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AEC Research and
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**NUCLEAR ANALYSIS
OF VARIOUS SPERT-III
CRITICAL EXPERIMENTS**

ALCO PRODUCTS, INC.

NUCLEAR POWER ENGINEERING DEPARTMENT
P. O. BOX 414, SCHENECTADY 1, N. Y.

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CRITICAL EXPERIMENTS**

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Approved by:
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Issued: April 27, 1961 ✓

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ABSTRACT

The calculational models employed in the nuclear analysis of the SM-2 were applied to various critical experiments performed upon the SPERT-III core. The purpose of this analysis was to ascertain the accuracy of the models employed by Alco Products in predicting the reactivity of APPR-type cores, both at room temperature and under operating conditions. The wide variety of experiments performed at SPERT-III (employing fuel elements similar to those of the APPR-type) provided an excellent opportunity to determine the degree of accuracy with which reactivities may be calculated by the particular analytical models employed.

Results of this analysis indicated that the calculated reactivities underestimated the experimental values for SPERT-III by approximately 2% to 4% $\Delta\rho$. Efforts to improve the models to reduce this discrepancy are being investigated further under the Army PWR Support and Development Program.

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SUMMARY

This report contains results of a study performed under the SM-2 program, Contract No. AT-(30-3)-326, to ascertain the accuracy of calculational models employed to predict core reactivity, both at room temperature and under operating conditions.

During the SPERT-III project, experiments were performed upon both cold and hot clean cores employing fuel elements similar to those of the SM-2. In addition, various critical water height experiments were performed. In the present study, the reactivities of these cores were calculated by the same models as used in the nuclear analysis of the SM-2 core, and compared to experimental values. In addition, calculated results were compared to those obtained by SPERT-III personnel as reported in reference (12).

Results of this analysis indicate that the calculated reactivities underestimate SPERT-III experimental values by 2% to 4% $\Delta\rho$. It was not possible to determine accuracy of the experimental values due to uncertainties in actual material content of the fuel elements, rod calibration, etc.; however, improvements in the method of calculating the fast nuclear parameters (in the two-group theory employed) would further reduce the discrepancy between calculated and measured reactivities by approximately 2% $\Delta\rho$. This is being investigated further under the Army PWR Support and Development Program.

Discrepancies between predicted reactivities as calculated by Alco Products and those calculated and reported in reference (12) may be explained by differences in the analytical models employed; in addition to use of different cross section files for stainless steel, as used by the MUFT-III, IBM-650 code to calculate the fast group nuclear parameters. Based upon the two-group model, reactivities calculated by Alco underestimated experimental values by 2% to 4% $\Delta\rho$ while those calculated and reported in reference (12) overestimated experimental values by 3% to 4% $\Delta\rho$.

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1.0 INTRODUCTION

The SPERT-III core employs fuel elements which are fairly similar to the APPR-type (SM-1, SM-1A, PM-1A, and SM-2). Thus, the SPERT-III core affords an excellent opportunity to check the analytical models used for SM-2 reactivity calculations under operating conditions.

SPERT-III is a heterogeneous, light water-cooled and -moderated reactor, designed for operation at pressures up to 2500 psig and temperatures up to 670°F. The SPERT project is operated by Phillips Petroleum Company at the National Reactor Testing Station as a part of the Atomic Energy Commission's reactor safety program.

The cold-to-hot reactivity change for the clean SM-2 reactor has been analytically and experimentally determined as $6.2\% \rho^{(1)}$ and $6.7\% \rho^{(1)}$ respectively, for a coolant temperature change from 68°F to 510°F. ⁽¹⁾ Two-group, two-dimensional diffusion theory (PDQ) calculations were performed on the SPERT-III core using the IBM-704, to confirm the accuracy of the analytical methods used to determine the SM-2 cold-to-hot reactivity change. Two initial critical core configurations were examined at 80°F and 650°F, and at 2500 psig. In addition, three other critical core arrays, as determined by critical water height experiments, were investigated analytically.

The five initial critical cores mentioned above provide a good test of the SM-2 calculational model since there are fewer inherent uncertainties when dealing with this type of core, relative to cores having large excess reactivities.

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2.0 SPERT-III CORE

Figures 1 and 2 present cross-sectional views of the cold (80°F) and hot (650°F) SPERT-III (2) cores, respectively, taken through the reactor centerline. A cruciform rod composed of a poison section and stainless steel follower is located in the geometrical center of the reactor. The purpose of this transient rod is to initiate reactor excursions by the rapid ejection of the poison section from the core. The poison section of the transient rod is normally out of the core. Eight fuel-poison control elements are located in the off-axis positions, and provide normal operating control.

Most of the fuel positions in the core are filled by Type 1S elements, as typified in Fig. 3. In order to provide space for the transient rod, the four central fuel elements are of a smaller (Type 2S) size. The control rod fuel elements are similar to the Type 2S fuel elements. Stainless steel filler pieces in the form of hollow boxes occupy all non-fueled core positions.

The SPERT-III fuel element is a plate-type, stainless steel unit consisting of fuel plates, side plates, upper and lower end-boxes, and a center reinforcing plate. The reinforcing plate increases the strength of the element and minimizes vibration of the fuel plates. The larger of the two sizes of fuel elements (Type 1S) is 2.95 in. x 2.95 in. with 38 fuel plates while the smaller is 2.476 in. x 2.476 in. with 32 fuel plates. This compares with 2.863 in. x 2.844 in. for the SM-2 stationary fuel element. (1) These fuel elements contain 638 gm and 442 gm of U-235, respectively, whereas the SM-2 stationary fuel elements contain 833 gm. (1) The active core height is 35.5 in., while the SM-2 height is 22 in. (1)

The meat of the fuel plates consists of 93.2% enriched UO_2 dispersed in stainless steel, and clad on each side with stainless steel.

The poison sections of the control rods consist of hollow boxes of B-10-bearing, stainless steel alloy. Flux suppressors are provided in the region between the end of the poison box and the top of the fuel plates. The control rod fuel elements consist of 32 fuel plates similar to those in the Type 2S fuel element and each element contains 414 gm of U-235.

Surrounding the core is a 1/2-in. thick stainless steel skirt and four thermal shields. The four thermal shields have thicknesses of 7/8-in., 1-in., 1-1/4-in., and 2-in. travelling radially outward, and associated water gaps of 1/2-in., 7/16-in., 7/16-in., and 1/2-in., respectively.

Table 1 contains a tabulation of the SPERT-III and SM-2 fuel element data.

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SPERT-III INITIAL CRITICAL CORE

($T = 80^{\circ}\text{F}$, $E_N = 0.0319 \text{ ev.}$)

28 FUEL ELEMENTS

FIGURE 1

SPERT - III INITIAL CRITICAL CORE

($T = 650^{\circ}\text{F}$, $E_N = 0.0663$ ev.)

52 FUEL ELEMENTS

CONTROL
RODS

FUEL
ASSEMBLIES

STAINLESS STEEL
FILLER PIECES

FIGURE 2

SPERT-III TYPICAL FUEL ELEMENT

FIGURE 3

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TABLE 1
SPERT-III AND SM-2 ELEMENT DATA

DATA		SPERT-III			SM-2	
		1S	2S TRANSIENT	2S CONTROL ROD	STATIONARY	CONTROL
Active length	(in.)	35.5	35.5	35.5	22	21.5
Overall element width	(in.)	2.950	2.476	2.476	2.863	2.618
Overall element thickness	(in.)	2.950	2.476	2.476	2.844	2.624
Cell size	(in.)	3 x 3	2.576 x 2.576	2.576 x 2.576	2.9375 x 2.9375	2.9375 x 2.9375
No. of fuel plates		38	32	32	18	16
Water channel thickness	(in.)	0.128	0.128	0.128	0.123	0.123
Plate thickness	(in.)	0.0300	0.0300	0.0300	0.04	0.04
Meat thickness	(in.)	0.0215	0.0211	0.0212	0.03	0.03
Clad thickness	(in.)	0.0043	0.0045	0.0044	0.005	0.005
Plate width	(in.)	1.4450	1.2080	1.2080	2.839	2.589
Meat width	(in.)	1.0465	0.8508	0.8562	2.65	2.40
Fuel		U-235	U-235	U-235	U-235	U-235
Uranium enrichment	(A/O)	93.2	93.2	93.2	93.07	93.07
Meat composition:	(W/O)					
U-235		15.4	15.9	14.75	20.28	20.28
UO ₂		18.8	19.4	18.0	24.87	24.87
S.S., Type 304		81.2	80.6	82.0	74.24	74.24
Total U-235 per fuel element	(gm)	638	442	414	833	656

2.1 CORE CONFIGURATIONS

Two initial clean critical cores (2) were analytically treated by the IBM-704 two-dimensional, two-group, diffusion theory code (PDQ) to predict reactivities.

The cold (80°F) critical core contained a total of 28 fuel elements, whereas the hot (650°F) critical core contained a total of 52 fuel elements. Each of the cores operated at a pressure of 2500 psig and included eight control rod fuel elements.

Table 2 tabulates some of the pertinent properties of both critical cores. Appendix A contains a tabulation of the number densities of the materials comprising the SPERT-III cores; the number densities of the materials comprising the SM-2 core are also given for comparison.

TABLE 2
SPERT-III INITIAL CRITICAL CORES

<u>Properties (2)</u>		<u>Cold (80°F)</u>	<u>(Hot 650°F)</u>
Total no. of fuel elements (including 8 control rod fuel elements)		28	52
Total weight of U-235 in core	(Kg)	15.3	30.6
Active core height	(in.)	35.5	35.5
8 Rod critical bank position*	(in.)	31.7	27.5
Excess reactivity }	(% ρ)	~ 0	1.8
	(K_{eff})	~ 1.00	1.018

* Measured from bottom of active core to top of active meat in control rod fuel elements.

2.2 CALCULATIONAL MODEL

The calculational model employed in analyzing the cold (80°F) and hot (650°F) SPERT-III cores consisted of the IBM-704 code, PDQ, (3), (4), (5) employing two-group theory. PDQ is a two-dimensional, reactor-design code which finds a discrete numerical approximation to the few-group, time independent, neutron diffusion equations for a heterogeneous reactor.

For calculational purposes, the cores were broken down into the following various regions:

- a. fuel cells
- b. reflector cells (stainless steel filler pieces plus associated water)
- c. transient rod follower, its associated water, and the end pieces of the cruciform portion of the stainless steel transient section enclosure
- d. outer region (skirt, thermal shields, and associated water).

The reflector cells, transient rod follower section, and outer region were directly volume - averaged. The compositions of these non-fuel bearing regions are given in Table 3.

TABLE 3
COMPOSITIONS OF NON-FUEL BEARING REGIONS

<u>Region</u>	<u>Volume Fraction of Stainless Steel</u>	<u>Volume Fraction of Water</u>
Reflector cells	0.17287	0.82713
Transient rod follower	0.48134	0.51866
Outer region	0.82007	0.17993

Meat dimensions were computed based on the U-235 loadings and corresponding meat compositions given in Table 1. It was assumed that the dimensional variations were similar to those obtained in SM-2 fuel plates, where variations as large as 6% were obtained. ⁽⁶⁾

Fuel cell properties were obtained by flux-volume weighting of the detailed cell. This homogenization procedure is discussed further in Section 2.2.1.

The fueled sections of the control elements were considered completely inserted into the core in their respective guide tubes. Three regions were used in the mock-up of the control rod fuel elements:

1. fuel cell - the region within the guide tube
2. guide tube - pure stainless steel
3. annulus of water surrounding the guide tube

2.2.1 Nuclear Parameters

Effective neutron temperatures for determining Maxwellian-averaged thermal neutron cross sections were based upon the Type 1S fuel cell, considering coolant temperatures of 80°F and 650°F. Effective neutron temperatures of 0.0319 eV and 0.0663 eV, corresponding to coolant temperatures of 80°F and 650°F, respectively, were determined using, ⁽¹⁾

$$\frac{T_n}{T_m} = 1 / 0.91 \frac{\sum_a}{\xi \sum_s} \quad (1)$$

where,

T_n = effective neutron temperature

T_m = moderator (or coolant) temperature

$\sum_a$ = cell macroscopic absorption cross section evaluated at T_m

$\xi \sum_s$ = cell slowing-down power evaluated at T_m

Microscopic thermal neutron cross sections were evaluated ⁽⁷⁾ at these effective temperatures, and are tabulated in Table 4.

TABLE 4
THERMAL MICROSCOPIC CROSS SECTIONS

Effective Neutron Temperature (ev)	Nuclide	$\bar{\sigma}_s$	$\bar{\sigma}_a$	$\bar{\sigma}_{tr}$	$\bar{\nu}\bar{\sigma}_f$
0.0319	U-235	9.0	518.351	527.325	1082.172
	U-238	9.0	2.139	11.114	-
	H	42.649	0.2604	34.223	-
	O	3.9	0	3.738	-
	S. S.	10.442	2.326	12.643	-
0.0663	U-235	9.0	342.511	351.485	714.472
	U-238	9.0	1.4847	10.460	-
	H	27.686	0.1808	19.110	-
	O	3.9	0	3.738	-
	S. S.	10.442	1.615	11.932	-

Macroscopic one-group fast constants were computed using the MUFT-III code (8), (9) with the P-1 Goertzel-Selengut slowing-down approximation. The fast macroscopic cross sections for the cold (80°F) and hot (650°F) cores are presented in Tables 5 and 6, respectively.

TABLE 5
FAST MACROSCOPIC CROSS SECTIONS
(T = 80°F, E_n = 0.0319 ev)

Region	D ₁ (cm)	$\Sigma_{a1}(\text{cm}^{-1})$	$\Sigma_{s1}(\text{cm}^{-1})$	$\nu\Sigma_{f1}(\text{cm}^{-1})$
Outer region	1.1734	0.008880	0.005650	0
1S Fuel cell	1.3605	0.008458	0.03117	0.009962
2S Control fuel cell	1.3660	0.007790	0.03202	0.008854
Stainless steel	1.49870	0.008155	0	0
Water	1.5796	0.000611	0.04708	0
Transient follower	1.2274	0.005952	0.02274	0
2S Transient fuel cell	1.3662	0.008081	0.03180	0.009412
Reflector cell	1.4039	0.002568	0.03836	0

TABLE 6
FAST MACROSCOPIC CROSS SECTIONS
(T = 650°F, E_n = 0.0663 ev)

Region	D ₁ (cm)	Σa_1 (cm ⁻¹)	Σ_{sl_1} (cm ⁻¹)	$\nu \Sigma_{f_1}$ (cm ⁻¹)
Outer region	1.2590	0.008336	0.003120	0
1S Fuel cell	1.8631	0.007521	0.01787	0.008827
2S Control fuel cell	1.8970	0.006926	0.01845	0.007863
Stainless steel	1.4455	0.008163	0	0
Water	2.5678	0.0003277	0.02869	0
Transient follower	1.4925	0.005543	0.01361	0
2S Transient fuel cell	1.8795	0.007188	0.01830	0.008354
Reflector cell	1.9934	0.002248	0.02325	0

Thermal macroscopic cross sections were calculated using an IBM-650, two-dimensional, P-3 superposition code⁽¹⁰⁾ written for the APPR type fuel cell geometry. This code computes flux-weighted cross sections for a cell in which equal dead region thicknesses exist on each side of the active region, and thus is not directly applicable to any SPERT-III fuel cell. In order to utilize this code, the central dead region was converted so that its thickness was made to be twice that of each of the other two dead regions, by linearly averaging the original thicknesses. As a result, this new SPERT-III fuel cell consisted of two identical APPR type fuel cells.

Slowing-down sources for the thermal group calculations were taken to be the volume fractions of water in any cell region.

The thermal macroscopic cross sections for the fuel cells used as input for the two-group, two-dimensional problems are given in Tables 7 and 8. Included in these tables are the assumed dead region thicknesses and slowing-down sources.

TABLE 7
FUEL THERMAL MACROSCOPIC CROSS SECTIONS
(T = 80°F, E_n = 0.0319 ev)

Fuel Element	Dead Region Thickness (cm)	Dead Region Source	Active Region Source	D ₂ (cm)	Σ_{a_2} (cm ⁻¹)	$\nu \Sigma_{f_2}$ (cm ⁻¹)
1S	0.57817	0.680	0.800	0.14804	0.20010	0.29914
2S Transient	0.55523	0.690	0.810	0.14846	0.19454	0.28711
2S Control	0.54818	0.688	0.814	0.14885	0.18716	0.27173

TABLE 8
FUEL THERMAL MACROSCOPIC CROSS SECTIONS
(T = 650°F, E_n = 0.0663 ev)

<u>Fuel Element</u>	<u>Dead Region Thickness (cm)</u>	<u>Dead Region Source</u>	<u>Active Region Source</u>	<u>D₂(cm)</u>	<u>$\frac{\sum a_2(\text{cm}^{-1})}{\sum f_2(\text{cm}^{-1})}$</u>	
1S	0.57817	0.680	0.800	0.33678	0.13738	0.21212
2S Transient	0.55523	0.690	0.810	0.33802	0.13347	0.20358
2S Control	0.54818	0.688	0.814	0.33988	0.12773	0.19157

The non-fuel regions were volume homogenized. The non-fuel thermal macroscopic cross sections are tabulated in Tables 9 and 10.

TABLE 9
NON-FUEL THERMAL MACROSCOPIC CROSS SECTIONS
(T = 80°F, E_n = 0.0319 ev)

<u>Region</u>	<u>D₂(cm)</u>	<u>$\sum a_2(\text{cm}^{-1})$</u>
Outer region	0.25224	0.16592
Stainless steel	0.30898	0.19848
Water	0.13731	0.017514
Transient follower	0.18744	0.10462
Reflector cell	0.15190	0.048798

TABLE 10
NON-FUEL THERMAL MACROSCOPIC CROSS SECTIONS
(T = 650°F, E_n = 0.0663 ev)

<u>Region</u>	<u>D₂(cm)</u>	<u>$\sum a_2(\text{cm}^{-1})$</u>
Outer region	0.33683	0.11433
Stainless steel	0.32740	0.13779
Water	0.38773	0.007409
Transient follower	0.35614	0.070166
Reflector cell	0.37576	0.029948

Data for the SM-2 core⁽¹⁾ which was based upon the FINK-I code,⁽¹⁴⁾ was used to obtain the reflector savings for the SPERT-III core. The reflector savings at 650°F was obtained by extrapolation from 510°F. Reflector savings of 6.60 cm and 9.35 cm were obtained for the cold (80°F) and hot (650°F) cores, respectively. The axial and radial reflector savings were assumed to be equal, for both temperatures. The total bucklings, B_T^2 , were calculated using the relation:

$$B_T^2 = \left(\frac{\pi}{H + 2S_H} \right)^2 + \left(\frac{2.405}{R + S_R} \right)^2 \quad (2)$$

where,

- H = active core height
- R = active core radius
- S_H = axial reflector savings
- S_R = radial reflector savings

The calculated total bucklings, B_T^2 , are given in Table 11, in addition to the axial buckling, B_Z^2 , and radial buckling, B_R^2 .

TABLE 11

CALCULATED BUCKLINGS FOR COLD (80°F) AND HOT (650°F) CORES

Core	$S_R = S_Z$ (cm)	B_Z^2 (cm ⁻²)	B_R^2 (cm ⁻²)	B_T^2 (cm ⁻²)
Cold (80°F)	6.60	0.000924	0.00671	0.00764
Hot (650°F)	9.35	0.000833	0.00355	0.00438

The axial bucklings, B_Z^2 , are required as input parameters for the PDQ problems.

2.3 PDQ vs. EXPERIMENTAL RESULTS

Table 12 gives a comparison between the calculated (PDQ) and measured values of K_{eff} .

TABLE 12

PDQ vs. EXPERIMENTAL K_{eff} FOR COLD (80°F) AND HOT (650°F) CORES

<u>Critical Core</u>	<u>PDQ</u>	<u>Experimental</u>	<u>%$\Delta\rho$</u>
Cold (80°F)	0.977	1.000	-2.4
Hot (650°F)	0.983	1.019	-3.6

As shown in Table 12, the PDQ code underestimates the experimental value for K_{eff} by approximately 2% to 4% $\Delta\rho$, based upon the fast and thermal models as used by Alco.

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3.0 WATER HEIGHT EXPERIMENTS

Clean critical rod-free experiments ⁽¹¹⁾ have been conducted on a number of light water-moderated and -reflected slab configurations containing only SPERT-III Type 1S fuel elements. Reactor control was provided entirely by variation of the water height. No control rods were used for control purposes. All critical experiments were performed in the temperature range between 55 and 60°F.

The cores were built up with SPERT-III Type 1S fuel elements spaced on 3-in. centers. This was identical to the SPERT-III lattice spacing.

3.1 CORE CONFIGURATIONS

Three rod-free critical cores, as determined by the water height experiments, were analytically investigated by the PDQ code, using two neutron groups.

Core A is a 3-row, 5-column slab array with the fuel plates parallel to the row axis, i. e., the long dimension of the slab. Core B is identical to Core A except that the fuel plates are normal to the row axis. Core C consists of two 3 x 4 slabs separated by a 3-in. water gap, with the fuel plates parallel to the row axis. Appendix B contains a tabulation of the number densities of the materials comprising the critical water height cores; the number densities of the materials comprising the SM-2 core are also given for comparison.

3.2 CALCULATIONAL MODEL

The PDQ-code was used to determine the reactivities of the three critical cores described in Section 3.1. The three cores, as seen by the PDQ code, are presented in Fig. 4, 5, and 6. Only a quarter section of each core is shown due to symmetry. A typical fuel element consists of two homogenized, active regions, and three homogenized, dead (non-fuel) regions.

3.2.1 Nuclear Parameters

The thermal and fast macroscopic cross sections used to describe each of the three critical cores are given in Tables 13 and 14, respectively.

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DIAGRAM OF CRITICAL WATER HEIGHT CORE A AS SEEN BY THE PDQ (X,Y) CODE

REFLECTOR
REGION

FIGURE 4

DIAGRAM OF CRITICAL WATER HEIGHT CORE B AS SEEN BY THE PDQ (X,Y) CODE

ACTIVE REGION

DEAD REGION

REFLECTOR
REGION

FIGURE 5

DIAGRAM OF CRITICAL WATER HEIGHT CORE C AS SEEN BY THE PDQ (X,Y) CODE

REFLECTOR
REGION

FIGURE 6

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TABLE 13
THERMAL MACROSCOPIC CROSS SECTIONS
(T = 80°F, E_n = 0.0319 ev)

<u>Region</u>	<u>D₂ (cm)</u>	<u>Σ_{a2} (cm⁻¹)</u>	<u>νΣ_{f2} (cm⁻¹)</u>
Active	0.139368	0.268478	0.463193
Dead	0.167001	0.075424	0
Reflector (pure H ₂ O)	0.137310	0.017510	0

TABLE 14
FAST MACROSCOPIC CROSS SECTIONS

<u>Region</u>	<u>D₁ (cm)</u>	<u>Σ_{a1} (cm⁻¹)</u>	<u>Σ_{sf1} (cm⁻¹)</u>	<u>νΣ_{f1} (cm⁻¹)</u>
Active	1.387505	0.010165	0.031406	0.013980
Dead	1.303414	0.004206	0.030930	0
Reflector (pure H ₂ O)	1.579620	0.000611	0.047078	0

The axial bucklings, B_Z², used as input parameters to the PDQ code, were obtained using the relation,

$$B_Z^2 = \left(\frac{\pi}{H_C + S_Z} \right)^2 \quad (3)$$

where H_C is the critical water height and S_Z is the total axial reflector savings. These values, for the three critical water-height cores, are reported in Table 15.

TABLE 15
AXIAL BUCKLINGS, B_Z², FOR CRITICAL WATER-HEIGHT CORES

<u>Critical Core</u>	<u>Critical Water Height, H_C (cm)</u>	<u>Total Axial Reflector Savings, S_Z (cm)</u>	<u>Axial Buckling, B_Z² (cm⁻²)</u>
A	59.65	9.41	0.002070
B	62.32	9.41	0.001915
C	71.90	9.41	0.001492

The total axial reflector savings, S_Z , was obtained from the relation, (11)

$$H_C = \left(2 \pi^2 M^2 \right)^{1/3} \left(\frac{d\rho}{dH_C} \right)^{-1/3} - S_Z \quad (4)$$

where M^2 is the migration area, and $\frac{d\rho}{dH_C}$ is the differential water height worth. A plot of H_C vs. $\left(\frac{d\rho}{dH_C} \right)^{-1/3}$, as measured by Phillips Petroleum, implied a linear variation for the three critical cores under investigation. The value of S_Z was then obtained by extrapolation.

3.3 PDQ vs. EXPERIMENTAL RESULTS

Table 16 gives a comparison for K_{eff} as obtained by the PDQ code and experimental results.

TABLE 16

PDQ vs. EXPERIMENTAL K_{eff} FOR CRITICAL WATER HEIGHT CORES

<u>Critical Core</u>	<u>PDQ</u>	<u>Experimental</u>	<u>%$\Delta\rho$</u>
A: plates parallel to row axis	0.9644	1.0000	-3.7
B: plates normal to row axis	0.9639	1.0000	-3.7
C: with 3-inch water gap	0.9739	1.0000	-2.7

As indicated by Table 16, the PDQ code underestimates the experimental value for K_{eff} by approximately 2% to 4% $\Delta\rho$. Again, this is consistent with Alco's past experience.

3.4 COMPARISON OF ANALYTICAL RESULTS

A comparison was made between Alco's results and those reported in reference (12) in analyzing the critical water-height cores using the PDQ code. A comparison of the analytical results for K_{eff} is given in Table 17.

TABLE 17
COMPARISON OF ANALYTICAL RESULTS FOR K_{eff}
OF CRITICAL WATER HEIGHT CORES
(Experimental $K_{eff} = 1.0$)

Critical Core	Alco	Reference (12)	$\% \Delta \rho$
A	0.9644	1.014	5.1
B	0.9639	-	-
C	0.9739	1.022	4.8

Table 17 indicates that the results reported in reference (12) are greater by approximately $5\% \Delta \rho$. An attempt was made to resolve this difference.

The results for K_{eff} reported in reference (12) (given in Table 17) were based upon a four-group model using the PDQ code. Their analysis indicated that the two-group scheme yielded eigenvalues which were 2% greater than the four-group eigenvalues. Thus, in comparing the calculated results on a two-group basis, the latter's results are greater by approximately $7\% \Delta \rho$.

There exist three principal differences which account for the $7\% \Delta \rho$ discrepancy. The first is the fact that Alco's fast constants were based upon the P-1, Goertzel-Selengut approximation while those reported in reference (12) were calculated using a consistent P-1 approximation. The latter method has been found to yield greater reactivities by approximately $2\% \Delta \rho$ (13).

The second difference is that Alco included the effect of thermal self-shielding parallel to the direction of the fuel plates whereas it was not considered in reference (12). The effect of self-shielding was found to decrease the reactivity by approximately $2.5\% \Delta \rho$, using the relation,

$$\frac{\Delta K_{eff}}{K_{eff}} = \frac{\Delta f}{f} \cdot \beta \quad (5)$$

where $\Delta K_{\text{eff}} / K_{\text{eff}}$ is the fractional change in K_{eff} due to self-shielding, $\Delta f/f$ is the fractional change in thermal utilization due to self-shielding, and β is the fraction of total fissions which are thermal. The values for $\Delta f/f$ and β were calculated to be 0.028 and 0.82, respectively. The quantity $\frac{\Delta K_{\text{eff}}}{K_{\text{eff}}}$ was then found to be approximately $2.5\% \Delta \rho$, using the relation,

$$\rho = \frac{K_{\text{eff}} - 1}{K_{\text{eff}}} \quad (6)$$

The third, and remaining difference, was found to be due to differences in the fast stainless steel file (elastic and inelastic parameters) as used by the MUFT-III, IBM-850 code, which contributed approximately $2.5\% \Delta \rho$ to the overall $7\% \Delta \rho$ difference. Thus, the results of Alco and those reported in reference (12) can be shown to be in excellent agreement in calculating reactivities of the five cores considered.

4.0 CONCLUSIONS

The primary purpose of this report was to ascertain the accuracy of calculational models employed in the nuclear analysis of the SM-2 to predict core reactivity under both cold (68°F) and hot (510°F) conditions. Based on results of the study of the SPERT-III cores, as presented in this report, the following conclusions are given:

1. The calculated reactivities are about 2.4% ρ and 3.6% ρ less than the measured values for the cold and hot full size cores studied, respectively.
2. The experimental value of the reactivity of the hot core studied is somewhat uncertain due to the reactivity controlled by the one partially inserted rod.
3. The calculated reactivities are uncertain due to:
 - a. Uncertainties of material composition
 - b. Large PDQ problem size due to lack of core symmetry
 - c. Necessity to homogenize certain core regions due to the problem size.
4. Hot-to-cold SM-2 predictions may be low by about 1% ρ .
5. SPERT-III is a more difficult core to calculate than the SM-2 core due to the size of the core and the element design.
6. Comparison of Alco and SPERT files indicates that epithermal absorption in Alco's steel file may be in error and may need revision.
7. The consistent P-1 approximation appears to give better agreement with experiment than the P-1 S-G approximation.

Based upon these conclusions, the following recommendations are given:

1. Continued analysis should be made of SPERT-III due to its similarity to the basic SM and PM core designs. In addition, SPERT-III offers excellent data on hot cores with little excess reactivity.
2. Alco's MUFT-III files need to be reviewed in light of the now available MUFT-V files.
3. Checks of SPERT-III should be repeated using the new MUFT-V files.

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APPENDIX A

NUMBER DENSITIES FOR SPERT-III CRITICAL CORES (cold and hot)

Table A.1 tabulates the number densities of the various regions in the SPERT-III critical core at 80°F ($E_n = 0.0319$ ev) as seen by the PDQ code. The fuel elements were completely homogenized within their associated cell spaces.

The number densities for the SPERT-III critical core at 650°F ($E_n = 0.0663$ ev) is reported in Table A.2. The temperature change was assumed to affect only the hydrogen and oxygen number densities of water.

TABLE A.1

NUMBER DENSITIES (ATOMS/CC X 10^{-24}) FOR SPERT-III CRITICAL CORE
(T = 80°F, E_n = 0.0319 ev)

REGIONS

<u>Atom</u>	<u>Outer Region</u>	<u>Type 1S Fuel Cell</u>	<u>Type 2S Control Fuel Cell</u>	<u>Stainless Steel</u>	<u>Water</u>	<u>Transient Follower</u>	<u>Type 2S Transient Fuel Cell</u>	<u>Reflector Cell</u>
H	0.012102	0.051364	0.051929	-	0.06726	0.034885	0.05193	0.055633
O	0.0060509	0.026352	0.026554	-	0.03363	0.017443	0.026595	0.027816
SS	0.069977	0.018982	0.018407	0.08533	-	0.041072	0.018334	0.014751
U-235	-	0.0003122	0.000275	-	-	-	0.0002935	-

TABLE A.2

NUMBER DENSITIES (ATOMS/CC X 10^{-24}) FOR SPERT-III CRITICAL CORE
(T = 650°F, E_n = 0.0663 ev)

REGIONS

<u>Atom</u>	<u>Outer Region</u>	<u>Type 1S Fuel Cell</u>	<u>Type 2S Control Fuel Cell</u>	<u>Stainless Steel</u>	<u>Water</u>	<u>Transient Follower</u>	<u>Type 2S Transient Fuel Cell</u>	<u>Reflector Cell</u>
H	0.0073734	0.031294	0.031639	-	0.040980	0.021255	0.0316395	0.033896
O	0.0036867	0.016317	0.015820	-	0.020490	0.010627	0.016450	0.016948
SS	0.069977	0.018982	0.018407	0.08533	-	0.041072	0.018334	0.014751
U-235	-	0.0003122	0.000275	-	-	-	0.0002935	-

APPENDIX B

NUMBER DENSITIES FOR SPERT-III CRITICAL WATER HEIGHT CORES

Table B.1 tabulates the number densities of the various regions in each of the three SPERT-III critical water height cores at 80°F, as seen by the PDQ code. Each of the cores was broken up into three distinct regions - the active and dead regions of the fuel cells, and the pure water reflector.

The number densities for the SM-2 reference core at 68°F are given in Table B.2 for purposes of comparison.

TABLE B.1

NUMBER DENSITIES (ATOMS/CC X 10^{-24}) FOR SPERT-III
CRITICAL WATER HEIGHT CORES
(T = 80°F, E_n = 0.0319 ev)

Atom	REGIONS		
	Active	Dead	Reflector (Pure Water)
H	0.0538454	0.0457368	0.067260
O	0.0278847	0.0228684	0.033630
SS	0.0153550	0.0273056	-
U-235	0.0004483	-	-

TABLE B.2

NUMBER DENSITIES (ATOMS/CC X 10^{-24}) FOR SM-2
REFERENCE CORE
(T = 68°F, E_n = 0.0331 ev)

Atom	REGIONS			
	Fixed Elements		Laminated Reflector	
	Active	Dead	SS	Water
H	0.050494	0.040823	-	0.066890
O	0.026880	0.020412	-	0.033445
SS	0.016938	0.033095	0.084922	-
U-235	0.00076	-	-	-