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STATISTICS OF WIND FLUCTUATIONS OVER COMPLEX TERRAIN

Final Report

By
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**The Pennsylvania State University
University Park, Pennsylvania**

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Solar Energy

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EXECUTIVE SUMMARY

This report deals with the properties of relative turbulence intensities, turbulent spectra and frequency distributions over uniform and complex terrain.

As to intensities of turbulence, the report concentrates on properties of σ_u/u_* , σ_v/u_* , σ_w/u_* , where the σ 's are the standard deviations of the longitudinal, transverse and vertical velocity components, respectively, and u_* is the friction velocity. Given these ratios, relative turbulent intensities (such as σ_u/U , where U is the wind speed) can be obtained with the standard wind profile formulae. Such formulae, though developed for homogeneous terrain, may turn out to be useful over complex terrain, provided u_* is taken to be the friction velocity at the surface.

The ratio σ_w/u_* behaves in the same manner over rolling terrain as over uniform terrain, even in air which has recently crossed a mountain range. On top of a hill this property is valid only if u_* is interpreted as the surface friction velocity. Here the ratio of σ_w to local u_* increases upward.

The ratios σ_u/u_* and σ_v/u_* are increased by rolling terrain and even double in the wake of a low mountain range. On top of hills σ_u and σ_v decrease upward, approximately proportionally to u_* . In contrast, in a low spot in rolling terrain, σ_u and σ_v increase upward.

On top of a steep escarpment, excessive vertical wind shears are found. These are approximately in equilibrium with intensities of local velocity fluctuations, so that relative turbulence intensities may reach 80% and more.

Models have been constructed for normalized logarithmic spectra of the velocity components over uniform terrain.

The spectra of vertical velocity are explained quite well by Monin-Obukhov scaling.

The spectra of the horizontal components in unstable air are composed of weighted sums of two functions: the low-frequency function scales by mixed-layer scaling, with the length scale proportional to the thickness of the planetary boundary layer, and the high-frequency function scales by Monin-Obukhov scaling, with height as length scale. In stable air, only the high-frequency function is needed. The limits for strong winds (mechanical turbulence) for the spectra are different when approached from the stable or unstable sides.

Integration of the spectra over all frequencies yields models for the ratios of standard deviation to friction velocities in good agreement with observations. The ratios approach the same limit for neutral conditions when approached from either the stable or the unstable sides.

Spectra of vertical velocity over rolling terrain are nearly the same as over uniform terrain. But the spectra of the horizontal velocity components possess more energy at the lowest frequencies analyzed.

A Gaussian patch model has been developed and tested for short-period changes of wind components. The kurtosis predicted by such a model approaches that which is observed for velocity changes; a value considerably larger than that predicted by Gaussian functions.

It is proposed to extend this work by obtaining additional data over complex terrain. In particular, spectra will be available over hills, escarpments and rolling terrain. The models described in this report will be generalized to cover such statistics.

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INTRODUCTION

This work is part of the total effort within the Design and Performance section of the Wind Characteristics Program Element at PNL to relate knowledge of the properties of wind fluctuations to requirements for design and performance of WECS. Characteristics of wind fluctuations over level terrain are fairly well understood and are needed as starting point for description of wind fluctuations over complex terrain. On the other hand little applicable description of wind fluctuation properties over complex terrain is available. Therefore a separate study on the statistics of wind fluctuations over complex terrain has been initiated. The first phase of this study is reported on herein.

Wind fluctuations affect both the design and operation of wind energy conversion systems (WECS). In particular, the mechanical integrity of WECS depends on the magnitude of high-frequency characteristics of wind changes, and the operation is particularly sensitive to the low-frequency components of wind fluctuations.

The natural wind and its fluctuations are extremely variable, depending on ground roughness, complexity of terrain, mean speed at a given height (or horizontal pressure gradient), and hydrostatic stability. In this report, we attempt to establish quantitative relationships between general "weather" characteristics and properties of wind fluctuations.

First, we consider standard deviations of the velocity components as general measures of turbulent intensity. However, this information cannot be applied directly to the design and operation of WECS because the effects of given turbulent intensities depend on their characteristics in frequency space. In common engineering practice, these characteristics are handled through the assumption of idealized "gust shapes". Such assumptions are

rather arbitrary and are in contradiction to the extremely variable nature of the frequency characteristics of the natural wind. We therefore prefer to make models of the spectra of wind components, as functions of the basic predictors. We then supplement the information on spectral behavior by information on frequency distributions of wind fluctuations, particularly for rapid changes of wind fluctuations.

In WECS operations and design, conditions of strong winds are important. The "mechanical" turbulence which is dominant under these conditions is relatively easy to characterize because it does not depend on vertical heat transfer or temperature distribution, but only on mean wind and terrain characteristics.

However, even in strong winds, the effects of heat convection on the turbulence cannot be neglected. The relative magnitude of heat convection to mechanical turbulence increases with height; thus, for example, heat convection is quite important at 50 m height on a sunny day with very strong winds. For large WECS, then, heat convection may be important.

Nevertheless, we will concentrate in this report on properties of mechanical turbulence. We will arrive at these properties by plotting various turbulence characteristics as functions of variables which describe the relative importance of heat convection and mechanical turbulence, and then we will interpolate these data for conditions of zero heat convection (mechanical turbulence only). But we will also describe quantitatively the dependence of the fluctuations on temperature stratification.

Most of this report describes the construction of models for characteristics of wind fluctuations over uniform terrain. But we also present a preliminary assessment of some of the changes in these characteristics over various

types of complex terrain: rolling farm land, the top of a hill, and the lee of a low mountain range. It is clear that different types of complex terrain require quite different modifications of the basic models. In this report, we describe some of these modifications qualitatively. In a follow-up period, September 1, 1979 to August 31, 1980, attempts at more quantitative formulations of turbulence characteristics over complex terrain will be made.

The relationships developed in this report refer to the so-called surface layer. This layer is that part of the "planetary boundary layer" (PBL) in which the stress does not vary significantly. It is usually taken as the lowest 10% of the PBL. In strong winds, the surface layer extends at least to 100 m, and often considerably higher.

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DISCUSSION

Chapter 1. The predictors

1.1 Summary of predictors

We use as predictors for the properties of velocity fluctuations: z_0 , the roughness length; u_* , the friction velocity; L , the Monin-Obukhov length, and z_1 , the "height of the lowest inversion". None of these quantities is usually immediately available from weather records. Therefore, in this chapter, we suggest methods for estimating these quantities.

1.1.1 The roughness length. The roughness length z_0 is a measure of local terrain roughness. It is proportional to "eddy size" at the surface. It is therefore, quite independent of general terrain features on horizontal scales larger than 100 m; instead, it is essentially a function of ground cover. It may depend on wind direction, because air coming from different directions may have traversed terrain with different ground cover. The turbulence at a given height is affected by the ground roughness in the upwind direction up to a distance about 10 times that height.

The following table gives typical roughness length for some types of ground cover.

Table 1.1 Typical roughness length, cm

Type of ground cover	roughness length
water	0.01
smooth grass	1
pasture	10
forest, cities	100

Thus, an estimate of z_0 can be obtained by inspection. More accurate values can be found if winds are given at two levels. Then the ratios U_2/U_1 (where U is wind speed* and the subscripts indicate the levels) are plotted as a function of $1/U_2$. On the abscissa, $1/U_2$ is defined negative in the daytime, positive at night. Then, U_2/U_1 is interpolated to $1/U = 0$. The wind ratio at this point is given by:

$$\frac{U_2}{U_1} = \frac{\log \frac{z_2}{z_0}}{\log \frac{z_1}{z_0}} \quad (1.1)$$

where z_2 and z_1 are the heights at which the winds are measured. This equation is then solved for z_0 .

A graph of this type has considerable scatter. Some of this scatter can be avoided, if U_2/U_1 is plotted as a function of $1/L$ (see next section), rather than $1/U_2$, and interpolated for $1/L = 0$. Equation(1.1) was originally derived for winds over uniform terrain, but it appears to be valid even over complex terrain, provided the winds are measured below 25 m or so.

1.1.2 Monin-Obukhov L. The Monin-Obukhov length L is a measure of the height below which heat convection and temperature stratification become relatively unimportant compared to the wind distribution and mechanical turbulence. It is defined to be negative for heating from below (daytime) and positive for cooling from below (nighttime). On windy days its numerical value is larger than on relatively calm days, so that often terms containing the reciprocal of L are small.

*More accurately, the mean wind component in the mean wind direction.

The Monin-Obukhov length is defined by:

$$L = \frac{-u_*^3 C_p \rho T}{kgH(1 + \frac{.07}{B})} \quad (1.2)$$

where u_* is the friction velocity; C_p the specific heat at constant pressure; ρ the air density; T the temperature; k the von Karman constant; g the acceleration of gravity; and B the Bowen ratio.* The evaluation of the factors in (1.2) requires much more sophisticated instrumentation than is usually available. Hence the quantities just listed will not be explained further at this point (although some will be discussed in detail later). Instead, some simple methods for estimating L will be suggested here.

The value of L can be estimated most simply (but least accurately) from a nomogram proposed by Golder (1972), reproduced here as Fig. 1.1. The Pasquill class, needed for this nomogram, is formed from rough estimates of wind and radiation (see Table 1.2).

If temperatures and wind speeds are available at two levels, estimates of L can be obtained from:

$$\frac{z}{L} = R_i \quad (\text{heating from below}) \quad (1.3)$$

$$\frac{z}{L} = \frac{R_i}{1 - 5R_i} \quad (\text{cooling from below}) \quad (1.4)$$

where R_i is the Richardson number, defined by:

$$R_i = \frac{g(\gamma_d - \gamma)}{T(\frac{\partial U}{\partial z})^2} \quad (1.5)$$

* Ratio of sensible to latent heat flux at the surface.

FIGURE 1.1. Monin-Obukhov length, L , as functions of roughness length and Pasquill class (see Table 1.2).

TABLE 1.2. RELATION OF TURBULENCE TYPES TO WEATHER CONDITIONS

A - Extremely unstable conditions
 B - Moderately unstable conditions
 C - Slightly unstable conditions

D - Neutral conditions*
 E - Slightly stable conditions
 F - Moderately stable conditions

Surface wind speed, m/sec	Daytime insolation			Nighttime conditions	
	Strong	Moderate	Slight	Thin overcast or $\geq 4/8$ cloudiness+	$\geq 3/8$ cloudiness
<2	A	A-B	B		
2	A-B	B	C	E	F
4	B	B-C	C	D	E
6	C	C-D	D	D	D
>6	C	D	D	D	D

* Applicable to heavy overcast, day or night.

+ The degree of cloudiness is defined as that fraction of the sky above the local apparent horizon which is covered by clouds.

where γ_d is the dry adiabatic lapse rate (decrease of temperature with height), $0.98^\circ\text{C}/100\text{ m}$, γ is the prevailing lapse rate, and T is temperature.

1.1.3 z_i , the height of the lowest inversion. It has been recognized since 1974 (see Wyngaard and Coté, 1974 or Panofsky et al., 1977) that the properties of the horizontal wind fluctuations in the surface layer are not only influenced by L (which depends on surface-layer properties) but also by z_i , which involves characteristics of the whole PBL.

On windy days, the atmosphere is well mixed up to a height of 1000 m or more, and temperature decreases with height in this region. Above the mixed layer, the temperature usually increases with height (or decreases more slowly). The height at which the lapse rate changes suddenly is z_i . The mixing depth h tends to be a little larger than z_i because the lowest portions of the "inversion" are usually turbulent.

The height z_i (or rather h) can be measured directly by acoustic sounders. Or, on a clear day, a morning temperature sounding can be used with a near-surface temperature measured at the time z_i is needed. Then, z_i is the intersection of the sounding and an adiabatic distribution constructed through the surface temperature.

Finally, on very windy days,

$$z_i = 0.04 \frac{U}{f \log_{10} \frac{z}{z_0}} \quad (1.6)$$

where U is the wind speed at level z , and f the Coriolis parameter defined, in sec^{-1} , by $1.46 \times 10^{-4} \sin \phi$, where ϕ is the latitude.

1.1.4 The friction velocity. The friction velocity u_* is defined by the equation

$$u_* = \sqrt{\tau/\rho} \quad (1.7)$$

where τ is the surface stress.

The friction velocity can be estimated from the equation for the wind profile:

$$u_* = \frac{kU}{\ln \frac{z}{z_0} - \psi\left(\frac{z}{L}\right)} \quad (1.8)$$

Values used for the von Karman constant vary from 0.35 to 0.40. Perhaps a compromise of 0.37 should be recommended at this time.

The function ψ is $-5 z/L$ in "stable" air (cooling from below, as at night). In unstable air, $\psi (z/L)$ is given by a complicated function, here tabulated in Table 1.3.

Although equation (1.8) was derived for conditions over uniform terrain, it gives good results for the surface friction velocity over hilly terrain.

If winds are available at two levels, then u_* can be found from

$$U_2 - U_1 = \frac{u_*}{k} \left[\ln \frac{z_2}{z_1} - \psi\left(\frac{z_2}{L}\right) + \psi\left(\frac{z_1}{L}\right) \right] \quad (1.9)$$

This equation avoids the necessity of estimating z_0 .

TABLE 1.3
The ψ -Function

Z/L	PSI	Z/L	PSI
0.00	-0.0000	-2.90	1.7180
-0.10	0.2836	-3.00	1.7391
-0.20	0.4613	-3.10	1.7595
-0.30	0.5945	-3.20	1.7794
-0.40	0.7023	-3.30	1.7988
-0.50	0.7934	-3.40	1.8176
-0.60	0.8725	-3.50	1.8360
-0.70	0.9427	-3.60	1.8540
-0.80	1.0059	-3.70	1.8715
-0.90	1.0634	-3.80	1.8886
-1.00	1.1162	-3.90	1.9054
-1.10	1.1651	-4.00	1.9218
-1.20	1.2107	-4.10	1.9378
-1.30	1.2533	-4.20	1.9535
-1.40	1.2934	-4.30	1.9688
-1.50	1.3313	-4.40	1.9839
-1.60	1.3672	-4.50	1.9987
-1.70	1.4013	-4.60	2.0131
-1.80	1.4339	-4.70	2.0273
-1.90	1.4649	-4.80	2.0413
-2.00	1.4947	-4.90	2.0550
-2.10	1.5232	-5.00	2.0684
-2.20	1.5507	-5.10	2.0816
-2.30	1.5771	-5.20	2.0946
-2.40	1.6026	-5.30	2.1074
-2.50	1.6272	-5.40	2.1200
-2.60	1.6510	-5.50	2.1323
-2.70	1.6740	-5.60	2.1445
-2.80	1.6964	-5.70	2.1565

Chapter 2. Standard deviations of the velocity components.

2.1 Theory

In this report, emphasis will be placed on the properties of the ratios $\sigma_i/u_* = \phi_i$, where σ_i is the standard deviation of the i_{th} velocity component. Given the properties of these nondimensional ratios, we can determine the properties of the components of turbulent intensity by combining these expressions with (1.8) to yield

$$\frac{\sigma_i}{U} = \frac{k\phi_i}{\ln \frac{z}{z_o} - \psi\left(\frac{z}{L}\right)} \quad (2.1)$$

If possible, standard deviations are computed over one-hour intervals, after linear trends have been removed from the data. However, published standard deviations do not necessarily satisfy these conditions, which introduces some systematic variations in ϕ_i .

2.1.1 Variances of vertical velocity. Variances of vertical velocity appear to obey Monin-Obukhov similarity theory in the surface layer:

$$\frac{\sigma_w}{u_*} = \phi_3\left(\frac{z}{L}\right) \quad (2.2)$$

Here σ_w is the standard deviation of vertical velocity, u_* is the friction velocity, z is the height, and L is the Monin-Obukhov length. The properties of ϕ_3 are reasonably well known and were the subject of a summary paper by Merry and Panofsky (1976). The unstable air, a reasonably good fit is provided by the equation

$$\phi_3 = 1.3[1 + 3 z/-L]^{1/3} \quad (2.3)$$

Based on the excellent data over Minnesota, Panofsky et al. (1977) proposed another empirical equation for ϕ_3 in unstable air:

$$\phi_3 = [1.6 + 2.9 (z/-L)^{2/3}]^{1/2} \quad (2.4)$$

Both (2.3) and (2.4) are plotted in Fig. 2.1. The two equations are different, but not significantly so. This figure shows that the equations also fit data over rolling terrain.

Of special interest is the value of $\sigma_w/u_* = \phi_w$ under neutral conditions. This value might lie between 1.2 and 1.3, with the smaller estimate for rough terrain (see Clarke et al., 1978). We will give a more precise estimate for this quantity later in this chapter.

In stable air, Merry and Panofsky observed an increase of the magnitude of ϕ_3 with z/L . However, both σ_w and u_* are small and uncertain, with particularly large fractional random errors in u_* . This behavior produces an apparent increase in the value of ϕ_3 with z/L which may not be real. At present, we cannot rule out the possibility that ϕ_3 in stable air is independent of z/L and equal to $\phi_3(0)$.

2.1.2 Variances of the horizontal velocity components. According to Panofsky et al. (1977), standard deviation of horizontal velocity components over uniform (homogeneous) terrain are given by

$$\frac{\sigma_H}{u_*} = \phi_H\left(\frac{z_1}{L}\right) \quad (2.5)$$

Fig. 2.1. Ratio σ_w/u_* of the standard deviation of the vertical wind component to the friction velocity versus z/L at Rock Springs, Pennsylvania. ($\bullet$ 2m., $\blacktriangle$ 5m., $\blacksquare$ 8m., non-mountain trajectories; $\circ$ 2m., $\triangle$ 5m., $\square$ 8m., mountain trajectories). The curve (2.3) is denoted by the dashed line and the curve (2.4) is denoted by the solid line.

where z_1 is the depth of the mixed layer approximated by the height of the lowest inversion, and σ_H is the standard deviation of either horizontal velocity component. They found that ϕ_H is well represented by

$$\frac{\sigma_H}{u_*} = [12 + 0.5 |z_1/L|]^{1/3} \quad (2.6)$$

There is considerable evidence, however, that the values of ϕ_H diverge for the longitudinal component, u , and the lateral component, v , for small $|z_1/L|$. In particular, $\phi_H(0)$ is usually given as about 1.9 for v and 2.4 for u . We will summarize these estimates later and study their dependence on terrain features.

Fig. 2.2 shows the distribution of σ_u/u_* at Rock Springs, PA using the following revised smooth-site form suggested for σ_u/u_* :

$$\frac{\sigma_u}{u_*} = (14 + 0.5 |z_1/L|)^{1/3} \quad (2.7)$$

For σ_v/u_* , we suggest

$$\frac{\sigma_v}{u_*} = (7 + 0.5 |z_1/L|)^{1/3} \quad (2.8)$$

2.2 Analysis of observations

There have been a number of studies of the characteristics of turbulent wind fluctuations in the planetary boundary layer that have produced data of good quality. To analyze the behavior of the statistics of the three wind

Fig. 2.2. Ratio of σ_u/u_* of the standard deviation of the longitudinal wind component to the friction velocity versus $-z_i/L$ at Rock Springs, Pennsylvania. (non-mountain trajectories - x; mountain trajectories - o). Also shown is the smooth-site curve $\sigma_u/u_* = (14 + 0.5(-z_i/L))^{1/3}$ suggested by Panofsky et al. (1977) with the neutral limit changed to 2.38.

components, we have brought together turbulent wind fluctuation measurements from as many recent sources as possible, including a group of new experiments for which the results are not yet published. We primarily discuss the neutral limits of the variances because these are the most useful statistics to wind power engineers.

2.2.1 Characteristics of terrain and observations. The characteristics of the sites and some details of the high-frequency data collection and processing are presented in Appendix A.

In dealing with observations from a variety of sites, we note that the data were collected with differing instrumentation, for varying sample lengths, and that the statistics were calculated by a number of methods (Appendix A). Because of this we are likely to find systematic discrepancies among different sets of data. Although we recognize this problem, we have not been able to normalize all of the data to a common system.

The study of McBean (1971) is important in this respect. The sample duration was only 12-17 minutes so that fluctuations with periods longer than about one-half the sample length cannot contribute to the variances. This can cause low estimates of the horizontal variances because the portion of the variance spectrum containing periods that are longer than the low frequency cutoff will be missing.

2.2.1.1 Characteristics of uniform sites. Some of the sites listed can be considered to be uniform or homogeneous. The authors associated with the experiments give more detailed explanations of their sites and

experimental procedures than shown here. The site at Risø Laboratory in Denmark has been described by Kristensen and Panofsky (1976) and the instrumentation is discussed by Norman et al. (1976).

2.2.1.2 Instrumentation and terrain at complex sites. The possibility that the turbulent statistics over non-uniform, or complex, sites have properties differing from those over uniform sites has been suggested by Högström (1974). In his study he notes that the variances of the horizontal wind components at a non-ideal site are significantly increased over typical values from a uniform site.

Högström (1974) discussed a field experiment in 1969 and 1970 near Uppsala, Sweden. The site has been described as a "typical" agricultural site with rolling terrain and with slopes on the order of about $\pm 5\text{m/km}$ (Smedman-Högström and Högström, 1973). The surface was covered with a variety of crops during the summer and by snow in the winter.

The instrumentation consisted of strain gauge anemometers and fine-wire resistance thermometers, along with supporting profile estimates. The high-frequency data were sampled at 30 Hz. Long-term trends were removed by taking two-minute running means over the sampling period.

The Department of Meteorology at The Pennsylvania State University conducted an experiment over complex terrain in central Pennsylvania in 1978. The site is located near Rock Springs, Pennsylvania in a broad valley bordered by ridges of the Allegheny Mountains. The fetch to the northeast and southwest is relatively flat with terrain variations of less than $\pm 5\text{m/km}$. Several hundred meters to the south and southeast, a tree-covered mountain ridge rises 220 meters above the site. To the north the terrain consists of rolling hills with scattered trees and houses about 500 meters

from the site. Figure 2.3 gives a general topographic survey of the site and its surroundings. The ground within 500 meters of the site was covered either with corn stubble or plowed fields ($z_0=0.8$ cm).

The instrumentation at the site, described by Norman et al. (1976), consisted of fast-response drag anemometers that were mounted at heights of 2.0, 5.0, and 8.0 meters. In addition, fast-response fine-wire resistance thermometers were mounted at each of the anemometer levels. Supporting profile measurements were made at four levels with Thornthwaite cup anemometers and ventilated platinum resistance thermometers. The depth of the mixed layer was monitored by acoustic sounders and by airplane flights.

The output from the high-frequency sensors was sampled at 20 Hz for periods of thirty minutes to one hour. The electronic signals from all the instruments were digitized on site and recorded on magnetic tape through a PDP 11/10 minicomputer. Final processing was done on the University's IBM 370/3033 computer. Long-term trends were removed from the data via linear regression and then estimates of spectral densities were computed.

Another site with non-uniform terrain characteristics has been instrumented by NOAA at Erie, Colorado. This site is known as the Boulder Atmospheric Observatory (BAO). The site and instrumentation have been described by Kaimal (1978a). The terrain varies with direction (Figure 2.4). The data used in our investigation correspond to a western fetch with gently rolling terrain with slopes less than $\pm 20\text{m/km}$.

The instrumentation used at BAO consists of three-axis sonic anemometers and fast-response platinum resistance thermometers mounted at levels of 10, 22, 50, and 100 meters. Also, wind and temperature profile measurements were obtained near the surface. The high-frequency data were sampled at 10 Hz for one-hour periods. Trends were removed manually by linear regression techniques.

Fig. 2.3. Topographic map of the Rock Springs area and location of the experimental site. Contours are height above sea level in feet.

Fig. 2.4. Topographic map of Boulder Atmospheric Observatory area and location of the experimental site. Contours show deviations from site elevation in meters.

Excellent observations at a completely different site in Australia have recently become available. E. F. Bradley (1979) instrumented a tower on a 170-meter hill (Black Mountain) near Canberra, Australia. Mean winds were measured at at least seven levels up to 89 m above the surface, and fluctuation statistics were measured at five levels using Gill and sonic anemometers. The statistics of the wind fluctuations represent 30-minute averages (apparently without trend removal). A contour map of the site is reproduced here as Fig. 2.5. Observations were also made on a lower tower located upwind of the mountain tower.

We have also begun the analysis of wind fluctuations measured on the top of a steep escarpment on the White Sands missile range. The escarpment runs basically N-S, with the steepest descent toward the West (see Fig. 2.6). So far, we have analyzed the behavior of flow from the West and East in strong winds.

Finally, some observations were available from a tower on Howard Knob, near Boone, N.C. This is a wooded site, on which a large windmill is being installed. Winds are available at three levels up to 70 m. Unfortunately, the anemometer at the central level usually was not operating, making the determination of roughness length uncertain. Wind fluctuations were determined at 1.7 second intervals, so that reliable spectra and variances are available at two levels. Some of the properties of the variances will be discussed in this report; spectra will be analyzed later.

Appendix A gives a summary of characteristics of observations at the complex sites used in this study.

Fig. 2.5. Contour plan of Black Mountain area. Base contour is 610 m above sea level, and contour interval is 61 m. The acceptance sector for wind direction is shown centered on tower.

Fig. 2.6. Terrain cross section at White Sands.

2.2.2 Ratios of the standard deviations to the friction velocity.

Tables 2.1 and 2.2 show ratios of standard deviations of velocity components to the friction velocities, for near-neutral (strong wind) conditions. The values given in these tables are for sites 1, 2, 5, 8, 10 and 15 (see table in Appendix A). For the remaining sites, the values were taken from the reports of the original investigators. Here u is the longitudinal velocity component, v the lateral component, and w the vertical component.

2.2.2.1 Uniform terrain. The σ_i/u_* ratios over uniform sites are listed in Table 2.1. In this table, the value of σ_w/u_* at Risø has been raised substantially over previous estimates. This is due to a correction for high-frequency loss caused by averaging the original data, taken every 0.01 seconds, in groups of 10. The correction was made by application of inertial-subrange theory. The best (average) estimates of the σ_i/u_* for strong winds are then:

$$\sigma_u/u_* = 2.40 \pm 0.03$$

$$\sigma_v/u_* = 1.96 \pm 0.05 \tag{2.9}$$

$$\sigma_w/u_* = 1.27 \pm 0.03$$

where the errors represent standard deviations of means, calculated by giving each set of observations equal weight. These results are consistent with recalculated values from an earlier summary by Ariel and Nadezhina (1976) and a summary of vertical-velocity statistics by Merry and Panofsky.

Clarke, et al. (1978) suggest that at urban sites in St. Louis there appears to be a weak relationship between the normalized standard deviations

Table 2.1. Ratios of standard deviations to friction velocity for neutral conditions over uniform terrain.

Site	σ_u/u_*	σ_v/u_*	σ_w/u_*
O'Neill, Nebraska	2.42	1.73	---
Sublette, Kansas	2.45	1.90	1.25
Lander, B.C., Canada	2.20	1.90	1.40
Beach Island, South Carolina	2.30	1.90	1.35
Donaldson, Minnesota	2.50	2.20	1.20
Guelph, [*] Ontario, Canada	2.50	1.85	1.22
St. Louis, ⁺ Missouri	2.39	1.79	1.26
Roskilde, ⁺⁺ Denmark	excessive scatter	2.18	1.25
Lincoln Ag. College, Christchurch, New Zealand	2.43	1.93	1.20
Hamburg	2.50	2.20	1.36

* 12-17 minute average, no trend removal

+ roughness lengths 0.7 - 1.7m

++ measurements over water

Table 2.2. Ratios of standard deviations to friction velocity for neutral conditions over undulating plains

Site	σ_u/u_*	σ_v/u_*	σ_w/u_*
Uppsala, Sweden	3.2	---	---
Erie, CO	2.85	2.00*	1.20
Rock Springs, PA			
a) non-mountain	3.20	2.90	1.24
b) mountain trajectory	4.50	3.80	1.24

* extremely uncertain

and the surface roughness length z_0 .

2.2.2.2 Rolling terrain. Lumley and Panofsky (1964) and Panofsky et al. (1977) suggest that the horizontal variances may be affected by mesoscale terrain features. In his study over non-uniform terrain, Högström (1974) found that the near-neutral stability (strong-wind) limit for σ_u/u_* was 3.2. This value is about 33% larger than the ratio of 2.40 generally observed over uniform terrain.

The values listed for Erie, Colorado were found from the observations plotted by Schotz (unpublished), and here reproduced as Figures 2.7 and 2.8. It is clear from these figures that interpolations for $z/L=0$ (strong winds) are quite uncertain; consequently the ratios σ_1/u_* listed in Table 2.2 have large possible errors.

Part of the difficulty lies in the fact that the observations of σ_1/u_* for April average much larger, and are much more scattered, than those for September. The reason for this difference is unknown. Both sets of data were analyzed in exactly the same manner, and the values quoted in Table 2.2 are underestimates of the averages for both months. In any case, the σ_u/u_* ratio of 2.8 for strong winds clearly is larger in Colorado than over uniform terrain. The situation for σ_v/u_* is much less clear.

The ratios σ_w/u_* , plotted as Fig. 2.9, show somewhat less scatter, but also exhibit the tendency for April observations to exceed those made in September. On the whole, the ratios are somewhat smaller than those found over uniform terrain (solid line). It is rather doubtful, though, whether this difference is significant.

Fig. 2.7. Ratio σ_u/u_* of the standard deviation of the longitudinal wind component to friction velocity plotted versus z/L at Boulder Atmospheric Observatory.

Fig. 2.8. Ratio σ_v/u_* of the standard deviation of the lateral wind component to the friction velocity plotted versus z/L at Boulder Atmospheric Observatory.

Figure 2.9. Ratio σ_w / u_* of the standard deviation of the vertical wind component to the friction velocity plotted versus z/L at Boulder Atmospheric Observatory (April 1978 - x, September 1978 - +). Also shown is the curve (2.3) for $z/L < 0$ and $\sigma_w / u_* = 1.3$ for $z/L \geq 0$.

The site near Rock Springs, Pennsylvania has produced a number of sets of data, the most recent of which we discuss in this report. Figure 2.2 shows the values of σ_u/u_* plotted as a function of z_1/L as suggested by Panofsky et al. (1977). Here z_1 is the mixed layer depth and L is the Monin-Obukhov Length. We note that the observations from Rock Springs for the horizontal components are systematically larger than those from homogeneous terrain. The curve shown on Figure 2.2 is the function derived by Panofsky et al. (1977) for uniform terrain in northern Minnesota. The value of σ_u/u_* interpolated for the near-neutral stability limit ($L \rightarrow \infty$), is about 3.2 for non-mountain trajectories (wind direction $< 240^\circ$). This magnitude of σ_u/u_* is about 35% larger than the homogenous terrain value. The magnitudes of σ_u/u_* for mountain trajectories (directions $< 240^\circ$) are generally larger and more widely scattered than the values for non-mountain trajectories. The average value for the mountain cases is about 4.5.

The lateral component shows behavior similar to the longitudinal component, with the values of σ_v/u_* , when plotted as a function of z_1/L , being systematically larger than those for homogenous terrain. Panofsky et al. (1978) show results from earlier Rock Springs experiments that are consistent with this observation. Figure 2.10 again presents the function suggested by Panofsky et al. (1977) for σ_v/u_* plotted with the Rock Springs data. A near-neutral limit of $\sigma_v/u_* = 2.9$ can be interpolated for non-mountain trajectories; this is about 50% larger than the magnitude found for homogeneous terrain. For air with mountain trajectories, the average value of the ratio σ_v/u_* was 3.8; but, as for σ_u/u_* , the data are widely scattered.

In contrast to the horizontal velocity components, the statistics of the vertical velocity component w are not affected by the mesoscale terrain at

Fig. 2.10. Ratio σ_v/u_* of the standard deviation of the lateral wind component to the friction velocity versus $-z_1/L$ at Rock Springs, Pennsylvania. (non-mountain trajectories - x; mountain trajectories - o). Also shown is the smooth-site curve $\sigma_v/u_* = (7 + 0.5(-z_1/L))^{1/3}$ suggested by Panofsky et al. (1977) with the neutral limit changed to 1.9.

Rock Springs. Figure 2.1 shows the values of σ_w/u_* for Rock Springs plotted as a function of z/L as suggested by Monin-Obukhov theory. We observed that the Rock Springs data fit the curve empirically derived by Merry and Panofsky (1976) for homogeneous sites. On the average, then, the mesoscale terrain does not affect the magnitude of σ_w/u_* in near-neutral conditions, with values of about 1.25 observed for both Rock Springs and homogenous sites. Further, the relation between σ_w/u_* and z/L seems universal, but for air with mountain trajectories there is an indication that the values of σ_w/u_* are much more scattered than those found for air with trajectories over slightly rolling fields.

The data from Rock Springs were processed with a number of filtering techniques. A one-hour linear regression was used to obtain the horizontal variances shown in Figures 2.2 and 2.10. A high-pass filter was applied to the data with a cutoff frequency of about 0.0013 Hz. The normalized longitudinal variances, σ_u/u_* , resulting from this filtering technique are plotted in Figure 2.11 as a function of z_1/L . The curve plotted in Figure 2.11 is the empirical curve (2.7) from Panofsky et al. (1977) that fits the Minnesota data. It is interesting to note that the variances computed with this type of high-pass filter agree well with the homogenous terrain observations, with the exception of extremely unstable conditions. This suggests that the contributions made to the variances by the complex terrain variations have periods longer than the thirteen minutes allowed by the filter and that the terrain effects are additive. But in order to discover the details of the contributions, we must study the logarithmic variance spectra.

The various ratios discussed so far were obtained from observations at the 2-meter height. Observations made at 5 m and 8 m suggest that there is no significant variation of σ_u/u_* and σ_v/u_* with height. But, on the average, σ_u , σ_v and u_* increased upward, by about 10%, from 2 m to 8 m.

Fig. 2.11. Ratio σ_v/u_* of the standard deviation of the lateral wind component to the friction velocity (after a high-pass filter has been applied to the data) versus $-z_i/L$ at Rock Springs, Pennsylvania. Also shown is the smooth-site curve $\sigma_v/u_* = (14 + 0.5 (-z_i/L))^{1/3}$ (non-mountain trajectories - x; mountain trajectories $- \square$).

2.2.2.3 Turbulence on hill tops. As air flows up hills, the velocity near the ground increases, by perhaps a few percent in the case of BAO (gentle hill) to a factor of 2 at Black Mountain (Bradley, 1979). These numbers are in good agreement with theories, particularly that by Jackson and Hunt (1975), even though this theory was developed for gentle terrain. Surface friction velocity u_* , σ_u , and σ_v seem to increase by about the same factor.

Conditions are relatively simple in an "inner layer" in which the wind profile (strong winds) is still approximately logarithmic. The intersection of the profile with the wind axis is the measure of the local roughness length, and the slope is a function of the surface stress. At the top of this inner layer, the wind speed reaches a maximum, then decreases slowly for a short distance before resuming its increase. The thickness of the inner layer depends on roughness length and hill geometry (for details see Jackson and Hunt, 1975). For Black Mountain, it is about 30 m above the effective surface, and at BAO it reaches almost 100 m.

At Black Mountain u_* , σ_u , and σ_v tend to decrease with height up to 87 m; only σ_w remains approximately constant, as seen in Table 2.3. Thus, the ratios σ_u/u_* and σ_v/u_* also decrease with height, if u_* is the surface friction velocity. However, if σ_u and σ_v are normalized by local values of u_* , then the ratios do not vary systematically (see Table 2.3). The numerical values for these ratios are not significantly different from the corresponding constants over uniform terrain (Table 2.1).

However, the relative intensities of horizontal turbulence decrease more rapidly with height in the inner layer over a hill than over uniform terrain. For

$$\frac{\sigma_i}{U} = \frac{k\phi_i}{\ln \frac{z}{z_0}} \quad (2.10)$$

Table 2.3

Standard deviations and friction velocities for strong winds
on Black Mountain, Canberra (after Bradley)

	$u_*, \text{m/sec}$	$\sigma_u, \text{m/sec}$	$\sigma_v, \text{m/sec}$	$\sigma_w, \text{m/sec}$	σ_u/u_*	σ_v/u_*	σ_w/u_*
9 m	0.85	1.94	1.39	1.11	2.3	1.6	1.3
16 m	0.47	0.95	0.81	0.54	2.0	1.7	1.1
25 m	0.42	1.18	1.31	0.80	2.8	3.1	1.9
<hr/>							
87 m	0.48	1.04	0.96	0.88	2.2	2.0	1.8
<hr/>							
Average					2.3	2.1	1.5

where ϕ_1 is the ratio of σ_1 to the surface friction velocity. And in contrast to the situation over uniform terrain, ϕ_1 decreases rapidly upward (see Table 2.3), for the horizontal wind components.

Analysis of the observations at White Sands has presented severe difficulties, partly because the basic measurements are 10-second averages, partly because there were only two levels of measurement, and partly because the mean vertical motion was far from zero.

For East winds, the profiles were consistent with each other, and eleven 43-minute runs could be combined to derive a roughness length of 3.5 cm. With this z_0 , surface friction velocities were computed from winds at 8 m and 16 m using the logarithmic wind profile equation, which should be valid at these levels. Some of the results are listed in Table 2.4.

Computation of the standard deviations presented considerable challenge because contributions from frequencies beyond 0.05 hertz had to be estimated. The scheme used for this purpose was to assume that the spectra for frequencies exceeding 0.05 Hz follow the Kolmogorov $-5/3$ law. This is consistent with spectra analyzed at other locations. Then the high-frequency contribution is given by $3/2 n_c S(n_c)$, where n_c is the Nyquist frequency, here 0.05 Hz. The estimate of $S(n_c)$ is affected by aliasing the effect of 10-second averaging. Approximately aliasing doubles the value, and averaging cuts it in half. Thus, the two effects were assumed to cancel. The true σ 's at 8 m may actually be larger, due to an underestimate of the high frequency correction.

In spite of the difficulties in arriving at the values listed in Table 2.4, the various quantities are quite reasonable and agree with conclusions derived from Bradley's measurements. First, the corrections of the observed standard deviations due to high-frequency loss decrease with height. Second, the observed standard deviations decrease with height. Third, the

Table 2.4. Standard deviations at White Sands, for strong east winds

Run (Feb. 6)	u_{*o}	8 m					16 m				
		U	Observed	Corrected*	Corrected		U	Observed	Corrected*	Corrected	
		m/sec	σ_u m/sec	σ_u m/sec	$\frac{\sigma_u}{U}$	$\frac{\sigma_u}{u_{*o}}$	m/sec	σ_u m/sec	σ_u m/sec	$\frac{\sigma_u}{U}$	$\frac{\sigma_u}{u_{*o}}$
2	0.67	9.1	1.13	1.57	0.17	2.3	10.2	0.79	1.08	0.11	1.2
3	0.72	9.7	1.17	1.64	0.17	2.3	11.0	0.86	1.11	0.10	1.5
4	0.72	9.7	1.27	1.73	0.18	2.4	10.8	0.95	1.31	0.12	1.8
5	0.76	10.3	1.24	1.68	0.16	2.2	11.2	1.06	1.31	0.12	1.4
6	0.76	10.2	1.35	1.93	0.19	2.6	11.4	.95	1.26	0.11	1.7

* Corrected $\sigma_u = \sigma_{u,o}^2 + \frac{3}{2} n_c (S(n_c))$

where $\sigma_{u,o}$ is the observed value.

corrected ratios of the σ 's to the surface friction velocities are about 2.4 at 8 m (quite consistent with the usual ratios), and the ratios decrease upward. Also, of course, the relative intensity of turbulence σ_u/U decreases rapidly with height.

Conditions for west winds are completely different, because the tower stands right at the edge of the steep escarpment (Fig. 2.6). Table 2.5 shows some of the properties of westerly flow. The most striking feature is the enormous vertical wind shear from 8 m to 16 m, with winds almost doubling from the lower to the upper level. Along with this large shear go huge values of σ_u , which at the 8-meter level, result in relative intensities of turbulence on the order of 80%. At 16 m, the standard deviations are typically 20% lower, and the relative intensities are much lower primarily due to the much larger wind speed. But even at 16 m, the relative intensity of turbulence approaches 50%. Clearly, if WECS are to be erected on escarpments similar to that studied here, the blades should not reach down to levels as low as 16 m. Comparison of spectra of the u-components observed with west and east winds shows that the excessive turbulent energy occurs primarily at high frequencies, so this turbulence is likely to be quite destructive.

Again, as in the case of east winds, it was assumed that the region below 16 m was within the inner layer, and that a logarithmic profile was valid. Combining four runs in this way resulted in a z_0 of 3.5 m, which is 100 times that determined from easterly flow. This enormous value for z_0 can be thought of as an "effective" value, and a measure of the surface turbulence produced by the escarpment.

Table 2.5. Turbulence characteristics at White Sands, for strong west winds.

z = 3 meters							z = 16 meters				
Run	u_{*0}	Observed		Corrected*			Observed		Corrected*		
		$\bar{U}$	σ_u	σ_u	$\frac{\sigma_u}{\bar{U}}$		$\bar{U}$	σ_u	σ_u	$\frac{\sigma_u}{\bar{U}}$	
1	2.55	5.46	3.63	5.62	1.03	2.20	10.34	3.44	4.94	0.477	1.94
16	3.18	6.81	3.85	5.80	0.851	1.82	12.70	4.13	5.60	0.441	1.76
17	2.89	6.18	4.12	5.36	0.868	1.86	11.81	3.68	5.41	0.458	1.87
18	3.25	6.95	3.59	5.13	0.739	1.58	11.16	3.78	5.33	0.478	1.64

* Corrected $\sigma_{u,o} = \sigma_{u,o}^2 + \frac{3}{2} n_c (S(n_c))$

where $\sigma_{u,o}$ is the observed value.

"Effective" surface friction velocities were computed using the 3.5-meter roughness length, and the σ_u 's were corrected in the same way they were for the east winds. Table 2.5 lists the ratios of the corrected σ_u 's to u_{*0} . They are surprisingly close to the standard value for uniform terrain; and the true surface value of the ratio would be even closer because σ_u/u_{*0} decreases with height. The behavior of this ratio suggests that the large σ 's are nearly in equilibrium with the large wind shears.

Some standard deviations and surface friction velocities have also been computed for Howard Knob (Boone, N.C.), and are listed in Table 2.6.

Since winds near the surface are needed to determine reliable roughness lengths and surface friction velocities from wind profiles, the roughness length at Howard Knob is somewhat more uncertain than elsewhere. From the winds given, a roughness length of 1 m was estimated, consistent with the ground cover. Even though this value is somewhat uncertain, it should not vary with time. Therefore, the relative magnitude of the friction velocities listed in Table 2.6 should be accurate.

The standard deviations decrease upward, which is consistent with observations in Australia and at White Sands. However, the data in Table 2.6 have one completely unexplained, and perhaps unexplainable feature: the winds, and therefore u_* , were almost twice as large on day 73 as on day 72, but the σ 's differ by only a small fraction. The ratios of σ_u/u_* and σ_v/u_* for the first period are quite consistent with ratios expected for strong winds close to the ground, and they display the usual decrease with height. But the ratios on the second day are inconsistent with measurements elsewhere.

Table 2.6.

Wind Statistics at Boone, N.C.

Day	Level Meters Above Effective Surface	U, m/sec	σ_u	σ_v	σ_u/u_{*0}	σ_v/u_{*0}
72.1	11	7.0	2.3	2.2	2.0	2.0
	69	11.6	2.0	2.3	1.8	2.1
72.2	11	8.2	2.5	2.2	1.9	1.7
	69	12.3	1.9	1.5	1.4	1.2
72.3	11	6.5	2.5	1.9	2.4	1.8
	69	11.2	2.0	1.7	2.0	1.7
73.1	11	14.9	3.3	3.1	1.3	1.3
	69	18.5	2.5	2.1	1.0	.83
73.2	11	11.4	3.2	3.0	1.4	1.3
	69	18.1	2.0	1.9	.88	.81
73.3	11	11.6	2.6	2.6	1.4	1.4
	69	14.6	1.6	1.8	.87	.97

Chapter 3. Turbulent Wind Spectra

3.1 Introduction

Spectra of horizontal wind components observed over complex terrain, particularly in stable conditions, are known to contain much more low-frequency energy than corresponding spectra obtained at smooth sites (Perry et al., 1978). Evidence of this increase can be found in Section 2 in which the values of the neutral standard deviations of the horizontal wind components were shown to be larger than those found typically at smooth sites. But in order to identify the length scales associated with the dimensions of the complex terrain, we must use the spectral decomposition of the variance and study its low-frequency region in detail. The spectra of both the longitudinal velocity u and lateral wind component v in unstable conditions contain much of their energy in the low-frequency portion, with a peak occurring at about 10^{-2} Hz for logarithmic spectra. Since terrain complexity adds spectral energy in this region, the shapes of the u - and v -spectra and the magnitudes (and perhaps the positions) of their peaks will be different over various types of sites. In stable stratification, the effect of terrain complexity should be even more important because spectra over uniform terrain contain less low-frequency energy.

The spectrum of vertical velocity w observed at uniform sites contains appreciable energy only at high frequencies, with the spectral peak occurring not far from the inertial subrange (Kaimal, 1978b). The frequency at the peak is inversely proportional to height. Because the form of the curve describing the ratio of the standard deviation σ_w to the friction velocity u_* in unstable air is the same over smooth and complex sites (Fig. 2.1) we expect that the form of the w spectra is the same over smooth and complex terrain.

In order to quantify the contribution of complex terrain to the horizontal wind spectra, we must formulate an hypothesis. We assume that the spectral form found in smooth sites is universal, and the effect of complex terrain simply adds energy at low frequencies. This is not an unreasonable assumption; as we have seen in Section 2, the curves of the σ_u/u_* and σ_v/u_* are approximately parallel to, but higher than those derived at smooth sites. Also, when the Rock Springs data are filtered so that all contributions to the horizontal velocity spectra with frequencies less than 0.008 Hz are removed, the resulting values of σ_u/u_* and σ_v/u_* follow the smooth site curves (Fig. 2.10). Thus we view smooth terrain as being the limiting case found when the complexity of a site, measured by some appropriate parameter, approaches zero.

In order to isolate the effects arising from complex terrain, and in order to see how this contribution varies with terrain complexity, we must describe accurately the u and v smooth-site spectra as functions of the length scales present at all sites, such as height z above the ground, depth z_1 of the boundary layer, Monin-Obukhov length L, and the ratio of the average horizontal wind speed $\bar{U}$ to the frequency n. The roughness length z_0 plays only an indirect role through its effect on u_* (see (1.8)).

The best atmospheric conditions for extracting power from the wind are those near neutral stability (strong winds) in which L approaches infinity. But we must study spectra obtained in stable and unstable air separately because the spectral forms for the neutral limit of stable air are different from those found in the neutral limit of unstable conditions (Kaimal et al., 1972). The neutral limit of the spectra when approached from unstable stratification contains most of its energy in low frequencies, but the neutral limit of the spectra when approached from stable conditions contains most of its variance at much higher frequencies.

In stable and unstable air, the high-frequency spectral densities of u and v depend only on the non-dimensional ratios $f=nz/U$ and z/L , and can be described by functional expressions similar to those applicable to the w spectrum (Kaimal, 1978b). At smooth sites in unstable air, the low-frequency part of the spectrum depends on the length scales $f_1=nz_1/U$ and z_1/L only. However, in the intermediate region labeled Region 2 by Kaimal (1978b) and centered near 10^{-1} Hz, the spectra are functions of all four scales, f , f_1 , z/L , and z_1/L . Kaimal (1978b) presented a scheme that describes the u and v spectra by using a low-frequency form for $n \leq 3U/2z_1$ (for u) and $n \leq 4U/z_1$ (for v), and a high-frequency expression for $n \geq U/2z$ (for u) and $n \geq U/z$ (for v). He interpolated linearly between the two limits to obtain a function for the intermediate values of n .

But Högström and Smedman-Högström (1974) noted that the u -spectra observed in unstable air at Marsta, Sweden contained two peaks, one at low and the other at high frequencies. The v -spectra from Minnesota also show this behavior. These observations suggest that the u - and v -spectra in unstable air are sums of two functions, one attaining its maximum value at low frequencies and the other reaching its largest magnitude at high frequencies.

As discussed earlier, we hypothesized that the observed form of the u - and v -spectra in complex terrain can be obtained empirically by adding a universal function to another function that depends on length scales related to the complex site. We will test the plausibility of our hypothesis by first developing a procedure that applies to smooth sites in unstable air. We will look for two functions whose sum will yield any smooth site spectrum. One expression will describe the high frequency portion that scales with f and z/L , and the other will give both the low-frequency part and the transition of

Region 2 centered near 10^{-1} Hz. The weighted sum of the two forms will yield a double-peaked spectrum for unstable air, and the high-frequency form alone will suffice for stable conditions, although different expressions for the magnitude and location of the peak will be needed.

3.2 Data sources

The smooth site that we primarily used to develop our empirical formulas was located near Donaldson, Minnesota where all relevant parameters, z , z_1 , L , U , u_* , and σ_H^2 , were available. Dr. J. C. Kaimal provided us with individual plots of the spectra during eleven 75-minute unstable runs and five stable runs observed at 4 m and 32 m height at this site. Details of the experiment are given in Appendix A and described in Readings et al. (1974) and Kaimal et al. (1976).

Values of the boundary layer depth z_1 are often not available. The magnitude of z_1 was measured at Minnesota by using MRU probes attached to a tethering cable of a balloon. Neither data for individual runs nor values of z_1 are available from the Kansas experiment; nevertheless we incorporate some results such as neutral limit values of parameters given in Kaimal et al. (1972), since z_1/L is then zero, independent of z_1 .

Data sources for which values of z_1 were measured include the Boulder Atmospheric Observatory (BAO) in Colorado (the terrain is slightly rolling west of the tower but more complex to the south-east) and Rock Springs, Pennsylvania (the terrain at Rock Springs has been described in Section 2). We have begun to analyze a few unstable 60-minute runs at 10, 22, 50, 100, 150, 200, 250, and 300m at BAO that were obtained in April 1978; we have included these observations into the design of the formulas for the vertical velocity spectra.

3.3 Empirical forms over smooth terrain

In this subsection we give the empirical formulas for the vertical and horizontal velocity spectra for both stable and unstable conditions. Some sample plots of spectra for near-neutral conditions are included. Details of their development are given in Appendix B. We also compare the spectrum with some observations. In future reports we will discuss the expected uncertainty of the spectral estimates.

3.3.1 The vertical velocity spectrum. From Table B.1 and (B.10)

we find that

$$\frac{n S_w(n)}{u_*^2} = \left(\frac{P_w}{u_*^2} \right) \left[\frac{2+3\gamma}{2+3\gamma(f/f_m)^\alpha} \right]^\beta \left(\frac{f}{f_m} \right)^\gamma \quad (3.1)$$

where f is the dimensionless frequency, nz/U .

The slopes γ , α , and β , position from the peak f_m and the magnitude of the peak P_w/u_*^2 all depend on z/L ; their expressions are given by

$$\gamma = \begin{cases} 0.7 - 0.06 (0.3 + |z/L|^{1/2})^{-1} & \text{for } z/L \leq 0 \\ 1 & \text{for } z/L > 0 \end{cases} \quad (3.2)$$

$$\beta = \ln \left[2.78 (f_m/\phi_\epsilon)^{2/3} \left(\frac{P_w}{u_*^2} \right) \right] / \ln [3\gamma/(2+3\gamma)] \quad (3.3)$$

$$\alpha = (2/3 + \gamma)/\beta \quad (3.4)$$

$$f_m = \begin{cases} 0.183 & \text{for } z/L < -0.7 \\ 0.482 + 0.437(z/L) & \text{for } -0.7 \leq z/L \leq 0 \\ 0.482 + 0.87(z/L) & \text{for } z/L > 0 \end{cases} \quad (3.5)$$

$$\phi_\epsilon = \begin{cases} 1 - z/L + 0.005(z/L)^2 & \text{for } z/L \leq 0 \\ [1 + 2.5(z/L)^{3/5}]^{3/2} & \text{for } z/L > 0 \end{cases} \quad (3.6)$$

$$\frac{R_w}{u_*^2} = \begin{cases} 0.4 + 0.2|z/L|^{1/2} - 0.35(z/L) & \text{for } z/L \leq 0 \\ -0.03(z/L)^2 & \\ 1.22(z/L) + 0.4 & \text{for } 0 \leq z/L < 0.18 \\ -0.057(z/L) + 0.64 & \text{for } z/L > 0.18 \end{cases} \quad (3.7)$$

These quantities are shown graphically in Figs. B.2-B.3, B. 10-B.11, and B.14-B.16. Three sample w-spectra for neutral, stable and unstable conditions are shown in Figs. 3.1-3.3.

3.3.2 The horizontal velocity spectrum in stable conditions. Based on the five runs at $z = 4$ m and $z = 32$ m in stable conditions at Minnesota (in which $0 \leq z/L \leq 1$), we find that the smooth site horizontal spectra scale with z/L and non-dimensional frequency f only; thus the u and v spectra are both given by (Table B.1)

$$\frac{nS(n)}{u_*^2} = F_s = \left[\frac{11}{8+3(f/f_m)^{6.5}} \right]^{0.14} \left(\frac{f}{f_m} \right)^{1/4} \left(\frac{P_s}{u_*^2} \right) \quad (3.8)$$

in which the peaks P_{us}/u_*^2 and P_{vs}/u_*^2 for the u and v spectra are (for $0 \leq z/L \leq 1$)

$$\frac{P_{us}}{u_*^2} = 1.5 - 0.9 (z/L) \quad (3.9)$$

$$\frac{P_{vs}}{u_*^2} = 0.9 - 0.4 (z/L) \quad (3.10)$$

and the position of the peak is for $(0 \leq z/L \leq 1)$

Fig. 3.1. The neutral limit ($z/L=0$) normalized, logarithmic w spectrum (3.1) as approached from unstable stratification, plotted against nondimensional frequency f .

Fig. 3.2. The normalized logarithmic w spectrum for $z/L=-0.10$ plotted against the nondimensional frequency f from Minnesota data (Run 2A1 at $z=4m$). The empirical form (3.1) is given by the solid curve.

Fig. 3.3. The normalized logarithmic w spectrum for $z/L=0.1$ plotted against the nondimensional frequency f from Kansas data (Kaimal et al., 1972). The empirical form (3.1) is given by the solid curve.

$$f_m = \left\{ \begin{array}{ll} 0.075 + 0.6 (z/L) & z/L < 0 \\ 0.3 + 1.6 z/L & 0 \leq z/L \leq 0.5 \\ 1.1 & z/L > 0.5 \end{array} \right\} \quad (3.11)$$

Sample spectra for run 2C2 at $z = 4$ m and 32 m from Minnesota for both u and v are given in Figs. 3.4-3.7; this run occurred in stable conditions that were closest to neutral. We note that a low-frequency slope must be incorporated into our model, but at frequencies about two orders of magnitude below the peak. In the next year, we intend to refine our model so that the total variance it predicts is correct. For now, we use run 2C2 to model the stable neutral limit with a slope of 2 added at low frequencies. The predicted neutral limits for u and v at $z = 4$ m and 32 m are shown in Figs. 3.8-3.9.

3.3.3 Horizontal velocity spectra in unstable air. When convection occurs, appreciable low-frequency contributions to the variance are added to the height-dependent, high-frequency portion also found in stable air. Thus we must add a low-frequency form that scales with boundary layer depth z_1 and $f_1 = n z_1/U$ to one whose general form fits the stable spectra. We accomplish this in a manner that incorporates all four scales z , z_1 , f , and f_1 into Region 2 that lies between the low-frequency and high-frequency peaks.

We define the weighting functions

$$\epsilon_1 = \left\{ \begin{array}{ll} 1 & \text{for } f < 0.8 z/z_1 \\ \log(2f)/\log(1.6 z/z_1) & \text{for } 0.8 z/z_1 \leq f \leq 0.5 \\ 0 & \text{for } f > 0.5 \end{array} \right.$$

Fig. 3.4. The normalized logarithmic u spectrum for $z/L = 0.034$ plotted against nondimensional frequency f from Minnesota data (Run 2C2 at $z = 4$ m). The empirical form (3.8) is given by the solid curve.

Fig. 3.5. The normalized logarithmic u spectrum for $z/L = 0.28$ plotted against nondimensional frequency f from Minnesota data (Run 2C2 at $z = 32$ m). The empirical form (3.8) is given by the solid curve.

Fig. 3.6. The normalized logarithmic v spectrum for $z/L = 0.034$ plotted against nondimensional frequency f from Minnesota data (Run 2C2 at $z = 4$ m). The empirical form (3.8) is given by the solid curve.

Fig. 3.7. The normalized logarithmic v spectrum for $z/L = 0.28$ plotted against nondimensional frequency f from Minnesota data (Run 2C2 at $z = 32$ m). The empirical form (3.8) is given by the solid curve.

Fig. 3.8. The normalized logarithmic u spectrum shown in Fig. 3.4 with the slope 2 correction added at low frequencies.

Fig. 3.9. The normalized logarithmic v spectrum shown in Fig. 3.6 with the slope 2 correction added at low frequencies

$$\epsilon_2 = \begin{cases} 0 & \text{for } f < 0.8 \, z/z_1 \\ 1 - [\log(2f)/\log(1.6 \, z/z_1)] & \text{for } 0.8 \, z/z_1 \leq f \leq 0.5 \\ 1 & \text{for } f > 0.5 \end{cases} \quad (3.13)$$

The u-spectrum takes the form

$$\frac{nS_u(n)}{u_*^2} = \epsilon_1 F_u + \epsilon_2 F_h \quad (3.14)$$

in which

$$F_u = \left[\frac{1.55}{0.05 + 6 f_1^{6.2}} \right]^{114} 1.4 f_1^{1.5} (1 + 0.09 |z_1/L|)^{2/3} \quad (3.15)$$

$$F_h = \left[\frac{2.5}{1 + 1.5(2f)^{4.6}} \right]^{0.36} 2f(0.03 + 0.4 |z/L|)^{1/3} \quad (3.16)$$

and $f_1 = n \, z_1/U$. For the v-spectrum we write

$$\frac{nS_v(n)}{u_*^2} = \epsilon_1 F_v + \epsilon_2 F_g \quad (3.17)$$

in which

$$F_v = \left[\frac{6}{1 + 7f_1^{2.5}} \right]^{0.5} 1.25f_1(0.45 + 0.09 |z_1/L|)^{2/3} \quad (3.18)$$

$$f_g = \left[\frac{11}{8 + 3(2f)^{6.5}} \right]^{0.14} 1.2f^{1/4}(0.07 + 0.65 |z/L|)^{1/3} \quad (3.19)$$

By comparing (3.8) and (3.19), we find that F_g and the stable spectrum form F_s are identical in form, but the magnitude and location of the peak are quite different. The effect of this is to cause the high-frequency expressions for P and f_m in stable and unstable air to be discontinuous at the neutral limit $z/L = 0$.

Use of the weights ϵ_1 and ϵ_2 in $n S_u(n)/u_*^2$ and $n S_v(n)/u_*^2$ provides a simple manner in which to incorporate the high- and low-frequency scales into Region 2 or the middle part of the spectrum. Kaimal (1978b) used a similar procedure, but the expressions (3.14) and (3.17) produce double-peaking when the stability is near neutral as is often observed (Högström and Smedman-Högström, 1974) (Figs. 3.10-3.17).

Sample plots of u - and v -spectra for the neutral limit and for Run 2A1, the closest near-neutral unstable run at Minnesota, are shown in Figs. 3.10-3.17; in these figures we also show Kaimal's (1978b) proposed expression. We note that the low-frequency slope is 1.5 for u and 1 for v ; a slope of 1 is expected from theory. Possibly the greater slope in the u -spectrum is due to the possible presence of longitudinal rolls, the effects of which would be reduced by the trend removal.

We combine the predicted neutral limits for stable and unstable u - and v -spectra in Figs. 3.18 and 3.19. We see that, although the areas under the stable and unstable curves are about the same, the distribution of the variance is markedly different. This difference was already recognized by Kaimal et al. (1972). The unstable spectra contain more energy at lower frequencies and they contain a double peak; the stable spectra contain one peak at an intermediate value. This is a significant result because it demonstrates that only specifying neutral conditions is not sufficient for wind power siting studies; knowledge of the stability history of the boundary layer is also needed in order to predict accurately the

Fig. 3.10. The neutral limit ($z/L = z_1/L = 0$) normalized logarithmic u spectrum for $z = 4$ m plotted against nondimensional frequency f . The solid curve is (3.14) and the dashed line is the curve given by Kaimal (1978b).

Fig. 3.11. The neutral limit ($z/L = z_1/L = 0$) normalized logarithmic u -spectrum for $z = 32$ m plotted against nondimensional frequency f . The solid curve is (3.14), and the dashed line is the curve given by Kaimal (1978b).

Fig. 3.12. The normalized logarithmic u spectrum for $z/L = -0.10$ as a function of nondimensional frequency f from Minnesota data (Run 2A1 at $z = 4$ m). The empirical form (3.14) is given by the solid curve, and the equation suggested by Kaimal (1978b) is given by the dashed line.

Fig. 3.13. The normalized logarithmic u spectrum for $z/L = -0.77$ as a function of nondimensional frequency f from Minnesota data (run 2A1 at $z = 32$ m). The empirical form (3.14) is given by the solid line and the equation suggested by Kaimal (1978b) is given by the dashed line.

Fig. 3.14. The neutral limit ($z/L = z_1/L = 0$) normalized logarithmic v spectrum for $z = 4$ m as a function of nondimensional frequency f . The solid line is (3.17) and the dashed line is the form suggested by Kaimal (1978b).

Fig. 3.15. The neutral limit ($z/L = z_i/L = 0$) normalized logarithmic v spectrum for $z = 32$ m as a function of nondimensional frequency f . The solid line is (3.17) and the dashed line is the form suggested by Kaimal (1978b).

Fig. 3.16. The normalized logarithmic v spectrum for $z/L = -0.10$ as a function of nondimensional frequency f from Mimesota data (Run 2A1 at $z = 4$ m). The empirical form (3.17) is given by the solid line and the form suggested by Kaimal (1978b) is given by the dashed line.

Fig. 3.17. The normalized logarithmic v spectrum for $z/L = -0.77$ as a function of nondimensional frequency f from Minnesota data (run 2A1 at $z = 32$ m). The empirical form (3.17) is given by the solid curve and the expression suggested by Kaimal (1978b) is given by the dashed line.

Fig. 3.18. The neutral limits of the unstable (solid line) and corrected stable (dashed line) normalized logarithmic u-spectra as predicted from (3.14) and (3.8).

Fig. 3.19. The neutral limits of the unstable (solid line) and corrected stable (dashed line) normalized logarithmic v-spectra as predicted from (3.17) and (3.8).

spectral distribution of the horizontal wind fluctuations in the boundary layer. We suggest the use of the stable limit at night and the unstable limit in the daytime, in strong winds.

For unstable spectra, we compare the frequency integral of $n S_u(n)/u_*^2$ and $n S_v(n)/u_*^2$ and the independently derived expressions (2.7) and (2.8) for σ_u/u_* and σ_v/u_* in Figs. 3.20-3.23. We find that the total area under our expressions for u- and v-spectra varies with $|z_i/L|$ in the correct manner. We note that our model predicts that σ_u/u_* and σ_v/u_* depend on height; this is not included in the expressions (2.7) and (2.8) for σ_u/u_* and σ_v/u_* .

The square root of the frequency integrals σ_u/u_* and σ_v/u_* of the stable u- and v-spectra are shown in Figs. 3.24-3.25. We note that the neutral limits of the predicted values of σ_u/u_* and σ_v/u_* of the stable and unstable forms are nearly the same. Thus the total variance changes continuously with stability, but the spectral distribution of the variance has a discontinuity (Figs. 3.18 and 3.19).

3.4 Complex terrain

The successful combination of two different forms to describe the u- and v-spectra in unstable air over uniform terrain suggests a manner in which we might add another expression that accounts for the additional variance produced by the complex terrain. We expect that horizontal spectra over complex terrain can be described by

$$\frac{n S_u(n)}{u_*^2} = \epsilon_1 G_u + \epsilon_2 G_h + \epsilon_3 G_a \quad (3.20)$$

$$\frac{n S_v(n)}{u_*^2} = \epsilon_1 G_v + \epsilon_2 G_g + \epsilon_3 G_b \quad (3.21)$$

Fig. 3.20. The square root σ_u/u_* of the frequency integral of the unstable u spectrum as a function of z_i/L . The neutral limit is $\sigma_u/u_* = 2.07$. Here we used $z = 4$ m, $z_i = 1000$ m and $U = 10$ m s⁻¹. The curve (2.7) is given by the solid line.

Fig. 3.21. The square root σ_u/u_* of the frequency integral of the unstable u spectrum as a function of z_i/L . The neutral limit is $\sigma_u/u_* = 1.83$. Here we used $z = 32$ m, $z_i = 1000$ m and $U = 10$ m s⁻¹. The curve (2.7) is given by the solid line.

Fig. 3.22. The square root σ_v/u_* of the frequency integral of the unstable v spectrum as a function of z_i/L . The natural limit is $\sigma_v/u_* = 1.86$. Here we used $z = 4$ m, $z_i = 1000$ m, and $U = 10$ m s⁻¹. The curve (2.8) is given by the solid line.

Fig. 3.23. The square root σ_v/u_* of the frequency integral of the unstable v spectrum as a function of z_1/L . The neutral limit is $\sigma_v/u_* = 1.71$. Here we used $z = 32$ m, $z_i = 1000$ m and $U = 10$ m s⁻¹. The curve (2.8) is given by one solid line.

Fig. 3.24. The square root σ_u/u_* of the frequency integral of the stable u spectrum (3.8) as a function of z_1/L . The neutral limit is $\sigma_u/u_* = 2.84$; the neutral limit of the corrected form shown in Fig. 3.18 is $\sigma_u/u_* = 2.59$. Here we used $z = 4$ m, and $U = 5 \text{ m s}^{-1}$.

Fig. 3.25. The square root σ_v/u_* of the frequency integral of the stable v spectrum (3.8) as a function of z_i/L . The neutral limit is $\sigma_v/u_* = 2.23$; the neutral limit of the corrected form shown in Fig. 3.19 is $\sigma_v/u_* = 2.13$. Here we used $z = 4$ m and $U = 5$ m s⁻¹.

in which G_u , G_h , G_v , and G_g have the same form as F_u , F_h , F_v , and F_g for unstable air, but in which $\epsilon_1 = 0$ for stable air. The weighting functions ϵ_1 and ϵ_2 for complex terrain will be similar to (3.11) and (3.12) for smooth terrain and the forms G_a and G_b will depend only on the length scales of the complex terrain.

This section will show a preliminary analysis of spectra at Rock Springs, PA. Data from this complex terrain site were obtained at heights of 2, 5, and 8 meters under both stable and unstable conditions. The data were then compared to the models for uniform terrain discussed in Section 3.3. The results for each wind component, u , v , and w , will be discussed in the following sections.

3.4.1 w-spectra. In Figs. 3.26-3.27 we show two sample w -spectra from Rock Springs taken during unstable and stable conditions. Superimposed on the data is the smooth terrain model (3.1). Plotting both the observed values of $nS(n)/u_*^2$ and (3.1) against $\log f$ we find no significant departures of the data from that predicted by the model. This would seem to indicate, from our preliminary analysis, that the complex terrain has little, if any, effect on w -spectra. This is substantiated by the results (Fig. 2.1) obtained for σ_w/u_* at Rock Springs which showed no significant departures from the values expected for σ_w/u_* over smooth terrain.

Fig. 3.26. Logarithmic w spectrum normalized by u_*^2 plotted against $\log f$ for Rock Springs, PA (X). Here $z = 5\text{m.}$, $z/L = -1.062$, $z_1/L = -224$. Equation (3.1) is denoted by the solid line.

Fig 3.27. Logarithmic w spectrum normalized by u_*^2 plotted against $\log f$ for Rock Springs, PA (X). Here $z = 5\text{m.}$, $z/L = +0.081$, $z_i/L = 37$. Equation (3.1) is denoted by the solid line.

3.4.2 Horizontal spectra for unstable cases. In figures 3.28-3.31 we show some sample spectra from Rock Springs for near-neutral, unstable conditions. A similar procedure was used as for the w-spectra. We plotted observed values of $nS(n)/u_*^2$ against $\log f$ together with the uniform terrain models for both the u- and v-components given by (3.14) and (3.17) (this is not an area-conserving plot but is convenient for fitting power laws). We found systematic departures of the data from the model only at low frequencies where observed values were consistently higher than those predicted by the models. However, the difference in magnitude does not appear significant when the data are displayed on a log-log scale.

Replotting (in Figs. 3.32-3.35) the low-frequency portion of the horizontal spectra with the area-conserving linear scales $VS(n)/(z_1 u_*^2)$ and f , we find that there is quite a large difference between the observations and the model, with the observations having significantly higher spectral densities than the smooth-terrain models. Thus the increase in the magnitudes of σ_u/u_* and σ_v/u_* from smooth to complex terrain is explained by the addition of variance due to terrain effects with wavelengths too long to be resolved. From Figs. 3.28-3.31 we see that the wavelengths beyond which the extra energy is added are greater than or equal to 1000 to 3000 meters. Thus the terrain scale is roughly of the same order of magnitude as the inversion height z_1 . However, we note that this scaling may be unique to Rock Springs, and investigation of data from other sites will be necessary to resolve this question.

3.4.3 Horizontal spectra in stable air. Similar analyses were performed on stable data taken from Rock Springs; these are shown in

Fig. 3.28. Logarithmic u spectrum normalized by u_*^2 plotted against $\log f$ for Rock Springs, PA (X). Here $z = 5\text{m.}$, $z/L = -0.030$, $z_1/L = -1.89$. The curve (3.14) is denoted by the solid line.

Fig 3.29. Logarithmic u spectrum normalized by u_*^2 plotted against $\log f$ for Rock Springs, PA (X). Here $z = 8\text{m.}$, $z/L = -0.049$, $z_1/L = -1.89$. The curve (3.14) is denoted by the solid line.

Fig. 3.30 Logarithmic v spectrum normalized by u_*^2 plotted against $\log f$ for Rock Springs, PA (X). Here $z = 5\text{m.}$, $z/L = -0.030$, $z_1/L = -1.89$. The curve (3.17) is denoted by the solid line.

Fig. 3.31. Logarithmic v spectrum normalized by u_*^2 plotted against $\log f$ for Rock Springs, PA (X). Here $z = 8\text{m.}$, $z/L = -0.049$, $z_i/L = -1.89$. The curve (3.17) is denoted by the solid line.

Fig. 3.32. Linear u spectrum multiplied by $V/(z_i u_*^2)$ plotted against f for Rock Springs, PA (X). Here $z = 5m$, $z/L = -0.030$, $z_i/L = -1.89$. The curve (3.14) is denoted by the solid line.

Fig. 3.33. Linear u spectrum multiplied by $V/(z_i u_*^2)$ plotted against f for Rock Springs, PA (X). Here $z = 8\text{m.}$, $z/L = -0.049$, $z_i/L = -1.89$. The curve (3.14) is denoted by the solid line.

Fig. 3.34. Linear v spectrum multiplied by $V/(z_i u_*^2)$ plotted against f for Rock Springs, PA (X). Here $z = 5\text{m.}$, $z/L = -0.030$, $z_i/L = -1.89$. The curve (3.17) is denoted by the solid line.

Fig. 3.35. Linear v spectrum multiplied by $V/(z_i u_*^2)$ plotted against f for Rock Springs, PA (X). Here $z = 8\text{m.}$, $z/L = -0.049$, $z_i/L = -1.89$. The curve (3.17) is denoted by the solid line.

Figs. 3.36-3.39. Once again, $nS(n)/u_*^2$ is plotted on a logarithmic scale as a function of $\log f$. For stable air, in the low-frequency region, the differences between the data and the model are even greater than they were for unstable air. At the high frequencies the fit is reasonably good.

The extra low-frequency energy in the spectra for stable conditions causes the stable complex-terrain spectra to appear similar in form to the unstable u- and v-spectra in which double peaking occurs. Thus the complex terrain observations in stable air differ quite markedly from those predicted by the smooth-terrain models because the latter depend only on height z and have only one peak towards higher frequencies. Over rolling terrain, another low-frequency peak appears whose magnitude and location is similar to the one in the unstable smooth-terrain models. Thus, it is clear from the analysis of the stable data that the added energy for complex terrain will depend on some terrain factor similar in magnitude to z_i . However, it is important to note that for other complex terrain sites, this terrain scale may be of a completely different order of magnitude, depending on the nature of the terrain. Upon comparing Figs. 3.28 and 3.36 we note that the neutral limit obtained by approaching from either stable or unstable stratification will be similar in the complex terrain case. But as we saw in Figs. 3.12-3.19, for uniform sites there are two distinctly different forms of the neutral limit when approached from stable and unstable conditions.

3.4.4 Concluding remarks. These results demonstrate the need and importance of obtaining uniform-site models first. With this procedure, we can readily identify the changes complex terrain might make in the observed forms. For example, the additional energy added at the lower frequencies for

Fig. 3.36. Logarithmic u spectrum normalized by u_*^2 plotted against $\log f$ for Rock Springs, PA (X). Here $z = 5\text{m.}$, $z/L = +0.081$, $z_i/L = 37$. The curve (3.8) is denoted by the solid line.

Fig. 3.37. Logarithmic u spectrum normalized by u_*^2 plotted against $\log f$ for Rock Springs, PA (X). Here $z = 8$ m., $z/L = +0.133$, $z_1/L = 37$. The curve (3.8) is denoted by the solid line.

Fig. 3.38. Logarithmic v spectrum normalized by u_*^2 plotted against $\log f$ for Rock Springs, PA (X). Here $z = 5\text{m.}$, $z/L = +0.081$, $z_1/L = 37$. The curve (3.8) is denoted by the solid line.

Fig 3.39. Logarithmic v spectrum normalized by u_*^2 plotted against $\log f$ for Rock Springs, PA (X). Here $z = 8\text{m.}$, $z/L = +0.133$, $z_i/L = 37$. The curve (3.8) is denoted by the solid line.

unstable conditions was not immediately apparent from Figs. 3.28-3.31 but was decidedly indicated by the stable data (Figs. 3.36-3.39). This was because the uniform-terrain unstable model scales at low frequencies with z_1 which tends to mask the dependence on the terrain. The uniform-terrain stable model does not depend on z_1 ; thus the effect of the terrain adding extra energy in this region becomes quite evident. In this way, we found that the terrain length scale for the models, at least for the Rock Springs case, is of the order of z_1 . This would not have been discovered had we not had the uniform site models first.

Therefore, in the year ahead, we plan to analyze additional complex terrain data at Rock Springs starting with stable conditions to see how the smooth-site models will have to be altered to include the terrain effects. We will try to express the spectra in the forms (3.20) and (3.21). The unstable cases will also be examined in a similar manner with emphasis on the neutral limits. We will then proceed to spectra over other types of complex terrain with emphasis on observations made on hilltops (Boone, N.C., and Canberra), and on escarpments (Christchurch and White Sands).

Chapter 4. Studies of the Probability Structure of Atmospheric Turbulence

Prediction of extreme loads is an important goal in modern studies of wind engineering applications. The structural integrity and the power of control systems both must be designed with specific probabilities of extreme and catastrophic loads in mind.

The three techniques available have involved use of ad hoc gust forms, spectra and coherences of the wind velocity time histories, and probability distributions of wind components or velocities. Assumed gust shapes allow estimates to be made, but do not accurately represent the complexity of actual wind forcing. Spectral quantities are of use primarily when the response is a linear integral of Gaussian forcing so that energy associated with a given frequency can be carried through via transfer function (or admittance function) theory to energy of response.

The only hope for developing effective estimation techniques for the non-Gaussian, nonlinear case lies in finding means to use the information contained in joint probability functions of the forcing variable.

For example, the number of times that a variable $y(t)$ will exceed the value y per unit time is given by

$$N(\dot{y}) = \int_0^{\infty} p(y, \dot{y}) d\dot{y} \quad (\dot{y} = dy/dt) \quad (4.1)$$

When the forcing is Gaussian and the response is linear, then it is easily shown that $N(y)$ for the response can be calculated directly in the form of a Gaussian exponential.

In more complex cases, it is necessary to calculate the probability distribution of the response from the known probability structure of the

forcing variable. Although the response is a deterministic function of the forcing, the actual calculation of the response probability structure presents practical difficulties in all but the Gaussian case.

The first step, then, is to ascertain whether the Gaussian assumption may be used for wind fluctuations. The result is that the velocities themselves are nearly Gaussian but their derivatives are not, having kurtosis that exceed significantly those of the Gaussian case. Thus $p(a, u)$ is not a joint Gaussian distribution, and even in the linear case, cannot easily be carried through to $N(y)$.

A two-step procedure is thus necessary. First we must find efficient means of modeling the probability functions of the wind fluctuations. Second, we must find ways to convert those models into models of the probability functions of the response.

The use of a sum of Gaussian distributions is explored in this report, reprinted from the Proceedings of the Conference and Workshop on Wind Characteristics and Wind Energy Siting 1979, June 19-21, 1979, in Portland, Oregon. The report shows that the Gaussian patch model is a good beginning toward modeling the observed distributions, but that refinements are necessary.

We plan to direct our efforts during the next year to developing prototype methods for calculating response statistics given models of the probability structure of the response. We anticipate that the combined study of the problems of representing wind distributions and of converting them into response statistics will be more effective for the present purposes than studying these topics separately.

4.1 Introduction

The two ingredients needed to construct models of atmospheric turbulence

useful in wind engineering applications are the kinetic energy spectra and the statistical structure of the fluctuating velocities.

Most of the attention in the last two decades has centered on the energy spectra of the velocities. From the viewpoint of engineering applications, this has been motivated by the fact that an assumption of a linear relation between forcing and response, along with an assumption that the turbulent forcing is Gaussian, is sufficient to produce all of the required statistical information about the response from knowledge of the turbulence spectra and the transfer (or admittance) function.

As engineering approaches become more sophisticated and the economic risks involved in wind engineering become more important, the validity of these two assumptions becomes increasingly critical. When either the linearization hypothesis or the assumption that the turbulent velocities are realizations of a Gaussian process is not valid, then explicit knowledge of both the spectra and the statistical structure of the turbulence is necessary, in principle, to calculate statistics of the response. It may be, however, that the resulting problem is quite intractable, and so reasonably accurate models that permit calculations to be made easily are especially valuable.

In this chapter, we report on the initial phase of an investigation into the statistical structure of turbulence in the atmospheric boundary layer. The theoretical basis is that the quantities of interest can be calculated if the joint distribution of $u(t)$ and $u(t + \Delta t)$ is known for various Δt . We propose a theoretical model for these joint distributions, and then demonstrate with calculations based on two samples of data that the model is worthy of further study.

4.2 The observed statistical structure

It is a common observation that the probability distribution of the fluctuating part of the wind is nearly Gaussian. There is some tendency, especially with the vertical velocity component, for the observed density functions to have longer tails than the Gaussian distribution does and to have values of the kurtosis somewhat greater than three (e.g., Dutton and Deaven, 1972). The departures from Gaussian, while perhaps eventually significant in theoretical considerations, would not seriously compromise the accuracy of engineering applications.

It is also a common observation in atmospheric, oceanic, and laboratory studies that the derivatives (or differences over small intervals Δt) of the velocities are markedly non-Gaussian. This fact demonstrates that although the velocities themselves may have a Gaussian distribution, they are not realizations of a Gaussian process in which the joint distributions of the velocity at different times are all Gaussian.

This conclusion invalidates the engineering method mentioned above, because the hypothesis that the forcing is a realization of a Gaussian process is used to conclude that the linear relation between forcing and response gives the response a Gaussian structure.

The contrast between the distributions of the velocities and their differences is illustrated by the two data samples considered here. Details about the observations are given in Table 4.1, and the distributions computed from the data samples are shown in Figures 4.1 and 4.2. Some statistical quantities associated with the data samples are given in Table 4.2.

TABLE 4.1 The Data Samples

Location	Altitude	Component	Instrument Sampling Rate	Length of Record
Risø Coastline of the Fjord	20 m	u	Sonic anemometer 10 Hz	2592 secs
North Sea Grosser Vogel- sand lighthouse	50 m	w	Sonic anemometer 10 Hz	6369 secs

TABLE 4.2 Statistical Characteristics (mks units)

Location and Quantity	Mean Wind	Intensity σ_u/U	Stabil- ity	Mean	Standard Deviation	Kurtosis
Risø	3.56	0.06	Stable			
u				3.56	0.22	2.99
Δu				0	0.13	4.53
($\Delta t = 1$ sec)						
North Sea	19.7	0.09	Neutral			
w				0	0.70	3.88
Δw				0	0.34	6.68
($\Delta t = 0.1$ sec)

FIG. 4.1 Probability distribution functions for the u component (left) of the Risø data and for the differences Δu at a lag of 1 second (right). The straight line is a Gaussian distribution.

FIG. 4.2 Probability distribution functions for the w component (left) of the North Sea w -component and for the differences Δw at lag of 0.1 second (right). The straight line is a Gaussian distribution.

4.3 A Gaussian patch model

The concept of intermittency in turbulent flows suggests that it might be useful to consider a model in which the patches of the flow with different intensities or dissipation rates have different probability distributions. Because of the ease of making calculations, it is worthwhile to see first whether the use of Gaussian distributions with different covariances will reproduce the observed probability structure. Some applications of this idea have been discussed by Dutton (1970).

The physical model is that the flow is composed of patches of turbulence, each with a Gaussian distribution but with different variances and covariance functions. The probability structure of the flow then becomes a mixture of Gaussian distributions.

In our study, we have used a mixture of two distributions on the argument that it should reveal whether the concept is useful without creating the difficulties encountered in more sophisticated and complicated mixing schemes. Thus the joint probability density function for the velocities $u(t)$ and $\tilde{u}(t) = u(t + \Delta t)$ is hypothesized to be of the form

$$p(u, \tilde{u}) = \alpha p_G(u, \tilde{u}; \sigma_1, r_1) + (1 - \alpha) p_G(u, \tilde{u}; \sigma_2, r_2) \quad (4.2)$$

in which p_G is a joint Gaussian distribution and σ and $r = r(\Delta t)$ denote the variance and the correlation function.

There are two implications of this hypothesis, which we mention here but do not discuss in detail. The first is that (4.2) implies that the kurtosis of the differences $\Delta u = \tilde{u} - u$ will be larger than the kurtosis of the velocity component u in the likely ranges of the parameters.

The second follows from the way in which we obtain marginal distributions for u and Δu from the joint distribution for $\tilde{u}$ and u . To obtain the distribution $p(u)$, we integrate at constant u over all values of $\tilde{u}$. To obtain the distribution $p(\Delta u)$, we integrate along the lines $\tilde{u} - u = \Delta u = \text{const.}$, which are parallel to the longer axis of the joint density function. If the joint density is broader along the direction of $\tilde{u} + u = \text{const.}$, then the marginal distribution for u may be nearly Gaussian, but the marginal density function for Δu will have longer tails than a Gaussian function. By mixing two Gaussian distributions, we can create this broadness of the joint density ellipses and have the freedom to model the case in which the velocities are nearly Gaussian but the differences, or derivatives, are not.

4.4 Optimum choice of parameters

The next step is to determine the parameter vector $\theta = (\alpha, \sigma_1, r_1, \sigma_2, r_2)$ in the Gaussian patch model so as to obtain a representation of the observed joint distributions that is optimum in a suitable sense.

We use here the maximum likeness estimate proposed by Barndorff-Nielsen (1977) in which we minimize

$$I(\theta) = \sum_{i,j} p_{i,j} \ln p_{i,j} / \hat{p}_{i,j}(\theta) \quad (4.3)$$

where $p_{i,j}$ is the observed probability density in the i,j^{th} cell of $(u, \tilde{u})$ space and $\hat{p}_{i,j}(\theta)$ is the value of the model density function (4.2) at the midpoint of the cell. The minimizing, or optimum, choice of the parameters was determined with a computer through use of a standard library routine. (With such a routine, we know that we have found a local minimum of (4.3), but we have not yet verified that we have actually found global minimum values of θ .)

Figures 4.3 and 4.4 show the observed joint density functions, the optimum Gaussian patch model density functions, and the equivalent Gaussian density functions obtained by using the correlation coefficient of the

Observed Joint Density Function

Prediction of Gaussian Patch Model

$\ddot{u}$

u

Equivalent Gaussian density formed
with correlation function of the
observed density.

FIG. 4.3 Joint probability density functions for the Risø u component at a lag of 1 sec for the first 600 sec of the record. (The lack of smoothness in the contours results from the method used in the available contouring routine.)

Observed Density Function

Gaussian Patch Model

$\bar{u}$

u

Equivalent Gaussian Density

FIG. 4.4 Joint probability density functions for the North Sea w component at a lag of 0.1 sec for the entire record.

observed distribution in a two-dimensional Gaussian function. The parameters that give minima of (4.3) are listed in Table 4.3.

It is clear from the figures that the equivalent Gaussian functions do not represent the observed joint density functions well, and that the observed function does expand along the axes $u + \tilde{u} = \text{const}$ (is broader than the Gaussian ellipse). It is also obvious, in the absence of a rigorous statistical test, that the Gaussian patch model is qualitatively similar to the observed density function.

As a quantitative test of the goodness of fit, we observe that the hypothesis (4.2) implies that the kurtosis, k , of the velocity is given by

$$k = 3[uu_1^4 + (1 - \alpha)u_2^4]/\sigma^4 \quad (4.4)$$

and that the kurtosis, k_Δ , for the difference is

$$k_\Delta = \frac{3[\alpha\sigma_1^4(1 - r_1)^2 + (1 - \alpha)\sigma_2^4(1 - r_2)^2]}{[\alpha\sigma_1^2(1 - r_1) + (1 - \alpha)\sigma_2^2(1 - r_2)]^2} \quad (4.5)$$

A comparison of the variances and kurtoses for the normalized data and those from (4.4) and (4.5) for the model is given in Table 4.4. Obviously the Gaussian patch model is superior to the equivalent Gaussian model (which would give $k = k_\Delta = 3$). Use of more terms in (4.2) would give additional freedom for the model to fit the observed distributions and would allow more moments than the variance of the velocity to be determined correctly.

TABLE 4.3 Optimum Values of the Parameters.

Case	Observed Correlation	α	σ_1^2	r_1	σ_2^2	r_2
Risø u	0.75	0.52	0.94	0.87	1.06	0.63
North Sea w	0.88	0.79	0.79	0.90	1.79	0.83

TABLE 4.4 Comparison of Moments

	Normalized Data		Model	
	σ^2	k	σ^2	k
Risø u	1.00	2.99	1.00	3.00
Δu	0.35	4.53	0.22	3.86
North Sea w	1.00	3.88	1.00	3.50
Δw	0.24	6.68	0.16	4.58

4.5 Conclusions

The concept that a mixture of Gaussian distributions will give an adequate representation of the observed joint distributions of turbulent velocities would seem to be validated by the preliminary results presented here. In any case, it is obviously worth examining a larger sample of data and this work is in progress.

The joint distribution we consider here is the key to determining the statistical quantities required in applications. Clearly, if we know $p(u, \dot{u})$ for a variety of time lags Δt , then we can calculate the joint distribution of u and $\Delta u = \dot{u} - u$, and for small Δt this will give an approximation to the joint distribution of u and its derivative $\dot{u}$. It is this distribution that determines the number $N(U)$ of times that the velocity $u(t)$ crosses the value U with positive slope.

For the case in which the output is a linear functional of forcing variables that are realizations of a Gaussian process, the quantity $N(Y)$ for the response $y(t)$ is directly related to the Gaussian density function of the forcing and can be derived from knowledge of the turbulence spectra and the transfer function alone. The fact, shown by Figures 4.1 and 4.2, that the turbulence is not a realization of a Gaussian process invalidates this approach.

But if the turbulence probability structure can be represented by a mixture of Gaussian distributions involving two or more components, then when linearization is valid, the accedence statistics ($N(y)$ or the output will be an appropriate mixture of the Gaussian accedence statistics.

Thus the present results offer hope that the current conceptually attractive approach can be replaced by a more accurate one that is not much

more difficult computationally. A full report on these matters will be given when our analysis of a larger sample of data is complete.

CONCLUSIONS

The ratios σ_u/u_* , σ_v/u_* and σ_w/u_* , from which the relative turbulence intensities can be found, have the following properties in strong winds:

1) Over uniform terrain, the ratios are independent of height in the surface layer; their numerical values are 2.4, 1.9 and 1.27, respectively.

2) On mountain and hilltops, the ratios σ_u/u_* and σ_v/u_* also are independent of height; σ_w/u_* increases with height. Near the surface, the ratios have the same values as over uniform terrain. However, σ_u , σ_v and u_* all decrease upward. Therefore the ratios of these σ 's to the surface friction velocity, on which the relative turbulence intensities depend, decrease upward.

3) Over rolling terrain, σ_u/u_* and σ_v/u_* are increased over their values over uniform terrain, but are approximately independent of height. In low spots, σ_u , σ_v and u_* separately increase upward. The σ 's are further increased in air which has traversed a low mountain. In contrast, σ_w/u_* appears to be unaffected by rolling terrain.

4) On top of a steep escarpment, the intensity of turbulence is enormous, corresponding to extremely strong wind shear; σ_u/U reaches values above 1 at 8 meters. Above that level the intensity of turbulence decreases rapidly.

Spectra of vertical velocity, over uniform and rolling terrain, follow Monin-Obukhov scaling. Formulae for these spectra are given in the body of the report.

Spectra of horizontal velocity components over homogeneous terrain in stable air obey Monin-Obukhov scaling. Formulae are given in the body of the report.

Spectra of the horizontal velocity components over homogeneous terrain in unstable air are best represented by weighted sums of two functions, a

"mechanical" and a "convective" function. The mechanical function is the same function that describes spectra in stable air; the convective function, described in detail in the body of the report, has the thickness of the boundary layer as length scale.

Rolling terrain adds energy at the low-frequency end of the spectra, but the effect is more dramatic in stable than unstable air. The additional energy is produced at wavelengths of 2 km, in a region of the spectrum in which detailed resolution of wavelengths has not been possible.

RECOMMENDATIONS

1. WECS should be built on the tops of gentle hills, not only because of the strong winds there, but also because of the relatively low turbulence levels some distances ($\sim 80\text{m}$) above the surface.
2. WECS should be built on hills with small z_0 .
3. WECS should probably not be built on top of steep hills or near the edges of escarpments due to excessive turbulence levels.
4. More studies are needed of spectra over various types of complex terrain especially hills. Such studies will be made in the continuation of the work described in this report.
5. Experiments should be made on two-dimensional ridges (e.g., in Pennsylvania) with well-instrumented curves on the ridge, the slope, and in the upwind valley, in order to verify and extend existing theories.

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APPENDIX A

Details regarding sites included in analysis

Table A.1. Characteristics of terrain and particulars of high frequency data for uniform sites

Authors	Location	Terrain Characteristics	Instrumentation	Heights	Sampling Frequency	Sample Duration
Haugen (1959)	O'Neill, Nebraska	homogeneous	3-component sonic anemometer thermometer	2.0 m	1 Hz	30 min
Izumi (1971)	Sublette, Kansas	less than $\pm .5\text{m/km}$ 18 cm wheat stubble	3-component sonic anemometer, fast response, Pt resistance thermometer	5.66 m 11.3 m 22.6 m	20 Hz	1 hr
McBean (1971)	Lander, British Col., Canada	homogeneous 4-8cm grass	3-component sonic anemometers, fast response Pt. resistance thermometer	2.0 m	80 Hz	12 min
Weber et al (1975)	Beach Island, South Carolina	relatively flat 60cm grass plowed fields	climate cup and Bivane, fast response thermistor	18.3 m 91.4 m	5 Hz and 10 Hz	40 min
Izumi and Caughey (1976)	Donaldson, Minnesota	flat featureless harrowed dirt wheat stubble	3-axis sonic anemo meter, Pt resistance thermometer, Rawinsonde	4.0 m 32.0 m	10 Hz	2.5 hr/ 75 min segments
Wesley (1978)	Guelph, Ontario, Canada	1-5 m corn	pressure sphere anemo-meters, fine-wire thermometer	2.4 m 4.1 m	20 Hz	30 min
Clarke et al (1978)	St. Louis, Missouri	urban area with roughness lengths 0.7-1.7 m	3-D propeller anemo-meters, fine-wire thermometer	30.0 m	2 Hz	1 hr
Vilardo and Lipschultz unpublished	Roskilde, Denmark	water 5km fetch	3-component drag anemometer, fine-wire thermometer	2.0 m 4.0 m 12.0 m	20 Hz	45 min
Flay unpublished	Christchurch, New Zealand	farmland, flat	bladed propellers		15 Hz	37-73 min
Wamser (1975)	Hamburg, Germany	city, one side - marsh, other side	bivanes, cups, sonics	6 levels 300m mast	10 Hz	1 1/2 hr 1 hour

Table A.2. Characteristics of terrain and particulars of high frequency data for uniform sites

Authors	Location	Terrain Characteristics	Instrumentation	Heights	Sampling Frequency	Sample Duration
Högström (1974)	Uppsala, Sweden	Rolling terrain + 5m/km crops and snow	Strain gauge anemo- meters, fine-wire resistance thermo- meters	?	30 Hz	?
Schotz Unpublished	Erie, Colorado	sloping terrain + 20 m/km western fetch	3-axis sonic anemo- meters, fast response Pt. resistance thermometers	10,20,50 and 100 m	10 Hz	1 hr
Lipschultz and Vilardo unpublished	Rock Springs, Pennsylvania	Rolling terrain + 5m/km 220 m mountain ridge trees, crops	3-component drag anemometer, fine- wire thermometer, sodar, airplane	2 m 5 m 8 m	20 Hz	30 min and 1 hr.
Bradley (1979)	Canberra, Australia	Hill, 170 m	bills; sonics, cups	7 levels to 124 m	5 Hz	90 min
Unpublished	White Sands New Mexico	Steep escarpment	bill propellers	8, 16 m	10 sec	45 min
Unpublished	Boone	hill	vanes	18, 46, 76 m	5/3 sec	~1 hr

APPENDIX B

Empirical Forms for Smooth Site Velocity Spectra

The expressions that we give in Section 3.3 to describe the velocity spectra all have the form

$$\frac{n S(n)}{u_*^2} = F = \frac{Ax^\gamma}{(1 + Bx^\alpha)^\beta} \quad (\text{B.1})$$

in which $x = f/f_m$ or $x = f_i/f_{im}$. Here f_m and f_{im} are the values of $f = n z/U$ and $f_i = n z_i/U$ at the peak F_m of F .

The maximum value F_m of F occurs at $x = 1$ where $\partial F/\partial x = 0$; this is satisfied if

$$B = \gamma(\alpha\beta - \gamma)^{-1} \quad (\text{B.2})$$

The coefficient A in the expression for F is determined by requiring that $F = F_m$ at $x = 1$:

$$A = F_m(1 + B)^\beta = F_m [\alpha\beta/(\alpha\beta - \gamma)]^\beta \quad (\text{B.3})$$

After inserting (B.3) into (B.1), we rewrite F as

$$F = F_m \left[\frac{\alpha\beta}{(\alpha\beta - \gamma) + \gamma x^\alpha} \right]^\beta x^\gamma \quad (\text{B.4})$$

For the applications of F to the high-frequency portions of the velocity spectra, we impose the additional constraint that F approach the known

inertial subrange expressions. In this frequency range, the velocity spectra scale with f and z so that $x = f/f_m$ in (B.4). The limiting form G of F in the inertial subrange is (Kaimal et al., 1972)

$$G \approx C(\phi_\epsilon/f)^{2/3} \quad (B.5)$$

in which the nondimensional dissipation rate ϕ_ϵ is defined by

$$\phi_\epsilon = k z \epsilon / u_*^3, \quad (B.6)$$

ϵ is the dissipation rate and k is the von Karman constant. In (B.5), $C = 0.36$ for the lateral v and vertical w velocity spectra but $C = 0.27$ for the longitudinal u spectrum.

Comparing (B.4) and (B.5), we find that

$$\alpha\beta - \gamma = 2/3 \quad (B.7)$$

After neglecting the constant term in the denominator of (B.4), we find by combining (B.4), (B.5), and (B.7) that F must obey

$$F \approx F_m [(2 + 3\gamma)/3\gamma]^\beta (f/f_m)^{-2/3} = C(f/\phi_\epsilon)^{-2/3} \quad (B.8)$$

in the inertial subrange; in (B.8) we have substituted f/f_m for x . We solve (B.8) for β and write

$$\beta = \frac{\ln [C(\phi_\epsilon/f_m)^{2/3}/F_m]}{\ln [(2 + 3\gamma)/3\gamma]} \quad (B.9)$$

For applications to the u-and v-spectra, we will find it convenient to approximate β by the neutral limit $z/L = 0$; this simplification does not introduce any appreciable error.

The high frequency form H of F is therefore

$$H = F_m \left[\frac{2 + 3\gamma}{2 + 3\gamma(f/f_m)^\alpha} \right]^\beta (f/f_m)^\gamma \quad (\text{B.10})$$

in which β is given by (B.9) once γ is chosen and α is determined from (B.7). The low-frequency form E of F will scale only with the boundary layer depth z_i and f_i . In this case we find from (B.4) that

$$E = F_m \left[\frac{\alpha\beta}{(\alpha\beta - \gamma) + \gamma(f_i/f_{im})^\alpha} \right]^\beta (f_i/f_{im})^\gamma \quad (\text{B.11})$$

We note that here α , β , and γ are free parameters with $\gamma < \alpha\beta$.

In order to use (B.9) to find β for (B.10) we must have expressions for ϕ_ϵ in stable and unstable air; ϕ_ϵ depends only on z/L and takes the form (Fig. B.1)

$$\phi_\epsilon = \begin{cases} 1 - z/L + 0.005 (z/L)^2 & z/L \leq 0 \\ [1 + 2.5 (z/L)^{3/5}]^{3/2} & z/L > 0 \end{cases} \quad (\text{B.12})$$

We used the Minnesota data reported by Kaimal (1978b) as well as BAO data to develop the formula for ϕ_ϵ in unstable conditions; the equation for ϕ_ϵ in a stable stratification is taken from Kaimal et al. (1972).

Expressions for the exponents and peak values for both the low- and high-frequency forms E and H of F are given in Table B.1, for the u, v, and w spectra

in both stable and unstable conditions. Figure numbers showing the empirical expressions along with observed data for many of the variables are indicated in Table B.1. Equation numbers are given when the expressions are too complicated to be displayed. The names of the empirical forms of the various velocity components and the stability type, denoted by "+" for stable and "-" for unstable, are also shown. Note that both low and high frequency expressions are required for the u- and v-spectra in unstable air; their combinations to describe the horizontal velocity spectra are discussed below. The expressions for all cases of velocity component and stability are given in Section 3.3.

The low-frequency portions of the unstable u- and v-spectra scale with the boundary layer depth z_1 and $f_1 = n z_1/U$, but the high-frequency parts depend on height z and f . We must combine these expressions in such a way that a double-peaked spectrum near neutral results (Högström and Smedman-Högström, 1974); we require that $n S_u(n)/u_*^2$ be given by F_u when $f_1 \leq f_{1m}$ and by F_h when $f \geq f_m$. We use a weighted sum of F_u and F_h for the spectral estimates in Region 2 between $f_1 = f_{1m}$ and $f = f_m$. The weighting functions ϵ_1 and ϵ_2 vary linearly on a logarithmic plot and are given by

$$\epsilon_1 = \begin{cases} 1 & \text{for } f < 0.8 z/z_1 \\ \log(2f)/\log(1.6 z/z_1) & \text{for } 0.8 z/z_1 \leq f \leq 0.5 \\ 0 & \text{for } f > 0.5 \end{cases} \quad (\text{B.13})$$

$$\epsilon_2 = \begin{cases} 0 & \text{for } f < 0.8 z/z_1 \\ 1 - [\log(2f)/\log(1.6 z/z_1)] & \text{for } 0.8 z/z_1 \leq f \leq 0.5 \\ 1 & \text{for } f > 0.5 \end{cases} \quad (\text{B.14})$$

in which we have used $f_1 = f z_1/z$. Thus, we find that (Table B.1)

Table B.1

Variables in (B.10) and (B.11) for F

Velocity Component	$F = \frac{nS(n)}{u_*^2}$	Freq. Range	Stability	$x_{\max}$	α	β	γ	F_m
u	F_{us}	High (B.10)	+	f_m eq (3.11); fig (B.12)	6.5	0.14*	0.25	$P_{us}/u_*^2 = 1.5 - 0.9(z/L)$ (fig. B.4)
u	F_h	High (B.10)	-	$f_m = 0.5$	4.6	0.36*	1	$P_h/u_*^2 = (0.03 + 0.4 z/L)^{1/3}$ (fig. B.5)
u	F_u	Low (B.11)	-	$f_{im} = 0.8$	6.2	0.25	1.5	$P_u/u_*^2 = (1 + 0.09 z_i/L)^{2/3}$ (fig. B.6)
v	F_{vs}	High (B.10)	+	f_m eq (3.11); fig (B.13)	6.5	0.14*	0.25	$P_{vs}/u_*^2 = 0.9 - 0.4 z/L$ (fig. B.7)
v	F_g	High (B.10)	-	$f_m = 0.5$	6.5	0.14*	0.25	$P_g/u_*^2 = (0.07 + 0.65 z/L)^{2/3}$ (fig. B.8)
v	F_v	Low (B.11)	-	$f_{im} = 0.8$	2.5	0.5	1	$P_v/u_*^2 = (0.45 + 0.09 z_i/L)^{2/3}$ (fig. B.9)
w	F_w	High (B.11)	+	f_m eq (3.5); fig (B.2)	$(0.67+\gamma)/\beta$	eq (B.9) fig B.14	1	P_w/u_*^2 eq. (3.7); fig. B.10
w	F_w	High (B.10)	-	f_m eq (3.5); fig (B.3)	$(0.67+\gamma)/\beta$	eq (B.9) fig B.15	eq (3.2)	P_w/u_*^2 eq. (3.7); fig. B.11

*Neutral limit of β used in (B.9).

$$\frac{n S_u(n)}{2 u_*} = \epsilon_1 F_u + \epsilon_2 F_h \quad (B.15)$$

A similar expression is used for the v-spectrum:

$$\frac{n S_v(n)}{2 u_*} = \epsilon_1 F_v + \epsilon_2 F_g \quad (B.16)$$

Thus the u- and v- spectra in Region 2 between the low and high frequency peaks depend on all four scales z , z_i , f and f_i . The weighted sums (B.15) and (B.16) provide a description of the observed double peaking in the horizontal velocity spectra near neutral (Figs. 3.10-3.17). We note also that the u- and v- spectra in unstable air are composed of a weighted sum of a low-frequency form and a high-frequency equation that is similar to the ones (F_{us} and F_{vs}) that describe the stable u- and v- spectra. Thus we can qualitatively say that the low-frequency contributions to the variance that arise from convective overturning within the boundary layer are superimposed on the high-frequency portion that is also present in stable air.

No low-frequency expression is needed to describe the horizontal spectra in stable air, except that a slope of two is needed for low enough frequencies. Thus the distribution of the variance is quite different in stable and unstable air and the neutral limits of the two forms are not the same, as observed by Kaimal et al. (1972) (Figs. 3.18 and 3.19).

Fig. B.1 Nondimensional dissipation rate ϕ_ϵ as a function of z/L for unstable cases. Minnesota data (+) and BAO data (·) are shown with the curve (3.6).

Fig. B.2. The position f_m of the peak of the logarithmic w spectrum in unstable air as a function of z/L based on Minnesota (+) and BAO (·) data. Also shown is the curve (3.5).

Fig. B.3. The position f_m of the peak of the logarithmic w spectrum in stable air as a function of z/L based on Minnesota data. The letters H, M, and L denote the largest, best guess, and lowest values respectively so that the distance between H and L is a measure of the uncertainty. Also shown is the curve (3.5).

Fig. B.4 The magnitude P_{us} of the peak of the logarithmic u spectrum in stable air normalized by u_*^2 as a function of z/L based on Minnesota data. Also shown is the curve (3.9).

Fig. B.5 The magnitude of P_h of the high-frequency peak of the logarithmic u spectrum in unstable air normalized by u_*^2 as a function of z/L based on Minnesota data. Also shown is the curve $P_h/u_*^2 = (0.03 \pm 0.4 |z/L|)^{1/3}$.

Fig B.6 The magnitude P_u of the low-frequency peak of the logarithmic u spectrum in unstable air, normalized by u_*^2 , as a function of z_i/L based on Minnesota data. The neutral limit is provided by Kaimal *et al.* (1972) (see text). Also shown is the curve $P_u/u_*^2 = (1 + 0.09 |z_i/L|)^{2.13}$

Fig. B.7 The magnitude P_{vs} of the peak of the logarithmic v spectrum in stable air normalized by u_*^2 as a function of z/L based on Minnesota data. Also shown is the curve (3.10).

Fig. B.8 The magnitude P_g of the high-frequency peak of the logarithmic v spectrum in unstable air normalized by u_*^2 as a function of z/L based on Minnesota data. Also shown is the curve $P_g/u_*^2 = (0.07 + 0.65 |z/L|)^{1/3}$

Fig. B.9 The magnitude P_v of the low-frequency peak of the logarithmic v spectrum in unstable air normalized by u_*^2 as a function of z_i/L based on Minnesota data. Also shown is the curve $P_v/u_*^2 = (0.45 + 0.09 |z_i/L|)^{2/3}$.

Fig. B.10 The magnitude of P_w of the peak of the logarithmic w spectrum in stable air normalized by u_*^2 as a function of z/L based on Kansas data in Kaimal et al. (1972). Also shown is the curve (3.7).

Fig. B.11 The magnitude P_w of the peak of the logarithmic w spectrum in unstable air normalized by u_*^2 as a function of z/L based on Minnesota (+) and BAO data (*). The neutral limit was provided by Kaimal et al. (1972). Also shown is the curve (3.7).

Fig. B.12 The position f_m of the peak of the logarithmic u spectrum for stable air as a function of z/L based on Minnesota data. Also shown is the curve (3.11).

Fig. B.13 The position f_m of the peak of the logarithmic v spectrum in stable air as a function of z/L based on Minnesota data. Also shown is the curve (3.11).

Fig. B.14 The exponent β_w (3.3) of the empirical form (3.1) for unstable conditions as a function of z/L for the logarithmic w spectrum.

Fig. B.15 The exponent β_w (3.3) of the empirical form (3.1) for stable conditions as a function of z/L for the logarithmic w spectrum.

Fig. B.16 The exponent γ (3.2) of the empirical form (3.1) for unstable conditions as a function of z/L based on Minnesota data for the logarithmic w spectrum.