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Evaluation of Scanning Procedures in Terms of Information Theory

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Abstract

While a number of efforts have been made to analyze scanning procedures in terms of information theory, the formulation of scanning resolution using modulation transfer has enlarged the scope and flexibility of the subject and provided additional avenues of approach. It becomes possible to compute the amount of information which can be transmitted in any space frequency component of the image in the presence of the statistical fluctuation of radioactive decay. Manipulation of scanning data by background subtraction, contrast enhancement, data blending, spatial filtering, et cetera, either reduces the information density in the scan or leaves it unchanged, but never increases it, and under conditions where viewing and read-out parameters are not controlling factors, the information density appears to be a basic limitation on the attainable scanning image quality. The basic formulation assumes an information cell in the image as one square wave length of the space frequency component concerned. The received message is defined as the difference between the counts collected in the high half of the cell: 0 to π ; and in the low half: π to 2π . The transmitted message, unperturbed by statistical fluctuation (noise) is the same difference expressed as a true mean count. Shannon's expression for the transmission of information in the presence of noise may then be applied: $I = (H(y) - H_x(y))$; that is, the transmitted information is equal to the entropy of y , the received message, minus the entropy of y , when x , the transmitted message, is known. This leads to a simple expression in mean count density, system modulation transfer function, and space frequency which is closely related to the statistical figure of merit proposed by Dewey and Sinclair, and by Beck. The response to a sine wave test

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pattern with constant average count density and unit modulation provides an instrumental performance criterion in which actual scans and computed information densities correlate in a meaningful way, and system optimization in terms of information density determines uniquely the space frequency, collimator radius of view, and mean count density. The extension of this method of analysis to real objects appears reasonable and possible.

Introduction

The basic purpose of a mathematical description of a practical engineering process is to weave together the associated common sense and intuitive notions into a quantifiable pattern that hopefully will generate further insight into the problem and permit further refinement of the process. Some basic notions in connection with scanning are:

1. A coarse focus detector that will depict a large organ such as the liver fairly well is not of much use for scanning a small organ such as the thyroid.
2. In a low count, low contrast situation such as parathyroid scanning, even though the object being scanned is very small, it is hopeless to use anything but a high sensitivity, coarse focus collimator.

In the present report an attempt is made to incorporate such widely diverse requirements into a consistent mathematical theory using the methods of the information theory as developed by Shannon [1,2]. While a number of efforts in this direction have been reported [3,4,5,6,7,8], the formulation of scanning resolution in terms of modulation transfer by Beck [5,9,13] has enlarged the scope and flexibility of the subject considerably and provided additional avenues of approach.

Mathematical Development

A general definition of information in the limited technical sense might be stated as follows: Prior to an observation a certain amount of uncertainty exists. Following the observation this uncertainty is reduced. The amount of information associated with the measurement is then related to the reduction in uncertainty brought about by the measurement. Quantitatively, information is represented in units similar to those of negative entropy, and a gain in information may be regarded as a gain in negative entropy. In mathematical terms, consider that prior to an observation, N possible alternatives exist each with its associated probability p_i , where

$i = 1, \dots, N$ and $\sum_{i=1}^{i=N} p_i = 1$. The entropy associated with this array of probabilities is $\sum_{i=1}^{i=N} p_i \log p_i$. In the case of equiprobable possibilities where all the p_i are the same, the expression for the negative entropy takes on a simple form: $\log N$, i.e., the logarithm of the number of possibilities. If we know before the observation that there are 64 equiprobable possibilities, and if after the observation the number of possibilities is just 1, i.e., the observed number, then we know precisely which possibility we are dealing with. We have dispelled the uncertainty associated with the negative entropy of the 64 equiprobable possibilities, or $\log 64$. If we take the logarithm to the base 2 this becomes, in familiar terms, 6 bits of information gained.

The relevant approach to the scanning problem would appear to be in terms of the transmission of information through a noisy channel. Here we present a known message to the scanning system input and subsequently compare this to the system output. If there is no difference, we say that all information presented to the system has been transmitted. However, if for example, we present to the system a 6 bit message containing one of 64 possibilities and if after transmission through the system, it is not possible to distinguish whether the number K was the original message, but with equal probability it might also have been, say, $K + 1$, then there is a residual uncertainty in the received message with an entropy of $\log 2$, and one bit is lost in transmission. In a scanning system the loss of information in the process of transmission is due to the addition of statistical noise to the original transmitted message. The radioactive decay process is thus included in the channel through which the nuclide distribution in the tissues is mapped onto the scan record. The loss of spatial resolution due to imperfect collimation may theoretically be completely corrected in the total absence of statistical noise when the system modulation transfer function is known and differs from zero. Consequently, we regard this loss of resolution as distortion rather than noise.

The performance criterion that we have chosen is the capacity of the system to transmit information, or system information capacity. This, as defined by Shannon [1], refers to the limiting capacity of the system to transmit information when the information is coded in an optimal fashion. Since there appears to be no way of altering the manner in which scanning information is coded, i.e., as random pulses at varying average rates, we use the term "system information capacity" in a restricted sense referring only to this coding. The information capacity for any scanning system increases with the count rate or rather the county density in the scan since this reduces the statistical noise. The information capacity under any given circumstances is identical with the information transmitted into the resulting image, and for a performance criterion it is necessary to standardize the conditions under which the criterion is applied by using a standard test pattern.

In most scanning situations, with the notable exception of coincidence scanning of positron annihilation radiation, the limiting factor is not the count rate capability of the equipment but the loss of detector efficiency due to collimation, and one of the central problems in scanning is how best to compromise in the case of the reciprocal relationship between collimator resolution and sensitivity. In order to reduce collimator distortion by sharpening the resolution, the count rate must be markedly reduced, increasing the statistical noise. In the other direction, reducing statistical noise by sacrificing resolution leads ultimately to loss of position data, which cannot be recovered [3]. This is one of the basic problems for which a properly formulated information capacity criterion might provide a solution.

In transmitting information through a noisy channel, Shannon [1] defines the amount of information transmitted as the negative entropy of the received message minus the negative entropy of the received message when the transmitted message is known. Defining I as the information transmitted, X as the transmitted message, and Y as the received message, this is written $I = H(Y) - H(Y|X)$ where H denotes the associated negative entropy. $H(Y|X)$, the negative entropy of (Y) given X , is associated with the statistical noise in the message. In the present formulation we consider the system noise to be limited to the randomness of the radioactive decay process, which is the principal source of uncertainty in the received message. It is necessary to devise a formulation that conforms to these definitions and takes account of location and density of the isotope distribution being scanned. As in other formulations the scan is divided into elements or cells and the counts observed in each element are regarded as containing messages. The information transmitted in all elements is then summed to give the total information transmitted in the scan or divided by the area of the cell to give the information density. The use of the modulation transfer function for the description of collimator-detector performance permits the formulation of resolution parameters in terms of the count rate differences observed when scanning sinusoidal plane wave patterns of activity, while collimator-detector sensitivity may be measured in terms of the same count rates averaged over such a pattern.

We were led by these considerations to define the picture elements or information cells in terms of the individual frequency components in the image. They thus vary in size depending on the wavelength of the particular frequency component under consideration. The case with which we are concerned particularly is the visual detection of a doubtful lesion, i.e., the low contrast case near the border of detectability. This low detectability may be associated with poor statistics, low object contrast, or low image contrast as a result of poor resolution. In any case, a barely detectable sine wave is not resolvable in detail and under these circumstances a reasonable approximation is to use the average amplitude of the sine wave (smaller by a factor of $\frac{2}{\pi}$) rather than the maximum amplitude as a parameter on which to base a resolution criterion. Using this approximation, we define an information cell in the image for a given sinusoidal plane wave as a square area oriented parallel to the wave front and

encompassing one cycle from 0 to 2π of the wave in question (Fig. 1). It thus has high and low count regions, 0 to π and π to 2π , containing N_1 counts and N_2 counts respectively as in Gregg's model [10]. We define the message received by the scanning system as the difference in the number of "visible" radioactive disintegrations in the two halves of the information cell, i.e., $N_1 - N_2$. Assuming for the moment a modulation transfer function of one, the original transmitted message is defined as $N_1 - N_2$ expressed in terms of true mean count rates. This "original" message is thus entirely free from the effect of the randomness of the radioactive decay process, i.e., "noise." When the transmitted message is known, the only uncertainty in the received message is that due to random radioactive decay, and the entropy of the received message under these circumstances is the entropy of the noise, i.e., $H_X(Y)$.

The statistical uncertainty associated with radioactive decay is characterized by a Poisson distribution (for practical purposes a Gaussian distribution with $\sigma = \sqrt{\text{mean}}$) in contrast to the transmitted message, which we consider to have a flat equiprobable distribution. With the transmitted message defined as the number $(N_1 - N_2)$ we presume that any number between $(N_1 - N_2)$ and zero could have been transmitted by the system equally well in the absence of noise. In other words, the information (negative entropy) associated with the transmitted message is assumed to be $\log(N_1 - N_2)$.

The negative entropy of the received message, containing both the transmitted message and statistical noise, must be computed in order to determine the information transmitted according to Shannon's definition. The random noise distribution is formed from the two numbers N_1 and N_2 each of which has an associated uncertainty characterized by a Poisson distribution, i.e., $\sigma_1 = \sqrt{N_1}$. The resultant standard deviation of the difference $(N_1 - N_2)$ is $\sigma_D = \sqrt{\sigma_1^2 + \sigma_2^2}$ or $\sqrt{N_1 + N_2}$. Since the message is the difference $(N_1 - N_2)$, the standard deviation of the difference is in a sense independent of the difference; that is, given only the difference, it is not possible to determine the two original numbers. This approximation appears to be quite valid in the low contrast case that we are considering, i.e., where $N_1 \sim N_2$. In the high contrast case where $N_1 \gg N_2$, then $\sigma_{(N_1 - N_2)}$ approaches σ_{N_1} and the signal and noise lose their independence.

The noise and transmitted message are thus considered as two independent statistical distributions of possibilities, which must be combined to form a distribution representing the possibilities in the received message. The transmitted message is assumed to be a flat, equiprobable distribution, while the noise is essentially Gaussian. Since, in the computation of transmitted information, we deal with a difference of entropies, or a difference of logarithms, the arguments of the logarithms appear as a ratio; hence, relative values may be used as amplitudes of these distributions in constructing the entropies of the received message and noise. The standard deviations of the signal and noise distributions may be combined in the usual way $\sigma = \sqrt{\sigma_s^2 + \sigma_n^2}$ since they are assumed to

be statistically independent. An alternative approach is to substitute for the Gaussian noise distribution a flat distribution of equal entropy [1] (width = $\sqrt{2\pi e} \sigma$). Then the full widths of the distributions may be used in computing the entropies. An entirely equivalent formulation is achieved by substituting for the flat distribution of the transmitted message, a Gaussian distribution of equal entropy, and then dealing with the σ 's of these distributions as in the first example. We have chosen the first of these alternative approximations as possibly approaching the experimental situation more closely. The standard deviation of the flat equiprobable

distribution of the transmitted message: 0 to $(N_1 - N_2)$ is $\sqrt{\frac{(N_1 - N_2)^2 - 1}{12}}$.

This differs relatively little from the other approximations where

$\sigma = \frac{N_1 - N_2}{\sqrt{2\pi e}}$. In general $N_1 - N_2 \gg 1$. The negative entropy of the

received message thus becomes: $\log \sqrt{\frac{(N_1 - N_2)^2}{12} + (N_1 + N_2)}$ and that of the noise $\log \sqrt{N_1 + N_2}$. The transmitted information is thus:

$$I = \log \sqrt{\frac{(N_1 + N_2) + \frac{(N_1 - N_2)^2}{12}}{(N_1 + N_2)}} = \log \sqrt{1 + \frac{(N_1 - N_2)^2}{12 (N_1 + N_2)}}.$$

This expression is identical in form with Shannon's [2] expression

$\log 1 + (\frac{s}{n})^2$ where s and n represent signal and noise amplitudes, and

$1 + (\frac{s}{n})^2$ is the "number of distinguishable states."

The density of transmitted information in the scan record is the parameter that appears to be most relevant to look at. This has the dimensions of information per square centimeter. Since the area of the cell in the image for which the information capacity is computed is λ^2 or $\frac{1}{v^2}$, the information density $\rho_I = v^2 I$. Clearly, the system with the highest information capacity will produce an image with the highest information density, other things being equal.

In any practical situation the system modulation transfer function is not one as assumed above, but varies with space frequency, reducing the contrast in the image. Consequently, we must express N_1 and N_2 in terms of image modulation: $m_1 = m_0 \cdot \text{MTF}$

Referring to Fig. 1 which represents an information cell in the image,

$$N_1 = \frac{\bar{n}}{2v^2} \left(1 + \frac{2}{\pi} \text{MTF} \cdot m_o \right)$$

Where:

$$N_2 = \frac{\bar{n}}{2v^2} \left(1 - \frac{2}{\pi} \text{MTF} \cdot m_o \right)$$

$\bar{n}$ = mean count density

m_o = object modulation

MTF = Modulation Transfer Function

Then

$$N_1 - N_2 = \frac{\bar{n}}{v^2} \frac{2}{\pi} \text{MTF} \cdot m_o$$

$$N_1 + N_2 = \frac{\bar{n}}{v^2}$$

and

$$\rho_I(v) = v^2 \log \sqrt{1 + \frac{\bar{n}}{12} \left(\frac{2}{\pi} \frac{\text{MTF} \cdot m_o}{v} \right)^2} \quad (1)$$

In addition since $Q = \frac{(N_1 + N_2)}{\tau} \text{MTF}^2 m_o^2$ where Q is the statisti-

cal figure of merit [9] for detection of the difference between the two halves of the information cell and τ the associated observation time,

$$\rho_I(v) = v^2 \log \sqrt{1 + \frac{\left(\frac{2}{\pi}\right)^2 Q \tau}{12}}.$$

We have investigated application of these expressions in the development of a generally applicable instrumental performance criterion for which it would seem reasonable to use a test pattern in which all relevant space frequencies are presented with unit modulation ($m_o = 1$) and uniform average count density. Information density can then be computed for each space frequency at various average count densities, and the resultant curves used to describe the performance of the instrument.

Fig. 2 shows the behavior of the expression for information density when plotted for different average count densities using the same collimator-detector for looking at such a unit modulation test pattern (with a frequency range of 0.15 to ∞ cycles/cm). At low space frequencies (long wave lengths) there is low information density because of the large area of the information cell. As the space frequency rises, the information density rises to a rather broad maximum and then falls as the MTF approaches zero. Higher mean count densities give correspondingly higher information densities with a shift of the maximum toward finer resolution.

Comparison With Images

Comparison with actual scans of a unit modulation sine wave test pattern corresponding to these three curves are shown in Fig. 3. Here we

must digress briefly to consider the complex question of image quality and interpretation. The object of scanning is to produce a picture that is best able to permit the physician with the aid of his optical psychophysiological processes to make a correct diagnosis. Various forms of digital and analogue image processing, i.e., smoothing and averaging, spatial frequency filtering, edge enhancement, dot shifting, inverse spread function operations, etc., are under study by many investigators. Some read-out parameters such as those that alter the overall system MTF are directly relevant to information theory analysis. Other phenomena such as the way in which the eye scans along a linear contour and integrates out the noise, "visual autocorrelation," are less accessible to formulation. Perception of the final image in addition depends on the parameters of the display system and the eye-brain system. In the case of scanning images in general the resolution is so poor and the pictures so noisy that viewing the scan record under suitable illumination and visual solid angle introduces negligible further degradation of the overall MTF and introduces no significant further statistical noise.

In the present report the visual presentations are those currently used in our nuclear medicine laboratory [11], that is, the scan data after pulse height analysis (window of 2 X FWHM of photopeak) are recorded without background erase or contrast enhancement using one diffused dot on photographic film per detected photon. This dot is considerably smaller than the collimator field of view at the focus. The density of each dot is adjusted according to the count density to give approximately the same average film density in the scan record. This rather pragmatic system, under the circumstances of the present study, gives pictures with approximately equivalent viewing parameters for different count densities so that the limiting factor in the quality of the resultant image should be related to the information density.

A question immediately arises in connection with the two-dimensional recording of data. Adjacent closely spaced recorded dots on the scan record, when they overlap or are diffused, cannot be distinguished as individual events. A similar situation exists in the case of photographic film [12]. The loss of resolution due to this phenomenon may be treated in a manner identical with that of the detection-collimator resolution, i.e., by using the spread function or MTF, and by expressing the data in the scan pattern in terms of space frequencies rather than as dot distributions and densities. The notion presented by R. C. Jones [12] that the overlapping of recorded dots represents inter-symbol influence does not appear to us to apply to the present situation, in the same sense that the probability of the occurrence of a given letter of the alphabet is influenced by the adjacent letters. It is true that, given a count density $\bar{n}$ in a given region of a scan, the most probable count density in an adjacent region is close to $\bar{n}$. This is true inter-symbol influence. However, it is a property concerned with the information content of the object, not with the information transmission capacity of the imaging system.

The mathematical formulation so far describes the system only up to the recording operation. The effect of the recording mechanism on the system information capacity requires further consideration. Recording

each registered photon as a very small dot corresponding to the position of the detector at the time of detection introduces much high frequency noise into the record. Enlarging or diffusing the dots and allowing them to overlap reduces this high frequency noise and also the high frequency signal components. It thus seems appropriate to attempt to formulate the effect of the read-out system in terms of its modulation transfer function.

If we are to consider the effect of the recording system MTF on the signal to noise ratio, we must express the noise in terms of its frequency spectrum or Wiener spectrum. Under these circumstances when we consider a single signal frequency component as above, we can filter out noise of all other frequencies, and the only relevant noise has the same frequency as the signal. The read-out MTF thus, in the context which we are considering, appears to reduce the signal and noise amplitudes equally, leaving the signal to noise ratio unchanged and hence the system information capacity is unchanged. This rather surprising conclusion may perhaps be supported by the fact that innumerable more or less effective read-out systems exist. The implication seems clear that the read-out system should introduce no noise into the picture, and beyond this it becomes a parameter having to do with perception rather than detection.

Presumably, anything that increases the system information capacity necessarily improves the possible image quality, since without information we have no possibility of any image. It does not follow however, that the system with the highest information capacity necessarily produces the best image from the point of view of interpretation. For instance, adding a constant (not random) background certainly reduces the image contrast and may make the picture quite invisible without altering the system information capacity. The system information capacity thus appears to represent a limiting factor on the image quality. In the following paragraphs we shall consider which readout procedures may be used without reducing the system information capacity, and which processes reduce the information capacity and must in consequence be used with greater circumspection.

To return to the comparison of information density curves with actual scans, in Fig. 3 the dotted lines indicate the location of the maximum information densities beyond which the pattern rapidly fades into invisibility with higher space frequency. A better comparison may perhaps be made by looking at a single cycle since the mathematical development is concerned with a single information cell. In Fig. 4 the same three scan patterns are compared in this way with the appropriate rescaled information density curves, since the abscissa (space frequency) scale is not linear in the pattern. The maximum is again indicated and appears to correspond to the highest frequency that can be seen clearly, and the point where the pattern disappears in all three cases corresponds roughly to $1/2$ bit per information cell. While this particular value may be related to the read-out system, the correspondence at two qualitatively different, critical points between the visual and statistical data over a wide range of count densities appears to give considerable practical justification for the use of information density, as defined here, as a measure of image quality

under circumstances where visual read-out parameters impose no substantial limitation.

How does the information density vary under circumstances that might be encountered in scanning procedures? Background subtraction does not affect the difference or the statistical variation of the difference and hence does not affect the information density as long as subtraction is limited truly to background, i.e., to constant components. Similarly, contrast enhancement which is a reversible distortion of the record has no effect upon the underlying statistics and causes no change in the information density as we define it here. Data blending eliminates noise and presumably unwanted high frequency components while spatial filtering removes other unwanted components. The information density thus appears to behave properly, i.e., manipulation of the data leaves the information unchanged or decreased but never increased.

The information density curves, as presented here, possess one curious property that deserves comment, particularly in the fine resolution, low count density cases. Because the information cell becomes very small at high space frequencies, the information density remains significantly high even when the information per cell is very small, so small in fact that the probability of detecting a difference between the two halves of a single cell becomes almost negligible. Here the visual autocorrelation process mentioned above is able to make use of this low level of information distributed over many cells, increasing the effective statistical reliability to perceive to some extent the underlying pattern. Fig. 3 as compared to Fig. 4 is an illustration of this point. In some formulations [2], a factor K has been introduced into the argument of the logarithm, i.e.:

$$\rho_I = v^2 \log K \sqrt{\frac{1 + \left(\frac{2}{\pi}\right)^2 \text{MTF}^2}{12 v^2}} \quad (2)$$

which determines the degree to which distinguishable states are to be considered distinguishable. This introduces an additive constant in the expression for information density with the effect that one might say "we will not consider information retrievable from the noise until it exceeds a certain critical value." If we use the observed value of 1/2 bit per information cell as the level at which the message becomes perceivable under the present circumstances, $K \approx 1/\sqrt{2}$.

System Optimization

Increasing the collimator sensitivity to reduce the statistical noise in the scan degrades the spatial resolution and vice versa, and one of the much discussed problems in scanning as mentioned above is how best to compromise in this situation. Since the information density is a monotonic increasing function of the figure of merit and the observation time for a given space frequency, maximizing the information density is equivalent to

maximizing the figure of merit when the observation time is kept constant. The formulation in terms of information density, however, ties things together in a different way than does the figure of merit formulation and gives added insight into some questions. For instance, varying the resolution R of a collimator while keeping the crystal diameter fixed and looking at a fixed spatial frequency leads to a maximum value of the figure of merit when $2R \approx \lambda$ [9]. However, varying the spatial frequency of the object while keeping resolution constant gives the highest figure of merit at zero space frequency. The information density, however, goes through a maximum in both situations. In Fig. 5, information density curves are compared with the corresponding scans of the unit modulation test pattern using collimators of varying resolution with constant crystal diameter. The full width at half maximum of the line spread function, measured under the same conditions, is used as the measure of resolution, and the sensitivity is proportional to the $(FWHM)^2$. The abscissa scale corresponds to the space frequency of the test pattern as in Fig. 4. As one can see, this gives a unique maximum value for information density associated with an optimal space frequency and an optimal collimator resolution. The "best" pictures seem clearly to be the ones associated with the highest information density curves. Using expression (2) with $K = 1/\sqrt{2}$ and summing the information density over the effective frequency range gives relative values for total information density, again demonstrating a maximum corresponding to the "best" picture (Fig. 6). In the case of high resolution and low count density, the effect of visual autocorrelation along the linear contours is particularly striking. If all portions of the picture are excluded from viewing except a single information cell, the information per cell at the point where the pattern disappears is, as in the previous experiment, about 1/2 bit. When the whole picture is viewed, the pattern can be distinguished down to regions containing .1 to .2 bits per cell.

At higher total gamma flux from the test pattern another set of curves is defined at a higher level of information density, with the maximum shifted toward higher space frequency and finer resolution. Thus, at the optimum point, space frequency, resolution and emitted gamma flux are defined and at least in these terms, we appear to have a solution to the problem of compromising between sensitivity and resolution under conditions where the information capacity of the system is the limiting factor, and we are looking at individual space frequency components. From the point of view of an instrumental performance criterion, we can thus say that for a given mean count density $\bar{n}$ and space frequency ν there is an optimal collimator-detector of definite resolution. Sensitivity parameters of the collimator-detector may be introduced: $\bar{n} = G(1 + P + S)\eta\psi\sigma$, where σ is the equivalent sheet source intensity, η is the photopeak efficiency of the crystal, ψ is the fraction of the photopeak pulses that fall within the window of the pulse height analyzer, G is the geometric efficiency of the collimator, P and S are the penetration and scatter fractions, respectively. The effects of penetration and scatter on the modulation transfer function are described by

$$\text{MTF}(v) = \frac{\text{MTF}_G + P \cdot \text{MTF}_P + S \cdot \text{MTF}_S}{1 + P + S}$$

where MTF_G , MTF_P , and MTF_S are the modulation transfer functions associated with the geometric properties of the collimator, the septum penetration, and the scattered radiation in the tissue [13].

Discussion

Information criteria, constituted as they are in terms of signal to noise ratio may be thought of as defining necessary conditions for imaging. Their limitation lies in that they do not define sufficient conditions for optimum perception. In attempting to devise a more general approach to the description of image quality than appears to be provided by system information capacity, some further criterion of correspondence between object and image seems necessary. A system with a negative MTF, for instance, as produced by a flat point spread function will produce spurious resolution, i.e., phase reversal of some frequency components. The information is preserved, and may be reconstructed knowing the MTF, but without further processing will produce a distorted picture that may interfere with interpretation. An extreme example of such a presentation of information in distorted form is the hologram. It would seem appropriate to suggest that the optimum collimator-detector for a given situation should be the one that transmits the most information, and that subsequent image processing techniques should be directed toward optimization in terms of a criterion such as that proposed by Linfoot [14], i.e., fidelity. By this approach an image arbitrarily close to an aberration free image would be produced within the limitations of noise. This image could then be further manipulated as desired to match various of its characteristics to the visual mechanism to produce the most easily interpreted picture. Some efforts in these directions are in progress in several laboratories [15,16]. The correction procedure proposed by Iinuma and Nagai is of particular interest. These authors, by a digital iterative technique, attempt to return the signal amplitude in the image to that of the undistorted image using the known collimator spread function. This procedure ideally produces the same image as would have been produced by a collimator with an MTF of one for all space frequencies. In the process, however, under ideal conditions, the noise amplitude is increased by the same factor as the signal amplitude, so that the signal to noise ratio at each space frequency is unchanged and in consequence similarly the information capacity. Thus the choice of an optimal collimator is not affected by this correction process since it remains the collimator with the highest figure of merit or information capacity.

The principal advance in electrical communications engineering resulting from the introduction of information theory lies in the concept of matching the capacity of the information channel to the information source. The collimator optimizations discussed above represent such matching. They imply however detailed specification of the object to be scanned

and the questions to be asked before a suitable collimator may be chosen. Such considerations certainly appear to conform with the common sense notions of scanning.

Directions to pursue in future investigations might well be along the lines of determining the frequency structure of objects to be scanned, and particularly, investigation of which frequency components are most important from the point of view of perception in the presence of noise, so that instruments may be designed with these factors in mind.

Summary

In summary, a partial criterion is proposed for imaging system performance based on considerations of information theory which appear to correlate qualitatively and quantitatively, at least in simpler tests, with perceptual observations, and which appear to be amenable to extension into detailed analysis of many problems in scanning. A general conclusion appears to be that assessment of the performance of an imaging system requires detailed specifications of the object, the amount of radiation to be detected and the questions to be asked of the system. With these parameters specified, a concise mathematical formulation is presented which appears to conform to the intuitive notions associated with scanning. Although the criterion presented appears necessary for imaging, it is not sufficient, and does not permit definition of the optimal conditions for perception which appear to depend on additional parameters concerned with image processing and viewing.

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Fig. 1.--Information cell for single space frequency component. Signal is the difference in total counts ($N_1 - N_2$) in two halves of cell. Noise is proportional to the standard deviation of this difference. (See text.)

Fig. 2.--Information density curves computed from Expression 1 for same collimator (Picker No. 2107) at different average count densities, assuming unit modulation for object frequency components.

1110 CTS/CM² 276 CTS/CM² 10.8 CTS/CM²

PHOTO

Fig. 3.—Scans of unit modulation test pattern corresponding to curves in Fig. 2.

Fig. 4.—Comparison of Figs. 2 and 3, using single cycle from Fig. 3 and replotting abscissa scale in Fig. 2 to match frequency scale of the pattern.

Fig. 5.--Scans of unit modulation test pattern using collimators of varying resolution as indicated by FWHM of line spread function. Average count density is proportional to $(FWHM)^2$. Highest information density curves clearly correspond to best pictures. Note correspondence between pattern and information density, particularly at high frequency cut-off points.

Fig. 6.--Information densities summed over all effective frequency components for curves shown in Fig. 5.