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FIELDS ON THE CONFINEMENT OF
RUNAWAY ELECTRONS AND THERMAL
ELECTRON ENERGY IN TOKAMAKS

BY

H. E. MYNICK AND J. D. STRACHAN

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INFLUENCE OF STOCHASTIC MAGNETIC FIELDS ON THE CONFINEMENT
OF RUNAWAY ELECTRONS AND THERMAL ELECTRON ENERGY IN TOKAMAKS

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ABSTRACT

The ratio of the runaway electron confinement to thermal electron energy confinement is derived for tokamaks where both processes are determined by free streaming along stochastic magnetic field lines. The runaway electron confinement is enhanced at high runaway electron energies due to phase averaging over the magnetic perturbations when the runaway electron drift surfaces are displaced from the magnetic surfaces. Comparison with experimental data from LT-3, ORMAK, PLT, ST, and TM-3 indicates that magnetic stochasticity may explain the relative transport rates of runaways and thermal electron energy.

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I. INTRODUCTION

In tokamaks, both the confinement of energetic runaway electrons and the confinement of the bulk electron energy are 10+100 times less than expected by purely classical processes. Certainly, if one microscopic phenomenon is responsible for both transport processes, then one would expect that a specific relationship would exist between these two confinement times. It is the purpose of this paper to examine the possibility that free streaming along perturbed magnetic field lines is the single underlying mechanism for both runaway electron transport and bulk electron energy transport.^{1,2}

We take as a basic premise that the runaway electron population is sufficiently dilute that it does not interact directly with the background plasma. This means that since the runaway electron is essentially collisionless, it is insensitive to the background plasma (i.e., its density, temperature, and perturbations). These energetic electrons are then interesting test particles that are influenced primarily by the electric and magnetic fields associated with the microturbulence.

We attempt to describe both loss processes by a single mechanism, which means ignoring the possibility that electron heat is described by one microturbulent process while runaway electron transport is described by a second microturbulent process, or even a slightly different aspect of the first. Further, we restrict our considerations to purely magnetic microturbulence whereby both the runaway electrons and the bulk electron heat are

lost by their high parallel velocities along the stochastic magnetic field structure. Purely electrostatic perturbations are also a plausible mechanism for the transport of one or both types of electrons, although we have not considered them here. We note that electrostatic $\tilde{E} \times B$ motion is unlikely to cause runaway electron transport since the large parallel velocity of the runaway electrons allows them to be displaced only small perpendicular distances while the bulk electrons would suffer large transport.

Previous theories^{1,2} of transport due to magnetic stochasticity assumed that an electron runs along a single perturbed magnetic field line. This assumption of negligible drifts led to the expectation that the runaway electrons should be transported much more effectively than the bulk electron energy due to their higher parallel velocity, whereas experiments tend to find a runaway electron confinement longer than the bulk electron energy confinement.

A refined theory^{3,4} which takes into account particle drifts and finite Larmor radius, predicts enhanced confinement due to two principal mechanisms:

(a) In the course of a gyroperiod, a particle performs an average over the magnetic perturbation which it sees. Thus, when the Larmor radius ρ_R of a runaway becomes comparable to the linear perpendicular scale length $\delta \sim k_\perp^{-1}$ of the perturbation, this mechanism appreciably reduces the effect of the perturbation on the particle.

(b) In the course of a poloidal transit time, the guiding-center of a particle performs an average over the perturbation structure, analogous to mechanism (a). The criterion for reduced coupling between the particle and the perturbation is now $\delta \sim d_R$, where d_R is the drift-orbit displacement of the particle. Since the drift-orbit displacement (d_R) is larger than the runaway electron gyroradius, (ρ_R), effect (b) is the stronger of the two.

In addition to the rough criterion $\delta \sim d_R$ just given for the onset of reduced coupling (and concomitant confinement time enhancement) at high particle energies, the scaling of confinement time with energy at low and intermediate energies depends significantly on whether the modes comprising the turbulence have even or odd parity about their rational surfaces. The energy scaling of runaway electron confinement may thus be viewed as a probe indicating not only the width of a typical mode in the turbulence, but also the relative proportions of even and odd parity modes in the turbulence.

Comparison of the theoretical predictions with experimental results from LT-3, ST, PLT, TM-3, and ORMAK indicates that the $d_R > \delta$ enhancement condition is satisfied (while $\rho_R < \delta$) and that the ratio of runaway electron confinement to bulk electron energy confinement is reasonably explained.

II. MAGNETIC PERTURBATION THEORY

Radial transport due to magnetic perturbations scales with the square of the coupling coefficients, g , which measure the extent to which a given perturbation affects any particular particle.^{3,4}

The strength of this coupling is dependent on the characteristics of the perturbation. Because drift and tearing modes have $k_{||} \ll k_{\perp}$, the perpendicular drifts of runaway electrons can cause these particles to perform a phase average over the mode structure and reduce the coupling to the modes.

We picture this reduction schematically in Fig. 1. There we have drawn a single mode whose amplitude is appreciable in the vicinity of the radius at which the particle is localized, together with the trajectory of the guiding center of the particle.

In Fig. 1a the mode is drawn in the (θ, ϕ) plane of the flux surface of the mode rational surface, where $k_{||} = 0$. Thus, the direction of the magnetic field runs parallel to the lines of constant phase of the mode. A particle without drifts would run along a constant-phase line, thereby receiving a radial kick from that mode in the same direction at all points along the trajectory of the particle. By contrast, the perpendicular drifts in the runaway trajectory give it the oscillatory character shown in Fig. 1a. This oscillation causes the particle to experience perturbations from the mode of varying phase in the course of a transit period. For $k_{\perp} d_R \geq \pi$, this phase average will be appreciably less than the maximum amplitude of the mode.

In Fig. 1b the same wave-particle interaction is visualized, this time projected into the (r, θ) plane. Here, in addition to the phase - averaging effect depicted in Fig. 1a, there is the fact that the mode amplitude falls off as one moves a mode width δ from the rational surface. Thus, for $d_R/\delta \geq 1$, the particle only senses the mode for a fraction of its transit period,

causing a further reduction in the coupling strength of the particle with that mode. This reduction in coupling occurs for each mode, and so the total transport, arising from the sum of the incoherent contributions of all the modes, is correspondingly reduced.

The radial diffusion coefficient D of a given particle, described by particle label p , may be written^{3,4}

$$D(p) = C_1 \langle [n_w g(w, p)]^2 \rangle. \quad (1)$$

Here C_1 is a numerical constant, independent of the particle or perturbations in questions, n_w is the toroidal mode number of a contributing mode, which is specified by mode label w (standing for "wave"), and g is the coupling between wave w and particle p . (Mode label w corresponds to the "quantum numbers" describing any mode, and p may be thought of as the set of invariants which determine a particle's orbit.) The angular brackets designate an average over all modes contributing to the transport of the particle in question.

From Eq. (1) we find an expression for the ratio of the runaway confinement time τ_R to the confinement time $\tau_{RO} = (\beta c/v_e) \tau_{E_e}$ a runaway would have if, as is the case for thermal electrons (velocity v_e , confinement time τ_{RO}) particle drifts played a negligible role. This ratio is equal to the ratio L_R/L_{E_e} of the distance a runaway must travel along a field line to escape, to the corresponding distance for a thermal electron. This ratio is

$$\tau_R / \tau_{RO} = L_R / L_{E_e} = \langle (n_w g_{E_e})^2 \rangle / \langle (n_w g_{E_e} G)^2 \rangle \quad (2)$$

$$\approx 1 / \langle G^2 \rangle ,$$

where

$$G(w, p) = g_R(w, p) / g_{E_e}(w) .$$

That is, G is the ratio of the coupling coefficient including the drifts associated with the runaway electrons to the "driftless" coupling coefficient which describes the stochastic transport of thermal electrons.

In general, the turbulence will be a mixture of modes having even and odd parities about their rational surfaces, in relative proportions f_+ and f_- . We thus write

$$\langle g^2 \rangle = f_+ \langle g_+^2 \rangle + f_- \langle g_-^2 \rangle , \quad (3)$$

with $f_+ + f_- = 1$. Both g_+^2 and g_-^2 are functions of the runaway energy E_R , which determines the size of the perpendicular drifts.

The scaling of g_+^2 and g_-^2 with E_R is quite different for low energies where the particle drift orbit excursion d_R is small compared with the mode width δ . For mode w to maximize the transport of a particle, the radius r about which the particle's drift motion occurs must be about equal to the radius r_w of the mode's rational surface. In this manner the mode-particle

resonance condition is satisfied. For odd parity modes, a particle having $r \approx r_w$ and $d_R \ll \delta$ spends its time in a region where the mode amplitude is about zero, and the particle is therefore only very weakly coupled to the mode. Mathematically,

$$g^2 \rightarrow 0, (d_R/\delta \rightarrow 0). \quad (4)$$

In contrast, a low-energy particle, resonant with an even parity mode, spends its time where the mode amplitude is greatest, and so one finds g^2 is greatest for $d_R/\delta \approx 0$. (These coefficients are derived in Appendix A, where explicit expressions for $g_{\pm}$ are obtained in both the large and small limits of d_R/δ .)

Using relations (3) and (4) in (2), then, one may write

$$D_R/D_{RO} = \tau_{RO}/\tau_R = \langle g_+^2(d_R) \rangle + (f_-/f_+) \langle g_-^2(d_R) \rangle,$$

where

$$g_{\pm}^2(d_R) \equiv \langle g_{\pm}^2(d_R)/g_{\pm}^2(d_R = 0) \rangle, \text{ and } D_{RO} \equiv (v_e/\beta c) \chi_e.$$

Numerically evaluating the coupling coefficients from the expressions in Appendix A, in Fig. 2 we plot L_R/L_{E_e} versus d_R/δ for a range of values of f_+ , from $f_+ = 1$ (purely even-parity turbulence), down to $f_+ \approx 0.01$, and taking PLT plasma parameters. One sees that as f_+ decreases, one must go to higher energies (i.e., larger d_R) before L_R/L_{E_e} becomes appreciably greater than unity. Since odd modes produce no perturbation on driftless

particles, these modes influence primarily those runaway electrons which have drift orbit displacements, d_R , about equal to the mode width, δ . For f_- greater than about 0.9 and runaway electrons with intermediate energy so that $d_R \approx \delta$, L_R/L_{E_e} can be less than unity. The slight waviness of these curves is an artifact of the simple form (cf. Appendix A) used to simulate the average over modes. We regard Fig. 2 as embodying the principal theoretical point of this paper. For both the drift and shear-Alfvén branches, one finds modes with both even and odd parity. However, if one uses the linear growth rates of those modes to gauge their expected amplitudes in the plasma, one expects the drift (shear-Alfvén) branches to be predominantly odd (even) in character.^{5,6} The fractions $f_{\pm}$ would then assume an added physical significance, viz. the fractions of the turbulence comprised of drift and tearing (i.e., Shear-Alfvén) modes.

The enhanced confinement of the runaway electrons is strongly influenced by the radial width, δ , of a typical mode comprising the turbulence. L_R/L_{E_e} begins rising rapidly above unity when the runaway electron energy, E_R , is large enough that the drift orbit excursion d_R becomes larger than δ . As explained at the beginning of this section, and as illustrated in Fig. 1, a particle having $d_R > \delta$ only spends a fraction of its total transit time inside the region of mode localization, and $g_R(w, p)$ accordingly falls off with larger E_R . One therefore expects L_R/L_{E_e} to depend on d_R/δ , roughly independent of how large d_R or δ are independently.

The actual physical extent (δ) of the mode width is not determined by the transport analysis, but is taken from linear theory. For this work, we take $\delta \approx 6\rho_i$ (the ion Larmor radius)

to be the average mode width as is expected for drift or tearing modes⁵ when making our comparison with experiment. For central PLT plasma parameters, this means that $d_R \sim \bar{\omega} \rho_i$ for runaway electrons of energy $\sim 1 \rightarrow 2$ MeV.

III. TABULATION OF EXPERIMENTAL DATA

We have tabulated the experimental data (Table 1) from various tokamaks⁷⁻¹¹ in order to establish the relationship between the bulk electron energy confinement and the runaway electron confinement. The discharges for these devices are expected to be representative of normal discharges with moderately low runaway electron levels and free of runaway electron induced instabilities. However, in order to obtain measurable levels of runaway electrons, usually low electron densities were used ($\bar{n}_e \sim 1 \rightarrow 2 \times 10^{13} \text{ cm}^{-3}$ with the exception of TM-3 which had $\bar{n}_e \sim 2 \times 10^{12} \text{ cm}^{-3}$). It is assumed that there is no direct interaction between the thermal plasma and the runaway electrons due to the diluteness of the runaway electron population; however, such a result is difficult to establish experimentally, and we cannot exclude that such a direct interaction might be taking place.

The bulk electron energy confinement time, τ_{E_e} , is taken as quoted by the various devices. It should be realized that often radiative losses are also large and not excluded when τ_{E_e} was determined. Thus, we have also considered that na^2 scaling applied for the anomalous electron thermal transport (Fig. 3) as an experimental limit on τ_{E_e} (na^2 normalized to PLT). In Table I we quote a length, $\delta_{E_e} \sim 0.03 \rightarrow 0.16 \text{ cm}$ which is the

perpendicular step size required to account for the observed electron energy confinement if the electron thermal confinement is determined by parallel heat conduction along perturbed magnetic field lines. Thus,

$$\chi_e = \delta_{E_e}^2 \frac{v_e}{qR} + \delta_{E_e} = \left(\frac{3q a^2 R}{8 \tau_{E_e} \langle v_e \rangle} \right)^{1/2}. \quad (6)$$

The parallel length that the electron thermal energy then covers during the confinement is simply

$$L_{E_e} \approx \langle v_e \rangle \tau_{E_e}. \quad (7)$$

The length δ_{E_e} is a "nonlinear" perpendicular scale length, i.e., it scales with the strength of the turbulence present, thus requiring a nonlinear theory for its calculation. In contrast, one notes that the perpendicular scale length δ emerging naturally from the theory,^{3,4} viz. the radial width of a typical mode, is a linear quantity, independent of the amplitude of the perturbing fields. This independence from nonlinear quantities (which are more difficult to calculate) arises, of course, from the fact that since both L_R and L_{E_e} individually are proportional to the reciprocal of the square of the perturbing field strength, their ratio is independent of it.

For simplicity, we have taken $\langle T_e \rangle \approx T_e(0)/2.5$ since such average values are not usually quoted whereas central values are.

The drift-orbit displacement, d_R , of the runaway electrons is simply

$$d_R = \frac{17}{2R} \frac{a^2 \sqrt{\gamma^2 - 1}}{I_\phi} \quad (8)$$

where γ is the relativistic γ as determined by the runaway electron energy, E_R .

The parallel velocity of the runaway electron is βc , so that the parallel length that the runaway electron traverses while being confined for a duration, τ_R , is

$$L_R = \beta c \tau_R. \quad (9)$$

The runaway electron confinement times have been determined by different methods for the various devices such as probe measurements on LT-3⁸, plasma bremsstrahlung measurements on ST⁷, photonuclear production rates on PLT¹⁰, and orbit shift experiments on ORMAK.¹¹ The TM-3 experimental results⁹ are particularly noteworthy since they involve a perturbation in the runaway electron population which was observed to evolve in order to determine the confinement. Each of these techniques is substantially different and moreover (due to the nature of the device or diagnostic technique) often involves different energy runaway electrons. The ORMAK values are quoted as giving two values of L_R/L_{E_e} since for the same experimental conditions they obtained different results on different discharges.

IV. DISCUSSION

The comparison between theory and experiment (Fig. 4) reveals that:

1) The enhancement of the runaway electron confinement at large drift orbit displacements can simultaneously explain the otherwise discordant experimental results of very long runaway electron confinement times on ORMAK and PLT and relatively short confinement times on ST and LT-3. Experiments which have observed long confinement times have measured only those runaway electrons with large energies and thus large drift orbit displacements. In the PLT experiment, photonuclear reaction rates were used which have thresholds ≥ 6 MeV so that drift orbit displacements of ≥ 4 cm are involved. In the ORMAK experiment, pulse height analysis of the limiter X-Ray bremsstrahlung was used so that energies ≥ 8 MeV and drift orbit displacements ≥ 10 cm were involved.

2) The data is explainable by mostly even-parity turbulence with average mode width $\delta = 6\rho_i$, as expected from magnetic perturbations due to shear-Alfvén or drift waves.⁵ The ORMAK experiment seemed to obtain different experimental results on subsequent discharges, possibly indicating that the proportions of even and odd parity had changed, or that a higher level of low-n MHD activity occurred for the smaller L_R/L_{E_e} case. Since for low-n modes $d_R/\delta < 1$,¹² the enhancement mechanism would not be effective, thus yielding a lower L_R/L_{E_e} .

3) In contrast to the other experimental data, TM-3 involved a perturbation to the runaway electron population which increased the perpendicular energy of the runaway electrons. One would thus expect the finite runaway electron gyroradius enhancement mechanism to further increase L_R/L_{E_e} for TM-3 above the d_R/δ -only effect present for the other devices, and for the theoretical curves (which took E_i to be negligible). and indeed, the TM-3 points in Fig. 4 lie at higher L_R/L_{E_e} (for the same d_R/ρ_i) than do the other points.

The TM-3 ECRF experiments are useful to further test the validity of the theory. An experiment of the TM-3 type where the power (and duration) of the ECRF were varied would clarify the energy dependence.

TM-3 also differs considerably from the other experiments in that n_e is about an order of magnitude lower, raising the possibility that the plasma pressure has been reduced sufficiently that coupling of the magnetic perturbations to density fluctuations is inoperative.¹³ However, the fact that the TM-3 energy confinement time scales like na^2 in relation to the other tokamaks (Fig. 3) is suggestive that the same bulk electron energy confinement mechanism applies even at the lower density.

4) Further theoretical results are required in order to determine more precisely the appropriate value of the average mode width δ , and the proper values of f_+ and f_- . In addition we envision that the self-consistent inclusion of $\tilde{E} \times B$ drifting due to electrostatic components would be useful.

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APPENDIX A: THE COUPLING COEFFICIENTS

We here provide some additional detail on the origin and evaluation of the wave particle coupling coefficients, $g(w,p)$. This work extends that of Ref. 4, where an analytic expression for $g(w,p)$ was found, strictly valid only in the small (d_R/δ) regime, and only for even parity modes. Here analytic expressions are given for both the small and large- (d_R/δ) regimes, and both even and odd parity modes, and these are checked against numerically-obtained g 's, valid for arbitrary d_R/δ .

As described in Ref. 4, the coupling coefficients $g_{\underline{\ell}}(w,p)$ are the Fourier components of the perturbing Hamiltonian

$$H_1 = - \sum_a (e/c) \underline{v}(z) \cdot \underline{A}^w[\underline{r}(z), t]$$

$$H_1(z, t) = \sum_w \sum_{\underline{\ell}} g_{\underline{\ell}}(w, p) \exp(i \underline{\ell} \cdot \underline{\Theta} - i \omega_w t) . \quad (A1)$$

Here, $\underline{r}(z)$ and $\underline{v}(z)$ are the position and velocity of a particle, given its position z in phase space, e is the charge of the particle, and $\underline{A} = \sum_w \underline{A}^w$ is the vector potential of the perturbation. The Fourier transform is from the coordinate space $\underline{\Theta} \equiv (\Theta_g, \Theta_b, \phi)$ of the three phase variables (gyrophase, bounce phase, and bounce-averaged toroidal angle ϕ , respectively) whose unperturbed evolution is linear in time, to the corresponding Fourier component $\underline{\ell} \equiv (\ell_g, \ell_b, \ell_\phi)$. In Ref. 4, it is shown that the only contributing components have $\ell_g = 0$, $\ell_\phi = -n_w$

(the toroidal mode number of mode w), with ℓ_b assuming any integral value. (This additional dependence on ℓ_b was suppressed in the main text.) Then g_{ℓ} may be written

$$g_{\ell}(w, p) \approx -(e/c) (2\pi)^{-1} J_0(k_{\perp} \rho) \oint d\theta_b \exp(-i\ell_b \theta_b) \\ \times \dot{\vec{R}} \cdot \vec{A}^w(r_p + \delta r, \delta \theta, \ell_{\phi}) \exp(i\ell_{\phi} \delta \phi) . \quad (A2)$$

Here $\dot{\vec{R}}$ is the guiding-center velocity, ρ is the gyroradius, and δr , $\delta \theta$, $\delta \phi$ are those portions of the guiding-center motion in the r , θ , and ϕ directions which are controlled by the bounce phase, θ_b .

Using expression (A2), and a suitable model for $\vec{A}^w$, one evaluates g_{ℓ} . $\vec{A}^w$ is taken to have the form

$$\vec{A}^w(\underline{x}) = \hat{B} A_w(r) \exp i[m\theta - n\phi + \int^r dr' k_r(r')] , \quad (A3)$$

with radial envelope $A_w(r)$ given by

$$A_{w+}(r) = \bar{A}_{w+} \exp(-x^2), \quad x \equiv (r - r_w)/\delta_w \quad (A4)$$

for an even-parity mode, and

$$A_{w-}(r) = \bar{A}_{w-} x \exp(-x^2) \quad (A5)$$

for an odd-parity mode.

In Figs. A1 and A2 are shown the results of numerical evaluation of $h_{\ell} \equiv |g_{\ell}/e(v_{||}/c)J_0(k_{\perp}\rho)|$ for even and odd parity, respectively, and for $\ell \equiv \ell_b - m$ equal to 0 and 1. For the limits of $d_R/\delta_w < 1$ or $d_R/\delta_w \gg 1$, one may evaluate (A2) analytically. In the low energy ($d_R/\delta_w < 1$) regime, one finds

$$g_{\ell+} \approx -e(v_{||}/c) \bar{A}_{w+} J_0(k_{\perp}\rho) J_{\ell}(y_1) \quad (A6)$$

for even parity, and

$$g_{\ell-} \approx -e(v_{||}/c) \bar{A}_{w-} J_0(k_{\perp}\rho) (d_R/2\delta_w) \exp(i\theta_1) J_{\ell-1}(y_1) \\ + \exp(-i\theta_1) J_{\ell+1}(y_1) , \quad (A7)$$

for odd parity. Here $y_1 \equiv (y_f^2 + y_r^2)^{1/2}$, where $y_f \equiv md_{\theta} - nd_{\phi}$ and $y_r \equiv k_r d_R$ are measures of the amount of phase averaging which a particle performs over the mode structure due to drifts in the flux surface (y_f) and radially (y_r). (In analogy with d_R , d_{θ} and d_{ϕ} are the amplitudes of the drift motion in the θ and ϕ directions.) The angle θ_1 is defined by $\cos\theta_1 = y_f/y_1$, $\sin\theta_1 = -y_r/y_1$. The Bessel functions J_{ℓ} in Eqs. (A6) and (A7) arise from the phase averaging due to finite gyroradius [$J_0(k_{\perp}\rho)$] and drift [$J_{\ell}(y_2)$] effects. One sees explicitly, from the presence of the factor (d_R/δ_w) in (A7), that for low energy electrons, $g_{\ell-} \rightarrow 0$, whereas $g_{\ell+}$, lacking this factor, will be large in this limit, for $\ell = 0$.

For high-energy particles ($d_R/\delta_w \gg 1$), one finds

$$g_{\ell+} \approx -e(v_{||}/c) \bar{A}_{w+} J_0(k_{\perp} \rho) \pi^{1/2} (\tau_w/\tau_b) \\ \times \left\{ i^{\ell} \exp(iy_f) \exp[-(\Omega_+ \tau_w)^2/4] + (-i)^{\ell} \exp(iy_f) \exp[-(\Omega_- \tau_w)^2/4] \right\} \quad (A8)$$

for even parity, and

$$g_{\ell-} \approx -e(v_{||}/c) \bar{A}_{w-} J_0(k_{\perp} \rho) (1/2) i \pi^{1/2} (\tau_w/\tau_b) \left\{ i^{\ell} \exp(iy_f) (\Omega_+ \tau_w) \right. \\ \left. \times \exp[-(\Omega_+ \tau_w)^2/4] - (-i)^{\ell} \exp(-iy_f) (\Omega_- \tau_w) \exp[-(\Omega_- \tau_w)^2/4] \right\} \quad (A9)$$

for odd parity. Here, $\Omega_{\pm} \equiv \ell \Omega_b \pm k_r v_r$, where Ω_b is the transit frequency and v_r is the radial velocity of the guiding center at the time of crossing the mode rational surface. The factors involving $\Omega_{\pm}$ arise from the phase averaging the particle performs during the time $\tau_w \equiv \delta_w/v_r$ it takes the particle to traverse the region of mode localization. For this high energy regime, τ_w is only a fraction of the total transit time τ_b . The falloff of g_{ℓ} with higher energies is principally governed by the factor τ_w/τ_b in Eqs. (A8) and (A9).

The agreement between these analytic expressions and the numerically-obtained results is shown in Figs. A3 and A4 for the representative case of odd parity and $\ell = 1$. A solid line shows the numerical result, and a dashed line the analytic one. The energy at which $d_R/\delta_w = 1$ is also indicated.

Finally, we discuss the manner in which the average over nearly-resonant modes in Eq. (1) was performed to obtain Fig. 1. The effect of this averaging will be to wipe out the oscillatory character of $|g_{\underline{\ell}}|$ or $|h_{\underline{\ell}}|$ seen in Figs. A1 or A2, leaving only the modulating envelope. Due to the interlacing nature of the $g_{\underline{\ell}}$'s which is apparent for $\ell = 0, 1$ in Figs. A1 and A2, this envelope is conveniently approximated as

$$\langle h_{\underline{\ell}}^2 \rangle \approx [h_{\underline{\ell}}^2(\ell = 0) + h_{\underline{\ell}}^2(\ell = 1)]^{1/2}. \quad (A10)$$

Similarly, we take $\langle J_O^2(k_{\perp\rho}) \rangle \approx [J_O^2(k_{\perp\rho}) + J_1^2(k_{\perp\rho})]^{1/2}$.

APPENDIX B: TOROIDAL COUPLING OF MODES

Up to now, we have assumed a dependence $\exp[i(m\theta - n\phi - \omega_w t)]$ of the coherent waves with which a runaway interacts. This dependence is strictly valid in a cylindrical geometry. For typical tokamak parameters, however, the poloidal symmetry of a cylinder is strongly broken, and the proper description of an eigenmode changes. For fixed n , instead of modes characterized by a single m , localized about the surface r_w where $q(r_w) \approx m/n$, the true eigenmodes of the system can be represented as a sum over the single- m modes, adding up to a radially non-localized form, which twists to keep $k_{||}$ approximately zero as one moves radially. We now ask what modification of the foregoing results one can expect, using this more appropriate model of the perturbing waves.

Replacing mode label w for the moment by its more explicit representation in terms of the "quantum" numbers (m,n) , we write the toroidal eigenmodes $\tilde{A}_n$ ($\tilde{A} \equiv$ perturbing vector potential) as a sum over the radially localized modes $\tilde{A}_{mn}$, as just described:

$$\tilde{A}_n(\underline{x}) = \sum_m \tilde{A}_{mn}(\underline{x}), \quad \tilde{A}_{mn}(\underline{x}) = \tilde{A}_{mn}(r) [\exp i(m\theta - n\phi)] . \quad (B1)$$

Therefore, since $g \propto \tilde{A}$,

$$g(n,p) = \sum_m g(m,n;p), \quad (B2)$$

where the $g(m,n;p)$ are the $g(w,p)$ we have dealt with up until

this section. Thus $|g(n)|^2$ will involve, in addition to the terms $|g(m,n)|^2$ also present in the context of radially localized eigenmodes, cross-terms $g^*(m',n) g(m,n)$.

One expects these cross-terms to make only a small contribution, however, for two reasons. First, a particle with drift surface centered about $r = r_p$ interacts with only a few (on the order of d_R/δ_w) components in the vicinity of r_p . And second, of these components, A_{mn} , with which the particle interacts at all, the secular, diffusive motion of a particle comes from only those components which have $\ell \equiv \ell_b - m$ near enough to zero that $g(m,n;p)$ is appreciable. In the low-energy, driftless limit $d_R/\delta_w \rightarrow 0$, a particle can resonate with only a single component A_{mn} of the full mode A_n , and the difference between the toroidally coupled and uncoupled eigenmode results vanishes altogether.

APPENDIX C: EFFECT OF $d_R/L_t \leq 1$

Finally, we consider the effect on τ_R/τ_{RO} of allowing the turbulence to be radially spiky, i.e., to vary appreciably in amplitude over a scale length L_t comparable to the drift-orbit excursion d_R of the higher-energy runaways. We visualize the radial dependence of the turbulent amplitude, ϵ , as illustrated in Fig. A5-a. Such variation in ϵ might be caused, for example, by the presence of lower- m instabilities, which appreciably alter the equilibrium profile in the region of their localization, and thereby modify the ability of that region to support micro-turbulence.

The size of the radial diffusion coefficient D at any radius $r = r_p$ is an appropriate average of $\epsilon(r)$ over the excursion $2d_R$ which a particular particle makes in the course of a bounce period. In Fig. A5-b, this average is shown, for a lower-energy electron having a drift excursion d_{Rl} which is much less than the spikiness scale length L_t . The appearance of D is almost the same as $\epsilon(r)$. In particular, D becomes very small in regions where $\epsilon(r)$ does, and it is accordingly D in these regions which controls the total lifetime of the lower-energy particles.

For higher-energy particles having $d_R = d_{Rh}$ comparable to the spikiness scale length, even when the transit-time averaged radius r_p of the particle is in a region of very low-amplitude turbulence, the particle still senses the much stronger turbulence at radii in the range $r_p \pm d_R$ which it experiences in a transit time. Thus, transport in these regions of small ϵ

will be comparable to that in regions of large ϵ , as illustrated in Fig. A5-c.

Accordingly, this effect tends to enhance τ_R/τ_{RO} for electrons having a drift excursion small compared to the spikiness scale length, relative to the more energetic electrons, whose transport rate is not strongly affected by the spikiness.

TABLE I

Experiment		Thermal Electrons					Runaway Electrons			
Device	Ref.	$T_e(0)$ (keV)	τ_{E_e} (ms)	$\langle \delta E_e \rangle$ (cm)	$\langle \rho_i \rangle$ (cm)	$\langle \frac{c}{\omega_{pe}} \rangle$ (cm)	E_R (keV)	τ_R (ms)	d_R (cm)	ρ_R (cm)
ST	7	2.5	2+5	0.04+0.06	0.03	0.2	$1+1.5 \times 10^3$	10+20	0.6+0.8	--
LT-3	8	0.1	0.5+1.2	0.05+0.07	0.04	0.3	$1+2 \times 10^2$	0.03	0.4+0.5	--
TM-3	9	0.1	0.2	0.2	0.03	0.7	15+60	2	0.3+0.5	3×10^{-2}
							60+100	9	0.5+0.8	6×10^{-2}
PLT	10	2.0	25	0.06	0.07	0.2	$8+12 \times 10^3$	20+80	4+6	--
ORMAK	11	0.5	5	0.08	0.07	0.2	$8+10 \times 10^3$	$10+10^2$	9+12	--
							$8+10 \times 10^3$	10^3	9+12	

(PPPL-802187)

Fig. 1. Representation of the averaging which a particle performs on the structure of a single mode, due to drifts both in a drift surface (a), and radially (b). For runaway electrons, this averaging appreciably changes the strength of coupling of the mode to the electron from its driftless value.

(PPPL-802192)

Fig. 2. Theoretical curves for the expected enhancement of runaway confinement time due to drifts and finite Larmor radius over the value expected ignoring these effects, as a function of runaway energy or, equivalently, of drift orbit excursion d_R . Each curve assumes different proportions of even and odd-parity modes comprising the turbulence. PLT-like parameters ($R = 150$ cm, particle radius $= 30$ cm, $B = 5.0T$, $q = 2.5$, $T_i = 1$ keV) were assumed. The parameters describing a typical mode are $\delta = 6\rho_i$, $k_r = 0.5\rho_i^{-1}$, $k_\theta = \rho_i^{-1}$.

(PPPL-803405)

Fig. 3. Plot of na^2 scaling of thermal electron energy confinement time, for the devices considered in the text.

(PPPL-802193)

Fig. 4. Comparison of theoretical curves (Fig. 2) for confinement time enhancement with experimental values from a number of different tokamaks. For the mode parameters assumed in Fig. 2, most of the experimental points lie near the curve for purely even-parity turbulence.

(PPPL-802189)

Fig. A1. Plot of the reduction h_{ℓ} due to particle drifts of the mode-particle coupling strength, for an even parity mode, using the same mode parameters as Fig. 2, and for $\ell = 0$ and 1 (see Appendix A for details). At low energies (drift displacement d_R small compared to mode width δ), the coupling is strongest, approaching the driftless value of 1.0 as $d_R/\delta \rightarrow 0$.

(PPPL-802191)

Fig. A2. Same as Fig. A1, but for an odd-parity mode. In the low-energy limit ($d_R/\delta \rightarrow 0$), the coupling strength vanishes, due to the particle's moving only near the mode-rational surface, where the mode has zero amplitude.

(PPPL-802190)

Fig. A3. Agreement between the numerically-obtained value of h_{ℓ} and the analytic expression (A7) given in Appendix A for the low-energy limit ($d_R/\delta < 1$), and for the representative case of odd parity, $\ell = 1$.

(PPPL-802186)

Fig. A4. Same as Fig. A3, but comparing the expression (A9) valid in the high-energy ($d_R/\delta > 1$) limit.

(PPPL-802188)

Fig. A5. Illustration of the effect on confinement time enhancement of a radially spiky turbulent amplitude, discussed in Appendix C. Particles perform an average of the strength of the turbulence $\epsilon(r)$ (part a) over a radial range of twice their drift excursions d_R . Less energetic particles, having drift excursion d_{Rl} small compared to the scale length L_t of the turbulence have diffusion coefficient D as in part b, closely following $\epsilon(r)$. Higher-energy particles, with drift excursion $d_{Rh} \geq L_t$, have a transport rate which is more nearly uniform (part c).