bottomhole pressure and temperature are 14.9 MPa and 295°C, respectively. The depth of the well is 1520 m, the molality of the brine is 4 and the drawdown factor $k=0.022 \text{ MPa}\cdot\text{s/kg}$; the diameter of the existing well is 0.178 m (7"). It is assumed that a pump driven by an electric motor can be installed below the flashing level of the well (~700m) to provide a pressure increase of 3.6 and 7.1 MPa, the latter designed to deliver liquid brine at the wellhead under moderate wellhead pressures. Technical considerations for the construction installation and maintenance of the pump, or problems of cavitation, were left out of account. On the other hand, a doubling of the well diameter was stipulated at a depth of 1000m (approximately 300m above flashing).

The computer program was run for various mass flow rates in the well and stopped when the choking was indicated at high mass flow rates. The geothermal fluid conditions were thus predicted at the wellhead and a computation of the net specific available

work was calculated; in the case of pumping this includes the work spent for the operation of the pump, as shown in the following eqn. which assumes ideal efficiencies:

$$w_n^o = e_{wh} - \Delta P / \rho . \quad (II.9.14)$$

The results are depicted in Fig. II.12, which shows that there is little difference in the specific net work obtained in the four cases, but that the pumped and stepped wells can operate at very high mass flow rates before choking occurs.

The effect on the exergy available for the geothermal plant is shown in Fig. II.13. It is seen that pumping increases substantially the exergy at the wellhead and the 7.1 MPa increase in pressure results in an increase in exergy by 150%. The highest exergy increase (it is almost quadrupled) stems from the doubling of the diameter at a height of 1000m. This available exergy is utilized in the geothermal installation with a certain amount of loss due to the irreversibilities in the energy conversion processes. For this reason an overall utilization factor of 60% was assumed, and the pump efficiency was taken as 50%. The net power produced by such a geothermal installation is depicted in Fig. II.14. as a function of the mass flow rate. It is evident that even when all losses are taken into account, pumped and stepped wells produce considerably more power.

Fig. II.12 Ideal net specific work

Fig. II.13 Effect of pumping or stepping the geothermal well
(Isentropic components: efficiency 100%)

Fig. II.14 Effect of stepping the geothermal well
(Efficiencies: 70%)

It is obvious from the above that the most important factor in the enhancement of the wellhead exergy is the mass flow rate of the well. Both the pumped and stepped wells introduce an improvement in the form of higher mass flow rates. It is thought advantageous to calculate the mass flow rate increase resulting from a given fractional increase of diameter or an increase in pressure with respect to the natural reservoir pressure (P_b). These calculations are shown in Fig. II.15, which is a combination of two diagrams ($\dot{m}$ vs. $\frac{\Delta D}{D_o}$ and $\dot{m}$ vs. $\frac{\Delta P}{P_b}$) with a common axis. It is seen that a doubling in the well mass flow rate ($\dot{m}=140\text{kg/s}$) may occur either from a 36% increase in diameter or from a 37% increase in the downhole pressure (corresponding to $\Delta P_p = 5.5 \text{ MPa}$). A cross-plot of these diagrams is shown in Fig. II.16, where the effects of pumping and stepping are depicted for the same mass flow rate. This provides a measure of the equivalence of pressure increase and diameter stepping. A 50% increase in pressure downhole is equivalent to a 55% increase in the diameter at the previously specified level of 1000m from the bottom. The economic advantages of the two systems can be best compared in the light of this diagram.

Fig. II.15 Bottomhole pressure increments and diameter increments in a geothermal well

Fig. II.16 Diameter increment vs. pressure increment

III. THE MIXING LENGTH MODEL

III.1 Model of the geothermal well.

The geothermal well is taken here as a long vertical pipe carrying pure water; this assumption is made in order to avoid complicated computations involving impurities, at least in the early stages of the development of the theory. All flow quantities of interest are taken as time averaged quantities, an assumption which is also made in lumped models. The flow conditions at the bottom of the well are known by geophysical methods and the wellhead conditions need to be predicted. Figure (III.1) shows a short element of the well between states (1) and (2), and the relevant coordinate axes r and z . Here, P is the prevailing static pressure, r_0 the radius of the pipe and τ_0 the frictional stress at the wall. The well is furthermore assumed adiabatic and unperforated.

Fig. III.1 An element of the geothermal well

The height of the well is by four orders of magnitude greater than the diameter ($L \gg r_0$). The flow in the well is one-dimensional in the z direction ($\bar{u}_r = 0$). Application of the above in the Navier-Stokes equations for variable-density pipe flow will yield the following equation:

$$-\frac{dP}{dz} + \frac{\partial}{\partial r} \left[\nu \frac{\partial \bar{u}_z}{\partial r} - u'_r u'_z \right] + \bar{\rho} g = 0 . \quad (\text{III.1.1})$$

and

$$P = P(z) , \quad (\text{III.1.2})$$

where $\bar{u}_z$ and $\bar{\rho}$ are the time-average velocity and density, ν the kinematic viscosity and g the gravitational acceleration; the prime (') denotes perturbations. The physical significance of the above equation lies in the fact that the radial pressure distribution is constant and the static pressure becomes a function of the height only; the acceleration terms disappear in eqn. (III.1.1) because the continuity equation is applied and the term $\partial \rho / \partial z$ is taken to be negligible, given the length of the pipe.

The equations for the mass flow rate through the pipe and the shear stress at radius r take the following form:

$$\dot{m} = 2\pi \int_0^{r_0} \rho u_r dr , \quad (\text{III.1.3})$$

for the mass flow rate, and for the shear

$$\tau = - 0.5 r dP/dz - g/r \int_0^r \rho r dr , \quad (\text{III.1.4})$$

where u is the axial time-average velocity, ρ the time-average density, τ the stress and g the gravitational acceleration.

Equation (III.1.4) is in effect a mechanical equilibrium equation without the acceleration terms, and may be transformed into the following form:

$$\tau = r/r_0 \left[\tau_0 + 0.5 g r_0 \rho_\ell \left(G(1) - G(r/r_0) \right) \right] \quad (\text{III.1.5})$$

where

$$\tau_0 = - \frac{r_0}{2} \frac{dP}{dz} - \frac{g}{r_0} \int_0^{r_0} \rho r dr , \quad (\text{III.1.6})$$

and

$$G \left(\frac{r}{r_0} \right) = 2 \left(\frac{r_0}{r} \right)^2 \int_0^{r/r_0} \frac{\rho}{\rho_\ell} r dr . \quad (\text{III.1.7})$$

It is evident that for horizontal (absence of gravity term) or constant density flow equation (III.1.6) gives a linear shear stress variation across the pipe.

Now τ_0 is the stress at the wall ($r=r_0$) and ρ_ℓ the density of the liquid phase (in contrast to ρ which is the local time averaged density of the two-phase mixture)*. It will be seen that

* It should be noted that henceforth for simplicity the bar (-) is dropped from the time averaged quantities. It must not be forgotten though that now P , u and ρ represent such averages.

this form of the momentum equation, which corresponds to the usual assumptions in the analysis of frictional pipe flows, makes the computations easier compared to other forms containing the acceleration term.

III.2 The choice of the density function.

It was observed from experimental data that the time-average density in two-phase flows is not constant across and along the pipe. Evaporation of the liquid phase accounts for the differences of the density profiles along the pipe; three dimensional effects and forces of aerodynamic nature are responsible for the higher concentration of bubbles near the center of the pipe and the apparent minimum of the density function there. As a result of this, the walls of the pipe appear always wetted, while the area near the center of the pipe carries most of the vapor phase with some entrained droplets. This phenomenon suggests that the function ρ for the time averaged density distribution must be monotonically increasing with r and attain the value ρ_ℓ at the wall ($r=r_0$), the shape of the curve being deduced from experimentation. Available experimental results suggest a power variation of the density function and the following expression is taken for this study as the most suitable:

$$\rho = \rho_\ell [1 + \beta(x^n - 1)] , \quad (\text{III.2.1})$$

where

$$x = \frac{r}{r_o}, \text{ and } \beta \text{ is a parameter.}$$

At $x = 0$ (center of the pipe), the density function attains its lowest value $\rho_{\min} = (1-\beta)\rho_\ell$. Near the flashing level β is small and $\rho_{\min}$ is close to ρ_ℓ . At the upper stages of the pipe though, β becomes higher and the minimum density is closer to the vapor density. The highest value β may attain is $(1-\rho_v/\rho_\ell)$ when the density at the center of the pipe becomes exactly equal to the density of the vapor at the prevailing temperature. In fact, β does not attain this value because of the presence of droplets at the center of the pipe (entrainment). In this case the maximum value β may attain is determined by the local dryness fraction of the steam $\gamma^*(z)$. The density at the center of the pipe is given by the following equation:

$$\frac{\rho_c}{\rho_\ell} = \frac{1}{1 - \gamma^* + \gamma^* \frac{\rho_\ell}{\rho_v}} = 1 - \beta_{\max}, \quad (\text{III.2.2})$$

and

$$\beta_{\max} = \frac{1 - \frac{\rho_v}{\rho_\ell}}{1 + \frac{1 - \gamma^*}{\gamma^*} \frac{\rho_v}{\rho_\ell}} = \left(1 - \frac{\rho_v}{\rho_\ell} \right) \phi^*, \quad (\text{III.2.3})$$

where $\phi^*(z)$ is the local void fraction and is less than unity.

The other parameter n is a positive number for which available experimental data suggest a value greater than 5. One of the two parameters β and n must be determined from experiment and the other can be calculated from the continuity equation. The resulting function for the density distribution is depicted in Fig. III.2 where curves of ρ/ρ_l are shown as functions of position, varying the parameter n from 1 to 15 and keeping β constant at 0.8.

The void fraction $\phi(x,z)$ is defined as the fraction of the elemental volume occupied by the vapor phase and is an important parameter in the two-phase flow calculations. According to the choice of density function described above, the local void fraction is given as:

$$\phi = \beta \frac{1-x^n}{1-\rho_v/\rho_l} , \quad (\text{III.2.4})$$

and

$$\rho = (1-\phi) \rho_l + \phi \rho_v . \quad (\text{III.2.5})$$

Then eqn. (III.1.7) can be written as:

$$G(x) = 1-\beta \left[1 - \frac{2}{n+2} x^n \right] , \quad (\text{III.2.6})$$

and

$$G(0) = 1-\beta , \quad (\text{III.2.7})$$

$$G(1) = 1-\beta \frac{n}{n+2} . \quad (\text{III.2.8})$$

Fig. III.2 The density profiles for different values of the parameter n

The spatially averaged density ρ_s is given by the following equation:

$$\rho_s = \int_0^1 \rho x dx \quad (\text{III.2.9})$$

or

$$\frac{\rho_s}{\rho_l} = G(1) = 1 - \beta \frac{n}{n+2} . \quad (\text{III.2.10})$$

Because of the different average velocities of the two phases, this quantity differs from the discharge density $\bar{\rho}$ of the pipe which will be defined later.

The above equations can now be substituted into the mechanical equilibrium equation to give an expression for the rate of change of pressure along the pipe,

$$- \frac{dP}{dz} = 2 \frac{\tau_o}{r_o} + g \rho_l G(1) = 2 \frac{\tau_o}{r_o} + g \bar{\rho}_s ; \quad (\text{III.2.11})$$

this will be related to the entropy production or dissipation, which is the primary target of the study (Section III.6).

III.3 The mixing length theory.

Prandtl developed the original mixing length hypothesis under the assumption of rectangular flow with velocity u parallel to the x axis and zero time average transverse components v and w ,

$$u = \tilde{u}(y) , v = \tilde{v} = 0 , w = \tilde{w} = 0. \quad (\text{III.3.1})$$

Although the time average transverse velocities vanish, their fluctuations v' and w' are different than zero and the Reynolds stresses

$$\tau_t = -\rho u'v' , \quad (\text{III.3.2})$$

remain different from zero.

Someone may visualize aggregates of liquid particles (lumps) moving from one lamina to the other transversely in the flow field and carrying with them their original momentum. Figure III.3 shows two such lumps of fluid coming from distances $y-\ell$ and $y+\ell$ which coalesce at the point y . The lump from point $y-\ell$ has velocity $u(y-\ell)$ and after transverse movement through length ℓ reaches the streamline at y with $u'>0$, where the velocity is $u(y)$.

Fig. III.3 The mixing length ℓ

The difference of the two velocities is:

$$\Delta u_1 = u(y) - u(y-\ell) \approx \ell \left(\frac{du}{dy} \right)_y . \quad (\text{III.3.3})$$

Similarly, another lump of fluid traverses from $y+\ell$ to y with $u' < 0$ and the difference of the two velocities is approximately

$$\Delta u_2 = u(y+\ell) - u(y) \approx \ell \left(\frac{du}{dy} \right)_y . \quad (\text{III.3.4})$$

The velocity differences caused by this transverse motion can be regarded as the turbulent fluctuation of the flow. Hence, the time average of this fluctuation in absolute value is:

$$|u'| = \frac{1}{2} \left(|\Delta u_1| + |\Delta u_2| \right) = \ell \left| \left(\frac{du}{dy} \right)_y \right| . \quad (\text{III.3.5})$$

In light of eqn. (III.3.5) the mixing length may be interpreted as the distance in the transverse direction, which must be covered by an agglomeration of fluid particles traveling with its original mean velocity, in order to make the difference between its velocity and the velocity in the new lamina equal to the mean transverse fluctuation in turbulent flow.

The two lumps of fluid at the lamina y will either coalesce (if the slower precedes the faster) or have a relative velocity equal to $2|u'|$ (if the faster precedes). In both cases they give rise to a transverse component v' of velocity which must

be of the same order of magnitude but of different sign (because lumps with the higher velocity $u(y+\ell)$ move with negative transverse velocity u' towards the wall and lumps with lower velocity $u(y-\ell)$ move with positive v'). Hence we can write for the time averages:

$$|v'| = c \cdot |u'| = c \cdot \ell \left| \frac{du}{dy} \right|, \quad (\text{III.3.6})$$

where c is a negative constant. We can incorporate this constant in the mixing-length parameter which is also unknown and write for the Reynolds stress

$$\tau = -\rho (u'v') = \rho \ell^2 \left| \frac{du}{dy} \right| \frac{du}{dy}, \quad (\text{III.3.7})$$

taking into account that the sign of τ must change if the flow is reversed.

This mixing length hypothesis (Prandtl, 1925) has served in establishing a relationship for turbulent flow similar to Newton's law for laminar flow, between stress and velocity gradient. Thus, in laminar flow we have:

$$\tau_\ell = \mu \frac{\partial u}{\partial y}, \quad (\text{III.3.8})$$

and in turbulent

$$\tau = A_t \frac{du}{dy}, \text{ where } A_t = \rho \ell^2 \left| \frac{du}{dy} \right|. \quad (\text{III.3.9})$$

In many cases it is possible to establish a relation between the mixing length ℓ and a characteristic length of the pipe. One working hypothesis is to take the mixing length ℓ proportional to the distance from the wall:

$$\ell = \chi y . \quad (\text{III.3.10})$$

This is the simplest form of a relationship between ℓ and y given that ℓ vanishes at the wall of the pipe. Some other function $\ell = f(y)$ with $f(0) = 0$ could have been taken, but this would have introduced unnecessary complications.

This transforms eqn. (III.3.7) into:

$$\tau_t = \rho \chi^2 y^2 \left| \frac{du}{dy} \right| \cdot \frac{du}{dy} , \quad (\text{III.3.11})$$

or

$$\tau_t = \rho \chi^2 \left| \frac{du}{d\eta} \right| \cdot \frac{du}{d\eta} \quad (\text{III.3.12})$$

where

$$\eta = \ln y . \quad (\text{III.3.13})$$

Experimentation is needed to determine the parameter χ ; it was found through many experiments that it has a constant value 0.4 in one-phase flows.

Eqn. (III.3.12) has been applied successfully to the study of one-phase turbulent flow along walls and free turbulent flow.

The results were very favorable as to the utility and the

validity of the mixing length theory, and by now there is no hesitation in applying the theory for predictions.

It is believed that the application of the same principles in two-phase flows will yield equally useful results, once the parameter γ^* for the entrainment is determined experimentally.

III.4 Transformation of the equations into boundary layer coordinates.

It is seen that in eqn. (III.3.13) the variable $\eta = \ln y (d\eta = dy/y)$ appears as the independent variable of the equation instead of y . It is, therefore, advantageous in the development of the theory to transform all the equations into the so-called "boundary layer" system of coordinates which is written for the case of pipe flow:

$$\eta = \ln \frac{y}{y_0} \quad (y = r_0 - r) , \quad (\text{III.4.1})$$

with

$$\eta_0 = \ln \frac{r_0}{y_0} , \quad (\text{III.4.2})$$

where y_0 is determined by the laminar sublayer and is a function of the Reynolds number and the roughness of the pipe. It was said before that the pipe wall is always wetted. Therefore, the laminar sublayer is composed of the liquid phase only and hence, one may use the experimental results of the one-phase flow studies to determine the quantity η_0 or y_0 . These results, modified for our purposes, lead to the following correlation for

η_o which was used extensively in one-phase flow and is proposed for the present calculation as well:

$$\eta_o = \ln \frac{r_o}{k} - \ln 0.111 - \ln \left(0.3 + \frac{1}{R^*}\right) . \quad (\text{III.4.3})$$

The derivation of this form of eqn. (III.4.3) will be given in Appendix II. Here, k is the roughness of the pipe and R^* the shear Reynolds number at the boundary layer. The shear velocity V^* for this dimensionless number is defined by the shear stress at the wall as follows:

$$V^* |V^*| = \tau_o / \rho . \quad (\text{III.4.4})$$

The characteristic length for this Reynolds number is taken as the roughness k of the pipe. With the viscosity of the liquid phase μ_ℓ this Reynolds number is defined as:

$$R^* = \frac{V^* \rho_\ell k}{\mu_\ell} . \quad (\text{III.4.5})$$

It is seen that the shear Reynolds number is in fact the ratio of the two important lengths in the boundary layer: the roughness k and the viscous length $\mu_\ell / \rho_\ell V^*$. This quantity is a dimensionless group characterizing the flow at the turbulent boundary layer only and does not contain information about the flow outside this layer. Furthermore, it does not represent the ratio of the inertia to viscous forces as the usual definition

of Reynolds number implies. The flow in the pipe is characterized by another Reynolds number RE which is defined in terms of the diameter D , the cross-sectional average velocity $\bar{u}$, and density ρ , as follows:

$$RE = \frac{\bar{u} \cdot \bar{\rho} D}{\mu_{\ell}} = \frac{4\dot{m}}{\pi D \mu_{\ell}} . \quad (\text{III.4.6})$$

In all one-phase flow applications the Reynolds number of the average flow is related to the shear Reynolds number through the friction factor f of the flow. A similar relation exists for the two-phase flow conditions: we can define the two-phase friction factor for the frictional pressure loss in the two-phase flow as follows:

$$\frac{dP}{dz} - \bar{\rho}_s g = f_{TP} \frac{\bar{\rho} \bar{u}^2}{2D} , \quad (\text{III.4.7})$$

in analogy with single-phase flow. Then eqn. (III.2.11) yields:

$$\tau_o = \frac{f_{TP}}{8} \bar{\rho} \bar{u}^2 , \quad (\text{III.4.8})$$

where f_{TP} is the two-phase friction factor.

The last four equations yield:

$$\left(\frac{\mu_{\ell} R^*}{k \rho_{\ell}} \right)^2 = \frac{1}{8} \frac{\rho_a}{\rho_{\ell}} f_{TP} \left(\frac{\rho_{\ell} RE}{2r_o \bar{\rho}} \right)^2 ,$$

or

$$R^* = \sqrt{\frac{f_{TP}}{32} \cdot \frac{\rho_{\ell}}{\bar{\rho}}} RE \frac{k}{r_o} . \quad (\text{III.4.9})$$

This equation for the two-phase flow is analogous to one-phase flow equations, after taking into account the density variation in the pipe.

Under the light of the new equations the expression for η_o becomes:

$$\eta_o = 3.4 - \log \left[\frac{k}{r_o} + \frac{18.9}{RE \sqrt{\frac{f_{TP}}{32} \frac{\rho_l}{\rho}}} \right] , \quad (\text{III.4.10})$$

or

$$\eta_o = 0.46 - \log \left[\frac{0.053k}{r_o} + \frac{1}{RE \sqrt{\frac{f_{TP}}{32} \frac{\rho_l}{\rho}}} \right] . \quad (\text{III.4.11})$$

Equation (III.4.1) can now be written in another form which will be used later, in the computational section:

$$\eta - \eta_o = \ln \frac{y}{r_o} = \ln(1-x) , \quad (\text{III.4.12})$$

or

$$x = 1 - \exp(\eta - \eta_o) . \quad (\text{III.4.13})$$

In connection with vertical pipe flow another velocity, the gravitational velocity V_g , can be defined and a dimensionless number F whose significance will be apparent later:

$$V_g = \sqrt{g \frac{r_o}{2}} = \sqrt{g \frac{D}{4}} , \quad (\text{III.4.14})$$

$$F = \frac{V_g}{V^*} . \quad (\text{III.4.15})$$

The definition of the gravitational velocity is reminiscent of a unit Froude number for this flow; another Froude number based on the average velocity will appear later. All the previously obtained quantities are used to simplify the expression for the stress to the following form:

$$\begin{aligned}\tau_t &= x \rho_\ell V_g^2 \left[\frac{1}{|F|F} - G(x) + G(1) \right] \\ &= \rho_\ell V_g^2 \left(1 - e^{\eta - \eta_0} \right) \left[\frac{1}{|F|F} - G(\eta) + G(0) \right], \quad (\text{III.4.16})\end{aligned}$$

The function G can be transformed into boundary layer coordinates to make all equations consistent in the same independent variables:

$$G(\eta) = 1 - \beta \left[1 - \frac{2}{n+2} \left(1 - e^{\eta - \eta_0} \right)^n \right]. \quad (\text{III.4.17})$$

The quantity η_0 is an order of magnitude greater than unity, and hence $e^{-\eta_0} \approx 0$.

Then

$$G(0) \approx 1 - \beta \left[1 - \frac{2}{n+2} \right], \quad (\text{III.4.18})$$

and

$$G(\eta) - G(0) = \frac{2\beta}{n+2} \left[\left(1 - e^{\eta - \eta_0} \right)^n - 1 \right]. \quad (\text{III.4.19})$$

III.5 The equations for two-phase flow.

The mixing length theory and the transformation of the independent variable y onto η yields an ordinary differential equation for the velocity of the flow. The local velocity u can be made dimensionless with reference to either the gravitational or shear velocity and the combination of eqns. (III.3.12), (III.4.16) and (III.4.18) will yield in the two cases:

$$\chi^2 \frac{du^+}{d\eta} \left| \frac{du^+}{d\eta} \right| = \frac{\rho_\ell}{\rho} \left(1 - e^{\eta - \eta_0} \right) \left[\frac{V^*}{|V^*|} - F^2 (G(\eta) - G(0)) \right] \quad (\text{III.5.1})$$

$$\text{with } u^+ = \frac{u}{|V^*|} \quad (\text{III.5.2})$$

or

$$\chi^2 \frac{du_f}{d\eta} \left| \frac{du_f}{d\eta} \right| = \frac{\rho_\ell}{\rho} \left(1 - e^{\eta - \eta_0} \right) \left[\frac{1}{F|F|} - G(\eta) + G(0) \right], \quad (\text{III.5.3})$$

with

$$u_f = \frac{u}{V_g} \quad (\text{III.5.4})$$

These differential equations can be solved in principle in order to give the velocity distribution function across the pipe. The boundary condition is that the velocity vanishes at $\eta=0$. The fact that the equations are nonlinear suggests that an analytic solution may not exist, but a numerical solution is, nevertheless, feasible. Once the velocity distribution is obtained, all other quantities of interest can be determined.

For the case of the pipe flow there are two average physical quantities of interest, namely the volumetric flow rate (or equivalently the cross-sectional average velocity $\bar{u}$) and the mass flow rate $\dot{m}$ (or the mass flux j). These quantities are related through the following equations:

$$\bar{u} = \frac{\dot{Q}}{A} , \quad (\text{III.5.5})$$

$$j = \frac{\dot{m}}{A} . \quad (\text{III.5.6})$$

The ratio of the two is the discharge density of the fluid, an expression of which was obtained in eqn. (III.2.10)

$$\bar{\rho} = \frac{\dot{m}}{\dot{V}} = \frac{u}{j} . \quad (\text{III.5.7})$$

The average velocity u is obtained by integration across the pipe as follows:

$$\bar{u} = 2 \int_0^1 x u(x) dx , \quad (\text{III.5.8})$$

or in the light of the boundary layer coordinates,

$$\bar{u} = \int_0^{\eta_0} \frac{du_f}{d\eta} \left(1 - e^{\eta - \eta_0} \right)^2 d\eta = \frac{V_g}{x} I_1 . \quad (\text{III.5.9})$$

The average flux is given similarly:

$$j = 2\rho_\ell \int_0^1 \frac{\rho}{\rho_\ell} u \cdot x dx =$$

$$\begin{aligned}
 &= 2\rho_\ell V_g \int_0^{\eta_0} u_f \frac{\rho}{\rho_\ell} \left(1 - e^{\eta - \eta_0}\right) e^{\eta - \eta_0} d\eta = \\
 &= \rho_\ell V_g \int_0^{\eta_0} \frac{du_f}{d\eta} G(\eta) \left(1 - e^{\eta - \eta_0}\right)^2 d\eta = \frac{\rho_\ell V_g}{\chi} I_2 . \quad (\text{III.5.10})
 \end{aligned}$$

The integrals I_1 and I_2 defined above are functions of η_0, β, F and n : out of these, only η_0 is known through equation (III.4.3). A relation of these integrals is now made to other quantities of the flow, such as the average discharge density and the friction factor for the flow in order to obtain a closed set of equations and determine the unknowns. Of course, the solutions of the equations is through numerical integration and involves an iteration. The relations which will help determine the unknowns are:

$$\frac{\bar{\rho}}{\rho_\ell} = \frac{I_2}{I_1} , \quad (\text{III.5.11})$$

and

$$f_{TP} = \frac{8\chi^2}{F|F|} \frac{1}{I_1 I_2} . \quad (\text{III.5.12})$$

In terms of the above integrals I_1 and I_2 , the ratio of the spatially averaged to the discharge density can be written as:

$$\frac{\bar{\rho}_s}{\bar{\rho}} = \frac{I_1}{I_2} \left(1 - \beta \frac{n}{n+2}\right) . \quad (\text{III.5.13})$$

It must be noted that the two phases move with different average velocities because of buoyancy effects. This is the cause

of the difference in the two densities, the one of which ($\bar{\rho}_s$) is the average over the cross-sectional area and the other, the density obtained from the mass flux. In the case of single-phase flow there is no difference between the two and the above ratio is unity. In this case $\beta=0$ and $I_1=I_2$.

Thus for the single-phase flow:

$$I_1 = I_2 = \int_0^{\eta_0} \frac{1}{F} \left(1 - e^{\eta - \eta_0} \right)^{5/2} d\eta \approx \frac{1}{F} \cdot (\eta_0 - 1.67) , \quad (\text{III.5.14})$$

and the friction factor for the single-phase flow is

$$f_{SP} = \frac{8\chi^2}{(\eta_0 - 1.67)^2} . \quad (\text{III.5.15})$$

Now the ratio M of the two-phase to the single-phase friction factor can be given as follows:

$$M = \frac{f_{TP}}{f_{SP}} = \frac{(\eta_0 - 1.67)^2}{F|F| \cdot I_1 \cdot I_2} . \quad (\text{III.5.16})$$

It can be seen from the above equations that the two integrals I_1 and I_2 play an important role in the computations; all the quantities of interest can be written in terms of them and this fact is enormously helpful in the calculations.

III.6 Entropy production in the pipe.

It was mentioned in Section 2 that the connection to equilibrium thermodynamic quantities must be made through the concept of entropy production rather than the traditional method of calculating the "pressure drop". This approach is thought to be advantageous in a situation where equilibrium thermodynamic quantities, such as temperature and entropy, play an important role because the final objective of the geothermal projects is the determination of exergy in geothermal fluids and its subsequent conversion into available work. The direct determination of the entropy production in the well will give us a detailed picture of the available work losses occurring inside the well and their causes. Furthermore, questions related to pumping in the well or the use of a stepped diameter will be answered with the aid of the entropy production function. The approach to the flow parameters through entropy production will also yield a clear indication of the onset of critical flow, because the entropy then becomes a maximum.

There are two causes of entropy production in the geothermal well: the dissipation due to the wall friction and that due to the difference of velocities between the two phases. The first is the usual type of shear at the wall met in all flows (single and two-phase). The second effect is due to buoyancy (and hence

to gravity) forces acting on the lighter phase which then acquires a higher velocity and lifts the heavier phase through slippage. Thus, there is an unbalanced force created between the two phases which creates the entropy production; this phenomenon is called holdup.

Once the entropy is determined at the well the pressure distribution can be deduced through the Gibbs equations. Throughout these calculations an energy conservation equation is taken for the adiabatic well case, since the heat transfer from the well to the surroundings is negligible:

$$\frac{dh^\infty}{dz} = 0 . \quad (\text{III.6.1})$$

where $h^\infty = h + \frac{1}{2} \bar{u}^2 + \bar{p}z$ is the static enthalpy of the fluid at rest in the reservoir. Under these conditions the rate of change of entropy is derived as follows:

$$\frac{T d\dot{S}}{dz} = T \dot{m} \frac{ds}{dz} = - 2\pi \int_0^{r_0} r \tau \frac{du}{dr} dr . \quad (\text{III.6.2})$$

Integrating the last equation by parts we obtain

$$T \dot{m} \frac{ds}{dz} = 2\pi \int_0^{r_0} u \frac{d(\tau r)}{dr} dr . \quad (\text{III.6.3})$$

Using eqn. (III.1.4) we are led to

$$T \dot{m} \frac{ds}{dz} = - 2\pi \int_0^{r_0} r u \frac{dp}{dz} dr - 2\pi \int_0^{r_0} g \rho u r dr . \quad (\text{III.6.4})$$

Substituting eqn. (III.2.11), we see that

$$T \frac{ds}{dz} = \frac{2\tau_o}{\bar{\rho}r_o} + g \left(\frac{\bar{\rho}_s}{\bar{\rho}} - 1 \right) . \quad (\text{III.6.5})$$

It is observed that in the last equation the entropy production is composed of two terms, the first due to the friction at the wall and the second to the holdup, as explained above. This equation may be written in terms of the friction factor as follows:

$$T \frac{ds}{dz} = f_{TP} \frac{\bar{u}^2}{2D} + g \left(\frac{\bar{\rho}_s}{\bar{\rho}} - 1 \right) . \quad (\text{III.6.6})$$

We can anticipate a Froude number for the two-phase flow defined as follows:

$$FR = \frac{\sqrt{gD}}{\bar{u}} \quad (\text{III.6.7})$$

and then eqn. (III.6.6) is written in its final form:

$$T \frac{ds}{dz} = g \left[\frac{f_{TP}}{2(FR)^2} + \left(\frac{\bar{\rho}_s}{\bar{\rho}} - 1 \right) \right] . \quad (\text{III.6.8})$$

III.7 The solution of equations.

It is observed that the evaluation of the entropy production rate involves the knowledge of the Froude number for the flow and the parameter β . Alternatively, the Froude number can be calculated from the dimensionless parameter F defined in equation (III.4.14) as follows:

$$FR^2 = f_{TP} \cdot \frac{F|F| \cdot \rho_\ell}{\bar{\rho}} . \quad (\text{III.7.1})$$

Now F and β are calculated from the integrals I_1 and I_2 and their relationships to the mass flow rate $\dot{m}$ and the density ratio $\bar{\rho}/\rho_1$ as given by equations (III.5.10) and (III.5.11). The calculation involves the numerical solution of two simultaneous nonlinear ordinary integral equations and may be accomplished with an electronic computer. The final equation for the entropy production rate is given below as a consequence of (III.6.8) and (III.7.1):

$$\frac{T}{g} \frac{d\dot{s}}{dz} = \frac{1}{F|F|} \frac{\rho_\ell}{\bar{\rho}} + \frac{\bar{\rho}_s}{\bar{\rho}} - 1 , \quad (\text{III.7.2})$$

with the first term giving the entropy production at the wall due to friction and the second two terms to the production due to the holdup.

The pressure loss resulting from the entropy production, the gravitational effect and the acceleration of the component phases can now be calculated starting from the condition that the total enthalpy of the fluid is conserved:

$$h^\infty = h + gz + \frac{\bar{u}^2}{2} = \text{const} , \quad (\text{III.7.3})$$

with $\bar{u}^2/2$ being the kinetic energy term and h the static enthalpy of the fluid. An implicit assumption is made here, that $\overline{u^2} \approx \bar{u}^2$,

or that the variance of the velocity distribution is negligible compared to the square of the average velocity.

Then

$$Tds + \frac{1}{\rho} dp + g dz + cdc = 0 , \quad (\text{III.7.4})$$

and

$$- \frac{dp}{dz} = \rho \left[T \frac{ds}{dz} + g + cdc \right] . \quad (\text{III.7.5})$$

Substitution from (III.6.4) yields the final expression for the pressure change:

$$- \frac{dp}{dz} = \frac{g\rho_\ell}{F|F|} + g\bar{\rho}_s + \bar{\rho}_\ell cdc . \quad (\text{III.7.6})$$

This expression provides a measure of the pressure gradient at every station along the geothermal well in terms of quantities that can be evaluated. In this case the static pressure may be computed numerically at any point of the well and the condition of thermodynamic equilibrium will yield all quantities of interest such as temperature, quality, enthalpy of the fluid, etc.

The temperature of the fluid is uniquely determined by the pressure. The quality (dryness fraction) of the two-phase mixture is thus computed from the initial stagnation enthalpy h^∞ and the static enthalpy at the given point from the equation:

$$\gamma = \frac{h^\infty - gz - \frac{\bar{u}^2}{2} - h_\ell(T_s)}{h_{fg}(T_s)} , \quad (\text{III.7.7})$$

where $h_\ell(T_s)$ and $h_{fg}(T_s)$ are the enthalpy of the liquid at the saturation temperature T_s of the given point and the heat of vaporization at the same temperature. Then any specific static extensive property ϕ of the two-phase mixture at the saturation temperature T_s is given as:

$$\phi(T_s) = \phi_\ell(T_s) [1-\gamma] + \phi_v(T_s) \cdot \gamma \quad (\text{III.7.8})$$

where the symbols ℓ and v denote the liquid and vapor phase correspondingly.

It was indicated above that the key to obtain the rate of entropy production is the simultaneous solution of the two nonlinear, integral equations (III.5.10) and (III.5.11) for the values of β and F . For that matter, an iterative numerical procedure was adopted which is based on trial and error. A first pair of the values of β and F is selected (F_0, β_0) I_1 and I_2 are evaluated with the value of n being taken constant and equal to 7. The average density of the fluid $\bar{\rho}$ is computed according to the void fraction definition

$$\bar{\rho} = \rho_\ell (1-\phi) + \rho_v \phi, \quad (\text{III.7.9})$$

where ϕ is estimated from the non-slip condition:

$$\phi = \frac{1}{1 + \frac{1-\gamma}{\gamma} \frac{\rho_v}{\rho_\ell}} \quad (\text{III.7.10})$$

The values of $\bar{\rho}$ and j are computed according to equation (III.5.10) and they are compared to the values obtained from the mass flow rate in ($j = \dot{m}/A$) and equation (III.7.9). The comparison yields another pair of values for β and F (β_1, F_1) and the procedure is repeated again until the values of j and $\bar{\rho}$ obtained by the integral equations and those obtained by the mass flow rate and equation (III.7.9) agree within 2%. This error margin gave values of β and F which agreed with their actual values within 0.8%. The whole procedure is programmed on an IBM-370 computer as a subroutine of a main program, which is designed to calculate the properties of the water substance and to compute stepwise the pressure and the other equilibrium thermodynamic quantities along the geothermal well; a flow diagram of the program is given in Fig. III.4 and the whole program in Appendix 3.

At high elevations the pressure of the two-phase mixture drops considerably and high values dryness fraction and void fraction appear; consequently, the value of β rises and may reach its upper limit $(1 - \rho_v / \rho_\ell)$. The above computational procedure is physically meaningless after that point and for this reason, the following method was adopted: The value of β is computed as above until it reaches a predesignated limit $\beta_{\max}$ which may be less than $(1 - \rho_v / \rho_\ell)$ due to the fact that some droplets are always present in the center of the pipe and increase the average density there, above the density of the pure vapor. When

Fig. III.4 A flow diagram of the mixing length computer program

the step-by-step procedure reaches this limit the method is altered and keeps β constant at the $\beta_{\max}$ value while it changes n in order to solve the equations. Thus, the outcome of the procedure is now n and F and these are used for the calculation of the entropy production and the other thermodynamic quantities. The whole change in procedure corresponds physically to the fact that due to the increased amount of steam present in the pipe, the liquid phase is "pushed" towards the wall, thus being limited to a thinner region.

The velocity of the mixture is taken according to the specified mass flow rate and the discharge density:

$$\bar{u} = \frac{\dot{m}}{\rho A} \quad . \quad (\text{III.7.11})$$

The elevation from the flashing level is calculated from the inverse of the ratio dP/dz . The procedure takes a step of the static pressure ΔP (1 bar) and the finite change of elevation Δz is estimated from equation (III.7.6). This method has the advantage over the alternative of choosing a step of Δz , that the variation of the thermodynamic quantities between two steps is smaller along the well for a given number of steps, and hence, the accuracy obtained for the wellhead pressure is higher. The program ends when the elevation from the flashing level is equal to the estimated depth of the two-phase region in a given geothermal

well or when the flow becomes critical and the ratio dP/dz is higher than 0.5 bars/m. At this stage the specific entropy of the fluid approaches a maximum value (turning point at the Fanno line).

IV.1 Comparison of results.

The predictive power of the models described in Chapters II and III was tested against experimental results obtained in the IID-1 well (Imperial Irrigation District). The results were also compared to the results supplied by the Denver Research Institute (DRI). The latter emanate from a homogeneous model which is based on a certain correlation for the holdup (Hughmark 1964).

The prevailing bottomhole pressure and temperature of the above well are 14.9 MPa and 295°C, respectively. The one-phase flow regime is calculated according to the theory presented in Section II.1, and the flash-point is located at a place where the static pressure is equal to the saturation pressure. The models were used to predict the wellhead conditions and the latter were checked against existing data. A mass-flow rate equal to 50 kg/s was assumed for all the models; two roughness values, namely $0.002 r_o$ and 0.000 (smooth pipe) were taken for the mixing-length model. The value of the void fraction at the center of the pipe ϕ^* was taken as 0.9 in order to account for the entrainment of droplets.

The results are depicted in Fig. IV.1 where the two broken-line curves represent the results from the DRI model and the pattern model while the solid curves are for the mixing-length model. The DRI curve was estimated for the given conditions from a handbook of prepared two-phase flow graphs (Geothermal Well Design Handbook). The circle represents the experimentally determined wellhead pressure from field observations. It is seen that the pattern model and the smooth-pipe case of the mixing-length model predict the wellhead conditions quite accurately. The DRI model gives a good approximation to the pressure distribution but fails to account for the higher pressure losses in the top part of the well. The rough-pipe case of the mixing-length model overestimates the pressure losses and predicts the onset of critical conditions below the wellhead. The last curve for a uniform-density model does not take into consideration two-phase flow effects and fails to predict the pressure distribution along the well.

The predictive power of the three models was tested for higher and lower mass-flow rates (65 and 30 kg/s). The corresponding results are shown in Fig. IV.2. For this purpose a smooth pipe was stipulated for the mixing-length model. It is seen that the mixing-length model agrees fairly well (4.0% accuracy) with the experimental point at the lowest rate (30 kg/s) but indicates critical conditions below the wellhead at the highest

Fig. IV.1 Comparison of the available models for a geothermal well

Fig. IV.2 Comparison of the pattern and the mixing-length models

rate (65%) and results in a 12% underestimation of the well depth. The pattern model shows good agreement at the higher and lower flow-rates (1% and 5% accuracy) and predicts the onset of critical conditions well. Finally, the DRI model (not plotted in this diagram) shows reasonably good agreement (7%) at the lower mass-flow rate, but fails to predict the wellhead conditions at the higher mass-flow rate. The reason for this is that the DRI model does not account for the precritical conditions encountered at the top of the pipe. The results for the pattern and the mixing length models are seen plotted in Fig. IV-2. The first is drawn with solid lines and the latter with broken lines; the experimental data are shown again as circles.

A cross-plot of the last figure is shown in Fig. IV-3, which depicts the mass-flow rate curves against the wellhead pressure, for the given depth of the well (1520 m). The three curves show the experimental results and those predicted by the two models described in Chapters II and III. It transpires again that the pattern model describes the well flow accurately, especially at the higher mass-flow rates, while the mixing-length model is accurate at the medium mass-flow rates. The good agreement of the two models with the experimental values suggests that both enjoy a predictive power and confirms the usefulness of both.

Fig. IV.3 The experimental and theoretical results for the flow rate of the geothermal well

IV.2 Conclusions.

It is apparent from the previous section that the pattern model and the mixing-length model for the two-phase flow give sufficiently accurate results for the pressure distribution along the geothermal well. The pattern model predicts the mass flow rate curve very well, especially in the near-critical range where the available power at the wellhead is high. The mixing-length model also predicts this portion of the curve well; it is believed that the predictive power of the latter model can be enhanced by a more accurate knowledge of the density distribution and the relevant parameters. It must be pointed out though, with respect to comparisons, that the measurements in geothermal wells are subject to a high degree of uncertainty.

The pattern flow model is easier to use and the computer algorithm associated with it is much easier to understand physically and to adapt to a given set of conditions. In addition, it involves straightforward calculations and takes less computer time. Therefore, it can be applied with confidence to any geothermal well.

The mixing-length model is more difficult to use in practical cases because of the numerous iterations; furthermore, it takes more computer time and some parameters employed in it must be determined very accurately. However, it gives fairly

accurate results and it is thought that improved accuracy in the parameters would enhance its predictive power. The most important parameter for this purpose is the quality of the steam at the center γ^* , as mentioned in Section IV.3. A sure determination of this parameter requires further experimentation with actual one-component, two-phase flows. The density distribution function across the pipe also needs to be determined accurately. It is believed that two parameters (β and n) are sufficient to describe the distribution and that one of them must be supplied by experiment. The other will be determined by the continuity equation. An accurate determination of the density distribution will yield in turn an accurate expression for the velocity distribution. These two distributions are extremely important in the analysis of two-phase flow. Another topic of obscurity at the present time is the importance of three-dimensional effects (vortices). Studies in this field will give a better understanding of the liquid film surrounding the walls of the well. It is believed that the mixing-length hypothesis will provide a very useful model for the analysis of two-phase flows when the above improvements are completed.

References - Bibliography

- [1] Bankoff, S.G. A variable density single-fluid model for two-phase flow with particular reference to steam-water flow. Trans. ASME J. of Heat Transfer, November 1960, pp. 265-272.
- [2] Bergles, A.E. and Rosenow, W.M. Theoretical motion of forced convection surface boiling heat transfer. Trans. ASME Ser C J. of Heat Transfer, Aug. 1964, p. 365.
- [3] Bilicki, Z. Analiza energetyczna struktur przepływu dwufazowego w adiabatycznych kanałach pionowych. Institute of Fluid Flow Machinery, Polish Academy of Science, Gdansk, 44/914/78, 1978, Vol. I and II.
- [4] Bilicki, Z., Kestin J. and Michaelides, E. Theoretical model of geothermal wells. Brown University Report No. GEOFLO/5. To be published.
- [5] Cole, R. Motion of vapor bubbles in saturated liquids. A.I.Ch.E. Journal, 13, 2, March 1967, pp. 403-404.
- [6] Considine, D.M., Editor-in-chief. Energy technology handbook. McGraw-Hill Book Co.
- [7] Duckler, A.E., Wicks, M. and Cleveland, R.G. Frictional pressure drop in two-phase flow. A. A comparison of existing correlations for pressure loss and holdup. A.I.Ch.E. Journal, 10, 1, January 1964, pp. 38-43.
- [8] Duckler, A.E., Wicks, M. and Cleveland, R.G. Frictional pressure drop in two-phase flow. B. An approach through similarity analysis. A.I.Ch.E. Journal, 10, 1, January 1964, pp. 44-51.
- [9] Elliot, D.G. Production well flow. 4th Meeting of the Centers for the Analysis of Thermal/Mechanical energy conversion concepts. Brown University report No. CATMEC/3, COO-4051-4, Feb. 1977.
- [10] Facca, G. The structure and behaviour of geothermal fields. Geothermal Energy, ed. by H.C.H. Armstead, UNESCO, Paris, 1973.
- [11] Ginoux, J.J. (ed.) Two-phase flows and heat transfer. Series in thermal and fluids engineering, Hemisphere, New York, 1978.
- [12] Griffith, P. and Wallis, G.B. Two-phase slug flow. Trans. ASME, J. of Heat Transfer, August 1961, pp. 307-320.

- [13] Hewitt, G.F. and Hall-Taylor, N.S. Annular two-phase flow. Pergamon Press, Oxford, 1970.
- [14] Hildebrand, J. H. and Scott, R. L. Regular solutions. Prentice-Hall, Englewood Cliffs, N.J., 1962.
- [15] Hsu, Y.Y. and Graham, R.W. Transport processes in boiling and two-phase systems. Hemisphere, Washington, 1976.
- [16] Hughmark, G.A. Holdup in gas liquid flow. Chem. Eng. Progress, 58, 4 April 1962, pp. 62-65.
- [17] Kakac, S. and Mayinger, F. Two-phase flows and heat transfer vol. I. Hemisphere, Washington, 1977.
- [18] Keenan, F.G., Keyes, F.G., Hill, P.G. and Moore, J.G. Steam tables (International edition-metric units). Wiley, 1969.
- [19] Kestin, J. A course in thermodynamics. Blaisdell Publishing Co.
- [20] Kestin, J., 1978. Available work in geothermal energy. Brown University report No. CATMEC/20, D.O.E. No. COO-4051-25, Providence, RI.
- [21] Kruger, P. and Otte, C. Geothermal energy, Resources - Production - Stimulation. Stanford Univ. Press.
- [22] Kutateladze, S.S. Boiling heat transfer. Int. J. of Heat and Mass Transfer, 4, 1961, p. 31.
- [23] Levy, S. Prediction of two-phase critical flow rate. Trans. ASME, J. of Heat Transfer, February 1965, pp. 53-58.
- [24] Levy, S. Prediction of two-phase pressure drop and density distribution from mixing length theory. Trans. ASME, J. of Heat Transfer, May 1963, pp. 137-152.
- [25] Levy, S. Steam slip - theoretical prediction from momentum model. Trans. ASME, J. of Heat Transfer, May 1960, pp. 113-124.
- [26] Madejski, J. Wymiana ciepła przy wrzeniu i przepływy dwufazowe, 2, Information bureau of nuclear energy, Warsaw, 1973.
- [27] Mendelson, H.D. The prediction of bubble terminal velocities from wave theory. A.I.Ch.E. Journal, 13, 2, March 1967, pp. 250-253.

- [28] Milora, S.L. and Tester, J.W. Geothermal energy as a source of electric power. The MIT Press, Cambridge, Mass., 1976.
- [29] Moissis, R. and Griffith, P. Entrance effects in a two-phase slug flow. Trans. ASME, J. of Heat Transfer, February 1962, pp. 29-39.
- [30] Moissis, R. The transition from slug to homogeneous two-phase flow. Trans. ASME, J. of Heat Transfer, November 1963, pp. 366-370.
- [31] Moore, M.J. and Sieverding, C.H. Two-phase steam flow in turbines and separators. Hemisphere, Washington, 1976.
- [32] Prandtl, L. Über die ausgebildete turbulenz. ZAMM 5, 136-139 (1925).
- [33] Schlichting, E. Boundary layer theory, transl. by J. Kestin, 6th ed., McGraw-Hill.
- [34] Taitel, Y. and Duckler, A.E. A model for predicting flow regime transitions in horizontal and near horizontal gas-liquid flow. A.I.Ch.E. Journal 22, 1, January 1976, pp. 47-55.
- [35] Van Driest, E.R. On turbulent flow near a wall. Paper 12 Proc. 1955 Heat Trans. Fluid Mech. Inst., University of California at Los Angeles, 1955.
- [36] Wahl, E.F. Geothermal energy utilization. J. Wiley and Sons, New York, 1977.
- [37] Wallis, G.B. Use of the Reynolds flux concept for analysing one-dimensional two-phase flow. J. of Heat and Mass Transfer, 11, pp. 445-472.
- [38] Wehlage, E.F. The basics of applied geothermal engineering. Geoth. Information Services, 1975.
- [39] White, D.E. Characteristics of geothermal resources; in Geothermal Energy ed. by P. Kruger and C. Otte, Stanford University Press, 1973.
- [40] Zuber, N. and Hench, J. Steady state and transient void fraction of bubbling systems and their operating limits. G.E. Rpt. No. 62GL100, 1962.

Appendix I.

The first 19 lines give the boundary conditions at the bottom and characteristics of the stratum. Some values are redundantly specified in case the program runs many times. Rmf is the mass flow rate, sdhl the initial enthalpy loss, q heat loss, b1 and b2 the temperature slopes of the stratum, sq the total heat loss, pstep the incremental step in pressure, t the bottom temperature, d the diameter, gcr the value for critical flow, k the number denoting the flow regime, z the height, x the quality, acc and acco the acceleration components of the pressure drop.

DZ=0.0

M=0.

RMF=40.

2 SDHL=0.0

4 Q=0.0

B1=0.022222

B2=0.3

SQ=0.0

DEPTH=2433

PSTEP=1.

T=168

D=0.391

GCR=1000.

K=0

P=225.4-0.21*RMF

225.4 is the bottomhole pressure and 0.21 the drawdown

These values are subject to alteration for different wells

$$Z=0.0$$

$$X=0.0$$

$$RM=0.$$

$$ACC=0.0$$

the next 46 lines determine the properties of the fluid

$$5 \quad CP = [(0.2796E-6 * T - 0.122771E-3) * T + 0.20121E-1] * T + 3.1357$$

the specific heat of the liquid

$$DQT = DQTO * DZ / (CP * DEPTH)$$

drop of temperature due to heat losses

$$DHL = DHLO * DZ / DEPTH$$

drop of enthalpy

$$12 \quad SDHL = SDHL + DHL$$

IF (K,GE,1.) GO TO 7

$$T = T - DQT$$

$$DHL = 0.0$$

$$7 \quad PSAT = EXP(((0.21913E-6 * T - 0.17816E-3) * T + 0.653665E-1) * T - 4.96087)$$

saturation pressure

C P AND PSAT ARE IN BARS

$$R2 = PSAT * 100. / ((-0.1296E-2 * T + 0.6325) * T + 121.05)$$

$$9 \quad R1 = (-2.47524E-3 * T - 0.2030) * T + 1003.8$$

density of vapor and liquid

$$HFG = ((-0.3797E-4 * T + 0.11799E-1) * T - 4.04181) * T + 2582$$

latent heat

$$T0=273.15+T$$

$$DP=(8.315E-5*1.8*T0*RM)/((1./R2)-(1./R1))$$

depression of saturation pressure

$$PBS=PSAT-DP$$

$$DT=(1.4967E-2*T0**2.)*RM/HFG$$

elevation of saturation temperature

$$TH=647.27-T-273.15-DT$$

$$VO=(3.1975-0.315154*TH**0.3333-1.203374E-3*TH+7.48908E-13*TH**4.$$

$$C/(1.+0.1342489*TH**0.3333-3.946263E-3*TH)$$

$$RK=(-13.644+13.97*VO)*(3.1975**2.)/(3.1975*VO)**2.$$

$$PHIST=-167.219+448.55*VO-261.07*VO**2.$$

$$PPHI=PHIST+RK*SQRT(RM)$$

$$DM=(1000.+RM*58.44)*1000./(1000.*VO+RM*PPHI)$$

density of solution

$$30 \quad CORR=4.184/(1000.+RM*58.44)$$

$$X1=1000./(1000.+RM*58.44)$$

$$X2=1.-X1$$

$$PSI=[(-0.49E-4*RM**2.+0.71924E-3*RM+0.17965E-2)*T**3.+(0.24967E-1$$

$$C*RM**2.-0.36524*RM-0.90963)*T**2.+(-4.685607*RM**2.+68.577*RM+$$

$$C166.58)*T+(286.96*RM**2.-4080*RM-9633.6))*CORR$$

$$HLS=[(-0.83624E-3*T+0.16792)*T-25.9293]*T*4.184/58.44$$

$$HLW=[(0.12453E-T-0.45137E-2)*T+4.81155]*T-29.578$$

enthalpy of salt and water

$$HL=X1*HLW+X2*HLS+RM*PSI-SDHL$$

enthalpy of solution

$$HG = (-0.81257E-2 * T + 3.65228) * (T + 2388.4)$$

enthalpy of vapor

$$DH = HG - HL$$

$$SIGMA = ((0.5779 - T/647.15) ** 1.256) * (0.6388 - T/1035.4) * 0.2358$$

surface tension

$$20 \quad AC = 0.3324E-1 * RM + 0.3624E-2 * RM ** 2 - 0.1879E-3 * RM ** 3,$$

$$BC = ((-0.702E-3 * RM + 0.102E-1) * RM - 0.3961E-1) * RM$$

$$VIST = (((0.138598E-6 * T - 0.148587E-3) * T + 0.595723E-1) * T - 11.0489) * T$$

$$C + 927.7) * 1E-6$$

viscosity of water

$$VC = AC + BC * ALOG(VIST/1002E-6) / 2.30258$$

$$V = EXP(2.30258 * VC) * VIST$$

viscosity of solution

C V IS IN PA*SEC

IF (K.GE.1) GO TO 55

liquid flow

$$77 \quad X = 0.0$$

$$PHI = 0.0$$

void fraction

IF (PBS.GT.P) GO TO 50

$$HLTO = HL + 0.1$$

GO TO 190

$$50 \quad K = 1$$

```
WRITE(2,51)
51  FORMAT(4X,'TRANSITION TO BUBBLE FLOW')
55  X=(HLTO-HL)/DH
    WS1=RMF*(1.-X)/(DM*AR)
*liquid superficial velocity*
    IF (X.GE.1.)GO TO 216
    WS2=RMF*X/(R2*AR)
*vapor superficial velocity*
    GO TO (99,115,135,155) ,K
*goes to the appropriate flow regime*
*bubbly flow*
99  F1=(2.*SIGMA/(D*DM)+9.81*D/2.)**0.5
    F2=(1.-X)*F1*R2/(X*WS1*DM)
    F3=(1.+(1.-X)*R2/(X*DM))
    PHI=(-F3+(F3**2.+4.*F2)**0.5)/(2.*F2)
    AL=0.886/(PHI)**0.5-1.
    XI=0.023*(V/(D*WS1*DM))**0.25
    WS3=WS2/PHI-WS1/(1.-PHI)
    EL=0.2*(0.05*ABS(WS3)*(1.-PHI)**0.7/(WS1*(XI)**0.5))**1.5
*criterion for transition*
    IF (EL.GT.AL) GO TO 114
    GO TO 190
*slug flow*
114 K=2
    WRITE (2,113)
    FROUDE=WS2/(9.81*D)**0.5
```

```
113  FORMAT (4X,'BUBBLE TO SLUG TRANSITION')
      PHIF=PHI/(1.06/(1.2*(WS1/WS2+1.))+0.25/FROUDE)
115  FROUDE=WS2/(9.81*D)**0.5
      PHIR=1.06/(1.2*(WS1/WS2+1.))+0.25/FROUDE)
      PHI=PHIR*PHIF
      PHISL=PHI
      XSL=X
      WB=((0.3014*D*ABS(WS2/PHI-WS1-WS2)*(DM)**0.5/V**0.5)**0
C.666667)
      DELTA=1.5*(3*V*WB/(DM*9.81))**0.5
*first criterion*
      CRIT1=SQRT(SIGMA/(DELTA*R2))+WB-WS2
      IF (CRIT1.LE.0.0) GO TO 130
      WS3=ABS(WS2/PHI-WS1/(1.-PHI))
      XI=0.023*(V/(WS1*D*DM))**0.25
      EL=1.2*(0.05*WS3*(1.-PHI)**0.7/(WS1*(XI)**0.5))**1.5
      AL=7.*(1.-PHI)/PHI
*second criterion for transition*
      CRIT2=AL-EL
      IF (CRIT2.LE.0.0) GO TO 130
      GO TO 190
*froth flow*
130  K=3
      WRITE (2,131)
```

```
131  FORMAT (4X,'SLUG TO FROTH TRANSITION')
135  PHIAN=1./((R1**0.5*WS1)/(R2**0.5*WS2)+1.)
      PHINS=1./(((1.-X)*R2)/(X*DM)+1.)
      CONNS=(PHISL-PHIAN)/(PHINS-PHIAN)
      PHI=(1.-CON)*PHIAN+CON*PHINS
      XNS=X
      CRITFA=PHI*(9.81*D*(DM-R2)/R2)**0.5
      *criterion for transition*
      *F(CRITFA.LE.WS2) GO TO 150
      GO TO 190
*annular flow*
150  K=4
      WRITE (2,151)
151  FORMAT (4X,'FROTH TO ANNULAR TRANSITION')
155  PHIAN=1./((DM**0.5*WS1)/(R2**0.5*WS2)+1.)
      PHINS=1./(((1.-X*R2)/(X*DM)+1.
      CON2=CONNS*X/(XSL-1.)-CONNS/(XSL-1.)
      PHI=(1.-CON2)*PHIAN+CON2*PHINS
160  FORMAT (1H+,4X,'ANNULAR FLOW')
      GO TO 190
190  R=((1.-X)/(1.-PHI))**2
      *the two-phase friction factor ratio*
      AR=0.7854*D**2
      VEL=(RMF)/(AR*DM)
      ACC=VEL**2.*DM
```



```
IF (K.LT.1) GO TO 196
ACC=VEL**2.*DM*((1.-X)**2./(1.-PHI)+X**2.*R2/(PHI*DM))
196 RE=(D*VEL*DM/V)
*reynolds number*
FR=R*0.158*(VEL**2.)*DM/(D*(RE**0.25))
*friction factor from Blasius solution*
DENS=(1.-PHI)*DM+PHI*R2
*density of geothermal fluid at given point*
PF=(DENS*9.81+FR+ACC-ACCO)*1.E-5
C ACC-ACCO IS THE CHANGE OF MOMENTUM
C PF IS PRESSURE DROP PER METER IN BAR/M
ACCO=ACC
DZ=PSTEP/PF
Z=Z+DZ
IF (Z.GE.DEPTH) GO TO 210
P=P-PSTEP
TSTEP=PSTEP*TO*((1./R2)-(1./R1))*100./HFG
*temperature change from Clapeyron equation*
IF (K.LT.1) GO TO 191
T=T-TSTEP
*the next 8 lines calculate the critical flow for water*
10 VL=1/((-2.47524E-3*T-.2030)*T+1003.8)
VG=((-0.1296E-2*T+0.6325)*T+121.05)/(100.*P)
IF (VG.LE.0.0) GO TO 200
```

```
Y=SQRT(VG/VL)
GAMA=(CP*(VG-VL)*(T+273.15))/(HFG**2)
BETA=(VG-Y*VL)*Y*(Y*X+L-X)+Y*(Y-1)*(Y*VL+Y*X*VL+X*VG)
GCR=3.14159*D**2.*SQRT(1000*Y**2/(BETA*GAMA))/4.
IF (GCR.LT.RMF) GO TO 200

*the next 7 lines calculate the heat losses in dz*
191 TW=313.8-B1*Z
    IF (Z.LT.620) GO TO 230
    TW=300-B2*(Z-620.)
230 THICK=0.0508
    U=2*1.2*ALOG((D+THICK+0.0127)/(D+0.0127))/(D+0.0127)
    Q=U*DZ*3.14*(D+THICK+0.0127)*(T-TW)
    SQ=SQ+Q
C Q AND SQ ARE IN J/SEC
    WRITE (2,192) Z,P,T,X,FR,PF,DENS,PHI,DHL,SDHL,RMF,D
192 FORMAT (1X,F8.2,2X,F8.3,2X,F8.3,2X,F8.5,2X,F8.1,2X,
    CF8.4,2X,F8.3,2X,F8.4,1X,f10.4,1X,F10.3,1X,F5.1,1X,F9.5)

*this is the output of the program*
*additional variables can be added*
    GO TO 5
200 WRITE (2,302)
302 FORMAT (4X,'CRITICAL FLOW, CALCULATION STOPS')
210 WRITE (2,215)
215 FORMAT (4X,'WELL ENDS',/)
    GO TO 4
```

216 WRITE (2,217)

217 FORMAT (4X,'SUPERHEATED STEAM')

220 CONTINUE

STOP

END

Appendix II.

The velocity of the fluid adjacent to the wall is zero because of intermolecular forces (no-slip condition). Therefore, the velocity function attains small values and the flow regime is laminar. At some distance from the wall the laminar contribution to the shear stress is considerably higher than the turbulent contribution and the region is called the laminar sub-layer; here the velocity profile is linear and obeys the equation:

$$u^+ = \frac{yV^*}{\nu_1} . \quad (A-1)$$

At further distances a transition region develops where the turbulent effects are of the same order of magnitude as the laminar effects and even further purely turbulent friction predominates in the flow. For the latter the velocity distribution was found from experiments to be given by the following equations:

$$u^+ = 2.5 \ln \frac{yV^*}{\nu_1} + 5.5 \quad \text{for smooth pipes} , \quad (A-2)$$

and

$$u^+ = 2.5 \ln \frac{y}{k} + 8.5 \quad \text{for rough pipes} . \quad (A-3)$$

The onset of turbulence is characterized by the values η_0 and y_0 of eqn. (III.4.2), and the two formulae above yield for y_0 :

$$y_0 = 0.111 \frac{\nu_1}{V^*} \quad (\text{smooth}) , \quad (A-4)$$

$$y_0 = 0.0334 k \quad (\text{rough}) . \quad (A-5)$$

The two expressions when added give an expression for commercial pipes:

$$y_o = 0.111 k \left(\frac{1}{R^*} + 0.3 \right) , \quad (A-6)$$

where

$$R^* = \frac{kV^*}{v_1} .$$

It can be seen in the above equation that if $1/R^* \ll 0.3$, the roughness effects are predominant in the pipe which can be characterized as rough, while in the inverse case the pipe is smooth. This value of y_o when substituted in eqn. (III.4.2) will yield the appropriate equation for y_o :

$$\eta_o = \ln \frac{r_o}{y_o} = \ln \frac{r_o}{k} - \ln 0.111 - \ln \left(\frac{1}{R^*} + 0.3 \right) . \quad (A-7)$$

Appendix III.

FORTRAN IV PROGRAM FOR THE COMPUTATIONS

11 72 *This program is composed of three main parts: the first is the master program, contains information about the flow, the properties of the fluid and the characteristics of the well; the second is a subroutine designated to perform the iterative calculations for the determination of the VARIABLES $|F|$ AND $|BETA|$ the third part is a subroutine also called by the previous one and its function is to perform the numerical integrations for the solution of the equations.*

REAL L1

COMMON K,BE,FR,XO,L1,RMO,AO,M

V1(T) = ((-0.6608E-10*T+0.4557E-7)*T-10.69048E-6)*T+973E-6

H1(T) = ((0.12453E-4*T-0.45137E-2)*T+4.81155)*T-29.578

HFG(T)=((-0.3797E-4*T+0.11799E-1)*T-4.04181)*T+2582

R2(T)=P*100./((-0.1296E-2*T+0.6325)*T+121.05)

R1(T)=(-2.47524E-3*T-0.2030)*T+1003.8

RMF=50.

T=270.36

P=55.93

ACC1=0.

C0=0.

A0=1.

C=0.0

D=0.178

G=9.81
DEPTH=700.
Z=478.2
L1=7.
DS=0.1
DP=1.
ACC=0.0
HO=1316.3
RU=0.000
FR=4.63
FTP=0.02
N=0
X1=0.07754
PHI=0.6955
M=2
AR=3.14159*(D**2.)/4.
9 C=RMF*AO/(R1(T)*AR)
RD=R2(T)/R1(T)
10 RMO=RMF/(R1(T)*AR*(2.*G*D)**0.5)
RE=4.*RMF/(3.14159*D*V1(T))
AO=1./(1.-PHI+RD*PHI)
XO=3.4-ALOG(RU+18.9/(RE*(FTP*AR)**0.5))
CALL ITTE (SG,SW,A1,RM,FTP)
IF (L1.GT.100.) GO TO 120
IF (BE.GT.(0.8)) GO TO 112

```
11  DZ1=(DP*100000)-(R1(T)*(C-C0)/A0)
    DZ2=G*R1(T)*((1.-BE*L1/(L1+2.))+1./(FR**2.))
    DZ=DZ1/DZ2
    IF (DZ.LT.4.) GO TO 120
    Z=Z+DZ
    P=P-DP
    DT=-DP*1E2*(T+273.15)*(1./R2(T)-1.R1(T))/HFG(T)
    T=T+DT
    X1=(H0-H1(T)-G*Z/1000.)/HFG(T)
    C0=C
    PHI=1./((1.-X1)/X1*RD+1.)
    WRITE (2,70) Z,DZ,SW,SG,RE,XO,RD,PHI
70  FORMAT (8E14.6)
    WRITE (2,80) P,DP,T,DT,X1,RM,A1
80  FORMAT (7E14.6,/)
    IF (Z.GT.DEPTH) GO TO 110
    GO TO 9
110 WRITE (2,176)
176 FORMAT (3X,'WELL ENDS')
    GO TO 120
112  WRITE (2,86)
86  FORMAT (3X,'BMAX EXCEEDED')
    M=2
    GO TO 11
```


120 CONTINUE

STOP

END

SUBROUTINE ITTE

SUBROUTINE ITTE (SG,SW,A1,RM,FTP)

REAL L1

COMMON K,BE,FR,XO,L1,RMO,AO,M

GO TO (1,130),M

1 CONTINUE

N=0

L1=7.

AF=6.

BE=(1.-(1./AO))* (L1+2.)/L1

20 CALL SIMP (S1,S2)

RM=S2*2.5/8.**0.5

A1=S1/S2

65 FORMAT (3X,4E12.6)

IF (N.GT.20) GO TO 100

IF (ABS(A1-A0).GT.ABS(A1P-A0)) GO TO 30

A1P=A1

DA=ABS((A1-A0)/AO)

BE1=BE

IF (DA.LT.0.04) GO TO 30

```
25  BE=BE+(1.-BE)*(A0-A1)/(AF-A1)
    GO TO 20
30  FR=FR*(RM/RMO)**0.5
    CALL SIMP (S1,S2)
    RM=S2*2.5/8**0.5
    A1=S1/S2
    IF (ABS(RM-RMO).GT.ABS(RM1-RMO)) GO TO 25
    RM1=RM
    DRM=ABS((RM-RMO)/RMO)
    FR1=FR
    IF (DRM.LT.0.04) GO TO 40
    GO TO 30
40  DA=ABS((A1-A0)/A0)
    IF (DA.LT.0.02) GO TO 50
    GO TO 25
130 BE=0.8
140 L1=L1+1.
    CALL SIMP (S1,S2)
    RM=S2*2.5/8.**0.5
    A1=S1/S2
    IF (ABS(A1-A0).GT.ABS(A1P-A0)) GO TO 145
    A1P=A1
    GO TO 140
145 L1=L1-1.
```

```
FR=FR*(RM/RM0)**0.5
CALL SIMP (S1,S2)
RM=S2*2.5/8.**0.5
A1=S1/S2
IF (ABS(RM0-RM).GT.ABS.(RM1-RM0)) GO TO 50
RM1=RM
DRM=ABS((RM-RM0)/RM0)
FR1=FR
IF (DRM.LT.0.02) GO TO 50
GO TO 145
50  WRITE (2,70)
70  FORMAT (3X,'SUCCESSFUL ITTERATION')
    WRITE (2,80) FR,BE,RM,A1,L1
80  FORMAT (3X,'F=',E12.6,3X,'BE=',E12.6,3X,'MR=',E12.6,3X,'A='
    CE12.6,3X,'N=',E12.6)
    SW=A1/FR**2.
    SG=A1*(1.-(BE*L1/(L1+2.)))-1.
    FTP=1.28/(FR**2.*S1*S2)
    WRITE (2,90) SW,SG,FTP,S1,S2
90  FORMAT (5X,'SW=',E12.6,3X,'SG=',E12.6,3X,'FTP=',E12.6,2E15.6,/)
100 CONTINUE
    RETURN
    END
```

SUBROUTINE SIMP

SUBROUTINE SIMP (S1,S2)

REAL L1

COMMON K,BE,FR,XO,L1,RMO,AO,M

H1(X)=1.-EXP(X-XO)

H2(X)=1./(1.-BE*((1.-EXP(X-XO))**L1-1.))

H3(X)=1.-BE*(1.-2./(L1+2.)*(1.-EXP(X-XO))**L1)

H4(X)=1./(FR**2.)-(2.*BE/(L1+2.))*((1.-EXP(X-XO))**L1-1.)

H5(X)=(ABS(H2(X)*H4(X)*H1(X)))**0.5

H6(X)=SIGN(1.,H2(X)*H4(X))

F1(X)=H6(X)*H5(X)*H1(X)**2.

F2(X)=H6(X)*H5(X)*H1(X)**2.*H3(X)

2 CONTINUE

X=0.

S1=0.

S2=0.

5 DX=0.2

G1=DX/6.*(F1(X)+4.*F1(X+DX/2.)+F1(X+DX))

G2=DX/6.*(F2(X)+4.*F2(X+DX/2.)+F2(X+DX))

84 FORMAT (3E15.7)

A1=H1(X)

A2=H2(X)

A3=H3(X)

A4=H4(X)

A5=H5 (X)

A6=H6 (X)

A7=F1 (X)

A8=F2 (X)

95 FORMAT (8E14.6,/)

85 FORMAT (3X,3E15.6)

S1=S1+G1

S2=S2+G2

X=X+DX

IF (X.GE.(XO-DX-0.1)) GO TO 8

GO TO 5

8 CONTINUE

RETURN

END

Acknowledgements

I wish to express my deepest gratitude to my advisors at Brown University, Professors J. Kestin and P. F. Maeder, for their constant encouragement and help during the preparation of this thesis. A lot of thanks are due to Dr. Z. Bilicki who taught me tennis as well as the theory of two-phase flows. I am also indebted to the members of the geothermal energy research group, and especially to Professor Dobbins, Dr. DiPippo and Dr. Khalifa for the many helpful discussions. At this point I also wish to thank my teachers at all levels of education who contributed to my education and and professional training. Last but not least, I thank Mrs. J. Pion for patiently and competently typing the manuscript.